Information Fusion Analysis for Evaluating Educational Equity

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Abstract

The present study aims to analyze the situation of educational inequality in Ecuadorian rural students during the study period, by identifying the barriers and factors that affect their access to equitable and quality education. The results showed a marked digital educational gap between rural and urban students, aggravated by the lack of access to technological resources and connectivity in rural areas. The transition to virtual education during the pandemic exacerbated these inequalities, making it difficult for rural students to learn and limiting their integration into the education system. In conclusion, the need to implement comprehensive public policies to address these inequalities and promote educational equity is highlighted, considering the evaluations and weightings carried out through the MULTIMOORA method. For this, an investment in educational infrastructure, the provision of devices, and teacher training in educational technologies are required to improve access to quality education in rural areas. With this, the path towards a fairer and more inclusive future for all Ecuadorians is sought.

Keywords: Educational inequality; rural students, Ecuador; technological resources; equitable education.

1. Introduction

Access to quality education is a fundamental human right recognized internationally. However, in many countries, including Ecuador, this right faces significant challenges, especially in rural areas. One of the main obstacles is the lack of connectivity [1], which limits access to quality education and ultimately perpetuates social inequality.

Therefore, in rural areas, students' access to technological resources and connectivity for virtual learning in Ecuador is a significant concern in the educational sphere. The transition to virtual education during the COVID-19 pandemic has highlighted existing disparities in terms of access to technology and connectivity in rural areas of the country.

In many rural areas of Ecuador, internet access and the availability of electronic devices are limited. High connectivity costs and lack of adequate infrastructure have created a significant digital divide between urban and rural areas [2]. This directly affects rural students, who are disadvantaged by not being able to access the same educational opportunities as their urban counterparts.

Unequal access to technological resources and connectivity has negative consequences on the learning of rural students. Many of them face difficulties in participating in virtual classes, accessing online

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educational materials, and completing assignments or assessments online. This situation can lead to lower academic performance and increased school dropout in rural areas.
It is significant to mention that differences in academic performance between rural and urban students in Ecuador are influenced by several complex factors. These differences have become a relevant topic in the discussion on educational equity in the country. Concerning the differences in academic performance between rural and urban students in Ecuador, the following aspects are observed:

- Access to educational resources: Urban students generally have easier access to educational resources such as libraries, study centers, and up-to-date teaching materials. On the other hand, in rural areas, the availability of these resources can be limited, which directly affects the quality of learning.
- Educational infrastructure: In urban areas, there are more educational establishments with better infrastructure, equipped classrooms, laboratories, and spaces for extracurricular activities. In contrast, rural schools may have poor infrastructure, which influences the learning experience of students.
- Teacher training and education: In urban areas, teachers are more likely to receive ongoing training and have access to professional development resources. In contrast, in rural areas, teachers may face challenges in accessing these opportunities, which can impact their pedagogical skills.
- Socioeconomic conditions: Socioeconomic differences also play a significant role. Urban students, in general, may have greater access to basic goods and services, which creates a more conducive environment for learning. In rural areas, the lack of economic resources can affect nutrition, health, and the availability of educational materials.
- Technology and connectivity: The digital divide is a significant factor. Urban students may have easier access to technology and the internet, allowing them to access online educational resources [3]. Rural students, on the other hand, may face difficulties in accessing online education due to lack of connectivity.
- Cultural and linguistic context: Cultural and linguistic diversity in rural areas can influence academic performance. Students whose mother tongue is different from Spanish, which is the main language of instruction, may face additional barriers to their learning.

It is important to highlight that these differences are not due to any intellectual limitation or inherently lesser capacity among rural students. Rather, they are the result of structural inequalities and adverse conditions in which they are situated.
Therefore, the general objective of this study is to analyze the right to educational equality for rural Ecuadorian students as a constitutional right, by identifying the barriers and factors that affect their access to equitable and quality education. The specific objectives are established as follows:

- Analyze the impact of socioeconomic factors on educational inequality among rural students.
- Propose strategies and measures to improve educational equity in rural areas of Ecuador.

2. Methods for data processing

In Ecuador, educational inequality between rural and urban students represents a significant challenge to accessing equitable and quality education. During the study period, this issue was analyzed using Saaty's Analytic Hierarchy Process (AHP) and Neutrosophic MULTIMOORA methods, subsequently proposing the best strategy to improve educational equity in the rural areas of Ecuador.

3. Neutrosophic Methods

This section details the main concepts and techniques that will be used in the present study.

