Fuzzy Metric Space of Weak Fuzzy Complex Numbers and Plithogenic Metric Spaces

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Abstract

The main goal of this work is to define for the first time the concept of the metric space of weak fuzzy complex numbers, where we present a suitable metric defined over the ring of weak fuzzy complex numbers, and we study the open balls, closed balls, and torus generated from its structure. On the other hand, we study the symbolic 2-plithogenic and 3-plithogenic/4-plithogenic metric space and its ability to be generated from classical metrics, with many interesting properties related to its analytical structure. Also, many examples will be illustrated to clarify and explain the novelty of this work.

Keywords: Fuzzy number; weak fuzzy complex number; weak fuzzy complex metric space; plithogenic set.

1. Introduction and preliminaries

The weak fuzzy complex numbers are considered as two-dimensional generalizations of the real numbers and like that by which the complex numbers are constructed or split-complex numbers. Weak fuzzy numbers were first defined in [1], and many of their algebraic properties were studied. Then a weak fuzzy complex vector space was constructed over these numbers [2] and connected these new numbers to the plithogenic numbers [5-9].
In [5], the researchers studied some Diophantine equations associated with weak fuzzy complex numbers, where they found an efficient algorithm for calculating the triples and quadruples of the weak fuzzy complex Pythagorean [10].
On the other hand, these numbers were used in Computer Science and were programmed by the MATLAB environment [4], with an interesting application in representing some real vectorial equations [11].
As a generalization of fuzzy and neutrosophic sets, plithogenic sets have been widely used in the study of numerous mathematical and algebraic structures. Where in [12,16-17] we see concepts such as plithogenic vector spaces and plithogenic modules, and in [13-15] we see algebraic mappings defined in order to connect plithogenic rings with their neutrosophic analogs with wide applications in the classification of groups of units.

All this prompted us to study the metric fuzzy aspect associated with this numerical ring, where we used the set of weak fuzzy complex numbers to define a metric on it and then to construct a metric space whose elements are the weak fuzzy complex numbers. This new concept has led us to study many metric properties of these spaces, such as open balls, closed balls, and even their complements relative to the defined metric.
And also we studied the plithogenic metric spaces and compared them with weak fuzzy ones.

Definition ([1]). The *set of Weak Fuzzy Complex numbers* was defined as follows, where 'J' is the Weak Fuzzy Complex operator ($J \notin \boldsymbol{R}$):

$$\boldsymbol{F_J} = \{x_0 + x_1 J \; ; \; x_0 , x_1 \in \boldsymbol{R} , J^2 = t \in]0,1[\, \}$$

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Properties of the Weak Fuzzy Complex numbers: ([1])

Let $X = x_0 + x_1 J$, $Y = y_0 + y_1 J \in \boldsymbol{F}_J$, where $x_0, x_1, y_0, y_1 \in \boldsymbol{R}$

Addition is defined as follows: $+Y = (x_0 + y_0) + (x_1 + y_1)J$.

Multiplication is defined as follows: $.Y = (x_0 y_0 + x_1 y_1 t) + (x_0 y_1 + x_1 y_0)J$.

2. Main discussion

Definition:

Let $F_J = \{a + \mathrm{b_J}: a, b \in \mathbb{R}, J^2 = t \in]0,1[\ \}$ be the ring of weak fuzzy complex numbers, we define:

$d: F_J \times F_J \to [0, \infty[$ such that:

$d(a + b_J, c + d_J) = \sqrt{(c-a)^2 + t(b-d)^2}$

we call (d) a weak fuzzy complex metric, and (F_J, d) is called a weak fuzzy complex metric space.

Theorem:

Let (F_J, d) be the weak fuzzy complex metric space defined previously, then for $X = a + b_J, Y = c + d_J, Z = m + n_J \in F_J$, we have.

1] $d(x,x) = 0$, $d(x,y) \geq 0$

2] $d(x,y) = d(y,x)$

3] $d(x,y) + d(y,z) \geq d(x,z)$

Proof:

1] $d(x,x) = \sqrt{(a-a)^2 + t(b-b)^2} = 0$.

2] Also, $d(x,y) \geq 0$.

3]$d(x,y) + d(y,z) = \sqrt{(c-a)^2 + t(b-d)^2} + \sqrt{(m-c)^2 + (n-d)^2 t} \geq \sqrt{(m-a)^2 + t(n-b)^2} = d(x,z)$.

Remark:

If $d(x,y) = 0$, then:

$\begin{cases} c = a \\ b = d \end{cases}$ and $x = y$.

