



Computer purchasing using new type neutrosophic sets and its extension based on aggregation operators

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Abstract

This article discusses a new approach to multiple attribute decision-making (MADM) based on (ℓ_1, ℓ_2, ℓ_3) neutrosophic sets (NS). This is an extension of the NS. Neutrosophic weighted averaging (NWA), neutrosophic weighted geometrics (NWG), generalized neutrosophic weighted averaging (GNWA), and generalized neutrosophic weighted geometrics (GNWG) are the topics of this article. The flowchart we presented during our discussion showed an algorithm that used these operators. Numerical examples are provided for the extended Euclidean and Hamming distance measures. As part of this communication, we will also elaborate on the properties of neutrosophic sets, such as their idempotency, their boundness, their commutativity, and their monotonicity. They make it quicker, easier, and more convenient to find the best option. Thus, there is a stronger connection between (ℓ_1, ℓ_2, ℓ_3) and more precise conclusions. Some of the current models are compared with those that have been proposed in order to demonstrate their dependability and utility. The study also revealed fascinating and intriguing findings.

Keywords: Aggregating operator; decision making; NWA; NWG; GNWA and GNWG.

1 Introduction

The complexity of systems increases every day, making it more difficult for decision-makers to make the right decision. Single goals can be difficult, but they aren't impossible. Motivating employees, setting goals, and

establishing opinions were difficult for many organizations. Therefore, whether individual or committee decisions are made, a number of objectives must be considered simultaneously. In light of this reflection, it would seem that the criteria are solved flexibly, which makes it difficult for each decision maker to achieve an ideal solution optimal in each of the criterion involved. To determine the best alternative, decision-makers develop appropriate and reliable methods. Often, classical or crisp methods are not sufficient for solving decision-making (DM) problems involving uncertainty or ambiguity. To deal with the ambiguities, many uncertain theories including fuzzy set (FS),¹ intuitionistic FS (IFS),² Pythagorean FS (PFS)³ and spherical FS (SFS).⁴ An FS consists of elements with values ranging from 0 to 1 for their membership grade (MD); Later, Atanassov proposed the concept of an IFS that is divided into categories using non-membership grade (NMD), which cannot exceed one.² In cases where the combined MD and NMD grades are greater than 1, a single problem may be conveyed to the DM. The square sum of the MD and NMD of an IFS with a value less than one is classified as PFS, which can be characterized by Yager.³ According to Cuong et al.⁵ developed the picture FS concept with three pointers: positive MD, neutral MD and negative MD. Moreover, it is more beneficial than both PFS and IFS. An example of a generalization of picture FS with AOs is proposed by Liu et al.⁶ Liu et al.⁷ they proposed a generalized PFS with AO and applications. An analysis of AOs using PFS and interval values.⁸ As part of the DM approach challenge, the sum of the positive, neutral, and negative MDs rarely exceeds one. The concept of SFS is proposed by Ashraf et al.,⁴ whereby the square sum of positive, neutral, and negative grades is not greater than 1. Fatmaa et al.⁹ investigated the notion of SFS using TOPSIS models.

MADM issues cannot be solved without aggregation operators (AOs). Xu et al.¹⁰ discussed IFS data can be averaged using some IFS averaging operators. Additionally, Xu et al.¹¹ defined geometric operators derived from IFSs, such as weighted, ordered weighted, and hybrid operators. In 2002, Li et al.¹² proposed generalized ordered weighted averaging operators (GOWAs). Zeng et al.¹³ discussed ordered weighted distances are calculated using distance measures and AOs. Averaging and geometric AOs have been developed by Yager³ for PFS weighted, ordered weighted, and weighted power conditions. A fundamental PFS based on AOs properties was studied by Peng et al.¹⁴ As a result of using AOs, Liu et al.⁷ developed a generalised PFS under them. A fuzzy spherical Dombi AO was developed by Ashraf et al.²⁰ Further information about SFSs and T-SFSs can be found at.^{21,22} In 2022, Temel et al.²³ discussed its application to MADM based on Muirhead power normal SFS. Peng et al.²⁴ explore neutrosophic set with MADM using MABAC and TOPSIS approaches. A generalization of PFS using TOPSIS was presented by Lhang et al.²⁵ There are several algebraic structures and aggregation operators that can be applied in many applications, as discussed by Palanikumar et al.,^{26, 27, 28}

Throughout the remainder of this paper, the following structure is followed. An introduction is found in section 1. There was a discussion of PFS and NS in section 2. A definition of (ℓ_1, ℓ_2, ℓ_3) and some operations on it are discussed in section 3. There is a discussion of MADM in section 4 that uses the distance measure between (ℓ_1, ℓ_2, ℓ_3) NNs. In section 5 an interaction between MADM and some AOs is described based on some NNs for (ℓ_1, ℓ_2, ℓ_3) . A numerical example and an algorithm are discussed in section 6. Section 7 concludes with a conclusion. Based on the findings of the study, the following conclusions can be drawn:

1. To introduce a ED, HD and score values for NSs.
2. By using (ℓ_1, ℓ_2, ℓ_3) NNs, (ℓ_1, ℓ_2, ℓ_3) NWA, (ℓ_1, ℓ_2, ℓ_3) NWG, $G(\ell_1, \ell_2, \ell_3)$ NWA and $G(\ell_1, \ell_2, \ell_3)$ NWG operators are developed.
3. On the basis of AOs, MADM is explored using (ℓ_1, ℓ_2, ℓ_3) NSs.
4. The validity of the proposed approaches is demonstrated with a few numerical examples.
5. To compute the different ideal values for (ℓ_1, ℓ_2, ℓ_3) NWA, (ℓ_1, ℓ_2, ℓ_3) NWG, $G(\ell_1, \ell_2, \ell_3)$ NWA and $G(\ell_1, \ell_2, \ell_3)$ NWG.
6. We discuss the proposed and early studies in the context of comparative analysis.
7. DM results based on the positive integer (ℓ_1, ℓ_2, ℓ_3) .

2 Background

This section contains a number of important definitions that we must review for our further learning.

Definition 2.1. ${}^8 L \Delta$ be an universal. The PIVFS $\aleph = \left\{ \zeta, \left\langle \widetilde{\Psi}_{\aleph}^T(\zeta), \widetilde{\Psi}_{\aleph}^F(\zeta) \right\rangle \mid \zeta \in \Delta \right\}$, where $\widetilde{\Psi}_{\aleph}^T, \widetilde{\Psi}_{\aleph}^F : \Delta \rightarrow Int([0, 1])$ denote the MD and NMD of $\zeta \in \Delta$ to \aleph , respectively, and $0 \leq (\Psi_{\aleph}^T(\zeta))^2 + (\Psi_{\aleph}^F(\zeta))^2 \leq 1$. For convenience, $\aleph = \left\langle \left[\Psi_{\aleph}^T, \Psi_{\aleph}^T \right], \left[\Psi_{\aleph}^F, \Psi_{\aleph}^F \right] \right\rangle$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $\aleph = \left\{ x, \left\langle \Psi_{\aleph}^T(\zeta), \Psi_{\aleph}^I(\zeta), \Psi_{\aleph}^F(\zeta) \right\rangle \mid \zeta \in \Delta \right\}$, where $\Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F : \Delta \rightarrow [0, 1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Delta$, respectively and $0 \leq (\Psi_{\aleph}^T(\zeta)) + (\Psi_{\aleph}^I(\zeta)) + (\Psi_{\aleph}^F(\zeta)) \leq 2$. For $M = \langle \Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F \rangle$ is called a neutrosophic number (NN).

