Neutrosophic Binary Separation Axioms associated Neutrosophic Binary Kernel Set

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Abstract

The primary purpose of this report is to present an idea that is considered one of the main ideas in neutrosophic binary topological spaces. We plead them the axioms of neutrosophic binary separation axioms associated in the neutrosophic binary kernel. Also we studied its characteristics and the relationships between these new neutrosophic binary separation axioms and their relationships with some other properties.

Keywords: Neutrosophic Set; Neutrosophic Binary Separation; Neutrosophic Binary Kernel Set; binary topological spaces

Phrases: $\aleph EB.T.S\ (X,Y)$, $O(\aleph EB.T.S\ (X,Y))$, $C(\aleph BE.T.S\ (X,Y))$, $\aleph EBCL(P̣, Ɑ)$, $\aleph EBINT(P̣, Ɑ)$, $\aleph EBKER(P̣, Ɑ)$, $S\aleph EB-R_0.S$, $\aleph EB-R_0.S$, $\aleph EB-R_1.S$, $\aleph EB-R_2.S$, $\aleph EB-T_0.S$, $\aleph EB-T_1.S$, $\aleph EB-T_2.S$

1. Introduction

The neutrosophic set was introduced by Smarandache in 1998 [1] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. Salama, Bluey introduced a denotation of neutrosophic topological spaces in 2012 [3], and they introduced neutrosophic topological space as a generalization of the intuitive fuzzy topological space and topological space with the aspect of degree of membership, degree of indeterminacy degree of non-membership of each element. The connotation of separation axioms has always been important and indispensable in the branch of topology , research related to this concept has played fundamental and very important roles in many topological studies. Also, using the conception of neutrosophic topological space, many types of separation axioms are presented and studied [4], [5], [6], [7], [8], [9], [10] , [11]. S. N. Jothi introduced the topology between two universal sets whose is defined to be binary topology in 2011 [12]. The binary topology is a binary structure from X to Y which consists of ordered pairs (P̣ , Ɑ) wherever $P̣ \subseteq X$ and $Ɑ \subseteq Y$. So S. N. Jothi introduced and studied of the conception of binary separation axioms in 2012 [13], [14] , [15]. A neutrosophic binary structure from X to Y is defined as a neutrosophic set of ordered pairs (P, Q) where P̣ neutrosophic set in X and Ɑ neutrosophic set in Y. In continuation, S. S. Surekha, Sindhu G. and Broumi S. introduced, studied and developed the conception of neutrosophic binary Topological Space which consists of two universal sets and each universal set include its own reality, indeterminacy and false membership values in 2022 [16], [17]. Also A.G.Rose Venish and L. Vidyarani introduced along with some properties of neutrosophic binary crisp points in 2023 [18]. Imran et al. [19] , [20], [21] gave fresh insights into the concepts of

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topological groups, generalized alpha generalized continuity and generalized semi generalized closed sets that are weakly neutrosophic. In this Article, our primary motivation is to define and recognize the axioms of separation using neutrosophic binary which have implications for the set of neutrosophic binary kernels. But, before we get into that, we've prepared a few definitions and assumptions based on the neutrosophic binary sets and also on the neutrosophic binary kernel sets. In the second part of this article, we defined the weaker dualistic neutrosophic separation axioms and studied their properties and relationships, which we called neutrosophic binary R_L-Spaces, $L = 0,1,2$. In the third part, we give the definition of the dualistic neutrosophic separation axioms, we call them neutrosophic binary T_L-Spaces, $L = 0,1,2$, and we mention some of their properties. In this part, we also prove their relationship with the previous type. In the latter part we wrote conclusions for this article.

2. Preliminaries

A neutrosophic binary set $(Ꝑ, Ꝙ)$ symbolized $\aleph EB.S\,(Ꝑ, Ꝙ)$, $(X, Y, BT_{\aleph})$ is called neutrosophic binary topological spaces on $\aleph EB.S\,(X, Y)$ (represent as $\aleph EB.T.S\,(X, Y)$), the collection of each neutrosophic binary open set in $(X, Y, BT_{\aleph})$ symbolized by $O(\aleph EB.T.S\,(X, Y))$, the collection of each neutrosophic binary closed in $(X, Y, BT_{\aleph})$ is symbolized by $C(\aleph EB.T.S\,(X, Y))$, the binary neutrosophic closure, neutrosophic binary interior, neutrosophic binary kernel and neutrosophic binary complement of a $\aleph EB.S\,(Ꝑ, Ꝙ)$ in $\aleph EB.T.S\,(X, Y)$ is notation $\aleph EBCL(Ꝑ, Ꝙ)$, $\aleph EBINT(Ꝑ, Ꝙ)$, $\aleph EBKER(Ꝑ, Ꝙ)$ and $(Ꝑ, Ꝙ)^{C}$, respectively.

Definition 2.1 [14]: A neutrosophic binary topology is a binary structure consistence of two universal sets X and Y is where a family $BT_{\aleph} \subseteq P(X) \times P(Y)$, which satisfies the following three conditions:
(i) $(0_X, 0_Y)$ and $(1_X, 1_Y) \in BT_{\aleph}$.
(ii) $(Ꝑ_1 \cap Ꝙ_1,\ Ꝑ_2 \cap Ꝙ_2) \in BT_{\aleph}$ whereas $(Ꝑ_1 \cap Ꝙ_1) \in BT_{\aleph}$ and $(Ꝑ_2 \cap Ꝙ_2) \in BT_{\aleph}$.
(iii) If $\{(Ꝑ_{\ell}, Ꝙ_{\ell}) : \ell \in \Delta\}$ is a family of members of $BT_{\aleph}$, then $(\cup_{\alpha\in\Delta} Ꝑ_{\alpha}, \cup_{\alpha\in\Delta} Ꝙ_{\alpha}) \in BT_{\aleph}$.

The triplet $(X, Y, BT_{\aleph})$ is called $\aleph EB.T.S\,(X, Y)$. Every member $(Ꝑ, Ꝙ)$ of $\aleph EB.T.S\,(X, Y)$ belong to $O(\aleph EB.T.S\,(X, Y))$ and the complement of $(Ꝑ, Ꝙ)$ in $\aleph EB.T.S\,(X, Y)$ belong to $(\aleph EB.T.S\,(X, Y))$.

Definition 2.2 [14]:
a) The order $(0_X, 0_Y)$ can be defined as
(i) $0_X = \{\langle æ, 0, 0, 0\rangle : æ \in X\}$, $0_Y = \{\langle y, 0, 0, 0\rangle : y \in Y\}$
(ii) $0_X = \{\langle æ, 0, 0, 1\rangle : æ \in X\}$, $0_Y = \{\langle y, 0, 0, 1\rangle : y \in Y\}$
(iii) $0_X = \{\langle æ, 0, 1, 0\rangle : æ \in X\}$, $0_Y = \{\langle y, 0, 1, 0\rangle : y \in Y\}$
(iv) $0_X = \{\langle æ, 0, 1, 1\rangle : æ \in X\}$, $0_Y = \{\langle y, 0, 1, 1\rangle : y \in Y\}$
b) The order $(1_X, 1_Y)$ can be defined as
(i) $1_X = \{\langle æ, 1, 1, 1\rangle : æ \in X\}$, $1_Y = \{\langle y, 1, 1, 1\rangle : y \in Y\}$
(ii) $1_X = \{\langle æ, 1, 1, 0\rangle : æ \in X\}$, $1_Y = \{\langle y, 1, 1, 0\rangle : y \in Y\}$
(iii) $1_X = \{\langle æ, 1, 0, 1\rangle : æ \in X\}$, $1_Y = \{\langle y, 1, 0, 1\rangle : y \in Y\}$
(iv) $1_X = \{\langle æ, 1, 0, 0\rangle : æ \in X\}$, $1_Y = \{\langle y, 1, 0, 0\rangle : y \in Y\}$

Definition 2.3 [14] : The complement $(Ꝑ, Ꝙ)^{C}$ of the $\aleph EB.S\,(Ꝑ, Ꝙ) = \{\langle æ, \mu_{Ꝑ}(æ), \sigma_{Ꝑ}(æ), \gamma_{Ꝑ}(æ)\rangle : æ \in X, \langle y, \mu_{Ꝙ}(y), \sigma_{Ꝙ}(y), \gamma_{Ꝙ}(y)\rangle : y \in Y\}$ on a $\aleph EB.T.S\,(X, Y)$ can be defined as
(i) $(Ꝑ, Ꝙ)^{C} = \{\langle æ, 1 - \mu_{Ꝑ}(æ), \sigma_{Ꝑ}(æ), 1 - \gamma_{Ꝑ}(æ)\rangle : æ \in X, \langle y, 1 - \mu_{Ꝑ}(y), \sigma_{Ꝑ}(y), 1 - \gamma_{Ꝑ}(y)\rangle : y \in X\}$
(ii) $(Ꝑ, Ꝙ)^{C} = \{\langle æ, \gamma_{Ꝑ}(æ), \sigma_{Ꝑ}(æ), \mu_{Ꝑ}(æ)\rangle : æ \in X, \langle y, \gamma_{Ꝙ}(y), \sigma_{Ꝑ}(y), \mu_{Ꝙ}(y)\rangle : y \in Y\}$
(iii) $(Ꝑ, Ꝙ)^{C} = \{\langle æ, \gamma_{Ꝑ}(æ), 1 - \sigma_{Ꝑ}(æ), \mu_{Ꝑ}(æ)\rangle : æ \in X, \langle y, \gamma_{Ꝙ}(y), 1 - \sigma_{Ꝑ}(y), \mu_{Ꝙ}(y)\rangle : y \in Y\}$