Example:

For $J^2 = t = \frac{1}{9}$, consider: $x = 3 - 4J$, $y = 2 + J$, $z = 5 + 2J$

We have:

$$d(x,y) = \sqrt{(3-2)^2 + (-5)^2 . \tfrac{1}{9}} = \sqrt{1 + \tfrac{25}{9}} = \frac{\sqrt{34}}{3}$$

$$d(x,z) = \sqrt{(-2)^2 + \tfrac{1}{9} . (-6)^2} = \sqrt{4+4} = \sqrt{8}$$

$$d(y,z) = \sqrt{9 + \tfrac{1}{9} . (1)} = \sqrt{\tfrac{82}{9}} = \frac{\sqrt{82}}{3}$$

it is clear that $d(x,y) + d(y,z) \geq d(x,z)$

Definition:

The weak fuzzy complex open ball with radius $r > 0$: and $x = a + b_J$ as a center is:

$W_0 = \{y = c + d_J \in F_J;\ d(y,x) < r\}$.

The weak fuzzy complex closed ball with radius $r > 0$ and $x = a + b_J$ as a center is:

$W_c = \{y = c + d_J \in F_J;\ d(y,x) \leq r\}$

The weak fuzzy complex torus of center $X = a + b_J$ and $W_T = \{y = c + d_J \in F_J;\ d(y,x) = r\}$.

Theorem:

Consider the weak fuzzy complex metric space (F_J, d), then:

1] $W_o = (a + b_J, r)$ is equivalent to the points that located inside the cartesian ellipse $\frac{(x-a)^2}{r^2} + \frac{(y-b)^2}{\frac{r^2}{t}} = 1$

2] $W_T = (a + b_J, r)$ is equivalent to the points of the ellipse $\frac{(x-a)^2}{r^2} + \frac{(y-d)^2}{\frac{r^2}{t}} = 1$

3] $W_c = (a + b_J, r) = W_o = (a + b_J, r) \cup W_T = (a + b_J, r)$

Proof:

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Assume that $M = x + yJ \in W_o = (a + b_J, r)$, then:
$d^2(M, a + b_J) = (x - a)^2 + t(y - b)^2 < r^2$, hence $\frac{(x-a)^2}{r^2} + \frac{(y-b)^2}{\frac{r^2}{t}} < 1$, thus $W_o = (a + b_J, r)$ represents the points inside $\frac{(x-a)^2}{r^2} + \frac{(y-b)^2}{\frac{r^2}{t}} = 1$.

2] If $M = x + yJ \in W_T$, then $d^2(M, a + b_J) = r^2$, thus $\frac{(x-a)^2}{r^2} + \frac{(y-b)^2}{\frac{r^2}{t}} = 1$, and the proof holds.

3] let $y \in W_0 \cup W_t$, then $d(y, a + b_J) = r$ or $d(y, a + b_J) < r$, hence $y \in W_c$.
Conversely, if $y \in W_c$, then $d(y, a + b_J) \leq r$, so that $y \in W_0 \cup W_t$, which implies that $W_0 \cup W_t = W_c$.

Example:

For $J^2 = t = \frac{1}{2}$, let's consider $A = 4 + 6J$, $r = 2$, then

$W_0(A, 2) = \{(x, y) \in \mathbb{R}^2; \frac{(x-4)^2}{4} + \frac{(y-6)^2}{8} = 1\}$.

$W_t(A, 2) = \{(x, y) \in \mathbb{R}^2; \frac{(x-4)^2}{4} + \frac{(y-6)^2}{8} = 1\}$

Theorem:

Let $W_t(a + b_J, r)$ be a weak fuzzy complex tours, then it can be represented by the following parameters:

$$\begin{cases} x = r.\cos\theta + a \\ y = \frac{r}{\sqrt{t}} \sin\theta + b \end{cases}$$

Proof:

$W_t(a + b_J, r) = \{(x, y) \in \mathbb{R}^2; \frac{(x-a)^2}{r^2} + \frac{(y-b)}{\frac{r^2}{t}} = 1\}$, put

$$\begin{cases} \frac{x-a}{r} = \cos\theta \\ \frac{y-b}{\frac{r}{\sqrt{t}}} = \sin\theta \end{cases}, \text{ then } \begin{cases} x = r\cos\theta + a \\ y = \frac{r}{\sqrt{t}} \sin\theta + b \end{cases}$$

Example:

Let's find the parametric representation of the following weak fuzzy complex torus $W_T(3 + 2J, 4)$, $J^2 = t = \frac{1}{2}$.
$W_T(3 + 2J, 4) = \{(x, y) \in \mathbb{R}^2;\ x = 4\cos\theta + 3, y = 4\sqrt{2}\sin\theta + 2, \theta \in \mathbb{R}^2\}$