A. Neutrosophic AHP.

The development of the Neutrosophic Analytic Hierarchy Process (AHP) method, is based on the reference materials analyzed in the study's bibliography [4] [5] [6] [7] [8]. To develop the method, it has been necessary to apply neutrosophic values as represented in the following definition:

- The single-valued trapezoidal neutrosophic number [9] [10], $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, and falsehood membership functions are defined as follows [11].

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- Given $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$ two single-valued trapezoidal neutrosophic numbers and λ any non-null number in the real line [12].

Step 1. Select a group of experts.

Step 2. Structure the neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies, through the linguistic terms shown in Table 1.

Table 1: Saaty's scale translated to a neutrosophic triangular scale. Source: own elaboration.

Saaty's scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\tilde{1} = \langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$
7	Very strongly influential	$\tilde{7} = \langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9, 9, 9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1, 2, 3); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3, 4, 5); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5, 6, 7); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7, 8, 9); 0.85, 0.10, 0.15\rangle$

The neutrosophic scale is attained according to expert opinions. The neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies is described in Equation 6.

B. Neutrosophic MULTIMOORA.

The MULTIMOORA method for Single-Valued Neutrosophic Numbers (SVNNs) involves calculating the overall performance of each alternative as the difference between the sums of their normalized performances that belong to the cost and benefit criteria [13]. It is necessary to define all attributes and consider that all of these must be measurable, that is, they can be measured or valued with respect to each of the alternatives. Consequently, it is defined that $A = \{\rho_1, \rho_2, \ldots, \rho_m\}$ is a set of alternatives and $T = \{\beta_1, \beta_2, \ldots, \beta_m\}$ is a set of criteria, the following steps are carried out:

Step 1: For the development of the MULTIMOORA method, definitions are made through the Single-Valued Neutrosophic Set (SVNS). Therefore, the neutrosophic set is defined by the following elements: true α, indeterminate β, and false γ of x in G, respectively, and their images constitute standard or non-standard subsets within the range {0, 1}. For X from the universe of discourse, the single-valued neutrosophic set G over X is defined as an object in the representation $G = \{\langle x, \alpha_G(x), \beta_G(x), \gamma_G(x)\rangle : x \epsilon X\}$.

Where $\alpha_G(x), \beta_G(x), \gamma_G(x)$ satisfy the following condition $0 \le \alpha_G(x), \beta_G(x), \gamma_G(x) \le 3$ for all x∈X. In such a way that each Single-Valued Neutrosophic Number (SVNN) is expressed in the following form: $G = (o, i, z)$ for the modeling of the study.

Therefore, the true membership function corresponds to $o = \alpha_G(x)$, indeterminate corresponds to $i = \beta_G(x)$, and false corresponds to $z = \gamma_G(x)$. Meanwhile, to determine a point within the neutrosophic set Y(G) from a number (G), it proceeds to use the formula proposed by Smarandache or the formula proposed by Basset, according to equations (1) and (2).

Where the following condition is met for everything. So to define each unique value neutrosophic number (NNVU) it is expressed as follows: for the modeling of the study.$\alpha_G(x), \beta_G(x), \gamma_G(x) 0 \le \alpha_G(x), \beta_G(x), \gamma_G(x) \le 3 x \in XG = (o, i, z)$

Therefore, for the true-corresponds membership functions, indeterminate-corresponds membership and false-corresponds membership functions. To determine a point within the neutrosophic set Y (G) from a number (G), then the formula proposed by Smarandache or the formula proposed by Basset is used, according to equations (1) and (2) .$o = \alpha_G(x), i = \beta_G(x) z = \gamma_G(x)$

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$$Y(G) = o + z - i \quad (1)$$

$$Y(G) = \frac{o + z - i}{2} \quad (2)$$

For the modeling of methods and evaluation of criteria, definitions are made according to the scales shown in Table 2.

Table 2: Linguistic terms representing the weight of the importance of the criteria. Source: own elaboration.

Linguistic scale	**SVNN**(o, i, z)	**Effectiveness**
Very High (VH)	(0.91,0.15,0.11)	Very High (VH)
High (H)	(0.71,0.2,0.21)	High (H)
Medium (M)	(0.51,0.55,0.51)	Medium (M)
Low (L)	(0.31,0.8,0.81)	Low (L)
Very Low (VL)	(0.11,0.90,0.94)	Very Low (VL)

Whereas for evaluations concerning alternatives and criteria, a scale of importance is defined for each SVNN according to the scales shown in Table 3.

