Some Properties of $\alpha^{\wedge}$g-Closed Sets in Fuzzy Neutrosophic Topology

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Abstract

The present paper offers a new notion of sets known as fuzzy neutrosophic α^g-closed sets in fuzzy neutrosophic topology. It is an extended form of work conducted by Fatimah et.al. [1-3]. It investigates several properties of fuzzy neutrisophic α^ g- closed sets and explores a number of examples as to shed the light on the new characteristics of α^g-closed sets as well as some associated relations between them with other sets.

Keywords: fuzzy neutrosophic closed set; fuzzy neutrosophic α^g-closed sets; generalized α^ g-closed set; fuzzy neutrosophic α^g -generalized closed set; fuzzy neutrosophic topology.

1.Introduction

The concept of fuzziness had invaded almost all branches of mathematics since the definition of the concept by Zadeh [4]. The applications of fuzzy sets have been presents in many fields such as the theory of fuzzy topological spaces which was studied and developed by Chang [5]. Since then various concepts in general topology have been generalized. On the other hand, a generalization of fuzzy topological spaces was developed in many directions by the notion of neutrosophic sets where the term of neutrosophic set was defined with membership, non-membership and indeterminacy degrees by Smarandche [6] and topological spaces of neutrosophic sets were found out by A. Salama and S. A. Albowi [7]. After that a survey article of the developed areas of fuzzy neutrtrosophic topological spaces has been published by many authers(see, [8], [9-14]).

Hence, this study intended to introduce the concept of α^g -closed sets in the case of fuzzy neutrosophic topology and presents all the important definitions and theorems. It also, made some detailed comparisons with some examples.

2. Introduction And Preliminaries

In this section, we will review the basic concepts of $f$uzzy $n$eutrosophic $s$ets.

Definition 2. 1 [8]: Let F_N be a non-empty fixed set. The $f$uzzy $n$eutrosophic set (FNS), λ_N is an object having the form $\lambda_N = \{< f_n, \mu_{\lambda_N}(f_n), \sigma_{\lambda_N}(f_n), v_{\lambda_N}(f_n) >: f_n \in F_N\}$ = where the functions $\mu_{\lambda_N}, \sigma_{\lambda_N}, v_{\lambda_N}: F_N \rightarrow [0, 1]$ denote the degree of membership function (namely μ_{λ_N}(x)), the degree of indeterminacy function (namely $\sigma_{\lambda_N}(f_n)$) and the degree of non-membership function (namely $v_{\lambda_N}(f_n)$) respectively of each element $f_n \in F_N$ to the set λ_N and $0 \leq \mu_{\lambda_N}(f_n)+\sigma_{\lambda_N}(f_n)+v_{\lambda_N}(f_n) \leq 3$, for each $f_n \in F_N$.

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Remark 2. 2 [1]: FNS $\lambda_N = \{< f_n, \mu_{\lambda_N}(f_n), \sigma_{\lambda_N}(f_n), v_{\lambda_N}(f_n) >: f_n \in F_N\}$ can be identified

to an ordered triple $< f_n, \mu_{\lambda_N}, \sigma_{\lambda_N}, v_{\lambda_N} >$ in [0, 1] on F_N.

Lemma 2. 3 [3]: Let F_N be a non-empty set and the FNSs λ_N and β_N be in the form:

$\lambda_N = \{< f_n, \mu_{\lambda_N}(f_n), \sigma_{\lambda_N}(f_n), v_{\lambda_N}(f_n) >: f_n \in F_N\}$ and

$\beta_N = \{< f_n, \mu_{\beta_N}(f_n), \sigma_{\beta_N}(f_n), v_{\beta_N}(f_n) >: f_n \in F_N\} =$ on F_N. Then,

i. $\lambda_N \subseteq \beta_N$ iff $\mu_{\lambda_N}(f_n) \leq \mu_{\beta_N}(f_n)$, $\sigma_{\lambda_N}(f_n) \leq \sigma_{\beta_N}(f_n)$ and $v_{\lambda_N}(f_n) \geq v_{\beta_N}(f_n)$ for all $f_n \in F_N$,

ii. $\lambda_N = \beta_N$ iff $\lambda_N \subseteq \beta_N$ and $\beta_N \subseteq \lambda_N$,

iii. $1_N - \lambda_N = \{< f_n, v_{\lambda_N}(f_n), 1-\sigma_{\lambda_N}(f_n), \mu_{\lambda_N}(f_n)>: f_n \in F_N\}$,