Theorem:

Let $W_0(a + b_J, r_1)$, $W_0(a + b_J, r_2)$ be two weak fuzzy complex open balls. $W_T(a + b_J, r_1)$, $W_T(a + b_J, r_2)$, $W_c(a + b_J, r_1)$, $W_c(a + b_J, r_2)$ be the corresponding weak fuzzy complex toruses and closed balls, hence

$$\text{If } r_1 > r_2, \text{ then } \begin{cases} W_0(a + b_J, r_2) < W_0(a + b_J, r_1) \\ W_c(a + b_J, r_2) < W_c(a + b_J, r_1) \\ W_T(a + b_J, r_1) \cap W_T(a + b_J, r_2) = \emptyset \end{cases}$$

Proof:

Let $c + d_J \in W_0(a + b_J, r_2)$, then $d(a + b_J, c + d_J) < r_2 \leq r_1$, hence $c + d_J \in W_0(a + b_J, r_1)$, and $W_0(a + b_J, r_2) < W_0(a + b_J, r_1)$.
Also if $c + d_J \in W_c(a + b_J, r_2)$, then:
$d(a + b_J, c + d_J) \leq r_2 \leq r_1$, thus $c + d_J \in W_c(a + b_J, r_1)$, and $W_c(a + b_J, r_2) < W_c(a + b_J, r_1)$
on the other hand, if $c + d_J \in W_T(a + b_J, r_1) \cap W_T(a + b_J, r_2)$, then: $d(a + b_J, c + d_J) = r_1 = r_2$ which is a contradiction.
Let (F_J, d) be a weak fuzzy complex metric space with $J^2 = t \in]0,1[$, then if $m + n_J \in W_T(a + b_J, r_1) \cap W_T(c + d_J, r_1)$.

$$\text{We get } \begin{cases} r_1 = r_2 \\ (c - a)(2m - a - c) + t(d - b)(2n - b - d) = 0 \end{cases}$$

Proof:

Assume that $r_1, r_2 \in \mathbb{R}^2$, $a + b_J, c + d_J \in F_J$, and $W_T(a + b_J, r_1) \cap W_T(c + d_J, r_1) \neq \emptyset$, then there exists $m + n_J \in F_J$ such that:
$d(a + b_J, m + n_J) = r_1 = d(c + d_J, m + n_J)$, so that:
$(a - m)^2 + t(b - n)^2 = (c - m)^2 + t(d - n)^2$, hence
$(a - c)(2m - a - c) + t(d - b)(2n - b - d) = 0$.

Remark:

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If $r_1 \neq r_2$, then:

$$\begin{cases}(a-m)^2+t(b-n)^2={r_1}^2\\(c-m)^2+t(d-n)^2={r_2}^2\end{cases}$$

Thus: $(a-c)(2m-a-c)+t(d-b)(2n-b-d)=(r_1-r_2)(r_1+r_2)$.

Definition:

Let (F_J,d) be the weak fuzzy complex metric space, we defined

$\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\{\mathrm{c}+\mathrm{d_J}\in\mathrm{F_J};\ \mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})\geq\mathrm{r}\}$

$\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\{\mathrm{c}+\mathrm{d_J}\in\mathrm{F_J};\ \mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})>\mathrm{r}\}$.

Theorem:

Let (F_J,d) be a weak fuzzy complex metric space, then:

1] $W_o\cap\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\emptyset$

2] $W_c\cap\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\emptyset$

3] If $W_o(\mathrm{a}+\mathrm{b_J},r_1)\leq W_o(\mathrm{a}+\mathrm{b_J},r_2)$, then:
$\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},r_2)\leq\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},r_1)$.

4] If $W_c(\mathrm{a}+\mathrm{b_J},r_1)\leq W_c(\mathrm{a}+\mathrm{b_J},r_2)$, then:
$\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},r_2)\leq\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},r_1)$.

Proof:

1] Assume that $c+\mathrm{d_J}\in W_o\cap\mathrm{W_o}'$, then:

$$\begin{cases}\mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})<r_1\\\mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})\geq r_1\end{cases}$$

Which is a contradiction:

2] Assume that $c+\mathrm{d_J}\in W_c\cap\mathrm{W_c}'$, then:

$$\begin{cases}\mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})\leq r_1\\\mathrm{d}(\mathrm{a}+\mathrm{b_J},\mathrm{c}+\mathrm{d_J})>r_1\end{cases}$$

Which is another contradiction:

3] If $W_o(r_1)\leq W_o(r_2)$, then $r_1\leq r_2$, thus for any $c+\mathrm{d_J}\in\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},r_2)$, we get:
$d(\mathrm{a}+\mathrm{b_J},c+\mathrm{d_J})\geq r_2\geq r_1$, thus $c+\mathrm{d_J}\in\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},r_1)$.