Table 3: Linguistic terms representing the neutrosophic weight of the alternatives and criteria obtained. Source: own elaboration.

Linguistic term	**SVNN**
Extremely Good (EG)	(1,0,0)
Very Very Good (VVG)	(0.94,0.11,0.12)
Very Good (VG)	(0.84,0.21,0.22)
Good (G)	(0.74,0.31,0.32)
Moderately Good (MG)	(0.64,0.41,0.42)
Medium (M)	(0.54,0.51,0.52)
Moderately Bad (MB)	(0.44,0.61,0.62)
Bad (B)	(0.34,0.71,0.72)
Very Bad (VB)	(0.24,0.81,0.82)
Very Very Bad (VVB)	(0.14,0.91,0.92)
Extremely Bad (EB)	(0,0.95,1)

Step 2: Formulation of the final decision matrix (FDM).

The method begins with the identification of alternatives (A_n) and available criteria (Tx^n_{J+L}). Then, the decision-making matrix is constructed, which contains n rows representing the alternatives $A_1, \dots, A_n$ in the evaluation, and J+L columns representing the criteria under evaluation (J quantitative criteria and L qualitative criteria). In this way, the final decision matrix (FDM) is calculated by using equation (3).

$$\text{MDF==}[VO, VST] \begin{bmatrix} A^1 \\ A^2 \\ \vdots \\ A^n \end{bmatrix} \begin{bmatrix} t^1_1 & \cdots & t^1_J & t^1_{J+1} & \cdots & t^1_{J+L} \\ t^2_1 & \cdots & t^2_J & t^2_{J+1} & \cdots & t^2_{J+L} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t^n_1 & \cdots & t^n_J & t^2_{J+1} & \cdots & t^n_{J+L} \end{bmatrix} \quad (3)$$

Where A_i represents the alternatives, for $i = 1 \dots n$, and x^i_j represents the inputs of alternative i with respect to criterion j.

Step 3: Calculate the normalized decision matrix.

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It is feasible that the rating criteria are expressed in various units or measurement scales; therefore, normalization is carried out. Where the Euclidean norm is obtained in accordance with equation (4) for the criterion x_j.

$$|T_j| = \sqrt{\sum_{1}^{n} t_i^2} \tag{4}$$

Therefore, the normalization of each entry in the FDM is carried out in accordance with equation (5).

$$Nt_{ij} = \frac{t_{ij}}{|T_j|} \tag{5}$$

The results obtained by using equation (5) are dimensionless values that lack scale, allowing the operations among the criteria to be additive.

Step 4: Calculate the weighted normalized decision matrix.

Taking into account the different importance of the criteria, the weighted normalized ratings WNx_{ij} are calculated with equation (6).

$$g_{ij} = w_i \cdot Nt_{ij} \tag{6}$$

Step 5: Calculate the global score for each alternative, according to equation (7).

$$S(x_i) = \sum_{j=1}^{h} g_{ij} - \sum_{j=h+1}^{n} g_{ij} \tag{7}$$

Where, h is the number of responses to maximize, (n-h) the number of responses to minimize. While S(x_i) is the ranking of the i-th experimental case with respect to all responses. Consequently, the higher the value of S(x_i), the better the performance of multiple responses.

Step 6: The Tchebycheff metric is used to calculate the distance from the reference point. Subsequently, the distance between each alternative and the reference point $R[r_j]$ is measured. This reference point is constructed from the best evaluation to designate the ideal alternative, according to equations (8 and 9).

$$Dist_{(i;j)} = \{max_j|r_j - g_{ij}|\} \tag{8}$$

The alternatives are ordered according to the shortest distance.

$$min_i = \{max_j|r_j - g_{ij}|\} \tag{9}$$

Step 7: Calculate the complete multiplicative equation, U_i (Full multiplicative form), according to equation (10).

$$U_i = \frac{\prod_{j=1}^{n} X_{ij}^{w_j}}{\prod_{j=1}^{n} X_{ij}^{(1-w_j)}} \tag{10}$$

Where the numerator represents the benefit criteria and the denominator the cost criteria. Then, the alternatives are sorted in decreasing order, with the best alternatives being those with the higher value.

Step 8: Dominance Theory.

When applying the dominance theory, the rankings from the Reference Point, Full Multiplicative Form, and the overall score are combined. Weights are assigned to each index, and the ranks are summed to obtain the final ranking. Lastly, they are sorted in ascending order to obtain the final classification of the alternatives. It is considered that the three indices have the same importance to the decision-maker.