iv. $\lambda_N \cup \beta_N = \{< f_n, \text{Max}(\mu_{\lambda_N}(f_n), \mu_{\beta N}(f_n)), \text{Max}(\sigma_{\lambda_N}(f_n), \sigma_{\beta_N}(f_n)), \text{Min}(v_{\lambda_N}(f_n), v_{\beta_N}(f_n) >: f_n \in F_N\}$,

v. $\lambda_N \cap \beta_N = \{< f_n, \text{Min}(\mu_{\lambda_N}(f_n), \mu_{\beta N}(f_n)), \text{Min}(\sigma_{\lambda_N}(f_n), \sigma_{\beta N}(f_n)), \text{Max}(v_{\lambda_N}(f_n), v_{\beta_N}(f_n)) >: f_n \in F_N\}$,

vi. $0_N = < f_n, 0, 0, 1 >$ and $1_N = < f_n, 1, 1, 0 >$.

Definition 2. 4 [8]: Fuzzy $n$eutrosophic $t$opology (FNT) on a non-empty set F_N is a family τ of fuzzy neutrosophic subsets in F_N satisfying the following.

i. $0_N, 1_N \in \tau_N$,

ii. $\lambda_{N_1} \cap \lambda_{N_2} \in \tau_N$ for any $\lambda_{N_1}, \lambda_{N_2} \in \tau_N$,

iii. $\cup \lambda_{N_j} \in \tau_N, \forall \{\lambda_{N_j}: j \in J\} \subseteq \tau_N$.

The pair (F_N, τ_N) is called $f$uzzy $n$eutrosophic $t$opological space ($FNTS$). Every elements of τ are called $f$uzzy $n$eutrosophic-open sets (FN-open set). The complement of FN-open set in the FNTS (F_N, τ_N) is called $f$uzzy $n$eutrosophic -closed set (FN-closed set).

Definition 2. 5 [9]: Let (F_N, τ_N) is FNTS and $\lambda_N = \{< f_n, \mu_{\lambda_N}(f_n), \sigma_{\lambda_N}(f_n), v_{\lambda_N}(f_n) >: f_n \in F_N\}$ is FNS in F_N. Then the $f$uzzy $n$eutrosophic -closure (FNcl) and the $f$uzzy $n$eutrosophic -interior (FNint) of λ_N are defined by:

i. FNcl(λ_N) = $\cap \{\beta_N: \beta_N$ is FN-closed set in F_N and $\lambda_N \subseteq \beta_N\}$,
ii. FNint (λ_N) = $\cup \{\beta_N: \beta_N$ is FN-open set in F_N and $\beta_N \subseteq \lambda_N$.

Now, the FNcl(λ_N) is FN-closed set and FNint(λ_N) is FN-open set in F_N. And,

i. λ_N is FN-closed set in F_N iff FNcl(λ_N) = λ_N,
ii. λ_N is FN-open set in F_N iff FNint(λ_N) = λ_N.

Proposition 2. 6 [14]: Let (F_N, τ_N) is FNTS and λ_N, β_N are FNSs in F_N. Then the following properties is true:

i. FNint(λ_N) $\subseteq \lambda_N$ and $\lambda_N \subseteq$ FNcl(λ_N),
ii. $\lambda_N \subseteq \beta_N \Longrightarrow$ FNint(λ_N) $\subseteq$ FNint(β_N) and $\lambda_N \subseteq \beta_N \Longrightarrow$ FNcl(λ_N) $\subseteq$ FNcl(β_N),
iii. FNint(FNint(λ_N)) = FNint(λ_N) and FNcl(FNcl(λ_N)) = FNcl(λ_N),
iv. FNint($\lambda_N \cap \beta_N$) = FNint(λ_N) $\cap$ FNint(β_N) and FNcl($\lambda_N \cup \beta_N$) = FNcl(λ_N) $\cup$ FNcl (β_N),
v. FNint(1_N) = 1_N and FNcl(0_N) = 0_N.

Definition 2. 7 [12]: FNS λ_N in FNTS (F_N, τ_N) is called:

i. Fuzzy $n$eutrosophic semi-closed set (FNS-closed set) if FNint(FNcl(λ_N)) $\subseteq \lambda_N$.

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ii. Fuzzy $n$eutrosophic pre-closed set (FNP-closed set) if FNcl(FNint(λ_N)) ⊆ λ_N.

iii. Fuzzy $n$eutrosophic α-closed set (FNα-closed set) if FNcl(FNint(FNcl(λ_N)))⊆ λ_N.

iv. Fuzzy $n$eutrosophic $\alpha^{\wedge m}$ -closed set (FN $\alpha^{\wedge m}$-closed set) if FNint(FNcl (λ_N)) ⊆ U_N, wherever $\lambda_N \subseteq U_N$ and U_N is FN α -open set.

