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A Probabilistic Neutrosophic Hesitant Fuzzy Set for Waste Water Treatment Plants

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Abstract

Decision-makers at wastewater treatment plants must increase process efficiency and circularity while preserving economic performance. They must comply with increasing requirements about lowering emissions, sustainability, and human health safety. To operate and choose technologies to fulfil these expectations leads to complicated multi-objective issues. As a consequence, the water industry has developed several decision support systems. Multi-criteria decision-making (MCDM) is used to deal with various criteria in the evaluation process. The MCDM methodology integrated with the probabilistic neutrosophic hesitant fuzzy set (PNHFS) to deal with vague and incomplete information. The PNHFS used the VIKOR method to rank the alternatives and used the optimal wastewater treatment plants. The criteria weights are computed. The results show that safety is of the highest importance—the sensitivity analysis was conducted to show the different ranks under different cases. The main results show the different ranks are stable, and the suggested MCDM methodology is robust compared with other MCDM methods.

Keywords: Probabilistic Neutrosophic Hesitant Fuzzy Set; Waste Water Treatment; Multi-Criteria Decision Making; Plant; Evaluation

1. Introduction

Due to outdated and ineffective methods, the wastewater industry faces several difficulties, such as significant carbon emissions, excessive energy use, noncompliance with regulations, and dwindling public confidence. Unfortunately, the effects of urbanisation, population increase, and climate change are only making them worse[1], [2]. Water utilities are unable to make the necessary investment choices to transition towards sustainable wastewater treatment, despite the fact that industry and academics have created a multitude of solutions in recent years to fight these difficulties at the wastewater treatment plant (WWTP) level. In the water industry, particularly WWTPs, decision support systems (DSSs) have been utilised to facilitate complicated decision making with the goal of improving technology selection processes or process control to enhance operational performance[3]–[5].

Compared to decision makers in other sectors, wastewater decision makers have unique challenges since their decisions have social and regulatory ramifications in addition to the usual technical, economic, and environmental ones. Problems with social acceptability and public image often occur when wastewater streams are used and recycled to create resources. Because of the significance of water and sanitation services for business, society, and the environment, they are also heavily regulated and need to be preserved. Furthermore, it is proving challenging to develop markets for novel goods made from wastewater recovery[6]–[10].

Hence, in order to improve business performance, decision-makers at water utilities and WWTPs must comply with stricter regulations aimed at protecting human health, preserving the environment, and reducing emissions. They must also pursue greater circularity and revenue generation through resource recovery strategies. Due to this, managing and choosing technologies to improve WWTPs—which are typically labor-intensive, trial-and-error exercises that depend on operator judgment—becomes complicated multiobjective tasks[11]–[13].

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Due to the overwhelming need to capture very uncertain information, simple fuzzy sets (FS), intuitionistic FSs (IFS), and hesitant FSs (HFSs) are insufficient contexts in which to do so[14]–[17]. Examining the sophisticated fuzzy environments to be switched from HFS to probabilistic hesitant fuzzy sets (PHFSs) and from neutrosophic theory (NS)[18] to probabilistic neutrosophic hesitant fuzzy sets (PNHFSs), we find that PNHFS, albeit with configurable features[19], is the most appropriate environment to handle the uncertain information about parking algorithm[20]–[22].

2. Basic Notion of Neutrosophic Sets

This part introduces the basic notion of neutrosophic set[23].

Definition 2.1

The hesitant fuzzy set (HFS) presented as:

$$\xi : X \to \rho([0,1])$$

Where ρ presents the hesitant fuzzy

$$\xi_u : X \to \rho([0,1]) : \to \xi_u(y) = \bigcup u \in U\{u(y)\}$$

Definition 2.2

Probabilistic HF can be presented as:

$$r = \left\{\left(y, h_y(p_y)\right) \mid y \in X\right\}$$

Where $h_y = [0,1]$

Definition 2.3

Neutrosophic Set presented as:

$$\psi = \left\{\left(y, \varrho_\psi(y), \varsigma_\psi(y), \sigma_\psi(y)\right) \mid y \in X\right\}$$

$$-0 \leq \varrho_\psi(y) + \varsigma_\psi(y) + \sigma_\psi(y) \leq 3+$$

Definition 2.4

Single valued neutrosophic set presented as:

$$\psi = \left\{\left(y, \varrho_\psi(y), \varsigma_\psi(y), \sigma_\psi(y)\right) \mid y \in X\right\}$$