Definition 2.3. The NS $\aleph = \left\{ \zeta, \left\langle \Psi_{\aleph}^T(\zeta), \Psi_{\aleph}^I(\zeta), \Psi_{\aleph}^F(\zeta) \right\rangle \mid \zeta \in \Delta \right\}$, where $\Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F : \Delta \rightarrow [0, 1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Delta$ to \aleph , respectively and $0 \leq (\Psi_{\aleph}^T(\zeta))^2 + (\Psi_{\aleph}^I(\zeta))^2 + (\Psi_{\aleph}^F(\zeta))^2 \leq 2$. For all $\zeta \in \Delta$, $\sqrt{2 - ((\Psi_{\aleph}^T(\zeta))^2 + (\Psi_{\aleph}^I(\zeta))^2 + (\Psi_{\aleph}^F(\zeta))^2)}$ is called the grade of refusal of membership of ζ in \aleph . For convenience, $\aleph = \langle \Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F \rangle$ is called a Pythagorean neutrosophic number (PyNN).

Definition 2.4. ${}^{10} L \aleph_1 = (a_1, b_1) \in N$ and $\aleph_2 = (a_2, b_2) \in N$. Then the distance between \aleph_1 and \aleph_2 is defined as $\mathcal{D}(\aleph_1, \aleph_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

3 Operations for new type of (ℓ_1, ℓ_2, ℓ_3) NN

We discuss the concept of (ℓ_1, ℓ_2, ℓ_3) neutrosophic number (NN). As a result, the (ℓ_1, ℓ_2, ℓ_3) NN and its operations were defined.

Definition 3.1. The new type (ℓ_1, ℓ_2, ℓ_3) neutrosophic set $((\ell_1, \ell_2, \ell_3)$ NS) $\aleph = \left\{ \zeta, \left\langle \left(\Psi_{\aleph}^T(\zeta), \Psi_{\aleph}^I(\zeta), \Psi_{\aleph}^F(\zeta) \right) \right\rangle \mid \zeta \in \Delta \right\}$, where $\Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F : \Delta \rightarrow [0, 1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Delta$ to \aleph , respectively and $0 \leq (\Psi_{\aleph}^T(\zeta))^{\ell_1} + (\Psi_{\aleph}^I(\zeta))^{\ell_2} + (\Psi_{\aleph}^F(\zeta))^{\ell_3} \leq 1$. For convenience, $\aleph = \left\langle \left(\Psi_{\aleph}^T, \Psi_{\aleph}^I, \Psi_{\aleph}^F \right) \right\rangle$ is represent a new type of (ℓ_1, ℓ_2, ℓ_3) NN.

Definition 3.2. $L \aleph = \langle (\Psi^T, \Psi^I, \Psi^F) \rangle$, $\aleph_1 = \langle (\Psi_1^T, \Psi_1^I, \Psi_1^F) \rangle$ and $\aleph_2 = \langle (\Psi_2^T, \Psi_2^I, \Psi_2^F) \rangle$ be any three new type of (ℓ_1, ℓ_2, ℓ_3) NNs, and $\Theta > 0$. Then

$$\begin{aligned}
 1. \quad \aleph_1 \uplus \aleph_2 &= \left[\frac{\ell_1 \sqrt{(\Psi_1^T)^{\ell_1} + (\Psi_2^T)^{\ell_1} - (\Psi_1^T)^{\ell_1} \cdot (\Psi_2^T)^{\ell_1}}}{\ell_2 \sqrt{(\Psi_1^I)^{\ell_2} + (\Psi_2^I)^{\ell_2} - (\Psi_1^I)^{\ell_2} \cdot (\Psi_2^I)^{\ell_2}}, \frac{(\Psi_1^F)^{\ell_3} \cdot (\Psi_2^F)^{\ell_3}}{(\Psi_1^F)^{\ell_3} \cdot (\Psi_2^F)^{\ell_3}} \right], \\
 2. \quad \aleph_1 \circ \aleph_2 &= \left[\frac{(\Psi_1^T)^{\ell_1} \cdot (\Psi_2^T)^{\ell_1}}{\ell_2 \sqrt{(\Psi_1^I)^{\ell_2} + (\Psi_2^I)^{\ell_2} - (\Psi_1^I)^{\ell_2} \cdot (\Psi_2^I)^{\ell_2}}, \frac{(\Psi_1^F)^{\ell_3} \cdot (\Psi_2^F)^{\ell_3}}{\ell_3 \sqrt{(\Psi_1^F)^{\ell_3} + (\Psi_2^F)^{\ell_3} - (\Psi_1^F)^{\ell_3} \cdot (\Psi_2^F)^{\ell_3}} \right] \\
 3. \quad \Theta \cdot \aleph &= \left[\ell_1 \sqrt{1 - (1 - (\Psi^T)^{\ell_1})}^{\Theta}, \ell_2 \sqrt{1 - (1 - (\Psi^I)^{\ell_2})}^{\Theta}, ((\Psi^F)^{\ell_3})^{\Theta} \right], \\
 4. \quad \aleph^{\Theta} &= \left[((\Psi^T)^{\ell_1})^{\Theta}, \ell_2 \sqrt{1 - (1 - (\Psi^I)^{\ell_2})}^{\Theta}, \ell_3 \sqrt{1 - (1 - (\Psi^F)^{\ell_3})}^{\Theta} \right].
 \end{aligned}$$

4 Find new type of (ℓ_1, ℓ_2, ℓ_3) NN distance measure

We introduce ED and HD measures for new type of (ℓ_1, ℓ_2, ℓ_3) NNs and study mathematical properties.

Definition 4.1. For any two new type of (ℓ_1, ℓ_2, ℓ_3) NNs $\aleph_1 = \langle (\Psi_1^T, \Psi_1^I, \Psi_1^F) \rangle$ and $\aleph_2 = \langle (\Psi_2^T, \Psi_2^I, \Psi_2^F) \rangle$. Then

$$\mathcal{D}_E(\aleph_1, \aleph_2) = \frac{1}{2} \sqrt{\left[\frac{1 + (\Psi_1^T)^2 - (\Psi_1^I)^2 - (\Psi_1^F)^2}{2} \right]^2 + \frac{1}{2} \left[\frac{1 + (\Psi_2^T)^2 - (\Psi_2^I)^2 - (\Psi_2^F)^2}{2} \right]^2}$$

where $\mathcal{D}_E(\aleph_1, \aleph_2)$ is called the ED between \aleph_1 and \aleph_2 .

$$\mathcal{D}_H(\aleph_1, \aleph_2) = \frac{1}{2} \left[\begin{array}{c} \left| \frac{1 + (\Psi_1^T)^2 - (\Psi_1^I)^2 - (\Psi_1^F)^2}{2} \right| \\ - \left| \frac{1 + (\Psi_2^T)^2 - (\Psi_2^I)^2 - (\Psi_2^F)^2}{2} \right| \\ + \frac{1}{2} \left[\begin{array}{c} \left| \frac{1 + (\Psi_1^T)^2 - (\Psi_1^I)^2 - (\Psi_1^F)^2}{2} \right| \\ - \left| \frac{1 + (\Psi_2^T)^2 - (\Psi_2^I)^2 - (\Psi_2^F)^2}{2} \right| \end{array} \right] \end{array} \right]$$

where $\mathcal{D}_H(\aleph_1, \aleph_2)$ is called the HD between \aleph_1 and \aleph_2 .