Definition 2.4 [14]: Let $(Ꝑ_1, Ꝙ_1) = \{\langle \mu_{Ꝑ_1}(æ), \sigma_{Ꝑ_1}(æ), \gamma_{Ꝑ_1}(æ)\rangle, \langle \mu_{Ꝙ_1}(y), \sigma_{Ꝙ_1}(y), \gamma_{Ꝙ_1}(y)\rangle\}$ and $(Ꝑ_2, Ꝙ_2) = \{\langle \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_2}(æ)\rangle, \langle \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_2}(y)\rangle\}$ be two neutrosophic binary sets. Then
(i) $(Ꝑ_1, Ꝙ_1) \subseteq (Ꝑ_2, Ꝙ_2)$ if and only if $\mu_{Ꝑ_1}(æ) \le \mu_{Ꝑ_2}(æ)$, $\sigma_{Ꝑ_1}(æ) \le \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(æ) \ge \gamma_{Ꝑ_2}(æ)$ for each $æ \in X$, $\mu_{Ꝙ_1}(y) \le \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_1}(y) \le \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_1}(y) \ge \gamma_{Ꝙ_2}(y)$ for each $y \in Y$

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(ii) $(Ꝑ_1, Ꝙ_1) \subseteq (Ꝑ_2, Ꝙ_2)$ if and only if $\sigma_{Ꝙ_1}(y) \geq \sigma_{Ꝙ_2}(y), \mu_{Ꝑ_1}(æ) \leq \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_1}(æ) \geq \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(æ) \geq \gamma_{Ꝑ_2}(æ)$ for each $æ \in X$, $\mu_{Ꝙ_1}(y) \leq \mu_{Ꝙ_2}(y)$, $\gamma_{Ꝙ_1}(y) \geq \gamma_{Ꝙ_2}(y)$ for$\{\langle æ, \mu_{Ꝑ_1}(æ) \wedge \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_1}(æ) \wedge \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(y) \vee \gamma_{Ꝑ_2}(y)\rangle, \langle y, \mu_{Ꝙ_1}(y) \wedge \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_1}(y) \wedge \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_1}(y) \vee \gamma_{Ꝙ_2}(y)\rangle\}$ each $y \in Y$

(iii) $(Ꝑ_1, Ꝙ_1) \cap (Ꝑ_2, Ꝙ_2) =$

(iv) $(Ꝑ_1, Ꝙ_1) \cap (Ꝑ_2, Ꝙ_2) = \{\langle æ, \mu_{Ꝑ_1}(æ) \wedge \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_1}(æ) \vee \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(y) \vee \gamma_{Ꝑ_2}(y)\rangle, \langle y, \mu_{Ꝙ_1}(y) \wedge \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_1}(y) \vee \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_1}(y) \vee \gamma_{Ꝙ_2}(y)\rangle\}$

(v) $(Ꝑ_1, Ꝙ_1) \cup (Ꝑ_2, Ꝙ_2) = \{\langle æ, \mu_{Ꝑ_1}(æ) \vee \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_1}(æ) \vee \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(y) \wedge \gamma_{Ꝑ_2}(y)\rangle, \langle y, \mu_{Ꝙ_1}(y) \vee \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_1}(y) \vee \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_1}(y) \wedge \gamma_{Ꝙ_2}(y)\rangle\}$

(vi) $(Ꝑ_1, Ꝙ_1) \cup (Ꝑ_2, Ꝙ_2) = \{\langle æ, \mu_{Ꝑ_1}(æ) \vee \mu_{Ꝑ_2}(æ), \sigma_{Ꝑ_1}(æ) \vee \sigma_{Ꝑ_2}(æ), \gamma_{Ꝑ_1}(y) \wedge \gamma_{Ꝑ_2}(y)\rangle, \langle y, \mu_{Ꝙ_1}(y) \vee \mu_{Ꝙ_2}(y), \sigma_{Ꝙ_1}(y) \vee \sigma_{Ꝙ_2}(y), \gamma_{Ꝙ_1}(y) \wedge \gamma_{Ꝙ_2}(y)\rangle\}$.

Definition 2.5 [14]: Let $(X, Y, BT_{\aleph})$ be a $\aleph EB.T.S$, a $\aleph EB.S\ (Ꝑ, Ꝙ) = \{< æ, \mu_{Ꝑ}(æ), \sigma_{Ꝑ}(æ), \gamma_{Ꝑ}(æ) >: æ \in X, < y, \mu_{Ꝙ}(y), \sigma_{Ꝙ}(y), \gamma_{Ꝙ}(y) >: y \in Y\}$ is namely the neutrosophic binary point [shortly, $\aleph EB.P$] if and only if for each element $(æ^*, y^*) \in (X, Y)$

$$\begin{cases} \mu_{Ꝑ}(æ^*) = \alpha_1, \ \mu_{Ꝑ}(y^*) = \alpha_2 \\ \sigma_{Ꝑ}(æ^*) = \beta_1, \ \sigma_{Ꝑ}(y^*) = \beta_2 \quad \text{for } (æ^*, y^*) = (æ, y) \\ \gamma_{Ꝑ}(æ^*) = \delta_1, \ \gamma_{Ꝑ}(y^*) = \delta_2 \end{cases}$$

$$\begin{cases} \mu_{Ꝑ}(æ^*) = 0, \ \mu_{Ꝑ}(y^*) = 0 \\ \sigma_{Ꝑ}(æ^*) = 1, \ \sigma_{Ꝑ}(y^*) = 1 \quad \text{for } (æ^*, y^*) \neq (æ, y). \\ \gamma_{Ꝑ}(æ^*) = 1, \ \gamma_{Ꝑ}(y^*) = 1 \end{cases}$$

a $\aleph EB.S\ (Ꝑ, Ꝙ)$ will be denoted by $({Ꝑ^{æ}}_{\alpha_1,\beta_1,\delta_1}, {Ꝙ^{y}}_{\alpha_2,\beta_2,\delta_2})$ or $(æ_{\alpha_1,\beta_1,\delta_1}, y_{\alpha_2,\beta_2,\delta_2})$, for the neutrosophic binary point $(æ_{\alpha_1,\beta_1,\delta_1}, y_{\alpha_2,\beta_2,\delta_2})$, $(æ, y)$ will be called its support, The complement of the $(æ_{\alpha_1,\beta_1,\delta_1}, y_{\alpha_2,\beta_2,\delta_2})$ will be denoted by $(æ_{\alpha_1,\beta_1,\delta_1}, y_{\alpha_2,\beta_2,\delta_2})^C$.

Definition 2.6 [14]: Let $(X, Y, BT_{\aleph})$ be a $\aleph EB.T.S$ and a $\aleph EB.S\ (Ꝑ, Ꝙ) \subseteq (X, Y)$. The subset $(Ꝑ, Ꝙ)$ is namely neutrosophic binary neighborhood of a $\aleph EB.P\ (æ, y)$ if and only if there exists $(Ꝑ_0, Ꝙ_0) \in O(\aleph EB.T.S\ (X, Y))$ so that $(æ, y) \in (Ꝑ_0, Ꝙ_0) \subseteq (Ꝑ, Ꝙ)$.

Definition 2.7 [14]: Let $(X, Y, BT_{\aleph})$ be a $\aleph EB.T.S$ and $(Ꝑ, Ꝙ) \subseteq (X, Y)$,

(i) The ordered pair $({(Ꝑ, Ꝙ)_{\aleph}}^{1*}, {(Ꝑ, Ꝙ)_{\aleph}}^{2*})$ is called $\aleph EBCL(Ꝑ, Ꝙ)$ where
${(Ꝑ, Ꝙ)_{\aleph}}^{1*} = \cap \{Ꝑ_{\ell} : (Ꝑ_{\ell}, Ꝙ_{\ell}) \in C(\aleph EB.T.S\ (X, Y))\ \&\ (Ꝑ, Ꝙ) \subseteq (Ꝑ_{\ell}, Ꝙ_{\ell})\}$ and
${(Ꝑ, Ꝙ)_{\aleph}}^{2*} = \cap \{Ꝙ_{\ell} : (Ꝑ_{\ell}, Ꝙ_{\ell}) \in C(\aleph EB.T.S\ (X, Y))\ \&\ (Ꝑ, Ꝙ) \subseteq (Ꝑ_{\ell}, Ꝙ_{\ell})\}$.

(ii) The ordered pair $((Ꝑ, Ꝙ)^{1^\circ}, (Ꝑ, Ꝙ)^{2^\circ})$ is called $\aleph EBINT(Ꝑ, Ꝙ)$ where
${(Ꝑ, Ꝙ)_{\aleph}}^{1^\circ} = \cup \{Ꝑ_{\ell} : (Ꝑ_{\ell}, Ꝙ_{\ell}) \in O(\aleph EB.T.S\ (X, Y))\ \&\ (Ꝑ, Ꝙ) \supseteq (Ꝑ_{\ell}, Ꝙ_{\ell})\}$
${(Ꝑ, Ꝙ)_{\aleph}}^{2^\circ} = \cup \{Ꝙ_{\ell} : (Ꝑ_{\ell}, Ꝙ_{\ell}) \in O(\aleph EB.T.S\ (X, Y))\ \&\ (Ꝑ, Ꝙ) \supseteq (Ꝑ_{\ell}, Ꝙ_{\ell})\}$

Theorem 2.8 [14]: Let $(Ꝑ_1, Ꝙ_1)$, $(Ꝑ_2, Ꝙ_2) \subseteq (X, Y)$ and $(X, Y, BT_{\aleph})$ be $\aleph EB.T.S$. Then the following statements are fulfilled:

(i) $\aleph EBINT(Ꝑ_1, Ꝙ_1) \subseteq (Ꝑ_1, Ꝙ_1)$.