$$0 \leq \varrho_\psi(y) + \varsigma_\psi(y) + \sigma_\psi(y) \leq 3$$

Definition 2.5

PNHFS can be presented as:

$$\psi = \left\{\left(y, h(y) \mid p_h(y), t(y) \mid r_t(y), g(y) \mid q_g(y)\right)\right\}$$

$$0 \leq \varrho, \varsigma, \sigma \leq 1$$

$$0 \leq \varrho^+ + \varsigma^+ + \sigma^+ \leq 3$$

$$\left\{\begin{matrix} p_{\varrho} \in [0,1], r_{\varsigma} \in [0,1], q_{\sigma} \in [0,1], \\ \sum_{\varrho \in h} p_{\varrho} = 1\,, \\ \sum_{\varsigma \in h} p_{\varsigma} = 1\,, \\ \sum_{\sigma \in h} p_{\sigma} = 1\,, \\ \varrho \in h;\ \varsigma \in t;\ \sigma \in g \\ \varrho^{+} \in h^{+} = \cup_{\varrho \in h} \max\{\varrho\}; \\ \varsigma^{+} \in t^{+} = \cup_{\varsigma \in t} \max\{\varsigma\}; \\ \sigma^{+} \in g^{+} = \cup_{\sigma \in g} \max\{\sigma\}; \end{matrix}\right\}$$

3. Materials and Methods

This section introduces the steps of the neutrosophic VIKOR method. The optimal answer for real-world issues may be chosen using the multi-criteria decision-making (MCDM) method known as neutrosophic VIKOR. While Serafim Opricovic invented the VIKOR approach, which was eventually linked with Neutrosophic set theory to address ambiguity in decision-making and inaccurate numerical values, Zadeh established Neutrosophic set theory in 1965. By arranging all of the options according to a set of selection criteria, this approach suggests a middle-ground solution to the issue. Po-Lung Yu created the concept of a compromise solution in 1973. It was then improved upon to create the VIKOR approach and included in MCDM. These options are arranged according to the three crucial choice variables—S, R, and Q—in order of significance. Whereas "R" denotes the greatest criterion distance from the neutrosophic optimal value, "S" reflects the total of the criteria's distances from it. The term "neutronosophic best value" describes the ideal value for certain criteria. The Q value, which serves as the foundation for ranking alternatives, is then calculated using the S and R values as input. Figure 1 shows the steps of the neutrosophic VIKOR method[24], [25].

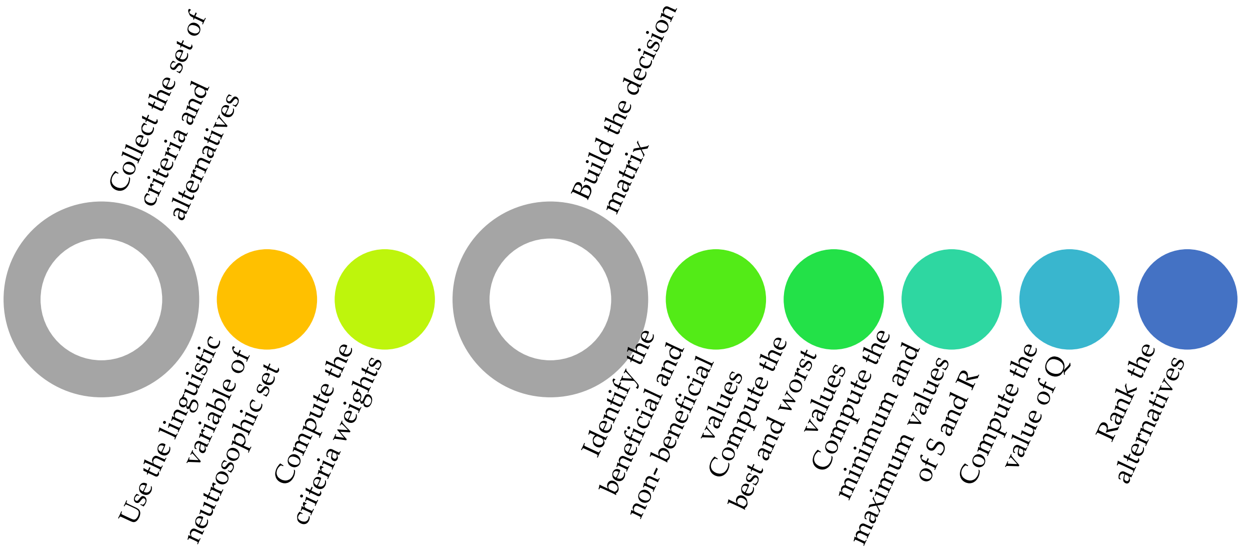

Figure 1: The steps of the neutrosophic VIKOR method.