The complement of Fuzzy neutrosophic semi-closed set is Fuzzy neutrosophic semi-open set, Fuzzy neutrosophic pre-closed set is Fuzzy neutrosophic pre-open set, Fuzzy neutrosophic α-closed set is Fuzzy neutrosophic α-open set and the complement of Fuzzy neutrosophic $\alpha^{\wedge m}$-closed set is Fuzzy neutrosophic neutrosophic $\alpha^{\wedge m}$ -open set respectively.

3. Fuzzy Neutrosophic α^g -Closed Sets.

Definition 3. 1. A subset K of FN-TS (F_N, τ_N) is called FN-α^g -closed set (briefly, FN-α^g -cs) if FN-cl(K) ⊆ U whenever, K ⊆ U and U is FN- α-open.

Example 3.2: Let F = $\{f\}$ define FN-Ss λ_N and β_NN in F as follows:

$$\lambda_N = \{(f, 0.7, 0.6, 0.5): f \in F\},\ \beta_N = \{(f, 0.4, 0.1, 0.8): f \in \mathrm{F}\}$$

and the family $\tau_N = \{0_N, 1_N, \lambda_N, \beta_N\}$ be FN-TS such that, FN-cl(β_N) = $\{(f, 0.5, 0.4, 0.7)\}$,

Therefore, FN-cl(K) ⊆ U, Where U = λ_N = $\{(f, 0.7, 0.6, 0.5)\}$ .Hence, β_N be FN- α^g-cs.

Definition 3.3: Let (F_N, τ_N) is FN-TS and λ_N = < $f_n, \mu_{\lambda_N}(f_n), \sigma_{\lambda_N}(f_n), v_{\lambda_N}(f_n)$ > be FN-S in F_N.Then, the fuzzy neutrosophic α^g-closure (FN-α^gcl) and the fuzzy neutrosophic α^g-interior (FN-α^g int) of λ_N are defined by:

i. FN-α^g cl(λ_N) = ∩ { β_N: β_N is FN-α^g -closed set in F_N and $\lambda_N \subseteq \beta_N$},

ii. FN-α^g int(λ_N) = ∪ { β_N: β_N is FN-α^g -open set in F_N and $\beta_N \subseteq \lambda_N$}.

Theorem 3.4: Let (F_N, τ_N) is FN-TS and λ_N, β_N are FN-Ss in F_N. Then, the following properties hold:

i. FN-α^g cl(0_N) = 0_N and FN-α^g cl (1_N) =1_N,

ii. λ_N ⊆ FN-α^g cl(λ_N),

iii. If $\lambda_N \subseteq \beta_N$. Then, FN-α^g cl($\lambda_N$) ⊆ FN-α^g cl($\beta_N$),

iv. λ_N is FN-α^g -closed set in F_N iff FN-α^g cl (λ_N) = λ_N,

v. FN-α^g cl(λ_N) = FN-α^g cl(FN-α^g cl(λ_N)).

Proof: i. By, Definition 3.3 (i). We have,

FN-α^gcl(0_N) = ∩ { β_N: β_N is FN-α^g -closed set in F_N and $0_N \subseteq \beta_N$} = 0_N.

And, FN-α^gcl(1_N) = ∩ { β_N: β_N is FN-α^g -closed set in F_N and $1_N \subseteq \beta_N$} =1_N.

ii. λ_N ⊆ ∩ { β_N: $\beta_N$is FN-α^g -closed set in F_N and $\lambda_N \subseteq \beta_N$ } = FN-α^gcl (λ_N).

iii. Suppose that $\lambda_N \subseteq \beta_N$. Then,

∩ { β_N: $\beta_N$is FN-α^-closed set in F_N and $\lambda_N \subseteq \beta_N$} ⊆ ∩ {$\eta_N$: η_N is FN-α^g -closed set in F_N and $\beta_N \subseteq \eta_N$ }.Therefore, FN-α^gcl(λ_N) ⊆ FN-α^gcl(β_N).

iv. If, λ_N is FN-α^g -closed set. Then,

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FN-α^gcl (λ_N) = ∩ { β_N: β_N is FN-α^g -closed set in F_N and $\lambda_N \subseteq \beta_N$}.

And, by (ii). We get, $\lambda_N \subseteq$ FN-α^gcl (λ_N) but, λ_N is necessarily to be the smallest set.

Thus, λ_N = ∩ { β_N: β_N is FN-α^g -closed set in F_N and $\lambda_N \subseteq \beta_N$}.

Therefore, λ_N = FN-α^gcl (λ_N).

