



Algorithms for Computing M-Plithogenic Eigen Values and Vectors for Some Special Values of M

Isra Al-Shbeil¹, Wael Mahmoud M. Salameh^{2,*}, Hossam Almahasneh³, Khalid Kaabneh⁴, Mutaz Shatnawi⁵, Khaled Al-Husban⁶

¹Department of Mathematics, Faculty of Sciences, University of Jordan, Amman 11942, Jordan

²Faculty of Information Technology, Abu Dhabi University, Abu Dhabi, UAE

³Assistant professor, IT faculty, Zarqa university, Jordan

⁴Faculty of Information Technology, Al-Ahliyya Amman University, Jordan

⁵Department of Mathematics, Faculty of Science and Technology, Irbid National University, Irbid 21110, Jordan,

⁶School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia 43600 UKM Bangi Selangor DE, Malaysia,

Emails: i.shbeil@ju.edu.jo; wael.salameh@adu.ac.ae; H.almahasneh@zu.edu.jo; k.alkaabneh@ammanu.edu.jo; m.shatnawi@inu.edu.jo; Khaledalhusban32@gmail.com

Abstract

Plithogenic matrices are considered as advanced mathematical generalizations of classical square matrices. One of the most research problems that is related to them is finding the eigen-values and vectors. This paper aims to present an easy algorithm to find all eigen-values and eigen-vectors for 10 different symbolic square n-plithogenic matrices with plithogenic real entries, where n is between 20 and 29. All algorithms are explained through clear theorems and proofs.

Keywords: plithogenic matrix; plithogenic eigen-value; plithogenic eigen vector; neutrosophic matrix.

1. Introduction

The theory of neutrosophic logic and plithogenic sets was studied widely by many authors, especially those applications that are related to algebra and algebraic structures [1-2].

In [20-23], we find many great efforts done by mathematicians to improve and to understand the algebraic behaviors of these algebraic structures. These structures diverse in many categories such as commutative structures [16-18], and non-commutative structures [3-4, 12-15].

The study of plithogenic matrices and their substructures began in [7], with many algebraic concepts such as determinants and Invertibility of 2-plithogenic matrices. Recently, many authors keep generalizing results to higher orders of plithogenic matrices. For example, 3-plithogenic and 4-plithogenic matrices were studied in [8,26]. Also, very large orders were discussed in different works, see [10-11, 27-28].

This prompted us to close an interesting research gap by providing algorithms to compute eigen-values and vectors of plithogenic matrices of finite orders between 20 and 29, which is considered as a continuation of the efforts done previously to generalize results for extended classes of plithogenic matrices.

2. Main Discussion

Definition:

The square 20-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$ be a symbolic 20-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{20} \delta_i P_i$ be a 20-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ be a 20-plithogenic square real matrix, then δ is called 20-plithogenic eigenvalue if and only if $\aleph \nabla = \delta \nabla$.

∇ is called 20-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{20} \delta_i P_i \in 20 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{20} \nabla_i P_i$ be a 20-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 20$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\begin{cases} \aleph_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \aleph_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 20 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 20$.

Definition:

The square 21-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$ be a symbolic 20-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{21} \delta_i P_i$ be a 21-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ be a 21-plithogenic square real matrix, then δ is called 21-plithogenic eigenvalue if and only if $\aleph \nabla = \delta \nabla$.

∇ is called 21-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{21} \delta_i P_i \in 21 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{21} \nabla_i P_i$ be a 21-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 21$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\begin{cases} \aleph_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \aleph_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 21 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 21$.

Definition:

The square 22-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$ be a symbolic 22-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{22} \delta_i P_i$ be a 22-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ be a 22-plithogenic square real matrix, then δ is called 22-plithogenic eigenvalue if and only if $\aleph \nabla = \delta \nabla$.

∇ is called 22-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{22} \delta_i P_i \in 22 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{22} \nabla_i P_i$ be a 22-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 22$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\left\{ \begin{array}{l} \aleph_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \aleph_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 22 \end{array} \right.$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 22$.

Definition:

The square 23-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{23} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{23} \aleph_i P_i$ be a symbolic 23-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{23} \delta_i P_i$ be a 23-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{23} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ be a 23-plithogenic square real matrix, then δ is called 23-plithogenic eigenvalue if and only if $\aleph \nabla = \delta \nabla$.

∇ is called 23-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{23} \delta_i P_i \in 23 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{23} \nabla_i P_i$ be a 23-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{23} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 23$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\left\{ \begin{array}{l} \mathfrak{N}_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \mathfrak{N}_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 23 \end{array} \right.$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \mathfrak{N}_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 23$.