Step 1. Collect the set of criteria and alternatives

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The alternatives and criteria are collected through the experts and decision makers who have expertise in the field of this problem. The alternatives and criteria are evaluated based on the views of the experts and decision makers.

Step 2. Use the linguistic variable of neutrosophic set

This paper used the neutrosophic linguistic variables to evaluate the criteria and alternatives.

Step 3. Compute the criteria weights.

Step 4. Build the decision matrix

The decision matrix is built based on the criteria and alternatives in the decision making model.

Step 5. Identify the beneficial and non- beneficial values

$$T_j^* = \begin{Bmatrix} \max_i y_{ij} & for \text{ beneficial criteria} \\ \min_i y_{ij} & for\ non - \text{beneficial criteria} \end{Bmatrix}$$

$$T_j^- = \begin{Bmatrix} \min_i y_{ij} & for \text{ beneficial criteria} \\ \max_i y_{ij} & for\ non - \text{beneficial criteria} \end{Bmatrix}$$

Where $i = 1,2, \dots m; j = 1,2, \dots n$

Step 6. Compute the best and worst values

$$S_i = \sum_{i=1}^{n} \frac{w_i (T_i^* - T_i)}{(T_i^* - T_i^-)}$$

$$R_i = \max \left(\frac{w_i (T_i^* - T_i)}{(T_i^* - T_i^-)} \right)$$

Step 7. Compute the minimum and maximum values of $S_i\ and\ R_i$

$$S^* = \min_i S_i$$

$$S^- = \max_i S_i$$

$$R^* = \min_i R_i$$

$$R^- = \max_i R_i$$

Step 8. Compute the value of Q_i

$$Q_i = \curlywedge \frac{S_i - S^*}{S^- - S^*} + (1 - \curlywedge) \frac{R_i - R^*}{R^- - R^*}$$

Step 9. Rank the alternatives

4. Application

This study proposed a cases study to evaluate the waste water treatment in plant. We used ten criteria and ten alternatives as shown in Figure 2. The experts and decision makers used the neutrosophic set to evaluate the criteria and alternatives

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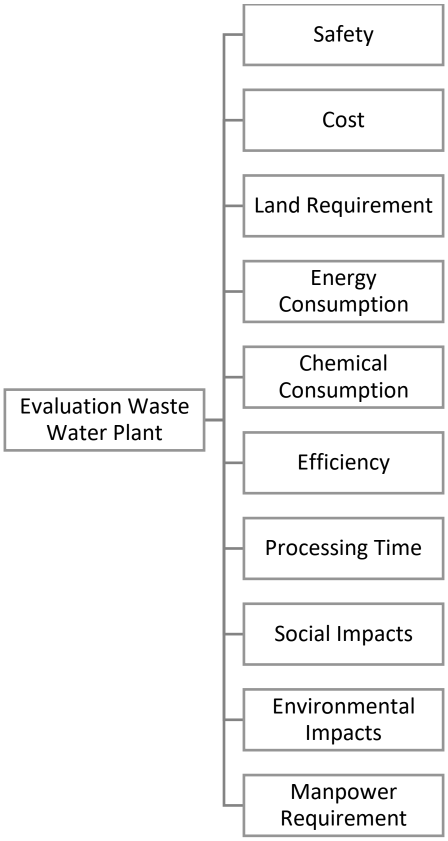

Figure 2: The list of criteria.

Step 1. Collect the set of criteria and alternatives

There are ten criteria and ten alternatives are used in this study as shown in Figure 2.

Step 2. Use the linguistic variable of neutrosophic set

This paper used the neutrosophic linguistic variables to evaluate the criteria and alternatives.

Step 3. Compute the criteria weights. The criteria weights are computed based on the average weight method. Figure 3 shows the criteria weights. The safety criterion has the highest importance in all criteria.