(ii) If $\aleph EBINT(Ꝑ_1, Ꝙ_1)$is neutrosophic binary open if and only if $\aleph EBINT(Ꝑ_1, Ꝙ_1) = (Ꝑ_1, Ꝙ_1)$.

(iii) If $(Ꝑ_1, Ꝙ_1) \subseteq (Ꝑ_2, Ꝙ_2)$, then $\aleph EBINT(Ꝑ_1, Ꝙ_1) \subseteq \aleph EBINT(Ꝑ_2, Ꝙ_2)$.

(iv) $\aleph EBINT(\aleph EBINT(Ꝑ_1, Ꝙ_1)) = \aleph EBINT(Ꝑ_1, Ꝙ_1)$.

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(v) $(\text{Ᵽ}_1, \text{Ⴓ}_1) \subseteq \aleph\mathrm{EBCL}(\text{Ᵽ}_1, \text{Ⴓ}_1)$.

(vi) If $(\text{Ᵽ}_1, \text{Ⴓ}_1) \in C\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big)$ if and only if $\aleph\mathrm{EBCL}(\text{Ᵽ}_1, \text{Ⴓ}_1) = (\text{Ᵽ}_1, \text{Ⴓ}_1)$.

(vii) If $(\text{Ᵽ}_1, \text{Ⴓ}_1) \subseteq (\text{Ᵽ}_1, \text{Ⴓ}_1)$, then $\aleph\mathrm{EBCL}(\text{Ᵽ}_1, \text{Ⴓ}_1) \subseteq \aleph\mathrm{EBCL}(\text{Ᵽ}_2, \text{Ⴓ}_2)$.

Definition 2.9: Let $(X, X, BT_\aleph)$ be a $\aleph\mathrm{EB.T.S}$ and $(\text{Ᵽ}, \text{Ⴓ}) \subseteq (X, Y)$. The ordered pair $((\text{Ᵽ}, \text{Ⴓ})^{1**}, (\text{Ᵽ}, \text{Ⴓ})^{2**})$ is called $\aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ})$ where
$(\text{Ᵽ}, \text{Ⴓ})^{1**} = \cap\{\text{Ᵽ}_\ell : (\text{Ᵽ}_\ell, \text{Ⴓ}_\ell) \in O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big) \,\&\, (\text{Ᵽ}, \text{Ⴓ}) \subseteq (\text{Ᵽ}_\ell, \text{Ⴓ}_\ell)\}$ and
$(\text{Ᵽ}, \text{Ⴓ})^{2**} = \cap\{\text{Ⴓ}_\ell : (\text{Ᵽ}_\ell, \text{Ⴓ}_\ell) \in O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big) \,\&\, (\text{Ᵽ}, \text{Ⴓ}) \subseteq (\text{Ᵽ}_\ell, \text{Ⴓ}_\ell)\}$.

For example: Let $X = \{p_1, p_2\}$ and $Y = \{q_1, q_2\}$ be the universe. Let $BT_\aleph = (0_X, 0_Y), (1_X, 1_Y), (\text{Ᵽ}, \text{Ⴓ})\}$ be the $\aleph\mathrm{EB.T.S}$. So that $(\text{Ᵽ}, \text{Ⴓ}) = \{\langle X, (0.7, 0.5, 0.3), (0.6, 0.5, 0.4)\rangle, \langle Y, (0.6, 0.5, 0.4), (0.7, 0.5, 0.3)\rangle\}$. Let $(\text{Ᵽ}_0, \text{Ⴓ}_0) = \{\langle X, (0.4, 0.5, 0.8), (0.3, 0.4, 0.7)\rangle, \langle Y, (0.5, 0.5, 0.6), (0.4, 0.3, 0.5)\rangle\}$. Then $\aleph\mathrm{EBKER}(\text{Ᵽ}_0, \text{Ⴓ}_0) = (\text{Ᵽ}, \text{Ⴓ})$

Theorem 2.10: Let $(X, Y, BT_\aleph)$ Be a $\aleph\mathrm{EB.T.S}$ and $(æ, y) \in X \times Y$, then $\aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ}) = \{(æ, y) \in X \times Y : \aleph\mathrm{EBCL}(\{æ\}, \{y\}) \cap (\text{Ᵽ}, \text{Ⴓ}) \neq (0_X, 0_Y)\}$.

Proof: Let $(æ, y) \in \aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ})$ & suppose that $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \cap (\text{Ᵽ}, \text{Ⴓ}) = (0_X, 0_Y)$. Hence $(æ, y) \notin \big[(X, Y) - \big(\aleph\mathrm{EBCL}(\{æ\}, \{y\})\big)\big]$ which is belong to $O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big)$ & containing $(\text{Ᵽ}, \text{Ⴓ})$, since $(æ, y) \in \aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ})$. This is a ambivalence.
Conversely, let $(æ, y) \in X \times Y$ be so that $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \cap (\text{Ᵽ}, \text{Ⴓ}) \neq (0_X, 0_Y)$. If possible, let $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \cap (\text{Ᵽ}, \text{Ⴓ}) \neq (0_X, 0_Y)$ & suppose that $(æ, y) \notin \aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ})$, then there exists $(\text{Ᵽ}_1, \text{Ⴓ}_1) \in O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big)$ & containing $(\text{Ᵽ}, \text{Ⴓ})$, $(æ, y) \notin (\text{Ᵽ}_1, \text{Ⴓ}_1)$. Let $\big(æ^*, y^*\big) \in \aleph\mathrm{EBCL}(\{æ\}, \{y\}) \cap (\text{Ᵽ}, \text{Ⴓ})$. Hence, $(\text{Ᵽ}_1, \text{Ⴓ}_1)$ is a neutrosophic binary neighborhood of $\big(æ^*, y^*\big)$ which $(æ, y) \notin (\text{Ᵽ}_1, \text{Ⴓ}_1)$. By this ambivalence $(æ, y) \in \aleph\mathrm{EBKER}(\text{Ᵽ}, \text{Ⴓ})$.

Theorem 2.11: Let $(X, Y, BT_\aleph)$ be a $\aleph\mathrm{EB.T.S}$ and a $\aleph\mathrm{EB.P}$ $(æ, y), \big(æ^*, y^*\big) \in (X, Y)$, then $(æ, y) \in \aleph\mathrm{EBKER}\big(\{æ^*\}, \{y^*\}\big)$ if and only if $\big(æ^*, y^*\big) \in \aleph\mathrm{EBCL}(\{æ\}, \{y\})$

Proof: Suppose that $(æ, y) \notin \aleph\mathrm{EBKER}\big(\{æ^*\}, \{y^*\}\big)$, then there exists $(\text{Ᵽ}, \text{Ⴓ}) \in O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big)$ so that $\big(æ^*, y^*\big) \in (\text{Ᵽ}, \text{Ⴓ})$. But $(æ, y) \notin (\text{Ᵽ}, \text{Ⴓ})$; therefore, we have $\big(æ^*, y^*\big) \notin \aleph\mathrm{EBCL}(\{æ\}, \{y\})$. By comparable procedure we can prove the opposite .

Theorem 2.12: If $(æ, y)$ and $\big(æ^*, y^*\big)$ each $\aleph\mathrm{EB.P}$ in the $\aleph\mathrm{EB.T.S}$ (X, Y), then $\aleph\mathrm{EBKER}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBKER}(\{æ^*\}, \{y^*\})$ if and only if $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBCL}(\{æ^*\}, \{y^*\})$.

Proof: Suppose that $\aleph\mathrm{EBKER}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBKER}(\{æ^*\}, \{y^*\})$, consequently there is $\big(æ_1, y_1\big) \in X \times Y$ so that $\big(æ_1, y_1\big) \in \aleph\mathrm{EBKER}(\{æ\}, \{y\})$ & $\big(æ_1, y_1\big) \notin \aleph\mathrm{EBKER}(\{æ^*\}, \{y^*\})$, it follows that $(\{æ\}, \{y\}) \cap \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big) \neq (0_X, 0_Y)$, from this we get $(\{æ\}, \{y\}) \in \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big)$. Now, $\big(æ_1, y_1\big) \notin \aleph\mathrm{EBKER}(\{æ^*\}, \{y^*\})$, we have $(\{æ^*\}, \{y^*\}) \cap \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big) = (0_X, 0_Y)$, since $(\{æ\}, \{y\}) \in \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big)$, $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \subset \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big)$ & $(\{æ^*\}, \{y^*\}) \cap \aleph\mathrm{EBCL}\big(\{æ_1\}, \{y_1\}\big) = (0_X, 0_Y)$, therefore $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBCL}(\{æ^*\}, \{y^*\})$.
Conversely, suppose that $\aleph\mathrm{EBCL}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBCL}(\{æ^*\}, \{y^*\})$, consequently there exist $\big(æ_2, y_2\big) \in X \times Y$ such that $\big(æ_2, y_2\big) \in \aleph\mathrm{EBCL}(\{æ\}, \{y\})$ & $\big(æ_2, y_2\big) \notin \aleph\mathrm{EBCL}(\{æ^*\}, \{y^*\})$, there exist $(\text{Ᵽ}, \text{Ⴓ}) \in O\big(\aleph\mathrm{EB.T.S}\,(X,Y)\big)$ including $\big(æ_2, y_2\big)$ & therefore $(æ, y)$ but not $\big(æ^*, y^*\big)$, namely $\big(æ^*, y^*\big) \notin \aleph\mathrm{EBKER}(\{æ\}, \{y\})$, hence $\aleph\mathrm{EBKER}(\{æ\}, \{y\}) \neq \aleph\mathrm{EBKER}(\{æ^*\}, \{y^*\})$.