Conversely; assume that λ_N = FN-α^gcl (λ_N) by using Definition 3.3 (i).

We get, λ_N is FN-α^g -closed set.

v. By, (iv). We get, λ_N = FN-α^gcl (λ_N). Then, FN-α^gcl (λ_N) = FN-α^gcl(FN-α^gcl(λ_N)).

Theorem 3.5: Let (F_N, τ_N) is FN-TS and λ_N, β_N are FN-Ss in F_N. Then, the following properties hold:

i. FN-α^g int(0_N) = 0_N and FN-α^g int (1_N) =1_N,

ii. FN-α^g int(λ_N) ⊆ λ_N,

iii. If, $\lambda_N \subseteq \beta_N$. Then, FN-$\alpha$^g int($\lambda_N$) ⊆ FN-$\alpha$^g int($\beta_N$),

iv. λ_N is FN-α^g -open iff λ_N = FN-α^g int (λ_N),

v. FN-α^g int(λ_N) = FN-α^g int(FN-α^g int(λ_N)).

Proof: i. By, Definition 3.3 (ii). We have,

FN-α^g int(0_N) = ∪ {β_N: β_N is FN-α^g -open set in F_N and $\beta_N \subseteq 0_N$} = 0_N.

And, FN-α^g int(1_N) = ∪ {β_N: β_N is FN-α^g -open set in F_N and $\beta_N \subseteq 1_N$} =1_N.

ii. Follows from Definition 3.3 (ii).

iii. FN-α^g int(λ_N) = ∪ {β_N: β_N is FN-α^g -open set in F_N and $\beta_N \subseteq \lambda_N$}.

Since, $\lambda_N \subseteq \beta_N$. Then, ∪ {$\beta_N$: β_N is FN-α^g -open set in F_N and $\beta_N \subseteq \lambda_N$}

⊆ ∪ {η_N: η_N: is FN-α^g -open set in F_N and $\eta_N \subseteq \beta_N$}.

Therefore, FN-α^g int(λ_N) ⊆ FN-α^g int(β_N).

iv. Suppose that λ_N is FN-α^g -open set in F_N.

Then, λ_N ⊆ FN-α^g int(λ_N) …… (1).

By using (ii). We get, FN-α^g int(λ_N) ⊆ λ_N…… (2).

From (1) and (2) we have, λ_N = FN-α^g int (λ_N).

Conversely; assume that λ_N = FN- α^g int (λ_N) by using Definition 3.3 (ii).

We get, λ_N is FN-α^g -open set in F_N.

v. By, (iv). We get, λ_N = FN- α^g int(λ_N). Then, FN- α^g int(λ_N) = FN-α^g int(FN-α^g int(λ_N)).

Proposition 3.6: i. Every FN-open set is FNα-open set.

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ii. Every FNα-closed set is FNα^g-closed set.

iii. Every FN-closed set is FNα^g-closed set.

iv. Every FN-semi-closed set is FN$\alpha^{\wedge}$ g-closed set.

v. Every FN-pre-closed set is FN α^g -closed set.

Proof: i. Let $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in F\}$ be FN-open set in FN-TS (F_N, τ_N).

Then, by Definition 3. 4 (ii). We get, λ_N = FNint(λ_N)…… (1).

But we have, $\lambda_N \subseteq$ FNcl(λ_N) and $\lambda_N \subseteq$ FNcl(FNint(λ_N)).

Then, FNint(λ_N) $\subseteq$ FNint(FNcl(FNint(λ_N))).

Therefore, by (1). We get, $\lambda_N \subseteq$ FNint (FNcl(FNint(λ_N))). Hence, λ_N is FNα-open set in (F_N, τ_N).

ii. Let $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in F\}$ be FNα-closed set in FN-TS (F_N, τ_N).

Then, FNcl(FNint(FNcl(λ_N))) $\subseteq \lambda_N$.

Now, let U_N is FN-open set such that, $\lambda_N \subseteq U_N$. By, proposition 3.6 If, U_N is FN-open set.

Then, it is FNα-open set.

But, FNcl(λ_N) $\subseteq$ FNcl(FNint(FNcl(λ_N))) $\subseteq \lambda_N \subseteq U_N$.

Therefore, FNcl(λ_N) $\subseteq U_N$. Hence, λ_N is FN$\alpha^{\wedge g}$-closed set in (F_N, τ_N).

iii. Let $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in F\}$ be FN-closed set in

FN-TS (F_N, τ_N).Then, by definition 3.14 (i). We get, λ_N = FNcl(λ_N)...... (1).

And we have, FNint(λ_N) $\subseteq \lambda_N$…… (2).