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Step 9: Calculate the Final Ranking. The ranks obtained in each index are added together and then arranged in ascending order.

3. Results.

It is noteworthy to highlight that socioeconomic factors play a significant role in the educational inequality of rural students in Ecuador. These factors can influence multiple aspects of the access and quality of education that students in these areas receive. Some of the most relevant socioeconomic factors include:

- Poverty and lack of resources: Rural communities often face higher levels of poverty, which limits access to basic resources such as food, housing, and health services [14]. This scarcity of resources also affects access to educational materials, transportation, and technology, potentially restricting the learning and academic development of students.
- Limited access to educational services: Geographical location and lack of infrastructure can hinder access to schools and educational centers in rural areas. The scarcity of educational institutions in these areas can force students to travel long distances to attend classes, affecting their attendance and participation in the educational process.
- Digital divide and connectivity: The lack of access to the internet and technology in rural communities can make accessing online educational resources and virtual learning programs challenging. This may disadvantage students compared to their urban counterparts who have access to a wider variety of educational tools.
- Quality of education: In some rural areas, the quality of education may be lower due to a lack of resources, teacher training, and adequate teaching materials. This can result in less effective education that does not meet the necessary standards to adequately prepare students.
- Access to educational opportunities: Opportunities to access higher education or technical and technological programs may be limited in rural areas. This can affect the educational and professional aspirations of students, limiting their chances for academic and career development.
- Family and cultural support: The family and cultural context can influence the motivation and support students receive in their education. In some rural communities, educational expectations may differ, and there may be less emphasis on the value of formal education.
- Inequality in job opportunities: The lack of job opportunities in rural areas can demotivate some students from continuing their education, as they may perceive that education will not provide them with significant advantages in the local labor market.

The interaction of these socioeconomic factors can lead to greater educational inequality between rural and urban students in Ecuador. To address this issue, it is essential to implement comprehensive public policies that consider the specific needs of rural communities by providing equitable access to educational resources. For this purpose, 6 strategies and/or measures to improve educational equity in rural areas of Ecuador are proposed (see Table 4).

Table 4: Strategies and/or measures to improve educational equity in rural areas. Source: Own elaboration.

Code	Strategies and/or measures	Scope
S1	Access to specialized educational programs	Ensure that rural students have access to specialized educational programs, such as bilingual and intercultural education, that recognize and value the cultural and linguistic diversity of these communities.
S2	Teacher training	Implement training and continuous training programs for teachers in rural areas, with emphasis on inclusive educational methodologies, effective use of technologies, and pedagogical approaches that address the specific needs of rural students.
S3	Scholarship programs and financial support	Establish scholarship and financial support programs for rural students with low economic resources, so that they can access quality education and continue their higher or technical studies.

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Code	Strategies and/or measures	Scope
S4	School feeding programs	Implement school feeding programs that ensure adequate nutrition for rural students, which can improve their health and well-being and contribute to better academic performance
S5	Improve and expand technological infrastructure and resources	Improve and expand school infrastructure in rural areas by providing well-equipped classrooms and access to technology, such as computers and internet access, to facilitate virtual learning and access to online educational resources. [15]
S6	Partnerships with the community	Foster alliances between educational institutions and the local community, by including parents and community leaders, to strengthen the participation and commitment to the education of rural students.

Below are 6 criteria for evaluating the proposed strategies and/or measures, each with an assigned weight within a scale of 0 to 1, based on the results obtained from Saaty's AHP (Analytic Hierarchy Process) method (see Tables 5, 6, and 7).

Table 5: Criteria for Evaluating Strategies and/or Measures. Source: Own elaboration.

Code	Criteria
F1	Cultural and linguistic relevance
F2	Effectiveness in teacher development
F3	Equity in access to opportunities
F4	Impact on school retention and dropout
F5	Community participation and commitment
F6	Impact on improving technological access

Table 6: Normalized Matrix. Source: Own elaboration.