Theorem 2.13: Let $(X, Y, BT_\aleph)$ be a $\aleph\mathrm{EB.T.S}$ and $(\text{Ᵽ}_1, \text{Ⴓ}_1)$, $(\text{Ᵽ}_2, \text{Ⴓ}_2) \subseteq (Y, Y)$. Then :

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i) $(\mathscr{x},y)\in\aleph EBKER(\mathcal{P}_1,\mathcal{Q}_1)$ if and only if $(\mathcal{P}_1,\mathcal{Q}_1)\cap(G,H)\neq(0_X,0_Y)$ for each $(G,H)\in O\big(\aleph EB.T.S\,(X,Y)\big)$ containing $(\mathscr{x},y)$.
ii) $(\mathcal{P}_1,\mathcal{Q}_1)\subseteq\aleph EBKER(\mathcal{P}_1,\mathcal{Q}_1)$ and $\aleph EBKER(\mathcal{P}_1,\mathcal{Q}_1)\subseteq(\mathcal{P}_1,\mathcal{Q}_1)$ if $(\mathcal{P},\mathcal{Q})\in O\big(\aleph EB.T.S\,(X,Y)\big)$
iii) If $(\mathcal{P}_1,\mathcal{Q}_1)\subseteq(\mathcal{P}_2,\mathcal{Q}_2)$, then $\aleph EBKER(\mathcal{P}_1,\mathcal{Q}_1)\subseteq\aleph EKER(\mathcal{P}_2,\mathcal{Q}_2)$.
Proof: Obvious.

Definition 2.14: Let $(\mathcal{P},\mathcal{Q}),(\mathcal{P}^*,\mathcal{Q}^*)\in O\big(\aleph EB.T.S\,(X,Y)\big)$, $(\mathcal{P},\mathcal{Q}),(\mathcal{P}^*,\mathcal{Q}^*)$ are called disjoint if $(\mathcal{P}\cap\mathcal{P}^*,\mathcal{Q}\cap\mathcal{Q}^*)=(0_X,0_Y)$.

3. Neutrosophic Binary R_L-Spaces, $L=0,1,2$

Definition 3.1: A $\aleph EB.T.S\,(X,Y)$is said to be:
(i) Neutrosophic Binary Sober $-R_0$ space [$S\aleph EB-R_0.S$] if $\bigcap_{(\mathscr{x},y)\in(X,Y)}\aleph EBCL(\{\mathscr{x}\},\{y\})=(0_X,0_Y)$.
(ii) Neutrosophic Binary$-R_0$ space [$\aleph EB-R_0.S$] if $(\mathcal{P},\mathcal{Q})\in O\big(\aleph EB.T.S\,(X,Y)\big)$ and $\aleph EB.P\,(\mathscr{x},y)\in(\mathcal{P},\mathcal{Q})$, then $\aleph EBCL(\{\mathscr{x}\},\{y\})\subseteq(\mathcal{P},\mathcal{Q})$.
(iii) Neutrosophic Binary $-R_1$ space [$\aleph EB-R_1.S$] if for each $\aleph EB.P\,(\mathscr{x},y),(\mathscr{x}^*,y^*)$ in (X,Y) with $\aleph EBCL(\{\mathscr{x}\},\{y\})\neq\aleph EBCL(\{\mathscr{x}^*\},\{y^*\})$, there are disjoint $(\mathcal{P},\mathcal{Q}),(\mathcal{P}^*,\mathcal{Q}^*)\in O\big(\aleph EB.T.S\,(X,Y)\big)$, so that $\aleph EBCL(\{\mathscr{x}\},\{y\})\subseteq(\mathcal{P},\mathcal{Q})$ and $\aleph EBCL(\{\mathscr{x}^*\},\{y^*\})\subseteq(\mathcal{P}^*,\mathcal{Q}^*)$.
(iv) Neutrosophic Binary$-R_2$ space [$\aleph EB-R_2.S$] if for each $\aleph EB.P\ (\mathscr{x},y)\notin(G,H)$ and $(G,H)\in C\big(\aleph EB.T.S\,(X,Y)\big)$ with $\aleph EBCL\big(B\aleph KER(\{\mathscr{x}\},\{y\})\big)\neq\aleph EBCL\big(B\aleph KER(G,H)\big)$, there are disjoint $(\mathcal{P},\mathcal{Q}),(\mathcal{P}^*,\mathcal{Q}^*)\in O\big(\aleph EB.T.S\,(X,Y)\big)$, so that $\aleph EBCL\big(B\aleph KER(G,H)\big)\subseteq(\mathcal{P},\mathcal{Q})$ and $\aleph EBCL\big(B\aleph KER(\{\mathscr{x}\},\{y\})\big)\subseteq(\mathcal{P}^*,\mathcal{Q}^*)$.

For example: Let $X=\{p_1,p_2\}$ and $Y=\{q_1,q_2\}$ be the universe. Let $BT_{\aleph}=(0_{\mathscr{x}},0_y),(1_{\mathscr{x}},1_y),(\mathcal{P},\mathcal{Q}),(\mathcal{P}_1,\mathcal{Q}_1),(\mathcal{P}_2,\mathcal{Q}_2)\}$ be the $\aleph EB.T.S.$ So that $(\mathcal{P}_0,\mathcal{Q}_0)=\{\langle X,(0.7,0.5,0.3),(0.6,0.5,0.1)\rangle,\langle Y,(0.6,0.5,0.4),(0.7,0.5,0.1)\rangle\}$. Let $(\mathcal{P}_1,\mathcal{Q}_1)=\{\langle X,(0.7,0.5,0.3),0_X\rangle,\langle Y,(0.6,0.5,0.4),0_Y\rangle\}$, $(\mathcal{P}_2,\mathcal{Q}_2)=\{\langle X,0_X,(0.6,0.5,0.1)\rangle,\langle Y,0_Y,(0.7,0.5,0.1)\rangle\}$ .Then (X,Y) is $\aleph EB-R_2.S$

Remark 3.2: Every $\aleph EB-R_L.S$ is an $\aleph EB-R_{L-1}.S$, $L=2,1$.

Theorem 3.3: A $\aleph EB.T.S\ (X,Y)$ is $S\aleph EB-R_0.S$ if and only if $\aleph EBCL(\{\mathscr{x}\},\{y\})\neq(X,Y)$ for each $(\mathscr{x},y)\in(X,Y)$.

Proof: presume that (X,Y) $S\aleph EB-R_0.S$ & that there is a $\aleph EB.P\ (\mathscr{x}^*,y^*)$ in (X,Y) so that $\aleph EBCL(\{\mathscr{x}^*\},\{y^*\})=(X,Y)$, then $(\mathscr{x}^*,y^*)\notin(\mathcal{P},\mathcal{Q})$ which $(\mathcal{P},\mathcal{Q})\subseteq O\big(\aleph EB.T.S\,(X,Y)\big)$, this implicate that to $(\mathscr{x}^*,y^*)\in\bigcap_{(\mathscr{x},y)\in(X,Y)}\aleph EBCL(\{\mathscr{x}\},\{y\})$, but and so on a ambivalence.
Consequently, $\aleph EBCL(\{\mathscr{x}\},\{y\})=(X,Y)$ for any $(\mathscr{x},y)\in(X,Y)$. If there exists a $\aleph EB.P\ (\mathscr{x}^*,y^*)$ in (X,Y) so that $(\mathscr{x}^*,y^*)\in\bigcap_{(\mathscr{x},y)\in(X,Y)}\aleph EBCL(\{\mathscr{x}\},\{y\})$, then any $(\mathcal{P},\mathcal{Q})\in O\big(\aleph EB.T.S\,(X,Y)\big)$ containing $(\mathscr{x}^*,y^*)$ must contain all $\aleph EB.P$ of (X,Y), this implicate that the $\aleph EB.T.S\,(X,Y)$ is the unique containing $(\mathscr{x},y)$ & belong to $O\big(\aleph EB.T.S\,(X,Y)\big)$, hence $\aleph EBKER(\{\mathscr{x}^*\},\{y^*\})=(X,Y)$ which is a ambivalence, therefore (X,Y) is $S\aleph EB-R_0.S$.