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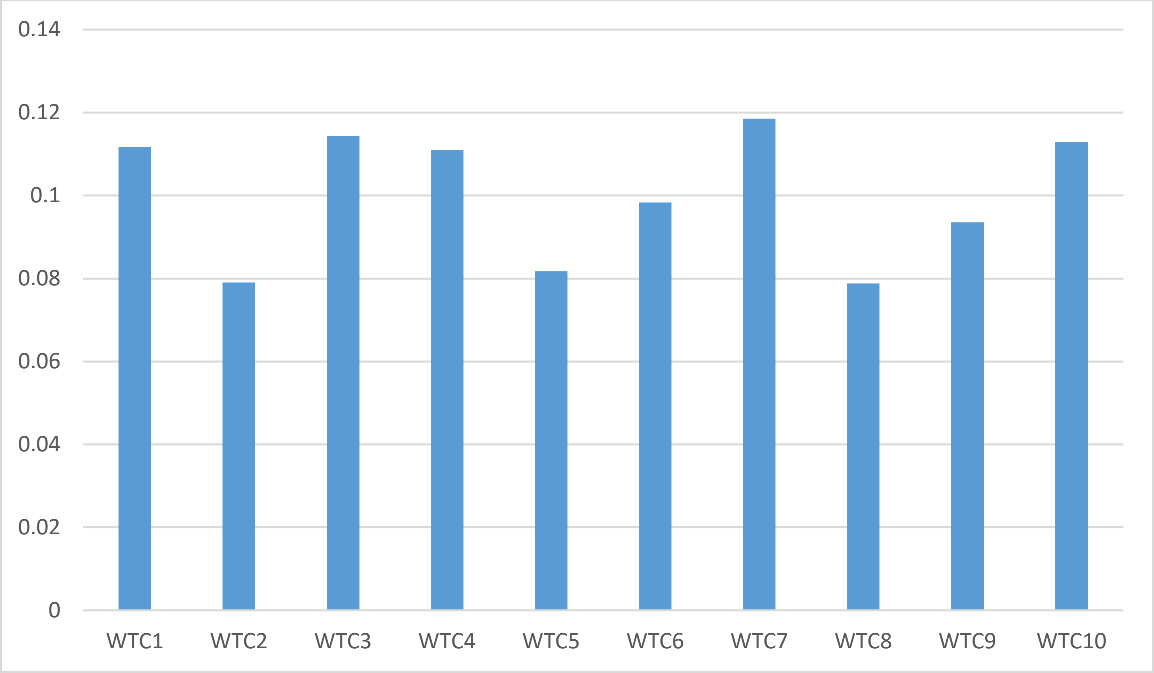

Figure 3: The criteria weights.

Step 4. Build the decision matrix

The decision matrix is built based on the criteria and alternatives in the decision making model.

Step 5. Identify the beneficial and non- beneficial values

Step 6. Compute the best and worst values as shown in Table 1.

Table 1: The values of weighted decision matrix.

	WTC_1	WTC_2	WTC_3	WTC_4	WTC_5	WTC_6	WTC_7	WTC_8	WTC_9	WTC_{10}
WTA_1	0.111725	0.079057	0.02003	0.058939	0.081772	0	0.072096	0.045767	0.00387	0.09601
WTA_2	0.048029	0.067577	0.059897	0.003705	0.050847	0.012682	0.072096	0.054568	0.093524	0.112887
WTA_3	0.000307	0.038337	0.114401	0.077294	0.050847	0.044761	0.089918	0.078834	0.01419	0.031844

WTA$_4$	0.047108	0.049817	0.052578	0.103732	0.050847	0.012682	0.089108	0.045767	0.036979	0
WTA$_5$	0.084857	0.012562	0.02003	0.077294	0.03857	0.098287	0.088028	0.032062	0	0.064007
WTA$_6$	0	0.032056	0	0	0.050847	0.012682	0.11854	0	0.079979	0.112887
WTA$_7$	0.067364	0.049817	0.084163	0.077698	0.050847	0.044201	0.075607	0.045767	0.036979	0.017992
WTA$_8$	0.051098	0.037579	0	0.055066	0.050847	0.097354	0.11854	0.065506	0	0.042989
WTA$_9$	0.040971	0	0.084163	0.077294	0.000232	0.024432	0	0.02917	0.047944	0.069102
WTA$_{10}$	0.067364	0.002274	0.084163	0.110973	0	0.076839	0	0.018231	0.03268	0.094577

Step 7. Compute the minimum and maximum values of S_i and R_i

Step 8. Compute the value of Q_i as shown in Figure 3.