Theorem 4.2. *If any three new type of (ℓ_1, ℓ_2, ℓ_3) NNS $\aleph_1 = \langle (\Psi_1^T, \Psi_1^I, \Psi_1^F) \rangle$ and $\aleph_2 = \langle (\Psi_2^T, \Psi_2^I, \Psi_2^F) \rangle$ and $\aleph_3 = \langle (\Psi_3^T, \Psi_3^I, \Psi_3^F) \rangle$, then $\mathcal{D}_E(\aleph_1, \aleph_2)$ satisfies the following valid statements.*

1. $\mathcal{D}_E(\aleph_1, \aleph_2) = 0$ if and only if $\aleph_1 = \aleph_2$.
2. $\mathcal{D}_E(\aleph_1, \aleph_2) = \mathcal{D}_E(\aleph_2, \aleph_1)$.
3. $\mathcal{D}_E(\aleph_1, \aleph_3) \leq \mathcal{D}_E(\aleph_1, \aleph_2) + \mathcal{D}_E(\aleph_2, \aleph_3)$.

Proof. It is straightforward to prove (1) and (2). Now, $(\mathcal{D}_E(\aleph_1, \aleph_2) + \mathcal{D}_E(\aleph_2, \aleph_3))^2$ implies

$$\frac{1}{4} \left((O - P)^2 + \frac{1}{2} (O - P)^2 \right) + \frac{1}{4} \left((P - Q)^2 + \frac{1}{2} (P - Q)^2 \right) + \frac{1}{2} \left(\sqrt{(O - P)^2 + \frac{1}{2} (O - P)^2} \times \sqrt{(P - Q)^2 + \frac{1}{2} (P - Q)^2} \right),$$

where

$$O = \frac{1 + (\Psi_1^T)^2 - (\Psi_1^I)^2 - (\Psi_1^F)^2}{2},$$

$$P = \frac{1 + (\Psi_2^T)^2 - (\Psi_2^I)^2 - (\Psi_2^F)^2}{2},$$

$$Q = \frac{1 + (\Psi_3^T)^2 - (\Psi_3^I)^2 - (\Psi_3^F)^2}{2}.$$

Hence, $(\mathcal{D}_E(\aleph_1, \aleph_2) + \mathcal{D}_E(\aleph_2, \aleph_3))^2$

$$\begin{aligned} &\geq \frac{1}{4} \left((O - P)^2 + \frac{1}{2} (O - P)^2 \right) + \frac{1}{4} \left((P - Q)^2 + \frac{1}{2} (P - Q)^2 \right) \\ &\quad + \frac{1}{2} \left((O - P) \times (P - Q) + \frac{1}{2} (O - P) \times (P - Q) \right) \\ &= \frac{1}{4} \left((O - P)^2 + (P - Q)^2 + 2(O - P) \times (P - Q) \right) \\ &\quad + \frac{1}{4} \left(\frac{1}{2} (O - P)^2 + \frac{1}{2} (P - Q)^2 + (O - P) \times (P - Q) \right) \\ &= \frac{1}{4} (O - P + P - Q)^2 + \frac{1}{8} (O - P + P - Q)^2 \\ &= \frac{1}{4} \left[(O - Q)^2 + \frac{1}{2} (O - Q)^2 \right] \\ &= \mathcal{D}_E(\aleph_1, \aleph_3)^2. \end{aligned}$$

Corollary 4.3. *L $\aleph_1 = \langle (\Psi_1^T, \Psi_1^I, \Psi_1^F) \rangle$ and $\aleph_2 = \langle (\Psi_2^T, \Psi_2^I, \Psi_2^F) \rangle$ and $\aleph_3 = \langle (\Psi_3^T, \Psi_3^I, \Psi_3^F) \rangle$ be the any three new type of (ℓ_1, ℓ_2, ℓ_3) NNS. Then*

1. $\mathcal{D}_H(\aleph_1, \aleph_2) = 0$ if and only if $\aleph_1 = \aleph_2$.

2. $\mathcal{D}_H(\aleph_1, \aleph_2) = \mathcal{D}_H(\aleph_2, \aleph_1)$.
3. $\mathcal{D}_H(\aleph_1, \aleph_3) \leq \mathcal{D}_H(\aleph_1, \aleph_2) + \mathcal{D}_H(\aleph_2, \aleph_3)$.

Proof. As a result of Theorem 4.2, the proof follows.

5 AOs based on new type of (ℓ_1, ℓ_2, ℓ_3) NN

Here we describe the AOs using (ℓ_1, ℓ_2, ℓ_3) NWA, (ℓ_1, ℓ_2, ℓ_3) NWG, $G(\ell_1, \ell_2, \ell_3)$ NWA, and $G(\ell_1, \ell_2, \ell_3)$ NWG.

5.1 (ℓ_1, ℓ_2, ℓ_3) NWA operator

Definition 5.1. Let $\aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs, $W = (\omega_1, \omega_2, \dots, \omega_n)$ be the weight of $\aleph_i, \omega_i \geq 0$ and $\sum_{i=1}^n \omega_i = 1$. Then (ℓ_1, ℓ_2, ℓ_3) NWA $(\aleph_1, \aleph_2, \dots, \aleph_n) = \uplus_{i=1}^n \omega_i \aleph_i$.

Theorem 5.2. Let $\aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNS. Then (ℓ_1, ℓ_2, ℓ_3) NWA $(\aleph_1, \aleph_2, \dots, \aleph_n)$

$$= \left[\begin{array}{l} \sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}}, \\ \sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}}, \odot_{i=1}^n \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \end{array} \right].$$

Proof. If $n = 2$, then (ℓ_1, ℓ_2, ℓ_3) NWA $(\aleph_1, \aleph_2) = \omega_1 \aleph_1 \uplus \omega_2 \aleph_2$, where

$$\omega_1 \aleph_1 = \left[\begin{array}{l} \sqrt[\ell_1]{1 - \left(1 - (\Psi_1^T)^{\ell_1}\right)^{\omega_1}}, \\ \sqrt[\ell_2]{1 - \left(1 - (\Psi_1^I)^{\ell_2}\right)^{\omega_1}}; \left((\Psi_1^F)^{\ell_3}\right)^{\omega_1} \end{array} \right]$$

$$\omega_2 \aleph_2 = \left[\begin{array}{l} \sqrt[\ell_1]{1 - \left(1 - (\Psi_2^T)^{\ell_1}\right)^{\omega_2}}, \\ \sqrt[\ell_2]{1 - \left(1 - (\Psi_2^I)^{\ell_2}\right)^{\omega_2}}; \left((\Psi_2^F)^{\ell_3}\right)^{\omega_2} \end{array} \right].$$