But, FNint(FNcl(λ_N)) $\subseteq$ FNcl(λ_N). Then, by (1). We get, FNint(FNcl(λ_N) $\subseteq \lambda_N$.

Now, let U_N be FN-open set such that, $\lambda_N \subseteq U_N$.

By, Proposition 3.6. If, U_N is FN-open set. Then, is FNα-open set.

Then, FNcl(λ_N) $\subseteq \lambda_N \subseteq U_N$. Therefore, FNcl($\lambda_N$) $\subseteq U_N$. Hence, λ_N is FN$\alpha^{\wedge}$ g-closed set in (F_N, τ_N).

iv. Let $\lambda_N = \{< x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) >: x \in F\}$ be FN-S-closed set in FNTS (F_N, τ_N).

Then, FNint(FNcl(λ_N)) $\subseteq \lambda_N$.

Now, let U_N be FN-open set such that, $\lambda_N \subseteq U_N$.

By, Proposition 3.6. If, U_N is FN-open set. Then, is FNα-open set.

Then, FNcl(λ_N) $\subseteq \lambda_N \subseteq U_N$. Therefore, FNcl ($\lambda_N$) $\subseteq U_N$. Hence, λ_N is FN$\alpha^{\wedge}$g-closed set in (F_N, τ_N).

v. Let $\lambda_N = \{< x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) >: x \in F\}$ be FN-pre-closed set in FNTS (F_N, τ_N).

Then, FNcl(FNint(λ_N)) $\subseteq \lambda_N$.

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Now, let U_N be FN-open set such that, $\lambda_N \subseteq U_N$.

By, proposition 3.6. If, U_N is FN-open set. Then, is FNα-open set.

Then, FNcl $(\lambda_N) \subseteq \lambda_N \subseteq U_N$. Therefore, FNcl $(\lambda_N) \subseteq \lambda_N \subseteq U_N$. Hence, λ_N is FNα^g-closed set in (F_N, τ_N).

Remark 3.7: The convers of proposition 3.6 is not true in general as shown by the following example:

Example 3.8: Let F $= \{f\}$

i. Define FN-Ss λ_N and β_N in F as follows:

$$\lambda_N = < f, 0.7, 0.6, 0.5 > \ with\ \beta_N = < f, 0.8, 0.9, 0.4 >.$$

And let the family, $\tau_N = \{0_N, 1_N, \lambda_N, \beta_N\}$ be FN-T. Now if, $\omega_N = < f, 0.8, 0.6, 0.5 >$.

Then, FNint$(\omega_N) = < f, 0.7, 0.6, 0.5 >$, FNcl(FNint($\omega_N$)) = 1_N and FNint(FNcl(FNint(ω_N))) =1_N.

So, $\omega_N \subseteq$ FNint(FNcl(FNint(ω_N))). Hence, ω_N is FNα-open set. But, not FN-open set.

iii. Define FN-Ss λ_N, β_N, η_N and Ψ_N in F as follows:

$$\lambda_N = < f, 1, 0.5, 0.7 >, \quad \beta_N = < f, 0, 0.9, 0.2 >,$$

$$\eta_N = < f, 1, 0.9, 0.2 > \text{ and } \Psi_N = < f, 0, 0.5, 0.7 >.$$

And the family, $\tau_N = \{0_N, 1_N, \lambda_N, \beta_N, \eta_N, \Psi_N\}$ is FN-T.

Now if, $\omega_N = < x, 0, 0.4, 0.8 >$ and $U_N = < x, 0, 0.5, 0.7 >$ where, U_N is FNα-open set such that,

$\omega_N \subseteq U_N$. Then, FNcl$(\omega_N) = < x, 0.7, 0.5, 0 >$.Therefore, FNcl $(\omega_N) \subseteq U_N$.

Hence, ω_N is FNα^g-closed set.

If, FNcl$(\omega_N) = < x, 0.7, 0.5, 0 >$ so,

FNint(FNcl(ω_N)) $= < x, 0, 0.5, 0.7 >$ and FNcl(FNint(FNcl(ω_N))) $= < x, 0.7, 0.5, 0 >$.

So, FNcl(FNint(FNcl(ω_N))) $\nsubseteq \omega_N$. Hence, ω_N is not FNα- closed set.

iii. Define FN-Ss A_{1_N} and A_{2_N} in F as follows:

$$A_{1_N} = \{f, (0.7, 0.6, 0.5): f \in F\}, \ A_{2_N} = \{f, (0.8, 0.9, 0.4): f \in F\}$$

and the family, $\tau_N = \{0_N, 1_N, A_{1_N}, A_{2_N}\}$ be FN-TS

Now if, $A_{3_N} = < f, (0.9, 0.5, 0.1) >$. Then, FN-cl($A_{3_N}$) =$1_N$. So, FN-cl $(\omega_N) \subseteq U_N$. Since, $1_N \subseteq 1_N$.