F	F1	F2	F3	F4	F5	F6
F1	1	〈(2,3,4);0.30,0.75, 0.70〉	〈(2,3,4);0.30,0.75, 0.70〉	〈(2,3,4);0.30,0.75, 0.70〉	〈(1,1,1);0.50,0.50, 0.50〉	〈(6,7,8);0.90,0.10, 0.10〉
F2	〈(2,3,4);0.30,0.75, 0.70〉	1	〈(2,3,4);0.30,0.75, 0.70〉	〈(2,3,4);0.30,0.75, 0.70〉	〈(1,1,1);0.50,0.50, 0.50〉	〈(6,7,8);0.90,0.10, 0.10〉
F3	〈(2,3,4);0.30,0.75, 0.70〉	〈(2,3,4);0.30,0.75, 0.70〉	1	〈(4,5,6);0.80,0.15, 0.20〉	〈(1,1,1);0.50,0.50, 0.50〉	〈(6,7,8);0.90,0.10, 0.10〉
F4	〈(2,3,4);0.30,0.75, 0.70〉	〈(2,3,4);0.30,0.75, 0.70〉	〈(4,5,6);0.80,0.15, 0.20〉	1	〈(2,3,4);0.30,0.75, 0.70〉	〈(6,7,8);0.90,0.10, 0.10〉
F5	〈(1,1,1);0.50,0.50, 0.50〉	〈(1,1,1);0.50,0.50, 0.50〉	〈(1,1,1);0.50,0.50, 0.50〉	〈(2,3,4);0.30,0.75, 0.70〉	1	〈(6,7,8);0.90,0.10, 0.10〉
F6	〈(6,7,8);0.90,0.10, 0.10〉	〈(6,7,8);0.90,0.10, 0.10〉	〈(6,7,8);0.90,0.10, 0.10〉	〈(6,7,8);0.90,0.10, 0.10〉	〈(6,7,8);0.90,0.10, 0.10〉	1

Table 7: Analysis of the consistency of the exercise. Source: Own elaboration.

Criteria	Weight (w)		Approximate eigenvalues	
F1	(0.11,0.90,0.94)	0.31	6.308383234	Eigenvalue= 6.5798 CI=0.12 RC=0.09<=0.10 Consistent
F2	(0.11,0.90,0.94)	0.66	6.51555855	
F3	(0.11,0.90,0.94)	0.43	6.004999755	
F4	(0.31,0.8,0.81)	1.26	6.869551834	
F5	(0.11,0.90,0.94)	0.42	6.612645786	

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F6	(0.91,0.15,0.11)	3.81	7.167921167	

Once the weights are calculated, the predominant criterion identified is the impact on improving technological access. To minimize the effects of the analyzed factors, it is necessary to improve educational equity in rural areas. Therefore, experts determined 6 alternatives to evaluate for the analyzed factors and to determine the resulting weight (tablesT8 to 11). For this purpose, the MULTIMOORA method is chosen to be used.

Table 8: Final decision matrix. Source: Own elaboration.

	F1	**F2**	**F3**	**F4**	**F5**	**F6**
Alternatives	**Max**	**Max**	**Max**	**Max**	**Max**	**Max**
S1	(0.74,0.31,0.32)	(0.64,0.41,0.42)	(0.54,0.51,0.52)	(0.74,0.31,0.32)	(0.64,0.41,0.42)	(0.74,0.31,0.32)
S2	(0.64,0.41,0.42)	(0.54,0.51,0.52)	(0.74,0.31,0.32)	(0.64,0.41,0.42)	(0.74,0.31,0.32)	(0.54,0.51,0.52)
S3	(0.54,0.51,0.52)	(0.74,0.31,0.32)	(0.64,0.41,0.42)	(0.64,0.41,0.42)	(0.54,0.51,0.52)	(0.74,0.31,0.32)
S4	(0.64,0.41,0.42)	(0.64,0.41,0.42)	(0.54,0.51,0.52)	(0.74,0.31,0.32)	(0.64,0.41,0.42)	(0.64,0.41,0.42)
S5	(0.84,0.21,0.22)	(0.84,0.21,0.22)	(0.74,0.31,0.32)	(0.94,0.11,0.12)	(0.84,0.21,0.22)	(0.74,0.31,0.32)
S6	(0.54,0.51,0.52)	(0.64,0.41,0.42)	(0.74,0.31,0.32)	(0.54,0.51,0.52)	(0.54,0.51,0.52)	(0.74,0.31,0.32)

Table 9: Normalized matrix. Source: own elaboration.