4] It can be proved by a similar argument.

Example:

Consider $J^2=t=\frac{1}{7}$, $2+3J\in\mathrm{F_J}$, hence:

$\mathrm{W_o}'(2+3J,2)=\{c+d_J\in F_J;d(2+3J\,,c+d_J)\geq 2\}=\{(c,d)\in\mathbb{R}^2;\ (c-2)^2+\frac{1}{7}(d-3)^2\geq 4\}$

$\mathrm{W_c}'(2+3J,2)=\{c+d_J\in F_J;d(2+3J\,,c+d_J)>2\}=\{(c,d)\in\mathbb{R}^2;\ (c-2)^2+\frac{1}{7}(d-3)^2>4\}$

Remark:

In any (F_J,d), we have the following results:

1] $\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})\cap\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})$

2] $\mathrm{W_o}(\mathrm{a}+\mathrm{b_J},\mathrm{r})\cup\mathrm{W_o}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\mathrm{F_J}$.

3] $\mathrm{W_c}(\mathrm{a}+\mathrm{b_J},\mathrm{r})\cup\mathrm{W_c}'(\mathrm{a}+\mathrm{b_J},\mathrm{r})=\mathrm{F_J}$.

3. Symbolic 2-plithogenic Metric spaces

Definition:

Let $X(P_1,P_2)$ be a nonempty symbolic 2-plithogenic set, let
$d\colon X\times X\to 2-SP_{\mathbb{R}}$ such that:

1] $d(A,B)\geq 0, d(A,B)=0\Rightarrow A=B$

2] $d(A,B)=d(B,A)$

3] $d(A,C)+d(C,B)\geq d(A,B);\ A,B,C\in X(P_1,P_2)$.

Then d is called symbolic 2-plithogenic, and $X(P_1,P_2)$ is called symbolic 2-plithogenic metric space.

Example:

Consider $2-SP_{\mathbb{R}}=\{x_0+x_1P_1+x_2P_2;\ x_i\in\mathbb{R}\}$, then:

Define: $d\colon 2-SP_{\mathbb{R}}\times 2-SP_{\mathbb{R}}\to 2-SP_{\mathbb{R}}$ such that:

$d(x_0+x_1P_1+x_2P_2,y_0+y_1P_1+y_2P_2)=|x_0-y_0|+P_1[|x_0+x_1-y_0-y_1|-|x_0-y_0|]+P_1[|x_0+x_1+x_2-y_0-y_1-y_2|-|x_0+x_1-y_0-y_1|]$

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$(2-SP_{\mathbb{R}}, d)$ is a symbolic 2-plithogenic metric space.

The following theorem explains how classical metrics can generate a symbolic 2-plithogenic metric.

Theorem:

Let T be a nonempty set with three metrics $d_1, d_2, d_3 : T \times T \to \mathbb{R}$, then there exists a symbolic 2-plithogenic metric as follows:

$d: T(P_1, P_2) \times T(P_1, P_2) \to 2 - SP_{\mathbb{R}}$ Such that:

$d(x,y) = d_1(x_0, y_0) + P_1[d_2(x_0 + x_1, y_0 + y_1) - d_1(x_0, y_0)]$

$+P_1[d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) - d_2(x_0 + x_1, y_0 + y_1)]; X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$

Proof:

First of all, it is clear that $d(x,y) = d(y,x)$.

$d(x,y) \geq 0$, that is because:

$$\begin{cases} d_1(x_0, y_0) \geq 0 \\ d_2(x_0 + x_1, y_0 + y_1) \geq 0 \\ d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) \geq 0 \end{cases}$$

Also, $d(x,y) = 0$ implies that:

$$\begin{cases} d_1(x_0, y_0) = 0 \Rightarrow x_0 = y_0 \\ d_2(x_0 + x_1, y_0 + y_1) = 0 \Rightarrow x_1 = y_1 \\ d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) = 0 \Rightarrow x_2 = y_2 \end{cases}$$

Hence $X = Y$

On the other hand, we have: for $x, y, z \in T$; $Z = z_0 + z_1P_1 + z_2P_2$

$$d(x,z) + d(z,y) \geq d_1(x_0, y_0) + P_1[d_2(x_0 + x_1, y_0 + y_1) - d_1(x_0, y_0)] + P_2[d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) - d_2(x_0 + x_1, y_0 + y_1)]$$

Remark:

We call (d) a symbolic 2-plithogenic metric generated by d_1, d_2, d_3.