Theorem 3.4: A $\aleph EB.T.S\ (X,Y)$ is an is $\aleph EB-R_0.S$ if and only if for each $(\mathcal{P},\mathcal{Q})\in C\big(\aleph EB.T.S\,(X,Y)\big)$ and $(\mathscr{x},y)\in(\mathcal{P},\mathcal{Q})$, then $\aleph EBKER(\{\mathscr{x}\},\{y\})\subseteq(\mathcal{P},\mathcal{Q})$.
Proof: Let a $\aleph EB.T.S\ (X,Y)$ be a $\aleph EB-R_0.S$ & $(\mathcal{P},\mathcal{Q})\in C\big(\aleph EB.T.S\,(X,Y)\big)$ & $(\mathscr{x},y)\in(\mathcal{P},\mathcal{Q})$. Then for each $(\mathscr{x}^*,y^*)\notin(\mathcal{P},\mathcal{Q})$ implicate $(\mathscr{x}^*,y^*)\in(\mathcal{P},\mathcal{Q})^C\in O\big(\aleph EB.T.S\,(X,Y)\big)$,then $\aleph EBCL(\{\mathscr{x}^*\},\{y^*\})\subseteq(\mathcal{P},\mathcal{Q})^C$, also $(\mathscr{x},y)\notin\aleph EBCL(\{\mathscr{x}^*\},\{y^*\})$. Hence $(\mathscr{x}^*,y^*)\notin\aleph EBKER(\{\mathscr{x}\},\{y\})$. Thus $\aleph EBKER(\{\mathscr{x}\},\{y\})\subseteq(\mathcal{P},\mathcal{Q})$.
Conversely: suppose for all $(\mathcal{P},\mathcal{Q})\in C\big(\aleph EB.T.S\,(X,Y)\big)$ & $(\mathscr{x},y)\in(\mathcal{P},\mathcal{Q})$, then $\aleph EBKER(\{\mathscr{x}\},\{y\})\subseteq(\mathcal{P},\mathcal{Q})$ & let $(U,V)\in O\big(\aleph EB.T.S\,(X,Y)\big)$, $(\mathscr{x},y)\in(\mathcal{P}_1,\mathcal{Q}_1)$ then for each $(\mathscr{x}^*,y^*)\notin(\mathcal{P}_1,\mathcal{Q}_1)$ implicate $(\mathscr{x}^*,y^*)\in\big((\mathcal{P}_1,\mathcal{Q}_1)\big)^C\in C\big(\aleph EB.T.S\,(X,Y)\big)$ implicate that $\aleph EBKER(\{\mathscr{x}^*\},\{y^*\})\subseteq(\mathcal{P}_1,\mathcal{Q}_1)^C$.

Therefore $(\mathfrak{x},y) \notin \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$ & $\left(\mathfrak{x}^*,y^*\right) \notin \aleph EBCL(\{\mathfrak{x}\},\{y\})$. Hence $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \subseteq (\mathfrak{P}_1,\mathfrak{Q}_1)$. That's mean$(X,Y)$ is an $\aleph EB-R_0.S$.

Corollary 3.5: A $\aleph EB.T.S$ (X,Y) is an $\aleph EB-R_0.S$ if and only if for each $(\mathfrak{P},\mathfrak{Q}) \in O(\aleph EB.T.S\,(X,Y))$ and $(\mathfrak{x},y) \in (\mathfrak{P},\mathfrak{Q})$, then $\aleph EBCL(\aleph EBKER(\{\mathfrak{x}\},\{y\})) \subseteq (\mathfrak{P},\mathfrak{Q})$.

Theorem 3.6: A $\aleph EB.T.S$ (X,Y) is a $\aleph EB-R_0.S$ if and only if for any $\aleph EB.P$ $(\mathfrak{x},y)$ in (X,Y), $\aleph EBCL(\{\mathfrak{x}\},\{y\}) = \aleph EBKER(\{\mathfrak{x}\},\{y\})$.

Proof: $\aleph EB.T.S$ (X,Y) is a $\aleph EB-R_0.S$. Now If $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \neq \aleph EBKER(\{\mathfrak{x}\},\{y\})$ for each $\aleph EB.P$ $(\mathfrak{x},y) \in (X,Y)$, then there are $\aleph EB.P$ $\left(\mathfrak{x}^*,y^*\right) \in (X,Y)$ so that $(\mathfrak{x},y) \neq \left(\mathfrak{x}^*,y^*\right)$ & $\left(\mathfrak{x}^*,y^*\right) \in \aleph EBCL(\{\mathfrak{x}\},\{y\})$, $\left(\mathfrak{x}^*,y^*\right) \notin \aleph EBKER(\{\mathfrak{x}\},\{y\})$ this implicate, there exist $(\mathfrak{P},\mathfrak{Q}) \in O(\aleph EB.T.S\,(X,Y))$, $\left(\mathfrak{x}^*,y^*\right) \not\subseteq (\mathfrak{P},\mathfrak{Q})$, from this we get $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \not\subseteq (\mathfrak{P},\mathfrak{Q})$ this is ambivalence, therefore $\aleph EBCL(\{\mathfrak{x}\},\{y\}) = \aleph EBKER(\{\mathfrak{x}\},\{y\})$.

Conversely, suppose that $\aleph EBCL(\{\mathfrak{x}\},\{y\}) = \aleph EBKER(\{\mathfrak{x}\},\{y\})$, then for all $(\mathfrak{P},\mathfrak{Q}) \in O(\aleph EB.T.S\,(X,Y))$, $(\mathfrak{x},y) \in (\mathfrak{P},\mathfrak{Q})$, let $\left(\mathfrak{x}^*,y^*\right) \in \aleph EBKER(\{\mathfrak{x}\},\{y\})$, then $(\mathfrak{x},y) \in \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$ & hence $\left(\mathfrak{x}^*,y^*\right) \in (\mathfrak{P},\mathfrak{Q})$. This implicate that $\aleph EKER(\{\mathfrak{x}\},,\{y\}) \subseteq (\mathfrak{P},\mathfrak{Q})$, thus $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \subseteq (\mathfrak{P},\mathfrak{Q})$, therefore (X,Y) is an $\aleph EB-R_0.S$.

Theorem 3.7: A $\aleph EB.T.S$ (X,Y) is an $\aleph EB-R_0.S$ if and only if for any two $\aleph EB.P$ $(\mathfrak{x},y)$, and $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) with $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \neq \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$, then $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \cap \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$.

Proof: let A $\aleph EB.T.S$ (X,Y) is an is $\aleph EB-R_0.S$, then for any two $\aleph EB.P$ $(\mathfrak{x},y)$ & $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) if $\aleph EKER(\{\mathfrak{x}\},\{y\}) \neq \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$, hence $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \neq \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$. In fact, $(\mathfrak{x}_0,y_0) \in \aleph EBKER(\{\mathfrak{x}\},\{y\}) \cap \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$, then we have $(\mathfrak{x},y)$, $\left(\mathfrak{x}^*,y^*\right) \in \aleph EBCL(\{\mathfrak{x}_0\},\{y_0\})$, we obtain that $\aleph EBCL(\{\mathfrak{x}\},\{y\}) = \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) = \aleph EBCL(\{\mathfrak{x}_0\},\{y_0\})$, which is impossible. Then $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \cap \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$

Conversely : assume that for any two $\aleph EB.P$ $(\mathfrak{x},y)$ & $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) & $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \cap \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$ then $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \cap \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$. In fact, $(\mathfrak{x}_0,y_0) \in \aleph EBCL(\{\mathfrak{x}\},\{y\}) \cap \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$, this implicate that $(\mathfrak{x},y)$ & $\left(\mathfrak{x}^*,y^*\right) \in \aleph EBKER(\{\mathfrak{x}_0\},\{y_0\})$, this means $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \cap \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) \neq (0_{\mathfrak{x}},0_y)$. Hence by hypothesis, we get $\aleph EBKER(\{\mathfrak{x}\},\{y\}) = \aleph EBKER(\{\mathfrak{x}_0\},\{y_0\})$, by similar way it follows that $\aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) = \aleph EBKER(\{\mathfrak{x}_0\},\{y_0\})$ which is a ambivalence. Then $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \cap \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$, therefore A $\aleph EB.T.S$ (X,Y) is $\aleph EB-R_0.S$.

Corollary 3.8: A $\aleph EB.T.S$ (X,Y) is an is $\aleph EB-R_0.S$ if and only if for any two $\aleph EB.P$ $(\mathfrak{x},y)$, and $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) with $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \neq \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$, then $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \cap \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) = (0_X,0_Y)$.

Theorem 3.9: If a $\aleph EB.T.S$ (X,Y) is $\aleph EB-R_1.S$, then (X,Y) is $\aleph EB-R_0.S$.

Proof: Let $(\mathfrak{x},y) \in (\mathfrak{P},\mathfrak{Q}) \in O(\aleph EB.T.S\,(X,Y))$, if $\left(\mathfrak{x}^*,y^*\right) \notin (\mathfrak{P},\mathfrak{Q})$,then since $(\mathfrak{x},y) \notin \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$, $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \neq \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$, hence there exist $\left(\mathfrak{P}^*,\mathfrak{Q}^*\right) \in O(\aleph EB.T.S\,(X,Y))$ so that $\aleph EBCL(\{\mathfrak{x}^*\},\{y^*\}) \subseteq \left(\mathfrak{P}^*,\mathfrak{Q}^*\right)$ & $(\mathfrak{x},y) \notin (\mathfrak{P}_1,\mathfrak{Q}_1)$, which implicate $\left(\mathfrak{x}^*,y^*\right) \notin \aleph EBCL(\{\mathfrak{x}\},\{y\})$, thus $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \subseteq (\mathfrak{P},\mathfrak{Q})$, therefore, (X,Y) is $\aleph EB-R_0.S$.

Theorem 3.10: A $\aleph EB.T.S$ (X,Y) is an $\aleph EB-R_1.S$ if and only if for any two $\aleph EB.P$ $(\mathfrak{x},y)$, and $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) with $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \neq \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$, then there exist $(G,H), (G^*,H^*) \in C(\aleph EB.T.S\,(X,Y))$, so that $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \subseteq (G,H)$, $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \cap (G^*,H^*) = (0_X,0_Y)$ and $\aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) \subseteq (G^*,H^*)$, $\aleph EBKER(\{\mathfrak{x}^*\},\{y^*\}) \cap (G,H) = (0_X,0_Y)$ *and* $(G,H) \cup (G^*,H^*) = (1_X,1_Y)$.