Step 9. Rank the alternatives. The alternative 9 is the best and alternative 7 is the worst.

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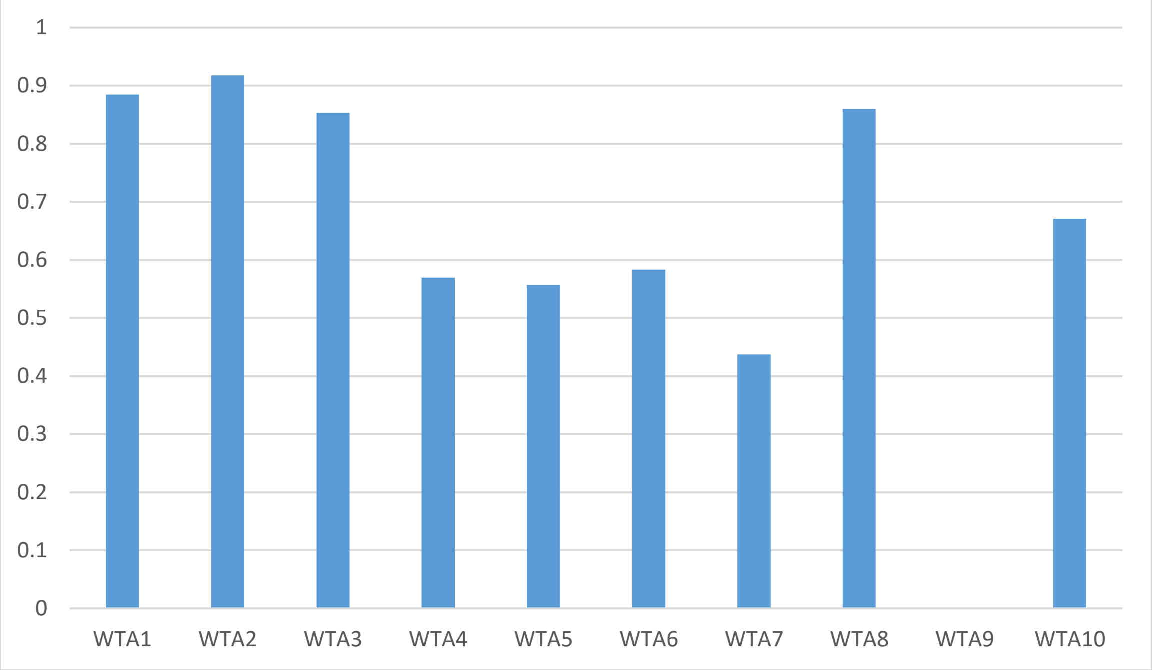

Figure 3: The values of Q_i.

5. Analysis

In this section, we change the value of ⅄ between 0.1 and 1 to show the rank of alternatives. The Qi values after this altering is shown in Figure 4. The results show the rank is stable under different cases. Figure 5 shows the rank of alternatives.