	F1	**F2**	**F3**	**F4**	**F5**	**F6**
Alternatives	**Max**	**Max**	**Max**	**Max**	**Max**	**Max**
S1	(0.44,0.61,0.62)	(0.34,0.71,0.72)	(0.34,0.71,0.72)	(0.34,0.71,0.72)	(0.34,0.71,0.72)	(0.34,0.71,0.72)
S2	(0.34,0.71,0.72)	(0.24,0.81,0.82)	(0.44,0.61,0.62)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.24,0.81,0.82)
S3	(0.24,0.81,0.82)	(0.44,0.61,0.62)	(0.34,0.71,0.72)	(0.24,0.81,0.82)	(0.24,0.81,0.82)	(0.34,0.71,0.72)
S4	(0.34,0.71,0.72)	(0.34,0.71,0.72)	(0.24,0.81,0.82)	(0.34,0.71,0.72)	(0.34,0.71,0.72)	(0.34,0.71,0.72)
S5	(0.44,0.61,0.62)	(0.44,0.61,0.62)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.44,0.61,0.62)	(0.34,0.71,0.72)
S6	(0.24,0.81,0.82)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.24,0.81,0.82)	(0.24,0.81,0.82)	(0.34,0.71,0.72)
w	**(0.11,0.90,0.94)**	**(0.11,0.90,0.94)**	**(0.11,0.90,0.94)**	**(0.31,0.8,0.81)**	**(0.11,0.90,0.94)**	**(0.91,0.15,0.11)**

Table 10: Normalized and weighted matrix. Source: own elaboration.

	F1	**F2**	**F3**	**F4**	**F5**	**F6**
Alternatives	**Max**	**Max**	**Max**	**Max**	**Max**	**Max**
S1	0.022	0.040	0.025	0.081	0.026	0.223
S2	0.019	0.034	0.032	0.068	0.029	0.173
S3	0.016	0.046	0.028	0.063	0.022	0.237
S4	0.019	0.039	0.024	0.078	0.025	0.203
S5	0.025	0.049	0.031	0.094	0.032	0.229
S6	0.017	0.039	0.032	0.058	0.022	0.231
r_j	(0.24,0.81,0.82)	(0.44,0.61,0.62)	(0.24,0.81,0.82)	(0.84,0.21,0.22)	(0.24,0.81,0.82)	(1,0,0)

Table 11: Ranking of the alternatives. Source: own elaboration.

Alternatives	**Rank Reference Point**	**Rank Full Multiplicative Form**	**Global Rank**
S1	0.014	36.3295	(0.71,0.2,0.21)
S2	0.064	38.3558	(0.51,0.55,0.51)
S3	0.031	55.1427	(0.71,0.2,0.21)
S4	0.034	45.2815	(0.71,0.2,0.21)
S5	0.008	17.7435	(0.91,0.15,0.11)
S6	0.036	56.2135	(0.71,0.2,0.21)

Doi: https://doi.org/10.54216/FPA.120106
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After evaluating each alternative according to the established criteria, the rankings of each alternative within the neutrosophic set are analyzed. For this, in this study, it is proposed to examine through MULTIMOORA the global rank, the reference point rank, and the full multiplicative form rank to determine the optimal solution to the identified factors (see Figures 1 to 3).

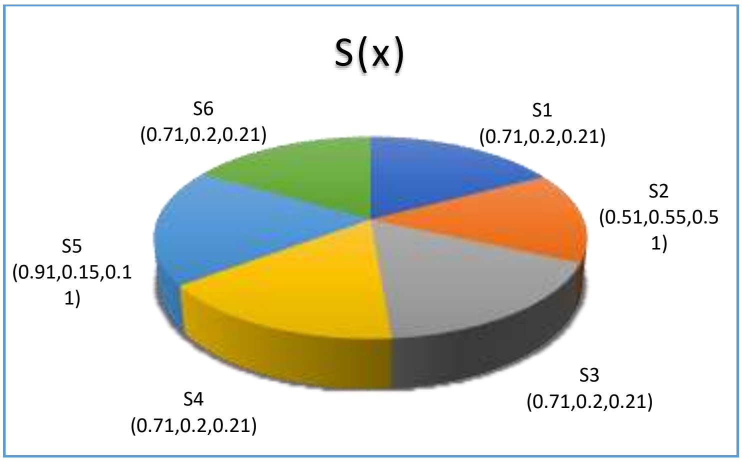

Figure 1: Overall score for each alternative $S(x)$. Source: own elaboration.

In Figure 1, the weighted importance values for each alternative are defined to obtain the global score. Thus, among the evaluated alternatives, it is evident that strategy S5 has a high effectiveness for solving the posed problem. While strategies S1, S3, S4, and S6 share the same subset of importance (high) in solving the problem. However, strategy S2 is far from being the optimal solution for improving educational equity in rural areas of Ecuador. Therefore, the distances of the alternatives with respect to the reference point $R[r_j]$ are calculated (see Figure 2).