Now,

$$\omega_1 \aleph_1 \uplus \omega_2 \aleph_2 = \left[\begin{array}{l} \sqrt[\ell_1]{\left(1 - \left(1 - (\Psi_1^T)^{\ell_1}\right)^{\omega_1}\right) + \left(1 - \left(1 - (\Psi_2^T)^{\ell_1}\right)^{\omega_2}\right)}, \\ \sqrt[\ell_2]{-\left(1 - \left(1 - (\Psi_1^I)^{\ell_2}\right)^{\omega_1}\right) \cdot \left(1 - \left(1 - (\Psi_2^I)^{\ell_2}\right)^{\omega_2}\right)}, \\ \left((\Psi_1^F)^{\ell_3}\right)^{\omega_1} \cdot \left((\Psi_2^F)^{\ell_3}\right)^{\omega_2} \end{array} \right]$$

$$= \left[\begin{array}{l} \sqrt[\ell_1]{1 - \left(1 - (\Psi_1^T)^{\ell_1}\right)^{\omega_1} \left(1 - (\Psi_2^T)^{\ell_1}\right)^{\omega_2}}, \\ \sqrt[\ell_2]{1 - \left(1 - (\Psi_1^I)^{\ell_2}\right)^{\omega_1} \left(1 - (\Psi_2^I)^{\ell_2}\right)^{\omega_2}}, \left((\Psi_1^F)^{\ell_3}\right)^{\omega_1} \cdot \left((\Psi_2^F)^{\ell_3}\right)^{\omega_2} \end{array} \right]$$

Hence, $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2)$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^2 \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}}, \\ \sqrt[\ell_2]{1 - \odot_{i=1}^2 \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}} ; \odot_{i=1}^2 \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \end{array} \right].$$

It valid for $n \geq 3$,

Thus, $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_l)$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^l \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}}, \\ \sqrt[\ell_2]{1 - \odot_{i=1}^l \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}} ; \odot_{i=1}^l \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \end{array} \right].$$

If $n = l + 1$, then $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_l, \aleph_{l+1})$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{\begin{array}{c} \uplus_{i=1}^l \left(1 - \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}\right) + \left(1 - \left(1 - (\Psi_{l+1}^T)^{\ell_1}\right)^{\omega_{l+1}}\right) \\ - \odot_{i=1}^l \left(1 - \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}\right) \cdot \left(1 - \left(1 - (\Psi_{l+1}^T)^{\ell_1}\right)^{\omega_{l+1}}\right) \end{array}}, \\ \sqrt[\ell_2]{\begin{array}{c} \uplus_{i=1}^l \left(1 - \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}\right) + \left(1 - \left(1 - (\Psi_{l+1}^I)^{\ell_2}\right)^{\omega_{l+1}}\right) \\ - \odot_{i=1}^l \left(1 - \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}\right) \cdot \left(1 - \left(1 - (\Psi_{l+1}^I)^{\ell_2}\right)^{\omega_{l+1}}\right) \end{array}}, \\ \odot_{i=1}^l \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \cdot \left((\Psi_{l+1}^F)^{\ell_3}\right)^{\omega_{l+1}} \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^{l+1} \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}}, \\ \sqrt[\ell_2]{1 - \odot_{i=1}^{l+1} \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}} , \odot_{i=1}^{l+1} \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \end{array} \right].$$

Theorem 5.3. $L\aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNS. Then $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n) = \aleph$ (idempotency property).

Proof. Since $\Psi_i^T = \Psi^T$, $\Psi_i^I = \Psi^I$ and $\Psi_i^F = \Psi^F$ and $\uplus_{i=1}^n \omega_i = 1$. Now, $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n)$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^T)^{\ell_1}\right)^{\omega_i}}, \\ \sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^I)^{\ell_2}\right)^{\omega_i}} ; \odot_{i=1}^n \left((\Psi_i^F)^{\ell_3}\right)^{\omega_i} \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \left(1 - (\Psi^T)^{\ell_1}\right)^{\uplus_{i=1}^n \omega_i}}, \\ \sqrt[\ell_2]{1 - \left(1 - (\Psi^I)^{\ell_2}\right)^{\uplus_{i=1}^n \omega_i}} ; \left((\Psi^F)^{\ell_3}\right)^{\uplus_{i=1}^n \omega_i} \end{array} \right]$$

$$= \left[\sqrt[\ell_1]{1 - \left(1 - (\Psi^T)^{\ell_1}\right)}, \sqrt[\ell_2]{1 - \left(1 - (\Psi^I)^{\ell_2}\right)}, (\Psi^F)^{\ell_3} \right]$$

$$= \aleph.$$

Theorem 5.4. $L\aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNS. Then $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n)$ where $\overleftarrow{\Psi^T} = \min \Psi_{ij}^T$, $\overrightarrow{\Psi^T} = \max \Psi_{ij}^T$, $\overleftarrow{\Psi^I} = \min \Psi_{ij}^I$, $\overrightarrow{\Psi^I} = \max \Psi_{ij}^I$, $\overleftarrow{\Psi^F} = \min \Psi_{ij}^F$, $\overrightarrow{\Psi^F} = \max \Psi_{ij}^F$ and where $1 \leq i \leq n$, $j = 1, 2, \dots, i_j$. Then, $\langle \overleftarrow{\Psi^T}, \overleftarrow{\Psi^I}, \overleftarrow{\Psi^F} \rangle$

$$\leq (\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n)$$

$$\leq \overrightarrow{\Psi^T}, \overrightarrow{\Psi^I}, \overrightarrow{\Psi^F} \rangle.$$

(Boundedness property).

Proof. Since, $\overleftarrow{\Psi}^{\mathcal{T}} = \min \Psi_{ij}^{\mathcal{T}}, \overrightarrow{\Psi}^{\mathcal{T}} = \max \Psi_{ij}^{\mathcal{T}}$ and $\overleftarrow{\Psi}^{\mathcal{T}} \leq \Psi_{ij}^{\mathcal{T}} \leq \overrightarrow{\Psi}^{\mathcal{T}}$. Now, $\overleftarrow{\Psi}^{\mathcal{T}} = \sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\overleftarrow{\Psi}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \leq \sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\Psi_{ij}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \leq \sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\overrightarrow{\Psi}^{\mathcal{T}})^{\ell_1})^{\omega_i}} = \overrightarrow{\Psi}^{\mathcal{T}}$.

Since, $\overleftarrow{\Psi}^{\mathcal{I}} = \min \Psi_{ij}^{\mathcal{I}}, \overrightarrow{\Psi}^{\mathcal{I}} = \max \Psi_{ij}^{\mathcal{I}}$ and $\overleftarrow{\Psi}^{\mathcal{I}} \leq \Psi_{ij}^{\mathcal{I}} \leq \overrightarrow{\Psi}^{\mathcal{I}}$. Now, $\overleftarrow{\Psi}^{\mathcal{I}} = \sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\overleftarrow{\Psi}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \leq \sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\Psi_{ij}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \leq \sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\overrightarrow{\Psi}^{\mathcal{I}})^{\ell_2})^{\omega_i}} = \overrightarrow{\Psi}^{\mathcal{I}}$.