Hence, A_{3_N} is FN-α^g -closed set. But, not FN-closed set.

iv. Take, iii.. Then, ω_N is FNα^g-closed set.

But, FNint(FNcl $(\omega_N)) \nsubseteq \omega_N$. Hence, ω_N is not FN-S-closed set.

v. Take, i Then, β_N is FNα^ g-closed set.

But, FNcl (FNint $(\beta_N)) \nsubseteq \beta_N$. Hence, β_N is not FN-pre-closed set.

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Remark 3.9: Every FN- α^g -cs is FN- α^m -cs but the opposite not need true. The following example show the case.

Example 3.11: Take Example 3.2. (i) if, $C_N = \{f, (0.6, 0.6, 0.6): f \in F\}$. then,

FN-cl(C_N) $=< f, 0.8, 0.9, 0.4 >$. So, FN-cl(C_N) $\nsubseteq U_N$. Hence, C_N is not FN- α^g - closed set.

But, FN-int(cl(C_N)) $=< f, 0.7, 0.6, 0.5 >$. So, FN-int(cl(C_N)) $\subseteq U_N$.

Then, C_N is a FN- α^m - closed set.

Proposition 3.12: Let (F_N, τ_N) be a FN-TS. K is FN-α^g - open set in F_N iff for each U is a FN-α^g -closed set such that $U \subseteq K$ and $U \subseteq$ FN-int(K).

Proof: Let K be a FN-α^g-open, then 1_{N_k}_K is FN-α^g -closed so, $1_{N_k} \subseteq U$, then FN-cl(1_{N_k}) $\subseteq U$.

Put $1_{N_k} = U$ and 1_N _FN-int(K) $\subseteq U$, for each $U \subseteq K$ and $U \subseteq$ FN-int(K). Then $1_{N_U} \subseteq$ FN-int(K).

⇐To prove 1_{N_k} is a FN-α^g-closed set. We take K be FN-α^g-open.

So, for each K is FN-α^g-closed set. Then, 1_N _FN-int(K) ⊆ 1_N _U therefore FN-cl(K) ⊆ 1_N _U for each

$K \subseteq U$, so 1_N _ K is FN-α^g–closed.

Theorem 3.13: K is a FN- α^g -cs iff FN-cl(K) $-$ K contains no non-empty FN-α^g -cs.

Proof. Necessity: Suppose that F ≠ φ be FN-α^g-cs and, F ⊆ FN-cl(K) where F ⊂ FN-cl(K) $-$ K.

Then, F ⊆ FN-cl(K) – K.That is, F ⊆ (FN-cl(K)) $\cap$ K^c. Therefore, F ⊆ (FN-cl(K)) and F ⊆ K^c.

Since, F^c is FN-α^g-open set and K is FN-α^ g-cs and FN-cl(K) ⊆ F^c

Thus, F ⊆ FN-cl$(K)^c$. Therefore, F⊆ FN-cl(K) ∩ FN-cl(K) = 0_N .

If, $F = 0_N$. ⇒ FN-cl(K) $-$ K contains no non-zero FN-α^g-cs.

Sufficiency: Let $K \subseteq U$ be FN-α^g -open set. Suppose that FN-cl(K) is not contained in U,

Then, FN-cl$(K)^c$is a non- empty FN-α^g-cs and contained in FN-(cl(K)) $-$ K. Which is a contradiction.

Therefore, FN-cl(K) $\subseteq U$ and Hence, K is a FN-α^ g-cs.

Theorem 3.14: If K is a FN-α^g- cs and $K \subseteq B \subseteq$ FN-cl(K) then, B is a FN-α^g-cs.

Proof. Let K be a FN-α^g-cs where as $K \subseteq B \subseteq$ FN-cl(K).

And let U be a FN- α^g- open set of F where as $B \subseteq U$.

Since K is a FN- α^ g-cs, we have FN-cl(K) $\subseteq U$ whenever, $K \subseteq U$.

So, $K \subseteq B$ and $B \subseteq$ FN-cl(K). Then, FN-cl(B) $\subseteq$ FN-int(FN-cl(FN-int(K))) ⊆ FN-cl(K) $\subseteq U$.

Therefore, FN-cl(B) $\subseteq U$. Thus, B is FN-α^g- cs in F.

Remark 3.15: The relationship between different sets in FNTS (F_N, τ_N) can be shown in the next diagram. But, the convers is not true in general.