Definition:

The square 24-plithogenic matrix is defined:

$\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{24} \mathfrak{N}_i P_i$; $(\mathfrak{N}_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{24} \mathfrak{N}_i P_i$ be a symbolic 24-plithogenic matrix, we have:

$$\det \mathfrak{N} = \det(\mathfrak{N}_0) + [\det(\sum_{i=0}^1 \mathfrak{N}_i) - \det(\mathfrak{N}_0)]P_1 + [\det(\sum_{i=0}^2 \mathfrak{N}_i) - \det(\sum_{i=0}^1 \mathfrak{N}_i)]P_2 + [\det(\sum_{i=0}^3 \mathfrak{N}_i) - \det(\sum_{i=0}^2 \mathfrak{N}_i)]P_3 + [\det(\sum_{i=0}^4 \mathfrak{N}_i) - \det(\sum_{i=0}^3 \mathfrak{N}_i)]P_4 + [\det(\sum_{i=0}^5 \mathfrak{N}_i) - \det(\sum_{i=0}^4 \mathfrak{N}_i)]P_5 + [\det(\sum_{i=0}^6 \mathfrak{N}_i) - \det(\sum_{i=0}^5 \mathfrak{N}_i)]P_6 + [\det(\sum_{i=0}^7 \mathfrak{N}_i) - \det(\sum_{i=0}^6 \mathfrak{N}_i)]P_7 + [\det(\sum_{i=0}^8 \mathfrak{N}_i) - \det(\sum_{i=0}^7 \mathfrak{N}_i)]P_8 + [\det(\sum_{i=0}^9 \mathfrak{N}_i) - \det(\sum_{i=0}^8 \mathfrak{N}_i)]P_9 + [\det(\mathfrak{N}) - \det(\sum_{i=0}^9 \mathfrak{N}_i)]P_{10} + [\det(\sum_{i=0}^{11} \mathfrak{N}_i) - \det(\sum_{i=0}^{10} \mathfrak{N}_i)]P_{11} + [\det(\sum_{i=0}^{12} \mathfrak{N}_i) - \det(\sum_{i=0}^{11} \mathfrak{N}_i)]P_{12} + [\det(\sum_{i=0}^{13} \mathfrak{N}_i) - \det(\sum_{i=0}^{12} \mathfrak{N}_i)]P_{13} + [\det(\sum_{i=0}^{14} \mathfrak{N}_i) - \det(\sum_{i=0}^{13} \mathfrak{N}_i)]P_{14} + [\det(\sum_{i=0}^{15} \mathfrak{N}_i) - \det(\sum_{i=0}^{14} \mathfrak{N}_i)]P_{15} + [\det(\sum_{i=0}^{16} \mathfrak{N}_i) - \det(\sum_{i=0}^{15} \mathfrak{N}_i)]P_{16} + [\det(\sum_{i=0}^{17} \mathfrak{N}_i) - \det(\sum_{i=0}^{16} \mathfrak{N}_i)]P_{17} + [\det(\sum_{i=0}^{18} \mathfrak{N}_i) - \det(\sum_{i=0}^{17} \mathfrak{N}_i)]P_{18} + [\det(\sum_{i=0}^{19} \mathfrak{N}_i) - \det(\sum_{i=0}^{18} \mathfrak{N}_i)]P_{19} + [\det(\sum_{i=0}^{20} \mathfrak{N}_i) - \det(\sum_{i=0}^{19} \mathfrak{N}_i)]P_{20} + [\det(\sum_{i=0}^{21} \mathfrak{N}_i) - \det(\sum_{i=0}^{20} \mathfrak{N}_i)]P_{21} + [\det(\sum_{i=0}^{22} \mathfrak{N}_i) - \det(\sum_{i=0}^{21} \mathfrak{N}_i)]P_{22} + [\det(\sum_{i=0}^{23} \mathfrak{N}_i) - \det(\sum_{i=0}^{22} \mathfrak{N}_i)]P_{23} + [\det(\sum_{i=0}^{24} \mathfrak{N}_i) - \det(\sum_{i=0}^{23} \mathfrak{N}_i)]P_{24}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{24} \delta_i P_i$ be a 24-plithogenic real number and $\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{24} \mathfrak{N}_i P_i$; $(\mathfrak{N}_i)_{n \times n}$ be a 24-plithogenic square real matrix, then δ is called 24-plithogenic eigenvalue if and only if $\mathfrak{N}\nabla = \delta\nabla$.

∇ is called 24-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{24} \delta_i P_i \in 24 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{24} \nabla_i P_i$ be a 24-plithogenic vector, then δ is eigen value of $\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{24} \mathfrak{N}_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \mathfrak{N}_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 24$.