Since, $\overleftarrow{(\Psi^{\mathcal{F}})^{\ell_3}} = \min(\Psi_{ij}^{\mathcal{F}})^{\ell_3}, \overrightarrow{(\Psi^{\mathcal{F}})^{\ell_3}} = \max(\Psi_{ij}^{\mathcal{F}})^{\ell_3}$ and $\overleftarrow{(\Psi^{\mathcal{F}})^{\ell_3}} \leq (\Psi_{ij}^{\mathcal{F}})^{\ell_3} \leq \overrightarrow{(\Psi^{\mathcal{F}})^{\ell_3}}$. We have, $\overleftarrow{(\Psi^{\mathcal{F}})^{\ell_3}} = \odot_{i=1}^n ((\overleftarrow{(\Psi^{\mathcal{F}})^{\ell_3}})^{\omega_i}) \leq \odot_{i=1}^n ((\Psi_{ij}^{\mathcal{F}})^{\ell_3})^{\omega_i} \leq \odot_{i=1}^n ((\overrightarrow{(\Psi^{\mathcal{F}})^{\ell_3}})^{\omega_i}) = \overrightarrow{(\Psi^{\mathcal{F}})^{\ell_3}}$.

Therefore,

$$\begin{aligned} & \frac{1}{2} \times \left[\frac{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\overleftarrow{\Psi}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \right)^2}{2} - \frac{\left(\sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\overleftarrow{\Psi}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \right)^2}{2} \right] \\ & \qquad \qquad \qquad + 1 - \frac{\left(\odot_{i=1}^n ((\overrightarrow{\Psi}^{\mathcal{F}})^{\ell_3})^{\omega_i} \right)^2}{2} \\ & \leq \frac{1}{2} \times \left[\frac{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\Psi_{ij}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \right)^2}{2} - \frac{\left(\sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\Psi_{ij}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \right)^2}{2} \right] + \\ & \qquad \qquad \qquad + 1 - \frac{\left(\odot_{i=1}^n ((\Psi_{ij}^{\mathcal{F}})^{\ell_3})^{\omega_i} \right)^2}{2} \\ & \leq \frac{1}{2} \times \left[\frac{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\overrightarrow{\Psi}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \right)^2}{2} - \frac{\left(\sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\overrightarrow{\Psi}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \right)^2}{2} \right] + \\ & \qquad \qquad \qquad + 1 - \frac{\left(\odot_{i=1}^n ((\overleftarrow{\Psi}^{\mathcal{F}})^{\ell_3})^{\omega_i} \right)^2}{2} \end{aligned}$$

Hence, $\langle \overleftarrow{\Psi}^{\mathcal{T}}, \overleftarrow{\Psi}^{\mathcal{I}}, \overrightarrow{\Psi}^{\mathcal{F}} \rangle \leq (\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n) \leq \langle \overrightarrow{\Psi}^{\mathcal{T}}, \overrightarrow{\Psi}^{\mathcal{I}}, \overleftarrow{\Psi}^{\mathcal{F}} \rangle$.

Theorem 5.5. $L \aleph_i = \langle (\Psi_{t_{ij}}^{\mathcal{T}}, \Psi_{t_{ij}}^{\mathcal{I}}, \Psi_{t_{ij}}^{\mathcal{F}}), (\ell_1, \ell_2, \ell_3) \rangle$ and $W_i = \langle (\Psi_{h_{ij}}^{\mathcal{T}}, \Psi_{h_{ij}}^{\mathcal{I}}, \Psi_{h_{ij}}^{\mathcal{F}}) \rangle$, be the (ℓ_1, ℓ_2, ℓ_3) NWAs. For any i , if there is $(\Psi_{t_{ij}}^{\mathcal{T}})^2 \leq (\Psi_{h_{ij}}^{\mathcal{T}})^2$ and $(\Psi_{t_{ij}}^{\mathcal{I}})^2 \leq (\Psi_{h_{ij}}^{\mathcal{I}})^2$ and $(\Psi_{t_{ij}}^{\mathcal{F}})^2 \geq (\Psi_{h_{ij}}^{\mathcal{F}})^2$ or $\aleph_i \leq W_i$. Prove that $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n) \leq (\ell_1, \ell_2, \ell_3)NWA(W_1, W_2, \dots, W_n)$, where $(i = 1, 2, \dots, n); (j = 1, 2, \dots, i_j)$ (monotonicity property).

Proof. For any i , $(\Psi_{t_{ij}}^{\mathcal{T}})^2 \leq (\Psi_{h_{ij}}^{\mathcal{T}})^2$.

Therefore, $1 - (\Psi_{t_{ij}}^{\mathcal{T}})^2 \geq 1 - (\Psi_{h_{ij}}^{\mathcal{T}})^2$.

Hence, $\odot_{i=1}^n (1 - (\Psi_{t_{ij}}^{\mathcal{T}})^2)^{\omega_i} \geq \odot_{i=1}^n (1 - (\Psi_{h_{ij}}^{\mathcal{T}})^2)^{\omega_i}$

and $\sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\Psi_{t_{ij}}^{\mathcal{T}})^{\ell_1})^{\omega_i}} \leq \sqrt[\ell_1]{1 - \odot_{i=1}^n (1 - (\Psi_{h_{ij}}^{\mathcal{T}})^{\ell_1})^{\omega_i}}$.

For any i , $(\Psi_{t_{ij}}^{\mathcal{I}})^{\ell_2} \leq (\Psi_{h_{ij}}^{\mathcal{I}})^{\ell_2}$.

Therefore, $1 - (\Psi_{t_{ij}}^{\mathcal{I}})^{\ell_2} \geq 1 - (\Psi_{h_{ij}}^{\mathcal{I}})^{\ell_2}$.

Hence, $\odot_{i=1}^n (1 - (\Psi_{t_{ij}}^{\mathcal{I}})^{\ell_2})^{\omega_i} \geq \odot_{i=1}^n (1 - (\Psi_{h_{ij}}^{\mathcal{I}})^{\ell_2})^{\omega_i}$.

This implies that $\sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\Psi_{t_{ij}}^{\mathcal{I}})^{\ell_2})^{\omega_i}} \leq \sqrt[\ell_2]{1 - \odot_{i=1}^n (1 - (\Psi_{h_{ij}}^{\mathcal{I}})^{\ell_2})^{\omega_i}}$.

For any i , $(\Psi_{t_{ij}}^{\mathcal{F}})^2 \geq (\Psi_{h_{ij}}^{\mathcal{F}})^2$ and $(\Psi_{t_{ij}}^{\mathcal{F}})^{\ell_3} \geq (\Psi_{h_{ij}}^{\mathcal{F}})^{\ell_3}$.

Therefore, $1 - \frac{(\odot_{i=1}^n \Psi_{t_{ij}}^{\mathcal{F}})^{\ell_3}}{2} \leq 1 - \frac{(\odot_{i=1}^n \Psi_{h_{ij}}^{\mathcal{F}})^{\ell_3}}{2}$.

$$\frac{1}{2} \times \left[\frac{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - (\Psi_{ii}^T)^{\ell_1} \right)^{\omega_i}} \right)^2 - \left(\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_{ii}^T)^{\ell_2} \right)^{\omega_i}} \right)^2}{+1 - \frac{(\odot_{i=1}^n (\Psi_{ij}^F)^{\ell_3})^2}{2}} \right] \\ \leq \frac{1}{2} \times \left[\frac{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - (\Psi_{hi}^T)^{\ell_1} \right)^{\omega_i}} \right)^2 - \left(\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_{hi}^T)^{\ell_2} \right)^{\omega_i}} \right)^2}{+1 - \frac{(\odot_{i=1}^n (\Psi_{hij}^F)^{\ell_3})^2}{2}} \right].$$

Hence, $(\ell_1, \ell_2, \ell_3)NWA(\aleph_1, \aleph_2, \dots, \aleph_n) \leq (\ell_1, \ell_2, \ell_3)NWA(W_1, W_2, \dots, W_n)$.