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2 > 0$ be a symbolic 2-plithogenic positive real number i.e. $r_0 > 0$, $r_0 + r_1 > 0$, $r_0 + r_1 + r_2 > 0$

Let $d: T(P_1, P_2) \times T(P_1, P_2) \to 2 - SP_{\mathbb{R}}$ be a symbolic 2-plithogenic metric generated by $d_1, d_2, d_3 : T \times T \to \mathbb{R}$, then:

1] $B(A, r) = \{b_0 + b_1P_1 + b_2P_2 \in T(P_1, P_2);\ d(b_0 + b_1P_1 + b_2P_2, A) < r\}$ is called a symbolic 2-plithogenic open ball, where $A = a_0 + a_1P_1 + a_2P_2 \in T(P_1, P_2)$.

2] $\overline{B}(A, r) = \{b_0 + b_1P_1 + b_2P_2 \in T(P_1, P_2);\ d(b_0 + b_1P_1 + b_2P_2, A) \leq r\}$ is called a symbolic 2-plithogenic closed ball, where $A = a_0 + a_1P_1 + a_2P_2 \in T(P_1, P_2)$.

3] $B_S(A, r) = \{b_0 + b_1P_1 + b_2P_2 \in T(P_1, P_2);\ d(b_0 + b_1P_1 + b_2P_2, A) = r\}$ is called a symbolic 2-plithogenic spherical surface.

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2, s = s_0 + s_1P_1 + s_2P_2 > 0$ with $r, s \in 2 - SP_{\mathbb{R}}$, then for $B_1(A, r), B_2(A, s)$, $A = a_0 + a_1P_1 + a_2P_2 \in T(P_1, P_2)$, we define

1] $min\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that:

$$d(X, A) < min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)]\}$$

2] $min\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that:

$$d(X, A) \leq min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)]\}$$

3] $max\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that:

$$d(X, A) < max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)]\}$$

4] $max\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that:

$$d(X, A) \leq max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)]\}$$

5] $\sim B_1 = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that: $d(X, A) \geq r_0 + r_1P_1 + r_2P_2\}$

6] $\sim \overline{B}_1 = \{X = x_0 + x_1P_1 + x_2P_2 \in T(P_1, P_2)\}$ such that: $d(X, A) > r_0 + r_1P_1 + r_2P_2\}$

Theorem:

The following properties are true:

1] $B_1 \cap (\sim B_1) = \overline{B}_1 \cap (\sim \overline{B}_1) = \emptyset$

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2] $min(B_1, B_2) \leq B_1 \cap B_2 \leq max\ (B_1, B_2)$
3] $min(\overline{B}_1, \overline{B}_2) \leq \overline{B}_1 \cap \overline{B}_2 \leq max\ (\overline{B}_1, \overline{B}_2)$.

Symbolic 3-plithogenic Metric spaces

Definition:

Let $X(P_1, P_2, P_3)$ be a non empty symbolic 3-plithogenic set, let
$d: X \times X \to 3 - SP_{\mathbb{R}}$ such that:
1] $d(A, B) \geq 0, d(A, B) = 0 \Rightarrow A = B$
2] $d(A, B) = d(B, A)$
3] $d(A, C) + d(C, B) \geq d(A, B);\ A, B, C \in X(P_1, P_2, P_3)$.
Then d is called symbolic 3-plithogenic, and $X(P_1, P_2, P_3)$ is called symbolic 3-plithogenic metric space.

Example:

Consider $3 - SP_{\mathbb{R}} = \{x_0 + x_1P_1 + x_2P_2 + x_3P_3;\ x_i \in \mathbb{R}\}$, then:
Define: $d: 3 - SP_{\mathbb{R}} \times 3 - SP_{\mathbb{R}} \to 3 - SP_{\mathbb{R}}$ such that:
$d(x_0 + x_1P_1 + x_2P_2 + x_3P_3, y_0 + y_1P_1 + y_2P_2 + y_3P_3) = |x_0 - y_0| + P_1[|x_0 + x_1 - y_0 - y_1| - |x_0 - y_0|] + P_2[|x_0 + x_1 + x_2 - y_0 - y_1 - y_2| - |x_0 + x_1 - y_0 - y_1|] + P_3[|x_0 + x_1 + x_2 + x_3 - y_0 - y_1 - y_2 - y_3| - |x_0 + x_1 + x_2 - y_0 - y_1 - y_2|]$
$(3 - SP_{\mathbb{R}}, d)$ is a symbolic 3-plithogenic metric space.
The following theorem explains how classical metrics can generate a symbolic 3-plithogenic metric.