Proof: Let a $\aleph EB.T.S$ (X,Y) be $\aleph EB-R_1.S$, then two $\aleph EB.P$ $(\mathfrak{x},y)$, and $\left(\mathfrak{x}^*,y^*\right)$ in (X,Y) with $\aleph EBKER(\{\mathfrak{x}\},\{y\}) \neq \aleph EBKER(\{\mathfrak{x}^*\},\{y^*\})$, this mean $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \neq \aleph EBCL(\{\mathfrak{x}^*\},\{y^*\})$, since (X,Y) is $\aleph EB-R_0.S$ then there exist disjoint $(\mathfrak{P},\mathfrak{Q})$, $\left(\mathfrak{P}^*,\mathfrak{Q}^*\right) \in O(\aleph EB.T.S\,(X,Y))$, so that $\aleph EBCL(\{\mathfrak{x}\},\{y\}) \subseteq (\mathfrak{P},\mathfrak{Q})$

& $\aleph EBCL(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$, also $(Ꝑ,Ɑ)^C$, $(Ꝑ^*,Ɑ^*)^C \in C(\aleph EB.T.S\,(X,Y))$ so that $(Ꝑ,Ɑ)^C \cup (1_X,1_Y)$. Now $(G,H) = (Ꝑ,Ɑ)^C$ & $(G^*,H^*) = (Ꝑ^*,Ɑ^*)^C$. Thus $(æ,y) \in (Ꝑ,Ɑ) \subseteq (G^*,H^*)$ & $(æ^*,y^*) \in (Ꝑ^*,Ɑ^*) \subseteq (G,H)$ implicate that $\aleph EBKER(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ) \subseteq (G^*,H^*)$ & $\aleph EKER(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*) \subseteq (G,H)$.

Conversely : suppose any two $\aleph EB.P\ (æ,y)$, $(æ^*,y^*) \in (X,Y)$ with $\aleph EBKER(\{æ\},\{y\}) \neq \aleph EBKER(\{æ^*\},\{y^*\})$, there exist (G,H), $(G^*,H^*) \in C(\aleph EB.T.S\,(X,Y))$, so that $\aleph EBKER(\{æ\},\{y\}) \subseteq (G,H)$, $\aleph EBKER(\{æ\},\{y\}) \cap (G^*,H^*) = (0_X,0_Y)$ & $\aleph EBKER(\{æ^*\},\{y^*\}) \subseteq (G^*,H^*)$, $\aleph EBKER(\{æ^*\},\{y^*\}) \cap (G,H) = (0_X,0_Y)$ & $(G,H) \cup (G^*,H^*) = (1_X,1_Y)$, then $(G,H)^C$, $(G^*,H^*)^C \in C(\aleph EB.T.S\,(X,Y))$ & $(G,H)^C \cap (G^*,H^*)^C = (0_X,0_Y)$. Now take $(G,H)^C = (Ꝑ^*,Ɑ^*)$ & $(G^*,H^*)^C = (Ꝑ,Ɑ)$. Thus $\aleph EBKER(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBKER(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$, $(Ꝑ,Ɑ) \cap (Ꝑ^*,Ɑ^*) = (0_X,0_Y)$,hence $(æ,y) \in (Ꝑ,Ɑ)$ & $(æ^*,y^*) \in (Ꝑ^*,Ɑ^*)$ implicate that $(æ,y) \notin \aleph EBCL(\{æ^*\},\{y^*\})$ & $(æ^*,y^*) \notin \aleph EBCL(\{æ\},\{y\})$, from this we get $\aleph EBCL(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$. Therefore a $\aleph EB.T.S\ (X,Y)$ is an $\aleph EB - R_1.S$.

Corollary 3.11: A $\aleph EB.T.S\ (X,Y)$ is an $\aleph EB - R_1$ if and only if for any two $\aleph EB.P\ (æ,y)$, and $(æ^*,y^*)$ in (X,Y) with $\aleph EBCL(\{æ\},\{y\}) \neq \aleph EBCL(\{æ^*\},\{y^*\})$, then there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ which $\aleph EBKER(\aleph EBKER(\{æ\},\{y\})) \subseteq (Ꝑ,Ɑ)$ and $\aleph EBCL(\aleph EBKER(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$.

Proof: Let a $\aleph EB.T.S\ (X,Y)$ is an $\aleph EB - R_1$ & then any two $\aleph EB.P\ (æ,y)$, and $(æ^*,y^*)$ in (X,Y) with $\aleph EBCL(\{æ\},\{y\}) \neq \aleph EBCL(\{æ^*\},\{y^*\})$, then there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ so that $\aleph EBCL(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$. Since for each $(æ,y) \in (X,Y)$ $\aleph EBCL(\{æ\},\{y\}) = \aleph EBCL(\aleph EBCL(\{æ\},\{y\}))$ & (X,Y) is an $\aleph EB - R_0$, then $\aleph EBCL(\{æ\},\{y\}) = \aleph EBCL(\aleph EBKER(\{æ\},\{y\}))$

Consequently $\aleph ECL(\aleph EBKER(\{æ\},\{y\})) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\aleph EBKER(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$.

Conversely: Assume any two $\aleph EB.P$ $(æ,y)$, $(æ^*,y^*) \in (X,Y)$ with $\aleph EBCL(\{æ\},\{y\}) \neq \aleph EBCL(\{æ^*\},\{y^*\})$, there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ such that $\aleph EBCL(\aleph EBKER(\{æ\},\{y\})) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\aleph EBKE(\{æ\},\{y\}) \subseteq (\aleph EBKER(\{æ\},\{y\}))\ R(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$. Also, then $\aleph EBCL(\{æ\},\{y\}) \subseteq \aleph EBCL(\aleph EBKER(\{æ\},\{y\}))$ for each $(æ,y) \in (X,Y)$. So we obtain $\aleph EBCL(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$. Consequently, (X,Y) is an $\aleph EB - R_1$.

Theorem 3.12: A $\aleph EB.T.S\ (X,Y)$ is $\aleph EB - R_1.S$ if and only if for each $\aleph EB.P\ (æ,y)$, and $(æ^*,y^*)$ in (X,Y) with $\aleph EBKER(\{æ\},\{y\}) \neq \aleph EBKER(\{æ^*\},\{y^*\})$ there exist $(Ꝑ,Ɑ)$,$(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ and it is disjoint so that $\aleph EBCL(\{æ\},\{y\}) \subseteq (Ꝑ,Ɑ)$ and $\aleph EBCL(\{æ^*\},\{y^*\}) \subseteq (Ꝑ^*,Ɑ^*)$.

Proof: Obviously.

Theorem 3.13: A $\aleph EB.T.S\ (X,Y)$ is $\aleph EB - R_2.S$ if and only if for each $(G,H) \in C(\aleph EB.T.S\,(X,Y))$ and $(æ,y) \notin (K,H)$ with $\aleph EBCL(\aleph EBKER(\{æ\},\{y\})) \neq \aleph EBCL(\aleph EBKER(K,H))$, then there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$, such that $(æ,y) \in (Ꝑ,Ɑ)$ and $(K,H) \subseteq (Ꝑ^*,Ɑ^*)$.

Proof: Let a $\aleph EB.T.S\ (X,Y)$ is $\aleph EB - R_2.S$ if and only if for all $(æ,y) \notin (K,H)$ & $(K,H) \in C(\aleph EB.T.S\,(X,Y))$ with $\aleph ECL(\aleph EBKER(\{æ\},\{y\})) \neq \aleph EBCL(\aleph EBKER(K,H))$, then there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ such that $\aleph EBCL(\aleph EBKER(K,H)) \subseteq (Ꝑ,Ɑ)$ & $\aleph EBCL(\aleph EBKER(\{æ\},\{y\})) \subseteq (Ꝑ^*,Ɑ^*)$ implicate that $(K,H) \subseteq (Ꝑ,Ɑ)$ & $(æ,y) \in (G,H)$.

Conversely**:** Assume that $(æ,y) \notin (K,H)$ & $(K,H) \in C(\aleph EB.T.S\,(X,Y))$ with $\aleph ECL(\aleph EBKER(\{æ\},\{y\})) \neq \aleph EBCL(\aleph EBKER(K,H))$, then there exist disjoint $(Ꝑ,Ɑ)$, $(Ꝑ^*,Ɑ^*) \in O(\aleph EB.T.S\,(X,Y))$ so that $(K,H) \subseteq (Ꝑ,Ɑ)$ & $(æ,y) \in (Ꝑ^*,Ɑ^*)$. Since $\aleph EBKER(K,H) \subseteq (Ꝑ,Ɑ)$, this is implicate to $\aleph EBKER(K,H) \cap (Ꝑ^*,Ɑ^*) = (0_X,0_Y)$, thus $(æ,y) \notin \aleph EBCL(\aleph EBKER(K,H))$, this explain $\aleph EBCL(\aleph EBKER(K,H)) \subseteq (K,H) = \aleph EBCL(K,H) =$

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ℵEBKER(K,H) & ℵEBCL({æ},{y}) = ℵEBKER({æ} , {y}) for each (æ,y) ∈ (X ,Y). Thus ℵEBCL(ℵEBKER(K, H)) ⊆ (Ꝑ, Ꝗ)& ℵEBCL(ℵEBKER({æ} , {y})) ⊆ $(Ꝑ^*, Ꝗ^*)$.