5.2 (ℓ_1, ℓ_2, ℓ_3) NWG operator

Definition 5.6. $L \aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs. Then $(\ell_1, \ell_2, \ell_3)NWG(\aleph_1, \aleph_2, \dots, \aleph_n) = \odot_{i=1}^n \aleph_i^{\omega_i}$.

Theorem 5.7. $L \aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs. Then $(\ell_1, \ell_2, \ell_3)NWG(\aleph_1, \aleph_2, \dots, \aleph_n)$

$$= \left[\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^T)^{\ell_2} \right)^{\omega_i}}, \sqrt[\ell_3]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^F)^{\ell_3} \right)^{\omega_i}} \right].$$

Proof. It follows from Theorem 5.2.

Theorem 5.8. $L \aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F), (\ell_1, \ell_2, \ell_3) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs and all are equal. Then $(\ell_1, \ell_2, \ell_3)NWG(\aleph_1, \aleph_2, \dots, \aleph_n) = \aleph$.

Proof. It follows from Theorem 5.3.

Remark 5.9. It has other properties, including boundedness and monotonicity, as well as having $(\ell_1, \ell_2, \ell_3)NWG$.

Proof. It follows from Theorem 5.4 and Theorem 5.5.

5.3 Generalized (ℓ_1, ℓ_2, ℓ_3) NWA ($G(\ell_1, \ell_2, \ell_3)$ NWA) operator

Definition 5.10. $L \aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F), (\ell_1, \ell_2, \ell_3) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NN. Then $G(\ell_1, \ell_2, \ell_3)$ NWA $(\aleph_1, \aleph_2, \dots, \aleph_n) = \left(\uplus_{i=1}^n \omega_i \aleph_i^\Theta \right)^{1/\Theta}$.

Theorem 5.11. $L \aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs. Then $G(\ell_1, \ell_2, \ell_3)$ NWA $(\aleph_1, \aleph_2, \dots, \aleph_n)$

$$= \left[\left(\sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - \left(\sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^T)^{\ell_1} \right)^{\ell_1}} \right)^{\omega_i}} \right)^{1/\ell_1}, \left(\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - \left(\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - (\Psi_i^I)^{\ell_2} \right)^{\ell_2}} \right)^{\omega_i}} \right)^{1/\ell_2}, \right. \\ \left. \sqrt[\ell_3]{1 - \left(1 - \left(\odot_{i=1}^n \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_i^F)^{\ell_3} \right)^{\ell_3}} \right)^{\omega_i} \right)^{\ell_3} \right)^{1/\ell_3}} \right].$$

Proof. We can prove this first by demonstrating that,

$$\begin{aligned} \uplus_{i=1}^n \omega_i \aleph_i^{\ell_1} &= \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^n \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_i}} ; \\ \sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_i}} ; \odot_{i=1}^n \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_i} \end{array} \right] . \\ \text{Put } n = 2, \omega_1 \aleph_1 \uplus \omega_2 \aleph_2 \end{aligned}$$

$$\begin{aligned} & \left[\begin{array}{c} \sqrt[\ell_1]{\left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_1^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_1}} \right)^{\ell_1} + \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_2^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_1}} \right)^{\ell_1}} , \\ \sqrt{- \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_1^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_1}} \right)^{\ell_1} \cdot \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_2^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_1}} \right)^{\ell_1}} \end{array} \right] , \\ & \left[\begin{array}{c} \sqrt[\ell_2]{\left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_1^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_1}} \right)^{\ell_2} + \left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_2^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_1}} \right)^{\ell_2}} , \\ \sqrt{- \left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_1^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_1}} \right)^{\ell_2} \cdot \left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_2^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_1}} \right)^{\ell_2}} \end{array} \right] , \\ & \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_1^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_1} \cdot \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_2^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_1} , \\ & \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^2 \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_i}} ; \\ \sqrt[\ell_2]{1 - \odot_{i=1}^2 \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_i}} ; \odot_{i=1}^2 \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_i} \end{array} \right] . \end{aligned}$$

Hence,

$$\uplus_{i=1}^l \omega_i \aleph_i^{\ominus} = \left[\begin{array}{c} \sqrt[\ell_1]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_i}} ; \\ \sqrt[\ell_2]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_i}} ; \odot_{i=1}^l \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_i} \end{array} \right] .$$

If $n = l + 1$, then $\uplus_{i=1}^l \omega_i \aleph_i^{\ominus} + \omega_{l+1} \aleph_{l+1}^{\ominus} = \uplus_{i=1}^{l+1} \omega_i \aleph_i^{\ominus}$.

Now, $\uplus_{i=1}^l \omega_i \aleph_i^{\ominus} + \omega_{l+1} \aleph_{l+1}^{\ominus} = \omega_1 \aleph_1^{\ominus} \uplus \omega_2 \aleph_2^{\ominus} \uplus \dots \uplus \omega_l \aleph_l^{\ominus} \uplus \omega_{l+1} \aleph_{l+1}^{\ominus}$

$$\begin{aligned} & \left[\begin{array}{c} \sqrt[\ell_1]{\left(\sqrt[\ell_1]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_i}} \right)^{\ell_1} + \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_{l+1}^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_{l+1}}} \right)^{\ell_1}} , \\ \sqrt{- \left(\sqrt[\ell_1]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_i}} \right)^{\ell_1} \cdot \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_{l+1}^{\mathcal{T}})^{\ell_1} \right)^{\ell_1} \right)^{\omega_{l+1}}} \right)^{\ell_1}} \end{array} \right] , \\ & \left[\begin{array}{c} \sqrt[\ell_2]{\left(\sqrt[\ell_2]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_i}} \right)^{\ell_2} + \left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_{l+1}^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_{l+1}}} \right)^{\ell_2}} , \\ \sqrt{- \left(\sqrt[\ell_2]{1 - \odot_{i=1}^l \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_i}} \right)^{\ell_2} \cdot \left(\sqrt[\ell_2]{1 - \left(1 - \left((\Psi_{l+1}^{\mathcal{I}})^{\ell_2} \right)^{\ell_2} \right)^{\omega_{l+1}}} \right)^{\ell_2}} \end{array} \right] , \\ & \odot_{i=1}^l \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_i} \cdot \left(\sqrt[\ell_3]{1 - \left(1 - (\Psi_{l+1}^{\mathcal{F}})^{\ell_3} \right)^{\ell_3}} \right)^{\omega_{l+1}} , \end{aligned}$$

$$\begin{aligned} \uplus_{i=1}^{l+1} \omega_i \aleph_i^{\ell_1} &= \left[\begin{array}{l} \sqrt[\ell_1]{1 - \odot_{i=1}^{l+1} \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\omega_i} \right)} ; \sqrt[\ell_2]{1 - \odot_{i=1}^{l+1} \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\omega_i} \right)} ; \\ \odot_{i=1}^{l+1} \left(\sqrt[\ell_3]{1 - \left(1 - \left((\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\omega_i} \right)} \right) ; \odot_{i=1}^{l+1} \left(\sqrt[\ell_3]{1 - \left(1 - \left((\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\omega_i} \right)} \right) \end{array} \right] \\ (\uplus_{i=1}^{l+1} \omega_i \aleph_i^{\Theta})^{1/\Theta} &= \left[\begin{array}{l} \left(\sqrt[\Theta]{1 - \odot_{i=1}^{l+1} \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\omega_i} \right)} \right)^{1/\ell_1} ; \\ \left(\sqrt[\Theta]{1 - \odot_{i=1}^{l+1} \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\omega_i} \right)} \right)^{1/\ell_2} ; \\ \sqrt[\Theta]{1 - \left(1 - \left(\odot_{i=1}^{l+1} \left(\sqrt[\Theta]{1 - \left(1 - \left((\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\omega_i} \right)} \right)^2 \right)} \right)^{1/\ell_3} \end{array} \right] \end{aligned}$$