Theorem:

Let T be a nonempty set with four metrics $d_1, d_2, d_3, d_4: T \times T \to \mathbb{R}$, then there exists a symbolic 3-plithogenic metric as follows:
$d: T(P_1, P_2) \times T(P_1, P_2) \to 3 - SP_{\mathbb{R}}$ Such that:
$d(x, y) = d_1(x_0, y_0) + P_1[d_2(x_0 + x_1, y_0 + y_1) - d_1(x_0, y_0)]$
$+P_2[d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) - d_2(x_0 + x_1, y_0 + y_1)] + P_3[d_4(x_0 + x_1 + x_2 + x_3, y_0 + y_1 + y_2 + y_3) - d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2)]$.

Proof:

First of all, it is clear that $d(x, y) = d(y, x)$.
$d(x, y) \geq 0$, that is because:

$$\left\{ \begin{array}{c} d_1(x_0, y_0) \geq 0 \\ d_2(x_0 + x_1, y_0 + y_1) \geq 0 \\ d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) \geq 0 \\ d_4(x_0 + x_1 + x_2 + x_3, y_0 + y_1 + y_2 + y_3) \geq 0 \end{array} \right.$$

Also, $d(x, y) = 0$ implies that:

$$\left\{ \begin{array}{c} d_1(x_0, y_0) = 0 \Rightarrow x_0 = y_0 \\ d_2(x_0 + x_1, y_0 + y_1) = 0 \Rightarrow x_1 = y_1 \\ d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) = 0 \Rightarrow x_2 = y_2 \\ d_4(x_0 + x_1 + x_2 + x_3, y_0 + y_1 + y_2 + y_3) = 0, implies\ x_3 = y_3 \end{array} \right.$$

Hence $X = Y$
On the other hand, we have: for $X, Y, Z \in T$;
$d(x, z) + d(z, y) \geq d(x, y)$.

Remark:

We call (d) a symbolic 3-plithogenic metric generated by d_1, d_2, d_3, d_4.

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2 + r_3P_3 > 0$ be a symbolic 3-plithogenic positive real number i.e. $r_0 > 0$, $r_0 + r_1 > 0$, $r_0 + r_1 + r_2 > 0, r_0 + r_1 + r_2 + r_3 > 0$
Let $d: T(P_1, P_2, P_3) \times T(P_1, P_2, P_3) \to 3 - SP_{\mathbb{R}}$ be a symbolic 3-plithogenic metric generated by $d_1, d_2, d_3, d_4: T \times T \to \mathbb{R}$, then:
1] $B(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 \in T(P_1, P_2, P_3);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3, A) < r\}$ is called a symbolic 3-plithogenic open ball, where $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 \in T(P_1, P_2, P_3)$.
2] $\overline{B}(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 \in T(P_1, P_2, P_3);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3, A) \leq r\}$ is called a symbolic 3-plithogenic closed ball, where $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 \in T(P_1, P_2, P_3)$.
3] $B_S(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 \in T(P_1, P_2, P_3);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3, A) = r\}$ is called a symbolic 3-plithogenic spherical surface.

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2 + r_3P_3, s = s_0 + s_1P_1 + s_2P_2 + s_3P_3 > 0$ with $r, s \in 3 - SP_{\mathbb{R}}$, then for $B_1(A, r), B_2(A, s)$, $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 \in T(P_1, P_2, P_3)$, we define

1] $min\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that:

$$d(X, A) < min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)] + P_3[min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - min(r_0 + r_1 + r_2, s_0 + s_1 + s_2)]\}$$

}

2] $min\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that:

$$d(X, A) \leq min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)] + P_3[min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - min(r_0 + r_1 + r_2, s_0 + s_1 + s_2)]\}$$

}

3] $max\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that:

$$d(X, A) < max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)] + P_3[max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - max(r_0 + r_1 + r_2, s_0 + s_1 + s_2)]\}$$

4] $max\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that:

$$d(X, A) \leq max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)] + P_3[max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - max(r_0 + r_1 + r_2, s_0 + s_1 + s_2)]\}$$

5] $\sim B_1 = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that: $d(X, A) \geq r_0 + r_1P_1 + r_2P_2 + r_3P_3\}$

6] $\sim \overline{B}_1 = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in T(P_1, P_2, P_3)\}$ such that: $d(X, A) > r_0 + r_1P_1 + r_2P_2 + r_3P_3\}$

Theorem:

The following properties are true:

1] $B_1 \cap (\sim B_1) = \overline{B}_1 \cap (\sim \overline{B}_1) = \emptyset$

2] $min(B_1, B_2) \leq B_1 \cap B_2 \leq max\ (B_1, B_2)$

3] $min(\overline{B}_1, \overline{B}_2) \leq \overline{B}_1 \cap \overline{B}_2 \leq max\ (\overline{B}_1, \overline{B}_2)$.