Theorem 3.14: A ℵEB. T. S (X , Y) is ℵEB − R_2. S if and only if for each (æ, y) ∉ (K, H) & (K, H) ∈ C(ℵEB. T. S (X, Y)) with ℵEBKER(G, H) ≠ ℵEBKER({æ} , {y}), then there exist (G_1, H_1) , (G_2, H_2) ∈ C(ℵEB. T. S (X, Y)) , such that ℵEBKER(G, H) ⊆ (K_1, H_1) , ℵEBKER(K, H) ∩ $(K_2, H_2) = (0_X , 0_Y)$ *and* ℵEBKER({æ} , {y}) ⊆ (K_2, H_2) , ℵEBKER({æ} , {y}) ∩ $(K_1, H_1) = (0_X , 0_Y)$ $(K_1, H_1) \cup (K_2, H_2) = (1_X , 1_Y)$.

Proof: Let a ℵEB. T. S (X , Y) is ℵEB − R_2. S if and only if for all (æ, y) ∉ (K, H) & (K, H) ∈ C(ℵEB. T. S (X, Y)) , then there exist (Ꝑ, Ꝗ), $(Ꝑ^*, Ꝗ^*)$ ∈ O(ℵEB. T. S (X, Y)) so that (K, H) ⊆ (Ꝑ, Ꝗ), (æ, y) ∈ $(Ꝑ^*, Ꝗ^*)$ & (Ꝑ, Ꝗ) ∩ $(Ꝑ^*, Ꝗ^*) = (0_X , 0_Y)$, then $(Ꝑ, Ꝗ)^C$ & $(Ꝑ^*, Ꝗ^*)^C$ ∈ C(ℵEB. T. S (X, Y)) such that $(Ꝑ, Ꝗ)^C \cup (Ꝑ^*, Ꝗ^*)^C = (1_X , 1_Y)$. Put $(K_2, H_2) = (Ꝑ, Ꝗ)^C$ & $(K_1, H_1) = (Ꝑ^*, Ꝗ^*)^C$, also ℵEBKER(K, H) ⊆ (Ꝑ, Ꝗ) ⊆ (K_1, H_1) , ℵEKER(K, H) ∩ $(K_2, H_2) = (0_X , 0_Y)$ & ℵEBKER({æ} , {y}) ⊆ $(Ꝑ^*, Ꝗ^*)^C \subseteq (K_2, H_2)$, ℵEKER({æ} , {y}) ∩ $(K_1, H_1) = (0_X , 0_Y)$ & $(K_1, H_1) \cup (K_2, H_2) = (1_X , 1_Y)$.
Conversely : (æ, y) ∉ (K, H) & (K, H) ∈ C(ℵEB. T. S (X, Y)) with ℵEBKER(K, H) ≠ ℵEBKER({æ} , {y}), then there exist (K_1, H_1) , (K_2, H_2) ∈ C(ℵEB. T. S (X, Y)) , such that ℵEBKER(G, H) ⊆ (K_1, H_1) , ℵEBKER(K, H) ∩ $(K_2, H_2) = (0_X , 0_Y)$ & ℵEBKER({æ} , {y}) ⊆ (K_2, H_2), ℵEBKER({æ} , {y}) ∩ $(K_1, H_1) = (0_X , 0_Y)$ & $(K_1, H_1) \cup (K_2, H_2) = (1_X , 1_Y)$. Then $(K_1, H_1)^C$ & $(K_2, H_2)^C$ ∈ C(ℵEB. T. S (X, Y)) so that $(K_1, H_1)^C \cap (K_2, H_2)^C = (0_X , 0_Y)$ and ℵEBKER(K, H) ∩ $(K_1, H_1)^C = (0_X , 0_Y)$, ℵEBKER({æ} , {y}) ∩ $(K_2, H_2)^C = (0_X , 0_Y)$, also (K, H) ⊆ $(K_2, H_2)^C$ & (æ, y) ∈ $(K_1, H_1)^C$. Thus (X , Y) is ℵEB − R_2. S.

4. Neutrosophic Binary T_L-Spaces , $L = 0, 1, 2$

Definition 4.1: A ℵEB. T. S (X, Y) is

(i) Neutrosophic Binary−T_0 space [ℵEB − T_0. S] if for each elements (æ, y) ≠ $(æ^*, y^*)$ in (X,Y) , there are (Ꝑ, Ꝗ), ∈ O(ℵEB. T. S (X, Y)) so that (æ, y) ∈ (Ꝑ, Ꝗ) and $(æ^*, y^*)$ ∉ (Ꝑ, Ꝗ) or (æ, y) ∉ (Ꝑ, Ꝗ) and $(æ^*, y^*)$ ∈ (Ꝑ, Ꝗ) .

(ii) Neutrosophic Binary−T_1 space [ℵEB − T_1. S] if for each elements (æ, y) ≠ $(æ^*, y^*)$ in (X ,Y) , there are (Ꝑ, Ꝗ), $(Ꝑ^*, Ꝗ^*)$ ∈ O(ℵEB. T. S (X , Y)) , so that (æ , y) ∈ (Ꝑ, Ꝗ) and $(æ^*, y^*)$ ∉ (Ꝑ, Ꝗ) or $(æ^*, y^*)$ ∈ $(Ꝑ^*, Ꝗ^*)$ and (æ, y) ∉ $(Ꝑ^*, Ꝗ^*)$.

(iii) Neutrosophic Binary−T_2 space [ℵEB − T_2. S] if for each elements (æ, y) ≠ $(æ^*, y^*)$ in (X ,Y) , there are disjoint (Ꝑ , Ꝗ), $(Ꝑ^*, Ꝗ^*)$ ∈ O(ℵEB. T. S (X, Y)) , so that (æ, y) ∈ (Ꝑ, Ꝗ) and $(æ^*, y^*)$ ∈ $(Ꝑ^*, Ꝗ^*)$.

For example: Let X = $\{p_1, p_2\}$ and Y = $\{q_1, q_2\}$ be the universe. Let $BT_{\aleph} = \{(0_{æ} , 0_y), (1_{æ} , 1_y), (Ꝑ, Ꝗ), (Ꝑ, Ꝗ)^C\}$ Be the ℵEB. T. S. So that (Ꝑ, Ꝗ) = {X ,⟨(1, 0, 0), (0, 1, 1)⟩ , ⟨Y, (0, 1, 1), (1, 0, 0)⟩}. Then A ℵEB. T. S (X, Y) is ℵEB − T_2. S .

Remark 4.2: Clearly, If a ℵEB. T. S (X, Y)is an ℵEB − T_L. S , then is an ℵEB − T_{L-1}. S , L = 1,2.

Theorem 4.3: A ℵEB. T. S (X, Y)is an ℵEB − T_0. S if and only if for each (æ, y) ≠ $(æ^*, y^*)$ in (X ,Y), either $(æ^*, y^*)$ ∉ ℵEBKER({æ} , {y}) or (æ, y) ∉ ℵEBKER($\{æ^*\}, \{y^*\}$).

Proof: Let (X ,Y) be an ℵEB − T_0. S & for each elements (æ, y) ≠ $(æ^*, y^*)$ in (X , Y) ,there are (Ꝑ, Ꝗ), ∈ O(ℵEB. T. S (X, Y)) so that (æ, y) ∈ (Ꝑ, Ꝗ) and $(æ^*, y^*)$ ∉ (Ꝑ ,Ꝗ)

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or $(œ, y) \notin (Ᵽ, Ɑ)$ and $(œ^*, y^*) \in (Ᵽ, Ɑ)$. Thus either $(œ, y) \in (Ᵽ, Ɑ)$ & $(œ^*, y^*) \notin (Ᵽ, Ɑ)$ implicate$(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$ or $(œ, y) \notin (Ᵽ, Ɑ)$ and $(œ^*, y^*) \in (Ᵽ, Ɑ)$ implicate $(œ, y) \notin \aleph EBKER(\{œ^*\}, \{y^*\})$.

Conversely: Suppose that $(œ, y) \neq (œ^*, y^*)$ in (X, Y) & $(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$ or $(œ, y) \notin \aleph EBK$. Then there exists $(Ᵽ, Ɑ), \in O(\aleph EB.T.S\,(X, Y))$ so that $(œ, y) \in (Ᵽ, Ɑ)$, $(œ^*, y^*) \notin (Ᵽ, Ɑ)$ *or* $(œ, y) \notin (Ᵽ, Ɑ)$, $(œ^*, y^*) \in (Ᵽ, Ɑ)$ $x \notin G$,. Thus $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_0.S$.

Theorem 4.4: A $\aleph EB.T.S\,(X, Y)$is an $\aleph EB - T_1.S$ if and only if for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y), then $(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$ and $(œ, y) \notin \aleph EBKER(\{œ^*\}, \{y^*\})$.