Remark 5.12. An operator modified from the $G(\ell_1, \ell_2, \ell_3)$ NWA operator to the (ℓ_1, ℓ_2, ℓ_3) NWA operator is performed if $\Theta = 1$.

Theorem 5.13. If all $\aleph_i = \langle (\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}), (\ell_1, \ell_2, \ell_3) \rangle$ and all are equal. Then $G(\ell_1, \ell_2, \ell_3)$ NWA($\aleph_1, \aleph_2, \dots, \aleph_n$) = \aleph .

Proof. There is a proof based on the Theorem 5.3.

Remark 5.14. In the $G(\ell_1, \ell_2, \ell_3)$ NWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 5.4 and Theorem 5.5.

5.4 Generalized (ℓ_1, ℓ_2, ℓ_3) NWG ($G(\ell_1, \ell_2, \ell_3)$ NWG) operator

Definition 5.15. $L \aleph_i = \langle (\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}), (\ell_1, \ell_2, \ell_3) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs. Then $G(\ell_1, \ell_2, \ell_3)$ NWG ($\aleph_1, \aleph_2, \dots, \aleph_n$) = $\frac{1}{\Theta} \left(\odot_{i=1}^n (\Theta \aleph_i)^{\omega_i} \right)$.

Theorem 5.16. $L \aleph_i = \langle (\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}) \rangle$ be the new type of (ℓ_1, ℓ_2, ℓ_3) NNs. Then $G(\ell_1, \ell_2, \ell_3)$ NWG($\aleph_1, \aleph_2, \dots, \aleph_n$)

$$= \left[\begin{array}{l} \sqrt[\ell_1]{1 - \left(1 - \left(\odot_{i=1}^n \left(\sqrt[\ell_1]{1 - \left(1 - \left((\Psi_i^{\mathcal{T}})^{\ell_1} \right)^{\omega_i} \right)} \right)^{\ell_1} \right)} \right)^{1/\ell_1} ; \\ \left(\sqrt[\ell_2]{1 - \odot_{i=1}^n \left(1 - \left((\Psi_i^{\mathcal{I}})^{\ell_2} \right)^{\omega_i} \right)} \right)^{1/\ell_2} ; \left(\sqrt[\ell_3]{1 - \odot_{i=1}^n \left(1 - \left((\Psi_i^{\mathcal{F}})^{\ell_3} \right)^{\omega_i} \right)} \right)^{1/\ell_3} \end{array} \right]$$

Proof. As a conclusion, we can say that Theorem 5.11 is based on this proof.

Remark 5.17. There is a conversion that takes place when $\Theta = 1$, which converts the $G(\ell_1, \ell_2, \ell_3)$ NWG into the (ℓ_1, ℓ_2, ℓ_3) NWG.

Remark 5.18. Boundness and monotonicity properties that are satisfied by $G(\ell_1, \ell_2, \ell_3)$ NWG operators.

Proof. The following proof builds on Theorem 5.4 and Theorem 5.5.

Theorem 5.19. If all $\aleph_i = \langle (\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}), (\ell_1, \ell_2, \ell_3) \rangle$ are equal. Then $G(\ell_1, \ell_2, \ell_3)$ NWG($\aleph_1, \aleph_2, \dots, \aleph_n$) = \aleph .

6 MADM approach based on (ℓ_1, ℓ_2, ℓ_3) NN

$L \aleph = \{\aleph_1, \aleph_2, \dots, \aleph_n\}$ be the set of n -alternatives, $C = \{C_1, C_2, \dots, C_m\}$ be the set of m -attributes, $w = \{w_1, w_2, \dots, w_m\}$ be the weights of attributes, for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$
 $\aleph_{ij} = \langle (\Psi_{ij}^T, \Psi_{ij}^I, \Psi_{ij}^F), (\ell_1, \ell_2, \ell_3) \rangle$ denote (ℓ_1, ℓ_2, ℓ_3) NN of alternative \aleph_i in attribute C_j . Since $\Psi_i^T, \Psi_i^I, \Psi_i^F \in [0, 1]$ and $0 \leq (\Psi_i^T)^{\ell_1} + (\Psi_i^I)^{\ell_2} + (\Psi_i^F)^{\ell_3} \leq 1$, where ℓ_1, ℓ_2, ℓ_3 are a positive integers. By combining n -alternatives with m -attributes, we get $\mathcal{D} = (\widehat{\aleph}_{ij})_{n \times m}$. A MADM problem involves collecting attributes with preferred weights to determine the most appropriate decision from finite alternatives. A decision is obtained by calculating the values of the ED, HD and score of each attribute given by (ℓ_1, ℓ_2, ℓ_3) NNs. It is possible to obtain the representation of the alternative by summing the positive and negative values of each attribute. A decision is reached using the following algorithm.

6.1 Algorithm for (ℓ_1, ℓ_2, ℓ_3) NN

Step-1: The decision values for (ℓ_1, ℓ_2, ℓ_3) NN should be entered.

Step-2: The aggregate value of each alternative should be found. On the basis of (ℓ_1, ℓ_2, ℓ_3) NN information aggregation operators, attribute C_j in $\widehat{\aleph}_i, \aleph_{ij} = \langle (\Psi_{ij}^T, \Psi_{ij}^I, \Psi_{ij}^F), (\ell_1, \ell_2, \ell_3) \rangle$ is aggregated into $\aleph_i = \langle (\Psi_i^T, \Psi_i^I, \Psi_i^F), (\ell_1, \ell_2, \ell_3) \rangle$.

Step-3: To calculate the positive and negative ideal values for each alternative, follow these steps: $\widehat{\aleph}^+ = \langle 1, 1, 0 \rangle, \widehat{\aleph}^- = \langle 0, 0, 1 \rangle$.

Step-4: To obtain the EDs between each alternative with two ideal values, follow the steps below:

$$\mathcal{D}_i^+ = \mathcal{D}_E(\widehat{\aleph}_i, \widehat{\aleph}^+); \mathcal{D}_i^- = \mathcal{D}_E(\widehat{\aleph}_i, \widehat{\aleph}^-).$$

Step-5: The following formula is used to compute relative closeness:

$$\mathcal{D}_i^* = \frac{\mathcal{D}_i^-}{\mathcal{D}_i^+ + \mathcal{D}_i^-}.$$

Step-6: Based on the output, we can determine that a value of $\max \mathcal{D}_i^*$ is optimal, and therefore we should choose that value as our optimal solution.