Symbolic 4-plithogenic Metric spaces

Definition:

Let $X(P_1, P_2, P_3, P_4)$ be a nonempty symbolic 3-plithogenic set, let $d: X \times X \to 4 - SP_{\mathbb{R}}$ such that:

1] $d(A, B) \geq 0, d(A, B) = 0 \Rightarrow A = B$

2] $d(A, B) = d(B, A)$

3] $d(A, C) + d(C, B) \geq d(A, B);\ A, B, C \in X(P_1, P_2, P_3, P_4)$.

Then d is called symbolic 4-plithogenic, and $X(P_1, P_2, P_3, P_4)$ is called symbolic 4-plithogenic metric space.

Example:

Consider $4 - SP_{\mathbb{R}} = \{x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_3;\ x_i \in \mathbb{R}\}$, then:

Define: $d: 4 - SP_{\mathbb{R}} \times 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$ such that:

$$d(x_0 + x_1P_1 + x_2P_2 + x_3P_3, y_0 + y_1P_1 + y_2P_2 + y_3P_3) = |x_0 - y_0| + P_1[|x_0 + x_1 - y_0 - y_1| - |x_0 - y_0|] + P_2[|x_0 + x_1 + x_2 - y_0 - y_1 - y_2| - |x_0 + x_1 - y_0 - y_1|] + P_3[|x_0 + x_1 + x_2 + x_3 - y_0 - y_1 - y_2 - y_3| - |x_0 + x_1 + x_2 - y_0 - y_1 - y_2|] + P_3[|x_0 + x_1 + x_2 + x_3 + x_4 - y_0 - y_1 - y_2 - y_3 - y_4| - |x_0 + x_1 + x_2 + x_3 - y_0 - y_1 - y_2 - y_3|]$$

$(4 - SP_{\mathbb{R}}, d)$ is a symbolic 4-plithogenic metric space.

The following theorem explains how classical metrics can generate a symbolic 4-plithogenic metric.

Theorem:

Let T be a nonempty set with five metrics $d_1, d_2, d_3, d_4, d_5: T \times T \to \mathbb{R}$, then there exists a symbolic 4-plithogenic metric as follows:

$d: T(P_1, P_2, P_3, P_4) \times T(P_1, P_2, P_3, P_4) \to 4 - SP_{\mathbb{R}}$ Such that:

$$d(x, y) = d_1(x_0, y_0) + P_1[d_2(x_0 + x_1, y_0 + y_1) - d_1(x_0, y_0)] + P_2[d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2) - d_2(x_0 + x_1, y_0 + y_1)] + P_3[d_4(x_0 + x_1 + x_2 + x_3, y_0 + y_1 + y_2 + y_3) - d_3(x_0 + x_1 + x_2, y_0 + y_1 + y_2)] + P_4[d_5(x_0 + x_1 + x_2 + x_3 + x_4, y_0 + y_1 + y_2 + y_3 + y_4) - d_4(x_0 + x_1 + x_2 + x_3, y_0 + y_1 + y_2 + y_3)].$$

Proof:

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It can be proved by a similar discussion of the 3-plithogenic case.

Remark:

We call (d) a symbolic 4-plithogenic metric generated by $d_1, d_2, d_3, d_4., d_5$

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2 + r_3P_3 + r_4P_4 > 0$ be a symbolic 3-plithogenic positive real number i.e. $r_0 > 0$, $r_0 + r_1 > 0$, $r_0 + r_1 + r_2 > 0$, $r_0 + r_1 + r_2 + r_3 > 0$, $r_0 + r_1 + r_2 + r_3 + r_4 > 0$

Let $d: T(P_1, P_2, P_3, P_4) \times T(P_1, P_2, P_3, P_4) \to 4 - SP_{\mathbb{R}}$ be a symbolic 4-plithogenic metric generated by $d_1, d_2, d_3, d_4., d_5: T \times T \to \mathbb{R}$, then:

1] $B(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 \in T(P_1, P_2, P_3, P_4);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4, A) < r\}$ is called a symbolic 3-plithogenic open ball, where $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4 \in T(P_1, P_2, P_3, P_4)$.

2] $\overline{B}(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 \in T(P_1, P_2, P_3, P_4);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4, A) \leq r\}$ is called a symbolic 3-plithogenic closed ball, where $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4 \in T(P_1, P_2, P_3, P_4)$.