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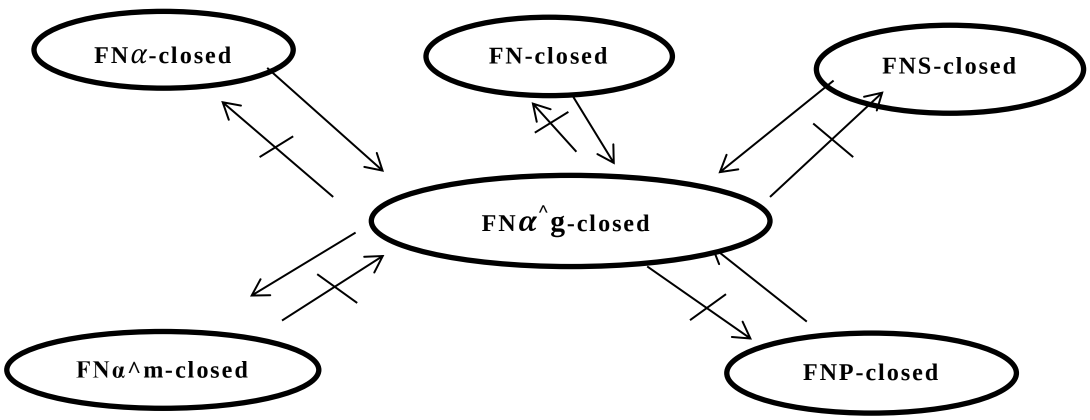

Diagram1

4. Fuzzy Neutrosophic Generalized α^g -Closed set and Fuzzy Neutrosophic α^g -Generalized Closed Set

Definition 4.1: Let (F_N, τ_N) be FN-TS and $\lambda_N = \{< f, \mu_{\lambda_N}(f), \sigma_{\lambda_N}(f), v_{\lambda_N}(f) \}$ be FN-S in F_N. Then, the fuzzy neutrosophic α^g - generalized closure (FN- α^g -G-cl) and the fuzzy neutrosophic α^g -interior (FN- α^g -G- int) of λ_N are defined by:

i. FN-α^g -G- cl(λ_N) = ∩ {β_N: β_N is FN-α^g -G-closed set in F_N and $\lambda_N \subseteq \beta_N$},

ii. FN- α^g -G-int(λ_N) = ∪ {β_N: β_N is FN- α^g -G-open set in F_N and $\beta_N \subseteq \lambda_N$}.

Example 4.2: In Example 3.2 (i). if, $C_N = \{f, (0.7, 0.7, 0.1): f \in F\}$. then,

i. FN-α^g -G- cl(C_N) = ∩{β_N: β_N is FN-α^g -G-closed set in F_N and $C_N \subseteq \beta_N$},

$$= \{f, (0.7, 0.7, 0.1): f \in F\} \subseteq \{f, (1, 1, 0): f \in F\} = 1_N$$

ii. FN- α^g-G-int(C_N) = ∪{β_N: β_N is FN- α^g -G-open set in F_N and $\beta_N \subseteq C_N$}.

$$= \{f, (0.7, 0.6, 0.5): f \in F\} \subseteq \{f, (0.7, 0.7, 0.1): f \in F\} = C_N$$

Theorem 4.3: Let (FN, τ_N) be a FN-TF. For each λ_N, the operator FN- α^g-G- cl satisfies the following statement:

i. FN- α^g-G- cl(0_N) = 0_N , FN- α^g-G- cl(1_N) = 1_N,
ii. $\lambda_N \subseteq$ FN- α^g-G- cl(λ_N),
iii. FN- α^g -G- cl(λ_N) ∪ FN- α^g -G- cl(μ) ⊆ FN- α^g-G- cl($\lambda_N \cup \mu$),
iv. FN- α^g-G- cl(FN- α^g -G- cl(λ_N)) = FN- α^g-G- cl(λ_N),
v. If λ_N is an FN- α^g-G-closed set, then FN- α^g -G- cl(λ_N)= λ_N,
vi. FN- α^g-G- cl (λ_N) ⊆ FN- α^g -cl(λ_N) ⊆ cl(λ_N).

Proof: Similarity of Theorem 3.4.

Theorem 4.4: Let (F_N, τ_N) be a FN-TF. For each λ_N, the operator FN- $\alpha^{\wedge g}$ -G- int, satisfies the following statement:

i. FN- α^g-G- int(0_N) = 0_N, FN- α^g -G- int(1_N) = 1_N,
ii. FN- α^g-G- int(λ_N) ⊆ λ_N,
iii. FN- α^g -G- int($\lambda_N \cap \mu$) = FN- α^g -G- int(λ_N) ∩ FN- α^g-G- int(μ),1
iv. FN- α^g -G- int(λ_N)= FN- α^g -G- int(FN- α^g -G- int(λ_N)).