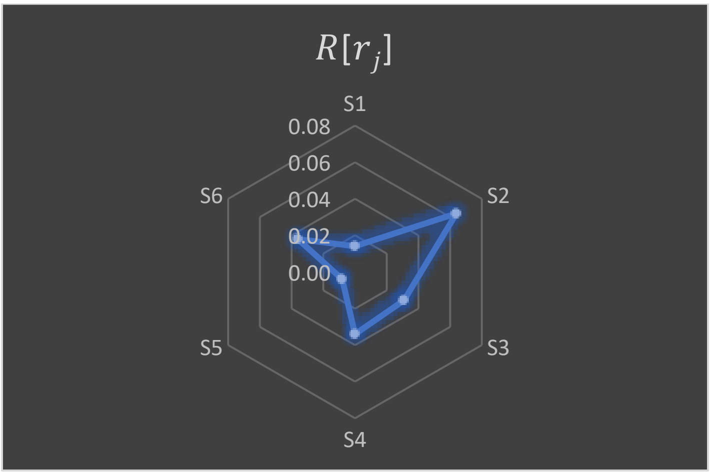

Figure 2: Distance between each alternative and the reference point $R[r_j]$. Source: own elaboration.

The results obtained and represented in Figure 2 define the radial solution sets within the established neutrosophic ranges. Thus, strategies with values close to the first radial constitute the best solutions that include the indeterminacy defined in the study. Therefore, once the reference point $R[r_j]$ is defined, the neutrosophic radials close to the ideal solution are analyzed:

- First neutrosophic radial: Alternatives S5 and S1 are part of the solution set. Within this radial, strategy S5 is dominant over S1.
- Second neutrosophic radial: Alternatives S3, S4, and S6 are located as part of the solution set. However, as a dominant strategy, S3 is visualized over the other strategies.
- Third neutrosophic radial: No strategies are visualized.

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- Fourth neutrosophic radial: Strategy S2 is located and thus the furthest from the ideal solution.

The analysis of the neutrosophic radials has clarified the results obtained in the global score $S(x)$ for each alternative. Thus, strategy S5 remains the optimal solution over the rest. As a second aspect to consider, it is observed that strategy S1 is outside the global subset S(x), where strategies S1, S3, S4, and S6 were located. Meanwhile, strategy S2 is far from being a timely solution to apply initially to improve educational equity in rural areas of Ecuador. Consequently, it is necessary to evaluate the full multiplicative form of the MULTIMOORA method to define the strength of the optimal solutions within the neutrosophic set and to define the components of the subsets S3, S4, and S6 (see Figure 3).

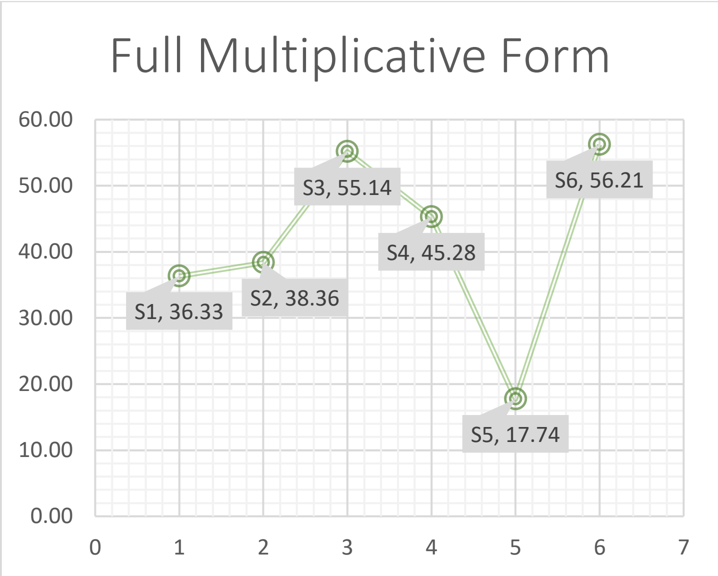

Figure 3: Full multiplicative form. Source: own elaboration.

The results represented in Figure 3 confirm that strategy S5 is dominant within the neutrosophic set for educational equity in rural areas of Ecuador. Meanwhile, strategy S1 continues among the optimal solutions to consider. In contrast, the following findings are visualized:

- Strategy S2, previously considered a distant solution, has moved up to third place.
- The subset consisting of S3, S4, and S6 shows uncertainties regarding the dominant strategy. There appears to be a rivalry between S4 and S3, while strategy S6 maintains a recessive position.

Therefore, a comprehensive evaluation of the rankings is required to determine the position of strategy S2 and to define the dominant strategy in the subsets S3, S4, and S6. To do this, the Reference Point ranking, the Full Multiplicative Form, and the global score are combined as shown in Figure 4.