Proof:

δ is an eigen value of \mathfrak{N} with ∇ as an eigen vector if:

$\mathfrak{N}\nabla = \delta\nabla$, thus:

$$\left\{ \begin{array}{l} \mathfrak{N}_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \mathfrak{N}_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 24 \end{array} \right.$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \mathfrak{N}_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 24$.

Definition:

The square 25-plithogenic matrix is defined:

$\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{25} \mathfrak{N}_i P_i$; $(\mathfrak{N}_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{25} \mathfrak{N}_i P_i$ be a symbolic 25-plithogenic matrix, we have:

$$\det \mathfrak{N} = \det(\mathfrak{N}_0) + [\det(\sum_{i=0}^1 \mathfrak{N}_i) - \det(\mathfrak{N}_0)]P_1 + [\det(\sum_{i=0}^2 \mathfrak{N}_i) - \det(\sum_{i=0}^1 \mathfrak{N}_i)]P_2 + [\det(\sum_{i=0}^3 \mathfrak{N}_i) - \det(\sum_{i=0}^2 \mathfrak{N}_i)]P_3 + [\det(\sum_{i=0}^4 \mathfrak{N}_i) - \det(\sum_{i=0}^3 \mathfrak{N}_i)]P_4 + [\det(\sum_{i=0}^5 \mathfrak{N}_i) - \det(\sum_{i=0}^4 \mathfrak{N}_i)]P_5 + [\det(\sum_{i=0}^6 \mathfrak{N}_i) - \det(\sum_{i=0}^5 \mathfrak{N}_i)]P_6 + [\det(\sum_{i=0}^7 \mathfrak{N}_i) - \det(\sum_{i=0}^6 \mathfrak{N}_i)]P_7 + [\det(\sum_{i=0}^8 \mathfrak{N}_i) - \det(\sum_{i=0}^7 \mathfrak{N}_i)]P_8 + [\det(\sum_{i=0}^9 \mathfrak{N}_i) - \det(\sum_{i=0}^8 \mathfrak{N}_i)]P_9 + [\det(\mathfrak{N}) - \det(\sum_{i=0}^9 \mathfrak{N}_i)]P_{10} + [\det(\sum_{i=0}^{11} \mathfrak{N}_i) - \det(\sum_{i=0}^{10} \mathfrak{N}_i)]P_{11} + [\det(\sum_{i=0}^{12} \mathfrak{N}_i) - \det(\sum_{i=0}^{11} \mathfrak{N}_i)]P_{12} + [\det(\sum_{i=0}^{13} \mathfrak{N}_i) - \det(\sum_{i=0}^{12} \mathfrak{N}_i)]P_{13} + [\det(\sum_{i=0}^{14} \mathfrak{N}_i) - \det(\sum_{i=0}^{13} \mathfrak{N}_i)]P_{14} + [\det(\sum_{i=0}^{15} \mathfrak{N}_i) - \det(\sum_{i=0}^{14} \mathfrak{N}_i)]P_{15} + [\det(\sum_{i=0}^{16} \mathfrak{N}_i) - \det(\sum_{i=0}^{15} \mathfrak{N}_i)]P_{16} + [\det(\sum_{i=0}^{17} \mathfrak{N}_i) - \det(\sum_{i=0}^{16} \mathfrak{N}_i)]P_{17} + [\det(\sum_{i=0}^{18} \mathfrak{N}_i) - \det(\sum_{i=0}^{17} \mathfrak{N}_i)]P_{18} + [\det(\sum_{i=0}^{19} \mathfrak{N}_i) - \det(\sum_{i=0}^{18} \mathfrak{N}_i)]P_{19} + [\det(\sum_{i=0}^{20} \mathfrak{N}_i) - \det(\sum_{i=0}^{19} \mathfrak{N}_i)]P_{20} + [\det(\sum_{i=0}^{21} \mathfrak{N}_i) - \det(\sum_{i=0}^{20} \mathfrak{N}_i)]P_{21} + [\det(\sum_{i=0}^{22} \mathfrak{N}_i) - \det(\sum_{i=0}^{21} \mathfrak{N}_i)]P_{22} + [\det(\sum_{i=0}^{23} \mathfrak{N}_i) - \det(\sum_{i=0}^{22} \mathfrak{N}_i)]P_{23} + [\det(\sum_{i=0}^{24} \mathfrak{N}_i) - \det(\sum_{i=0}^{23} \mathfrak{N}_i)]P_{24} + [\det(\sum_{i=0}^{25} \mathfrak{N}_i) - \det(\sum_{i=0}^{24} \mathfrak{N}_i)]P_{25}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{25} \delta_i P_i$ be a 25-plithogenic real number and $\mathfrak{N} = \mathfrak{N}_0 + \sum_{i=1}^{25} \mathfrak{N}_i P_i$; $(\mathfrak{N}_i)_{n \times n}$ be a 25-plithogenic square real matrix, then δ is called 25-plithogenic eigenvalue if and only if $\mathfrak{N}\nabla = \delta\nabla$.