3] $B_S(A, r) = \{b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 \in T(P_1, P_2, P_3, P_4);\ d(b_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4, A) = r\}$ is called a symbolic 4-plithogenic spherical surface.

Definition:

Let $r = r_0 + r_1P_1 + r_2P_2 + r_3P_3, s = s_0 + s_1P_1 + s_2P_2 + s_3P_3 > 0$ with $r, s \in 4 - SP_{\mathbb{R}}$, then for $B_1(A, r), B_2(A, s)$, $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4 \in T(P_1, P_2, P_3, P_4)$, we define

1] $min\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4)\}$ such that:

$$d(X, A) < min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)] + P_3[min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - min(r_0 + r_1 + r_2, s_0 + s_1 + s_2)] + P_4[min(r_0 + r_1 + r_2 + r_3 + r_4, s_0 + s_1 + s_2 + s_3 + s_4) - min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3)]\}$$

}

2] $min\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4)\}$ such that:

$$d(X, A) \leq min(r_0, s_0) + P_1[min(r_0 + r_1, s_0 + s_1) - min\ (r_0, s_0)] + P_2[min(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - min\ (r_0 + r_1, s_0 + s_1)] + P_3[min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - min(r_0 + r_1 + r_2, s_0 + s_1 + s_2)]P_4[min(r_0 + r_1 + r_2 + r_3 + r_4, s_0 + s_1 + s_2 + s_3 + s_4) - min(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3)]\}$$

}

3] $max\ (B_1, B_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4))\}$ such that:

$$d(X, A) < max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)] + P_3[max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - max(r_0 + r_1 + r_2, s_0 + s_1 + s_2)] + P_4[max(r_0 + r_1 + r_2 + r_3 + r_4, s_0 + s_1 + s_2 + s_3 + s_4) - max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3)]\}$$

4] $max\ (\overline{B}_1, \overline{B}_2) = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4))\}$ such that:

$$d(X, A) \leq max(r_0, s_0) + P_1[max(r_0 + r_1, s_0 + s_1) - max\ (r_0, s_0)] + P_2[max(r_0 + r_1 + r_2, s_0 + s_1 + s_2) - max\ (r_0 + r_1, s_0 + s_1)] + P_3[max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3) - max(r_0 + r_1 + r_2, s_0 + s_1 + s_2)] + P_4[max(r_0 + r_1 + r_2 + r_3 + r_4, s_0 + s_1 + s_2 + s_3 + s_4) - max(r_0 + r_1 + r_2 + r_3, s_0 + s_1 + s_2 + s_3)]\}$$

5] $\sim B_1 = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4))\}$ such that: $d(X, A) \geq r_0 + r_1P_1 + r_2P_2 + r_3P_3 + r_4P_4\}$

6] $\sim \overline{B}_1 = \{X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \in T(P_1, P_2, P_3, P_4))\}$ such that: $d(X, A) > r_0 + r_1P_1 + r_2P_2 + r_3P_3 + r_4P_4\}$

Theorem:

The following properties are true:

1] $B_1 \cap (\sim B_1) = \overline{B}_1 \cap (\sim \overline{B}_1) = \emptyset$

2] $min(B_1, B_2) \leq B_1 \cap B_2 \leq max\ (B_1, B_2)$

3] $min(\overline{B}_1, \overline{B}_2) \leq \overline{B}_1 \cap \overline{B}_2 \leq max\ (\overline{B}_1, \overline{B}_2)$.

5. Conclusion:

In this paper, we have defined the concept of the metric space of weak fuzzy complex numbers, where we provided a metric over the ring of weak fuzzy complex numbers, with some related concepts such as open balls and closed balls. On the other hand, we studied the symbolic 2-plithogenic metric space and its ability to be generated from classical metrics, with many interesting properties which are related to its analytical structure.

In the future, we aim to generalize our results to multi-dimensional weak fuzzy complex rings.

6. Future Applications:

We expect soon that this study we have carried out will be very useful in finding an expanded view on new types of metric spaces and their applications, and for this purpose we propose the following ideas that may enrich research in this field in the future:

[1] Define the concept of symbolic 4-plithogenic, m-plithogenic metric spaces, and try to study the different types of open and closed balls in these spaces.

[2] Try to define the Cauchy sequences in the weak fuzzy complex metric space, and some of the related concepts such as convergence and divergence sequences of weak fuzzy complex numbers.

[3] How can we define symbolic m-plithogenic Banach and Hillbert spaces, what are their properties?

[4] How can we define weak fuzzy complex Banach and Hillbert spaces, what are their elementary properties?.

[5] Try to build a fixed-point theorems and properties in m-plithogenic, and weak fuzzy complex spaces.

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