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Proof: Similarity of Theorem 3.5.

Definition 4.5: Let $(FN,\ \tau_N)$ be FN-TS. A $f$uzzy neutrosophic set K is called fuzzy neutrosophic α^g-generalized closed set "FN- α^g-G-cs" if FN- α^g -cl(K) $\subseteq U_N$, Wherever, $K \subseteq U_N$, U_N is FN- α^g-open set. K is called a FN- α^g -G-os iff 1_{N_k}- K is a FN- α^g -G-cs.

Theorem 4.6: Let (F_N, τ_N) be a FN-TS. A fuzzy neutrosophic set K is FN- α^g -G-os iff $U_N \subseteq$ FN- α^g-G- int (K) whenever $U_N \subseteq K$ and U_N is an FN- α^ g- closed set.

Proof: Let K be FN- α^g -G-open set in F_N and let U_N be any FN- α^g -closed set in F_N such that $U_N \subseteq K$ and U_N is an FN- α^g -open set such that $U_N \subseteq K$. So 1_N-K is an FN- α^g-G-closed set in F_N.

Hence, FN- α^g -cl(1_N -K) ⊆ 1_N_U_N and FN- α^g -cl(1_N -K) = 1_N - FN- α^g -G- int(K) ⊆ 1_N_U_N.

So, 1_(1_U_N) = U_N ⊆ 1_N_ FN- α^g -cl((1_N-K) = FN- α^g-G- int (K).

⇐ Let U_N be an FN- α^ g-closed set. such that 1_N - FN- α^g-G- int (K) and $U_N \subseteq K$.

Now, 1_N- int(K) ⊆1_N - U_N If K is a FN- α^ g-G-open set, this implies 1_N-K is an FN- α^g-G-closed set.
Thus, FN- α^g -cl(1_N -K) ⊆ 1_N- U_N,

So, by definition we get that,1_N_K is an FN- $\alpha^{\wedge g}$ -G-open set.

Remark 4.7: Every FN-closed set is FN- α^g -G-closed set but, the converse not need true. The following example show the case.

Example 4.8: Let F $= \{y\}$ and consider the family $\tau_N = \{0_N, 1_N, A_1, A_2, A_3\}$ where,

$A_1 = <(y, 0.3, 0.5, 0.8)>, A_2 = <(y, 0.2, 0.5, 0.8)>$ and $A_3 = <(y, 0.4, 0.5, 0.6)>$,

Then, A_1 is FN- α^g-G-closed set but, not FN-closed set.

Wherever,$U_N = 1_N$ and $A_1 \subseteq U_N, U_N$ is FN- α^ g-open set.

Theorem 4.9: If K is an FN- α^g-cs in (F_N, τ_N) then, K is a FN- α^g -G-closed set**.**

Proof: Suppose that K is a FN-α^ g-cs in F, and U is a FN- α^g-os in F such that $K \subseteq U_N$. Then,

FN- α^g-cl(K) $= K \subseteq U_N$. So, K is a FN- α^ g-G-closed set.

Remark 4.10: The concept of FN- α^ g-cs and FN- α^g-G-closed set are equivalent.

Example 4.11: Let $F_N = \{f\}$ define FN-Ss $A_{1_N}, A_{2_N}, A_{3_N}$ and A_{4_N} in F as follows:

$$A_{1_N} = \{f, (0.3, 0.4, 0.7): f \in \text{F}\}, \qquad A_{2_N} = \{f, (0.2, 0.6, 0.8): f \in \text{F}\},$$

$$A_{3_N} = \{f, (0.3, 0.6, 0.7): f \in \text{F}\} \text{ and } A_{4_N} = \{f, (0.2, 0.4, 0.8): f \in \text{F}\}.$$

With the family $\tau_N = \{0_N, A_{1_N}, A_{2_N}, A_{3_N}, A_{4_N}\}$ be FN-TS. Then, FN-cl$(A_{4N})^c \subseteq U_N$, where $A_{4N}{}^c \subseteq U_N$ and $U_N = 1_N$ is FN-α^g - closed set and $(A_{4N})^c$ is FN- α^g -G-closed set,

5. Conclusion

In this study, a new concept with respect to the theory of fuzzy neutrosophic sets has been defined, which is said to be fuzzy neutrosophic α^g-closed set. The work has proposed some characteristics of recently established concept. Some relations between the defined model with other sets has been clarified via of fuzzy neutrosophic topology.

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