∇ is called 25-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{25} \delta_i P_i \in 25 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{25} \nabla_i P_i$ be a 25-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{25} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:
 $\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 25$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\left\{ \begin{array}{l} \aleph_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \aleph_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 25 \end{array} \right.$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 25$.

Definition:

The square 26-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$ be a symbolic 26-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23} + [\det(\sum_{i=0}^{24} \aleph_i) - \det(\sum_{i=0}^{23} \aleph_i)]P_{24} + [\det(\sum_{i=0}^{25} \aleph_i) - \det(\sum_{i=0}^{24} \aleph_i)]P_{25} + [\det(\sum_{i=0}^{26} \aleph_i) - \det(\sum_{i=0}^{25} \aleph_i)]P_{26}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{26} \delta_i P_i$ be a 26-plithogenic real number and $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ be a 26-plithogenic square real matrix, then δ is called 26-plithogenic eigenvalue if and only if $\aleph \nabla = \delta \nabla$.

∇ is called 26-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{26} \delta_i P_i \in 26 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{26} \nabla_i P_i$ be a 26-plithogenic vector, then δ is eigen value of $\aleph = \aleph_0 + \sum_{i=1}^{26} \aleph_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 26$.

Proof:

δ is an eigen value of \aleph with ∇ as an eigen vector if:

$\aleph \nabla = \delta \nabla$, thus:

$$\left\{ \begin{array}{l} \aleph_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \aleph_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 26 \end{array} \right.$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \aleph_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 26$.

Definition:

The square 27-plithogenic matrix is defined:

$\aleph = \aleph_0 + \sum_{i=1}^{27} \aleph_i P_i$; $(\aleph_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\aleph = \aleph_0 + \sum_{i=1}^{27} \aleph_i P_i$ be a symbolic 27-plithogenic matrix, we have:

$$\det \aleph = \det(\aleph_0) + [\det(\sum_{i=0}^1 \aleph_i) - \det(\aleph_0)]P_1 + [\det(\sum_{i=0}^2 \aleph_i) - \det(\sum_{i=0}^1 \aleph_i)]P_2 + [\det(\sum_{i=0}^3 \aleph_i) - \det(\sum_{i=0}^2 \aleph_i)]P_3 + [\det(\sum_{i=0}^4 \aleph_i) - \det(\sum_{i=0}^3 \aleph_i)]P_4 + [\det(\sum_{i=0}^5 \aleph_i) - \det(\sum_{i=0}^4 \aleph_i)]P_5 + [\det(\sum_{i=0}^6 \aleph_i) - \det(\sum_{i=0}^5 \aleph_i)]P_6 + [\det(\sum_{i=0}^7 \aleph_i) - \det(\sum_{i=0}^6 \aleph_i)]P_7 + [\det(\sum_{i=0}^8 \aleph_i) - \det(\sum_{i=0}^7 \aleph_i)]P_8 + [\det(\sum_{i=0}^9 \aleph_i) - \det(\sum_{i=0}^8 \aleph_i)]P_9 + [\det(\aleph) - \det(\sum_{i=0}^9 \aleph_i)]P_{10} + [\det(\sum_{i=0}^{11} \aleph_i) - \det(\sum_{i=0}^{10} \aleph_i)]P_{11} + [\det(\sum_{i=0}^{12} \aleph_i) - \det(\sum_{i=0}^{11} \aleph_i)]P_{12} + [\det(\sum_{i=0}^{13} \aleph_i) - \det(\sum_{i=0}^{12} \aleph_i)]P_{13} + [\det(\sum_{i=0}^{14} \aleph_i) - \det(\sum_{i=0}^{13} \aleph_i)]P_{14} + [\det(\sum_{i=0}^{15} \aleph_i) - \det(\sum_{i=0}^{14} \aleph_i)]P_{15} + [\det(\sum_{i=0}^{16} \aleph_i) - \det(\sum_{i=0}^{15} \aleph_i)]P_{16} + [\det(\sum_{i=0}^{17} \aleph_i) - \det(\sum_{i=0}^{16} \aleph_i)]P_{17} + [\det(\sum_{i=0}^{18} \aleph_i) - \det(\sum_{i=0}^{17} \aleph_i)]P_{18} + [\det(\sum_{i=0}^{19} \aleph_i) - \det(\sum_{i=0}^{18} \aleph_i)]P_{19} + [\det(\sum_{i=0}^{20} \aleph_i) - \det(\sum_{i=0}^{19} \aleph_i)]P_{20} + [\det(\sum_{i=0}^{21} \aleph_i) - \det(\sum_{i=0}^{20} \aleph_i)]P_{21} + [\det(\sum_{i=0}^{22} \aleph_i) - \det(\sum_{i=0}^{21} \aleph_i)]P_{22} + [\det(\sum_{i=0}^{23} \aleph_i) - \det(\sum_{i=0}^{22} \aleph_i)]P_{23} + [\det(\sum_{i=0}^{24} \aleph_i) - \det(\sum_{i=0}^{23} \aleph_i)]P_{24} + [\det(\sum_{i=0}^{25} \aleph_i) - \det(\sum_{i=0}^{24} \aleph_i)]P_{25} + [\det(\sum_{i=0}^{26} \aleph_i) - \det(\sum_{i=0}^{25} \aleph_i)]P_{26} + [\det(\sum_{i=0}^{27} \aleph_i) - \det(\sum_{i=0}^{26} \aleph_i)]P_{27}.$$