6.2 Real life applications

People can use personal computers at home, in college, or in their businesses. Monitors, like screens, are built into computers to provide users with favorable colors. The screen has a higher resolution rate, which provides a clearer view. You can add printers, speakers, desktop scanners, and hard drives with more power to your personal computer. It is convenient to take a laptop anywhere, whether on business trips, vacations, or anywhere else. Today’s laptops are lightweight and portable to make transporting them easy. $\aleph = \{\aleph_a, \aleph_b, \aleph_c, \aleph_d, \aleph_e\}$ is the five-type laptop (alternative) that customers can choose from. Four attributes are considered, the performance of battery life (C_1), storage capacity (C_2), version of operating system (C_3), over all cost (C_4) and their corresponding weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. The objective is to decide which option is best by assessing experts against the criteria. There are many reasons why one might need to purchase a laptop. Consumers go through a number of stages before making a purchase. The following information is used to make decisions:

	C_1	C_2	C_3	C_4
\aleph_a	(0.65, 0.35, 0.55)	(0.5, 0.55, 0.45)	(0.55, 0.4, 0.65)	(0.65, 0.45, 0.6)
\aleph_b	(0.45, 0.4, 0.65)	(0.55, 0.6, 0.5)	(0.45, 0.65, 0.55)	(0.5, 0.65, 0.5)
\aleph_c	((0.6, 0.35, 0.65)	(0.65, 0.45, 0.6)	(0.6, 0.55, 0.45)	(0.5, 0.55, 0.65)
\aleph_d	(0.55, 0.35, 0.55)	(0.4, 0.55, 0.7)	(0.5, 0.45, 0.7)	(0.6, 0.45, 0.5)
\aleph_e	(0.4, 0.55, 0.6)	(0.45, 0.65, 0.55)	(0.55, 0.35, 0.75)	(0.45, 0.5, 0.7)

Aggregate information with (ℓ_1, ℓ_2, ℓ_3) NWA operators are as follows:

(ℓ_1, ℓ_2, ℓ_3) NWA operator (1, 1, 1)	
\mathfrak{N}_a	(0.5904, 0.4366, 0.5401)
\mathfrak{N}_b	(0.4870, 0.5480, 0.5660)
\mathfrak{N}_c	(0.6070, 0.4463, 0.5896)
\mathfrak{N}_d	(0.5049, 0.4463, 0.6146)
\mathfrak{N}_e	(0.4529, 0.5461, 0.6207)

To determine the ideal values of positive and negative alternatives, follow these steps:

$$\mathfrak{N}^+ = (1, 1, 0) \text{ and } \mathfrak{N}^- = (0, 0, 1)$$

Following are the ED values between alternative options with both ideal values:

\mathcal{D}_1^+	\mathcal{D}_2^+	\mathcal{D}_3^+	\mathcal{D}_4^+	\mathcal{D}_5^+
0.0410	0.1174	0.0546	0.0986	0.1465

\mathcal{D}_1^-	\mathcal{D}_2^-	\mathcal{D}_3^-	\mathcal{D}_4^-	\mathcal{D}_5^-
0.2652	0.1888	0.2516	0.2076	0.1597

As a result, the closeness values should be calculated as follows:

\mathcal{D}_1^*	\mathcal{D}_2^*	\mathcal{D}_3^*	\mathcal{D}_4^*	\mathcal{D}_5^*
0.8662	0.6165	0.8217	0.6779	0.5217

The following alternative rankings are provided:

$$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d > \mathfrak{N}_b > \mathfrak{N}_e.$$

Consequently, \mathfrak{N}_a is the most appropriate purchase option.

6.3 Analysis and discussion

As a result of the above information, we propose to apply the following strategies: (ℓ_1, ℓ_2, ℓ_3) NWA, NWG, GNWA, and GNWG based on ED, HD, and score values. Distances can be categorized as follows:

$(\ell_1 = 1, \ell_2 = 1, \ell_3 = 1)$	NWA	NWG	GNWA	GNWG
TOPSIS – Euclidean distance (proposed)	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$
TOPSIS – Hamming distance (proposed)	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$
Score – values (proposed)	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$	$\mathfrak{N}_a > \mathfrak{N}_c > \mathfrak{N}_d$ $\mathfrak{N}_b > \mathfrak{N}_e$

$\Upsilon = 1$	WA	WG	GWA	GWG
TOPSIS – Euclidean distance ²⁸	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$
TOPSIS – Hamming distance ²⁸	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$
Score – values ²⁸	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$	$\aleph_a > \aleph_c > \aleph_d$ $\aleph_b > \aleph_e$

Change the (ℓ_1, ℓ_2, ℓ_3) values from NWA approach. As a result, we obtain the following closeness values and orders:

(r)2-6 (ℓ_1, ℓ_2, ℓ_3)	Relative closeness values					Order
	\mathcal{D}_1^*	\mathcal{D}_2^*	\mathcal{D}_3^*	\mathcal{D}_4^*	\mathcal{D}_5^*	
(1, 1, 1)	0.8611	0.6087	0.8168	0.6731	0.5149	$\aleph_a > \aleph_c > \aleph_d > \aleph_b > \aleph_e$
(2, 1, 1)	0.8690	0.6179	0.8227	0.6812	0.5238	$\aleph_a > \aleph_c > \aleph_d > \aleph_b > \aleph_e$
(2, 2, 1)	0.8639	0.6101	0.8178	0.6763	0.5171	$\aleph_a > \aleph_c > \aleph_d > \aleph_b > \aleph_e$
(2, 2, 2)	0.9382	0.8279	0.9590	0.9114	0.7539	$\aleph_c > \aleph_a > \aleph_d > \aleph_b > \aleph_e$

A new order of alternative ranking is $\aleph_c > \aleph_a > \aleph_d > \aleph_b > \aleph_e$ if $(\ell_1 = 2, \ell_2 = 2, \ell_3 = 2)$. Therefore, \aleph_a should be changed to \aleph_c as the optimal alternative. As with the NWG operator, GNWA operator and GNWG operator, alternative rankings are determined according to (ℓ_1, ℓ_2, ℓ_3) .

7 Conclusion:

In this study, we established ED, HD and score values for new type of (ℓ_1, ℓ_2, ℓ_3) NSs, which also have the advantage of being mathematically simple. The ED, HD and score values are shown to be superior by employing appropriate. For (ℓ_1, ℓ_2, ℓ_3) NWA, (ℓ_1, ℓ_2, ℓ_3) NWG, $G(\ell_1, \ell_2, \ell_3)$ NWA and $G(\ell_1, \ell_2, \ell_3)$ NWG, we have proposed improved AO rules. In order to create these operators, we have also talked about a few aspects and provided a few algebraic operations. There is no doubt that the research contained in this article, which is still at a very early stage of its development, will provide a major benefit to future researchers in this field. It is a large field that is open to future academics who have an interest in it, therefore the ideas presented here will be beneficial to them in the future.

The following topics will be discussed in more detail:

- (1) The cubic FS and the IVPFS based on AOs.
- (2) There are three types of normal vague set, normal vague spherical set, and normal vague NS.
- (3) Complex NWAs, complex NWGs, complex GNWAs and complex GNWGs may be used to solve the problem.

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