Doi: https://doi.org/10.54216/FPA.120106
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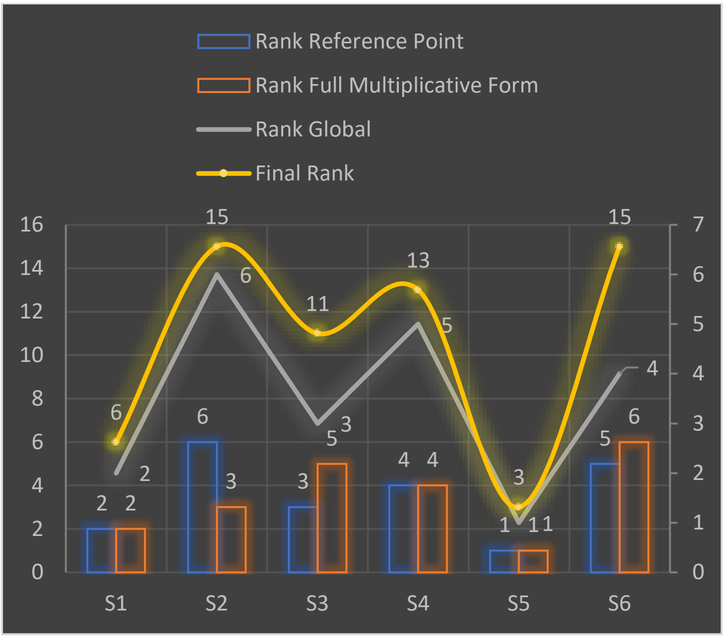

Figure 4: Combination of the Reference Point ranking, Full Multiplicative Form, and the global score Source: Own elaboration.

Based on the evaluation carried out for each ranking in the modeling of the MULTIMOORA method and visualized in Figure 4, the following results are defined:

- Strategy S5 is definitively identified as having the highest potential for developing education in rural areas of Ecuador.
- Strategy S1 is considered the second option due to its proximity to the ideal option.
- The radial subset formed by strategies S3-S4-S6 identifies S3 as the dominant strategy. However, there is a strong union between strategies S3 and S4 throughout the neutrosophic set. The explanation for this strong connection lies in the fact that S3 focuses on scholarship programs and financial support, while S4 highlights the need to create school feeding programs. This indicates that even if the objective proposed in strategy S3 is achieved, the implementation of school feeding programs for the well-being of students must be included. Therefore, both strategies should be developed simultaneously and not independently.
- It should be detailed that within this radial subset lies strategy S6, which shows a weak relationship with S3 and S4. Therefore, it does not constitute a dependent element of S3-S4 and is a distant strategy similar to S2.
- Strategy S2 is considered the last option due to its distance from the ideal option, indicating that the aforementioned strategies should be developed before continuing with S2.

It is worth noting that all strategies present a significant level of importance for improving educational equity in rural areas, but *improving and expanding infrastructure and technological resources* is key to mitigating educational inequalities in these areas. [16]

A project is proposed to enhance infrastructure and technological resources to improve education in rural areas of Ecuador. The main objective of this project is to improve access to education in rural areas of Ecuador through the strengthening of educational infrastructure and access to appropriate technological resources for students and teachers. The project is proposed to last three years. It is necessary to improve the quality and equity of education in rural areas of Ecuador through the implementation of suitable educational infrastructure and access to technological resources to support learning and teaching in virtual environments.

Teacher training in the effective use of educational technologies is also essential to optimize virtual learning and improve interaction with rural students. Furthermore, the development of accessible educational platforms tailored to the needs of these communities should be encouraged.

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It is important to highlight that reducing the educational gap in rural areas not only contributes to the academic development of students but will also have a positive impact on the socioeconomic growth of these communities. Among these impacts are promoting employment opportunities and improving the quality of life.

6. Conclusion

There is a noticeable digital educational gap between rural and urban students in Ecuador. The lack of access to technological resources and connectivity in rural areas limits academic development and access to educational opportunities, creating inequality in the learning process.

It is necessary to implement comprehensive public policies that address educational inequality in rural areas. These policies should include investment in educational infrastructure, provision of devices and access to the internet, as well as teacher training in the effective use of educational technologies.

Reducing the digital gap and improving access to education in rural areas can have a significant impact on the country's socioeconomic development. By providing equal educational opportunities, the development of skills and competencies in rural students is promoted, which in turn contributes to the economic and social growth of the country as a whole.

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