$$\det(\sum_{i=0}^{23} \mathfrak{K}_i)]P_{24} + [\det(\sum_{i=0}^{25} \mathfrak{K}_i) - \det(\sum_{i=0}^{24} \mathfrak{K}_i)]P_{25} + [\det(\sum_{i=0}^{26} \mathfrak{K}_i) - \det(\sum_{i=0}^{25} \mathfrak{K}_i)]P_{26} + [\det(\sum_{i=0}^{27} \mathfrak{K}_i) - \det(\sum_{i=0}^{26} \mathfrak{K}_i)]P_{27}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{27} \delta_i P_i$ be a 27-plithogenic real number and $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{27} \mathfrak{K}_i P_i ; (\mathfrak{K}_i)_{n \times n}$ be a 27-plithogenic square real matrix, then δ is called 27-plithogenic eigenvalue if and only if $\mathfrak{K}\nabla = \delta\nabla$.
 ∇ is called 27-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{27} \delta_i P_i \in 27 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{27} \nabla_i P_i$ be a 27-plithogenic vector, then δ is eigen value of $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{27} \mathfrak{K}_i P_i$ with ∇ as the corresponding eigen vector if and only if:
 $\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 27$.

Proof:

δ is an eigen value of \mathfrak{K} with ∇ as an eigen vector if:

$\mathfrak{K}\nabla = \delta\nabla$, thus:

$$\begin{cases} \mathfrak{K}_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \mathfrak{K}_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 27 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 27$.

Definition:

The square 28-plithogenic matrix is defined:

$\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{28} \mathfrak{K}_i P_i ; (\mathfrak{K}_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{28} \mathfrak{K}_i P_i$ be a symbolic 28-plithogenic matrix, we have:

$$\det \mathfrak{K} = \det(\mathfrak{K}_0) + [\det(\sum_{i=0}^1 \mathfrak{K}_i) - \det(\mathfrak{K}_0)]P_1 + [\det(\sum_{i=0}^2 \mathfrak{K}_i) - \det(\sum_{i=0}^1 \mathfrak{K}_i)]P_2 + [\det(\sum_{i=0}^3 \mathfrak{K}_i) - \det(\sum_{i=0}^2 \mathfrak{K}_i)]P_3 + [\det(\sum_{i=0}^4 \mathfrak{K}_i) - \det(\sum_{i=0}^3 \mathfrak{K}_i)]P_4 + [\det(\sum_{i=0}^5 \mathfrak{K}_i) - \det(\sum_{i=0}^4 \mathfrak{K}_i)]P_5 + [\det(\sum_{i=0}^6 \mathfrak{K}_i) - \det(\sum_{i=0}^5 \mathfrak{K}_i)]P_6 + [\det(\sum_{i=0}^7 \mathfrak{K}_i) - \det(\sum_{i=0}^6 \mathfrak{K}_i)]P_7 + [\det(\sum_{i=0}^8 \mathfrak{K}_i) - \det(\sum_{i=0}^7 \mathfrak{K}_i)]P_8 + [\det(\sum_{i=0}^9 \mathfrak{K}_i) - \det(\sum_{i=0}^8 \mathfrak{K}_i)]P_9 + [\det(\mathfrak{K}) - \det(\sum_{i=0}^9 \mathfrak{K}_i)]P_{10} + [\det(\sum_{i=0}^{11} \mathfrak{K}_i) - \det(\sum_{i=0}^{10} \mathfrak{K}_i)]P_{11} + [\det(\sum_{i=0}^{12} \mathfrak{K}_i) - \det(\sum_{i=0}^{11} \mathfrak{K}_i)]P_{12} + [\det(\sum_{i=0}^{13} \mathfrak{K}_i) - \det(\sum_{i=0}^{12} \mathfrak{K}_i)]P_{13} + [\det(\sum_{i=0}^{14} \mathfrak{K}_i) - \det(\sum_{i=0}^{13} \mathfrak{K}_i)]P_{14} + [\det(\sum_{i=0}^{15} \mathfrak{K}_i) - \det(\sum_{i=0}^{14} \mathfrak{K}_i)]P_{15} + [\det(\sum_{i=0}^{16} \mathfrak{K}_i) - \det(\sum_{i=0}^{15} \mathfrak{K}_i)]P_{16} + [\det(\sum_{i=0}^{17} \mathfrak{K}_i) - \det(\sum_{i=0}^{16} \mathfrak{K}_i)]P_{17} + [\det(\sum_{i=0}^{18} \mathfrak{K}_i) - \det(\sum_{i=0}^{17} \mathfrak{K}_i)]P_{18} + [\det(\sum_{i=0}^{19} \mathfrak{K}_i) - \det(\sum_{i=0}^{18} \mathfrak{K}_i)]P_{19} + [\det(\sum_{i=0}^{20} \mathfrak{K}_i) - \det(\sum_{i=0}^{19} \mathfrak{K}_i)]P_{20} + [\det(\sum_{i=0}^{21} \mathfrak{K}_i) - \det(\sum_{i=0}^{20} \mathfrak{K}_i)]P_{21} + [\det(\sum_{i=0}^{22} \mathfrak{K}_i) - \det(\sum_{i=0}^{21} \mathfrak{K}_i)]P_{22} + [\det(\sum_{i=0}^{23} \mathfrak{K}_i) - \det(\sum_{i=0}^{22} \mathfrak{K}_i)]P_{23} + [\det(\sum_{i=0}^{24} \mathfrak{K}_i) - \det(\sum_{i=0}^{23} \mathfrak{K}_i)]P_{24} + [\det(\sum_{i=0}^{25} \mathfrak{K}_i) - \det(\sum_{i=0}^{24} \mathfrak{K}_i)]P_{25} + [\det(\sum_{i=0}^{26} \mathfrak{K}_i) - \det(\sum_{i=0}^{25} \mathfrak{K}_i)]P_{26} + [\det(\sum_{i=0}^{27} \mathfrak{K}_i) - \det(\sum_{i=0}^{26} \mathfrak{K}_i)]P_{27} + [\det(\sum_{i=0}^{28} \mathfrak{K}_i) - \det(\sum_{i=0}^{27} \mathfrak{K}_i)]P_{28}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{28} \delta_i P_i$ be a 28-plithogenic real number and $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{28} \mathfrak{K}_i P_i ; (\mathfrak{K}_i)_{n \times n}$ be a 28-plithogenic square real matrix, then δ is called 28-plithogenic eigenvalue if and only if $\mathfrak{K}\nabla = \delta\nabla$.
 ∇ is called 28-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{28} \delta_i P_i \in 28 - SP_R$, $\nabla = \nabla_0 + \sum_{i=1}^{28} \nabla_i P_i$ be a 28-plithogenic vector, then δ is eigen value of $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{28} \mathfrak{K}_i P_i$ with ∇ as the corresponding eigen vector if and only if:
 $\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 28$.

Proof:

δ is an eigen value of \mathfrak{K} with ∇ as an eigen vector if:

$\mathfrak{K}\nabla = \delta\nabla$, thus:

$$\begin{cases} \mathfrak{K}_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \mathfrak{K}_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 28 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 28$.

Definition:

The square 29-plithogenic matrix is defined:

$\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{29} \mathfrak{K}_i P_i ; (\mathfrak{K}_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{29} \mathfrak{K}_i P_i$ be a symbolic 29-plithogenic matrix, we have:

$$\det \mathfrak{K} = \det(\mathfrak{K}_0) + [\det(\sum_{i=0}^1 \mathfrak{K}_i) - \det(\mathfrak{K}_0)]P_1 + [\det(\sum_{i=0}^2 \mathfrak{K}_i) - \det(\sum_{i=0}^1 \mathfrak{K}_i)]P_2 + [\det(\sum_{i=0}^3 \mathfrak{K}_i) - \det(\sum_{i=0}^2 \mathfrak{K}_i)]P_3 + [\det(\sum_{i=0}^4 \mathfrak{K}_i) - \det(\sum_{i=0}^3 \mathfrak{K}_i)]P_4 + [\det(\sum_{i=0}^5 \mathfrak{K}_i) - \det(\sum_{i=0}^4 \mathfrak{K}_i)]P_5 + [\det(\sum_{i=0}^6 \mathfrak{K}_i) - \det(\sum_{i=0}^5 \mathfrak{K}_i)]P_6 + [\det(\sum_{i=0}^7 \mathfrak{K}_i) - \det(\sum_{i=0}^6 \mathfrak{K}_i)]P_7 + [\det(\sum_{i=0}^8 \mathfrak{K}_i) - \det(\sum_{i=0}^7 \mathfrak{K}_i)]P_8 + [\det(\sum_{i=0}^9 \mathfrak{K}_i) - \det(\sum_{i=0}^8 \mathfrak{K}_i)]P_9 + [\det(\mathfrak{K}) - \det(\sum_{i=0}^9 \mathfrak{K}_i)]P_{10} + [\det(\sum_{i=0}^{11} \mathfrak{K}_i) - \det(\sum_{i=0}^{10} \mathfrak{K}_i)]P_{11} + [\det(\sum_{i=0}^{12} \mathfrak{K}_i) - \det(\sum_{i=0}^{11} \mathfrak{K}_i)]P_{12} + [\det(\sum_{i=0}^{13} \mathfrak{K}_i) - \det(\sum_{i=0}^{12} \mathfrak{K}_i)]P_{13} + [\det(\sum_{i=0}^{14} \mathfrak{K}_i) - \det(\sum_{i=0}^{13} \mathfrak{K}_i)]P_{14} + [\det(\sum_{i=0}^{15} \mathfrak{K}_i) - \det(\sum_{i=0}^{14} \mathfrak{K}_i)]P_{15} + [\det(\sum_{i=0}^{16} \mathfrak{K}_i) - \det(\sum_{i=0}^{15} \mathfrak{K}_i)]P_{16} + [\det(\sum_{i=0}^{17} \mathfrak{K}_i) - \det(\sum_{i=0}^{16} \mathfrak{K}_i)]P_{17} + [\det(\sum_{i=0}^{18} \mathfrak{K}_i) - \det(\sum_{i=0}^{17} \mathfrak{K}_i)]P_{18} + [\det(\sum_{i=0}^{19} \mathfrak{K}_i) - \det(\sum_{i=0}^{18} \mathfrak{K}_i)]P_{19} + [\det(\sum_{i=0}^{20} \mathfrak{K}_i) - \det(\sum_{i=0}^{19} \mathfrak{K}_i)]P_{20} + [\det(\sum_{i=0}^{21} \mathfrak{K}_i) - \det(\sum_{i=0}^{20} \mathfrak{K}_i)]P_{21} + [\det(\sum_{i=0}^{22} \mathfrak{K}_i) - \det(\sum_{i=0}^{21} \mathfrak{K}_i)]P_{22} + [\det(\sum_{i=0}^{23} \mathfrak{K}_i) - \det(\sum_{i=0}^{22} \mathfrak{K}_i)]P_{23} + [\det(\sum_{i=0}^{24} \mathfrak{K}_i) - \det(\sum_{i=0}^{23} \mathfrak{K}_i)]P_{24} + [\det(\sum_{i=0}^{25} \mathfrak{K}_i) - \det(\sum_{i=0}^{24} \mathfrak{K}_i)]P_{25} + [\det(\sum_{i=0}^{26} \mathfrak{K}_i) - \det(\sum_{i=0}^{25} \mathfrak{K}_i)]P_{26} + [\det(\sum_{i=0}^{27} \mathfrak{K}_i) - \det(\sum_{i=0}^{26} \mathfrak{K}_i)]P_{27} + [\det(\sum_{i=0}^{28} \mathfrak{K}_i) - \det(\sum_{i=0}^{27} \mathfrak{K}_i)]P_{28} + [\det(\sum_{i=0}^{29} \mathfrak{K}_i) - \det(\sum_{i=0}^{28} \mathfrak{K}_i)]P_{29}.$$

Definition

Let $\delta = \delta_0 + \sum_{i=1}^{29} \delta_i P_i$ be a 29-plithogenic real number and $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{29} \mathfrak{K}_i P_i ; (\mathfrak{K}_i)_{n \times n}$ be a 29-plithogenic square real matrix, then δ is called 29-plithogenic eigenvalue if and only if $\mathfrak{K}\nabla = \delta\nabla$.

∇ is called 29-plithogenic eigenvector.

Theorem:

Let $\delta = \delta_0 + \sum_{i=1}^{29} \delta_i P_i \in 29 - SP_R, \nabla = \nabla_0 + \sum_{i=1}^{29} \nabla_i P_i$ be a 29-plithogenic vector, then δ is eigen value of $\mathfrak{K} = \mathfrak{K}_0 + \sum_{i=1}^{29} \mathfrak{K}_i P_i$ with ∇ as the corresponding eigen vector if and only if:

$\sum_{i=0}^t \delta_i$ is eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as eigen vector with $0 \leq t \leq 29$.