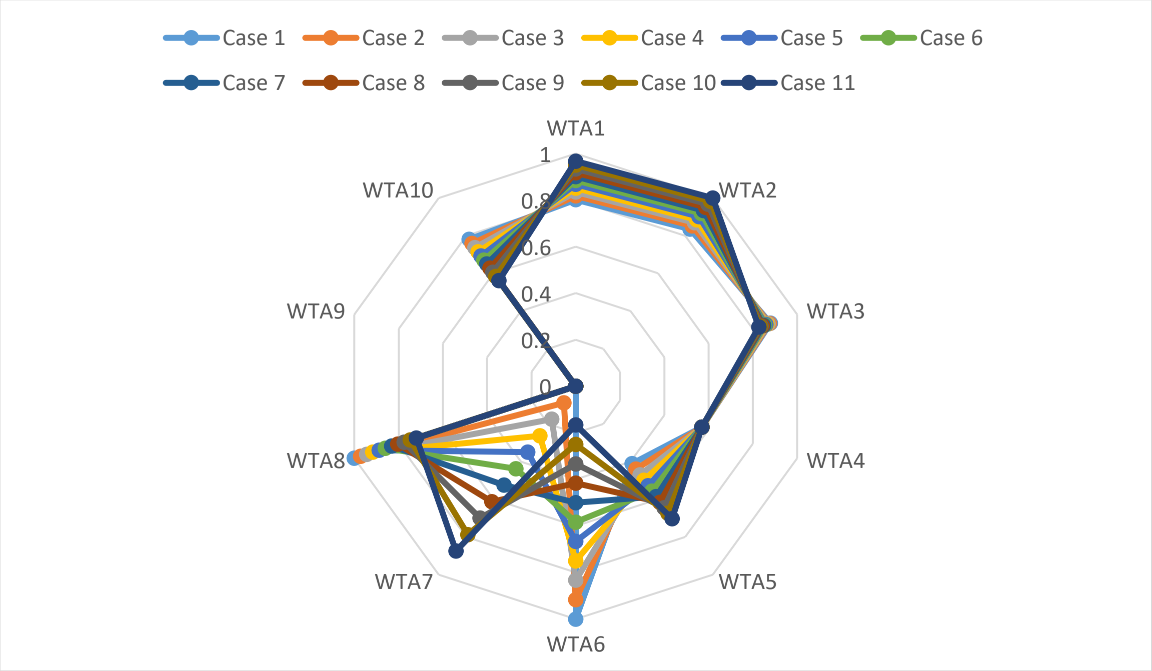

Figure 4: The Qi values under different cases in ⅄.

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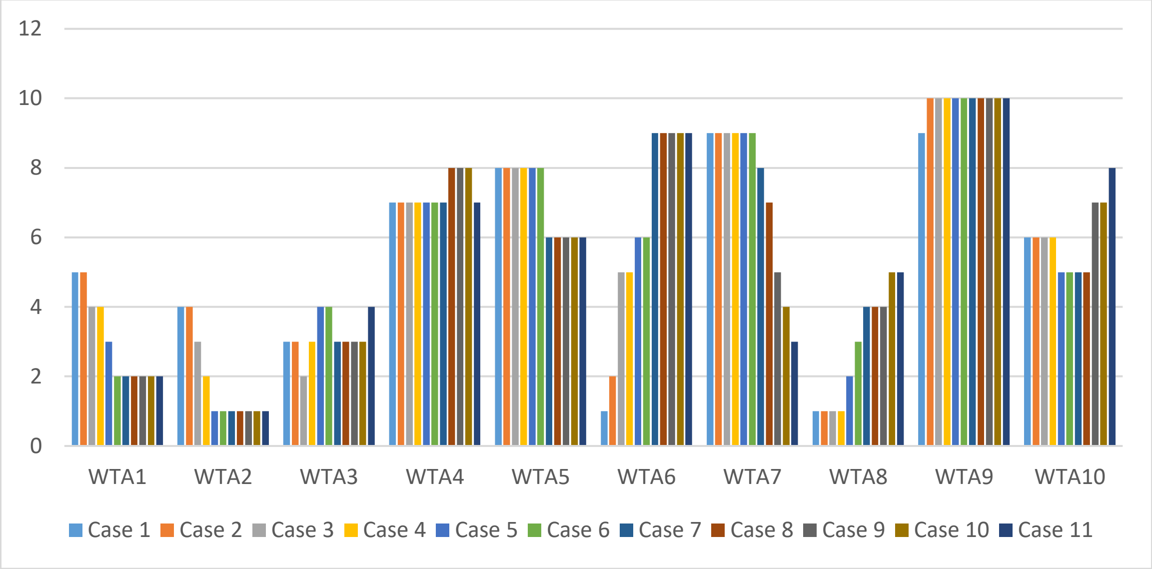

Figure 5: The rank of alternatives under different cases.

6. Conclusions

This study used the MCDM methodology to evaluate the wastewater treatment plants under decision-making and neutrosophic set. The VIKOR method is used as an MCDM methodology to rank the alternatives. The neutrosophic set is used in this paper to overcome the uncertainty in the evaluation process. In this study, ten criteria and ten alternatives are used. Then, ten criteria were chosen: social impacts, cost, safety, environmental impacts, and energy consumption. The experts and decision-makers evaluated the requirements and alternatives based on their opinions and views. The experts and decision-makers are invited based on their expertise in this field. The neutrosophic numbers are used to replace the opinions of experts and decision-makers. The mean method is used to compute the criteria weights. The results of this study show the safety criterion has the highest importance. This means the section's best treatment is related to safety and security at the first level. The neutrosophic VIKOR method shows the rank of alternatives. Alternative 9 is the best, and Alternative 7 is the worst. The sensitivity analysis is conducted in this study to show its robustness. The results show the ranks are stable.

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