Proof:

δ is an eigen value of \mathfrak{K} with ∇ as an eigen vector if:

$\mathfrak{K}\nabla = \delta\nabla$, thus:

$$\begin{cases} \mathfrak{K}_0 \nabla_0 = \delta_0 \nabla_0 \\ \sum_{i=0}^t \mathfrak{K}_i \sum_{i=0}^t \nabla_i = \sum_{i=0}^t \delta_i \sum_{i=0}^t \nabla_i ; 1 \leq t \leq 29 \end{cases}$$

Which is equivalent to:

$\sum_{i=0}^t \delta_i$ is an eigen value of $\sum_{i=0}^t \mathfrak{K}_i$ with $\sum_{i=0}^t \nabla_i$ as an eigen vector for all $1 \leq t \leq 29$.

3. Conclusion

In this paper we presented an easy algorithm to find all eigen-values and eigen-vectors for 10 different symbolic square n-plithogenic matrices with plithogenic real entries, where n is between 20 and 29. All algorithms are explained through clear theorems and proofs. In the future, we aim to generalize our results for plithogenic matrices of orders higher than 30.

References

- [1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
- [2] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.
- [3] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.
- [4] Albasheer, O., Hajjari, A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
- [5] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
- [6] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
- [7] Alfahal, A.; Alhasan, Y.; Abdulfatah, R.; Mehmood, A.; Kadhim, M. On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties. Int. J. Neutrosophic Sci. 2023, 21.
- [8] Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", Symmetry, MDPI, 2023.
- [9] Soueycatt, M., Charckekhandra, B., Abu Hakmeh, R., " On The Algebraic Properties Of Symbolic 6-Plithogenic Integers", Neutrosophic Sets and Systems, vol. 59, 2023.
- [10] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

- [11] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
- [12] Merkepci, M., and Abobala, M., " On Some Novel Results About Split-Complex Numbers, The Diagonalization Problem And Applications To Public Key Asymmetric Cryptography", Journal of Mathematics, Hindawi, 2023.
- [13] Abobala, M., Hatip, A., and Bal, M., " A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol.17, 2021.
- [14] Abobala, M., Ziena, M.B., Doewes, R.I., Hussein, Z. The Representation of Refined Neutrosophic Matrices By Refined Neutrosophic Linear Functions, *International Journal of Neutrosophic Science*, 2022, 19(1), pp. 342–349
- [15] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
- [16] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of neutrosophic Science, Vol. 16, pp. 72-79, 2021.
- [17] Soueycatt, M., Charckekhandra, B., Abu Hakmeh, R., " On The Foundations Of Symbolic 5-Plithogenic Number Theory", Neutrosophic Sets and Systems, vol. 59, 2023.
- [18] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.
- [19] avier Gamboa-Cruzado, Renatto Oyague Guerra, Enrique Condor Tinoco, Guillermo Paucar-Carlos, Juan Gamarra Moreno, Warshine Barry. (2024). A Study of the 16-Plithogenic and 17-Plithogenic Square Real Matrices and Their Properties. Journal of International Journal of Neutrosophic Science, 23 (3), 63-76 (Doi : <https://doi.org/10.54216/IJNS.230306>).
- [20] Smarandache F., and Abobala, M., "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.
- [21] Abobala, M., "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.
- [22] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.
- [23] Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.
- [24] Nader Mahmoud Taffach , Ahmed Hatip.," A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations ", Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.
- [25] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.
- [26] P. Prabakaran, Bilal Abdallah, Turayeva Dinara Tulkunovna. (2024). On Certain Algebraic Properties of Symbolic 3-Plithogenic Real Square Matrices. Journal of International Journal of Neutrosophic Science, 23 (4), 350-357 (Doi : <https://doi.org/10.54216/IJNS.230427>).
- [27] Necati Olgun. (2023). On The 15-Plithogenic Square Real Matrices. Galoitica: Journal of Mathematical Structures and Applications, 9 (2), 41-48 (Doi : <https://doi.org/10.54216/GJMSA.090205>).
- [28] Noor Edin Rabeh, Othman Al-Basheer, Sara Sawalmeh, Rozina Ali. (2023). An Algebraic Approach to n-Plithogenic Square Matrices For $18 \leq n \leq 19$. Journal of Neutrosophic and Fuzzy Systems, 7 (2), 08-23 (Doi : <https://doi.org/10.54216/JNFS.070201>)
- [29] Al-Husban, A., Al-Qadri, M. O., Saadeh, R., Qazza, A., & Almomani, H. H. (2022). Multi-fuzzy rings. *WSEAS Trans. Math*, 21, 701-706.