Proof: Let (X, Y) Be an $\aleph EB - T_1.S$ & for each elements $(œ, y) \neq (œ^*, y^*)$ in (X, Y), there exist $(Ᵽ, Ɑ)$, $(Ᵽ^*, Ɑ^*) \in O(\aleph EB.T.S\,(X, Y))$, so that $(œ, y) \in (Ᵽ, Ɑ)$, $(œ^*, y^*) \notin (Ᵽ, Ɑ)$ *or* $(œ^*, y^*) \in (Ᵽ^*, Ɑ^*)$, $(œ, y) \notin (Ᵽ^*, Ɑ^*)$

, implicate $(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$ & $(œ, y) \notin \aleph EBKER(\{œ^*\}, \{y^*\})$

Conversely: Suppose that $(œ, y) \neq (œ^*, y^*)$ in (X, Y) &$(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$ or $(œ, y) \notin \aleph EBKER(\{œ^*\}, \{y^*\})$. Then there exists $(Ᵽ, Ɑ), (Ᵽ^*, Ɑ^*) \in O(\aleph EB.T.S\,(X, Y))$ such that $(œ, y) \in (Ᵽ, Ɑ)$, $(œ^*, y^*) \notin (Ᵽ, Ɑ)$ & $(œ^*, y^*) \in (Ᵽ^*, Ɑ^*)$, $(œ, y) \notin (Ᵽ^*, Ɑ^*)$. Thus $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_1.S$.

Theorem 4.5: A $\aleph EB.T.S\,(X, Y)$is an $\aleph EB - T_1.S$ if and only if for each element $(œ, y)$ in (X, Y), then $\aleph EBKER(\{œ\}, \{y\}) = (\{œ\}, \{y\})$.

Proof: Let (X, Y) be an $\aleph EB - T_1.S$ & $\aleph EBKER(\{œ\}, \{y\}) \neq (\{œ\}, \{y\})$. Then there exists $(œ^*, y^*) \in \aleph EBKER(\{œ\}, \{y\})$ so that $(œ^*, y^*) \neq (œ, y)$. Hence a $\aleph EB.T.S\,(X, Y)$is not $\aleph EB - T_1.S$, this is ambivalence. Thus $\aleph EBKER(\{œ\}, \{y\}) = (\{œ\}, \{y\})$.

Conversely: Suppose that a $\aleph EB.T.S\,(X, Y)$is not $\aleph EB - T_1.S$ & $\aleph EBKER(\{œ\}, \{y\}) = (\{œ\}, \{y\})$ for each $(œ, y) \in (X, Y)$, then $(œ^*, y^*) \in \aleph EBKER(\{œ\}, \{y\})$ implicate $\aleph EBKER(\{œ\}, \{y\}) \neq (\{œ\}, \{y\})$, this is ambivalence. Thus a $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - T_1.S$.

Theorem 4.6: A $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - T_1.S$ if and only if for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y) implicate $\aleph EKER(\{œ\}, \{y\}) \cap \aleph EBKER(\{œ^*\}, \{y^*\}) = (0_X, 0_Y)$.

Proof : Let a $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_1.S$ .Then $\aleph EBKER(\{œ\}, \{y\}) = (\{œ\}, \{y\})$ & $\aleph EBKER(\{œ^*\}, \{y^*\}) = (œ^*, y^*)$. Thus $\aleph EBKER(\{œ\}, \{y\}) \cap \aleph EBKER(\{œ^*\}, \{y^*\}) = (0_X, 0_Y)$.

Conversely: Suppose that $\aleph EBKER(\{œ\}, \{y\}) \cap \aleph EBKER(\{œ^*\}, \{y^*\}) = (0_X, 0_Y)$ for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y) & let a $\aleph EB.T.S\,(X, Y)$is not $\aleph EB - T_1.S$ .Then for each for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y) implicate $(œ^*, y^*) \in \aleph EBKER(\{œ\}, \{y\})$ or $(œ, y) \in \aleph EKER(\{œ^*\}, \{y^*\})$ .Thus $\aleph EBKER(\{œ\}, \{y\}) \cap \aleph EBKER(\{œ^*\}, \{y^*\}) = (0_X, 0_Y)$, therefore ambivalence. Thus a $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_1.S$.

Theorem 4.7: A $\aleph EB.T.S\,(X, Y)$ is an

i) $\aleph EB - T_1.S$ if and only if is an $\aleph EB - T_0.S$ and $\aleph EB - R_0.S$

ii) $\aleph EB - T_2.S$ if and only if is an $\aleph EB - T_1.S$ and $\aleph EB - R_1.S$

Proof:

i) Let a $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - T_1.S$ & $(œ, y) \in (Ᵽ, Ɑ) \in O(\aleph EB.T.S\,(X, Y))$, then for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y), $\aleph EBKER\{œ\}, \{y\} \cap \aleph EBKER(\{œ^*\}, \{y^*\}) = (0_X, 0_Y)$ implicate that $(œ, y) \notin \aleph EKER(\{œ^*\}, \{y^*\})$ & $(œ^*, y^*) \notin \aleph EBKER(\{œ\}, \{y\})$. Thus $\aleph EBCL(\{œ\}, \{y\}) = (\{œ\}, \{y\})$, therefore a $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - R_0.S$.

Conversely, suppose that a $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_0.S$ & $\aleph EB - R_0.S$, then for each $(œ, y) \neq (œ^*, y^*)$ in (X, Y) there exists $(Ᵽ, Ɑ) \in O(\aleph EB.T.S\,(X, Y))$ so that $(œ, y) \in (Ᵽ, Ɑ)$ & $(œ^*, y^*) \notin (Ᵽ, Ɑ)$. Since (X, Y) is an $\aleph EB - R_0.S$, then $\aleph EBCL(\{œ\}, \{y\}) \subseteq (Ᵽ, Ɑ)$, there exists $(Ᵽ^*, Ɑ^*) \in O(\aleph EB.T.S\,(X, Y))$ so that $(œ^*, y^*) \in (Ᵽ^*, Ɑ^*)$ & $(œ, y) \notin (Ᵽ^*, Ɑ^*)$, therefore, (X, Y) is an $\aleph EB - T_1.S$.

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ii) Let a $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - T_2.S$, this results for each elements $(\oe, y) \neq (\oe^*, y^*)$ in (X, Y) there exists disjoin $(Ꝑ, Ꝗ), (Ꝑ^*, Ꝗ^*) \in C(\aleph EB.T.S\,(X, Y))$ so that $(\oe, y) \in (Ꝑ, Ꝗ)$ & $(\oe^*, y^*) \in (Ꝑ^*, Ꝗ^*)$ $(\oe, y) \notin \aleph EBKER(\{\oe^*\}, \{y^*\})$ & $(\oe^*, y^*) \notin \aleph EBKER(\{\oe\}, \{y\})$, then $\aleph EBCL(\{\oe\}, \{y\}) = (\{\oe\}, \{y\}) \subseteq (Ꝑ, Ꝗ)$ & $\aleph EBCL(\{\oe^*\}, \{y^*\}) = (\{\oe^*\}, \{y^*\}) \subseteq (Ꝑ^*, Ꝗ^*)$.
Conversely: suppose that a $\aleph EB.T.S\,(X, Y)$ is $\aleph EB - T_1.S$ & $\aleph EB - R_1.S$, then for each $(\oe, y) \neq (\oe^*, y^*)$ in (X, Y) there exists $(Ꝑ, Ꝗ), (Ꝑ^*, Ꝗ^*) \in O(\aleph EB.T.S\,(X, Y))$, so that $(\oe, y) \in (Ꝑ, Ꝗ)$ & $(\oe^*, y^*) \notin (Ꝑ, Ꝗ)$, $(\oe, y) \in (Ꝑ^*, Ꝗ^*)$, $(\oe, y) \notin (Ꝑ^*, Ꝗ^*)$ implicate that to $\aleph EBCL(\{\oe\}, \{y\}) \neq \aleph EBCL(\{\oe^*\}, \{y^*\})$, since (X, Y) is $\aleph EB - R_1.S$, there exist disjoint $(Ꝑ, Ꝗ), (Ꝑ^*, Ꝗ^*) \in O(\aleph EB.T.S\,(X, Y))$, so that $(\oe, y) \in (Ꝑ, Ꝗ)$ & $(\oe^*, y^*) \in (Ꝑ^*, Ꝗ^*)$, a $\aleph EB.T.S\,(X, Y)$ is an $\aleph EB - T_2.S$.

Corollary 4.8: A $\aleph EB - T_0.S\ (X, Y)$ is $\aleph EB - T_2.S$ if and only if for each $(\oe, y) \neq (\oe^*, y^*)$ in (X, Y) with $\aleph EBKER(\{\oe\}, \{y\}) \neq \aleph EBKER(\{\oe^*\}, \{y^*\})$, then there exist $(K, H), (K^*, H^*) \in C(\aleph EB.T.S\,(X, Y))$, so that $\aleph EBKER(\{\oe\}, \{y\}) \subseteq (K, H)$, $\aleph EBKER(\{\oe\}, \{y\}) \cap (K^*, H^*) = (0_X, 0_Y)$ and $\aleph EBKER(\{\oe^*\}, \{y^*\}) \subseteq (K^*, H^*)$, $\aleph EBKER(\{\oe^*\}, \{y^*\}) \cap (K, H) = (0_{\oe}, 0_y)$ & $(K, H) \cup (K^*, H^*) = (1_X, 1_Y)$
.

Proof: Obviously.

Theorem 4.9: A $\aleph EB - R_1.S\ (X, Y)$ is $\aleph EB - T_2.S$ if and only if one of the following satisfies:

i) for each $\aleph EB.P\ (\oe, y)$ in (X, Y), then $\aleph EBKER(\{\oe\}, \{y\}) = (\{\oe\}, \{y\})$

ii) for each two $\aleph EB.P\ (\oe, y)$ and $(\oe^*, y^*)$ in (X, Y) with $\aleph EBKER(\{\oe\}, \{y\}) \neq \aleph EBKER(\{\oe^*\}, \{y^*\})$, then $\aleph EBKER(\{\oe\}, \{y\}) \cap \aleph EBKER(\{\oe^*\}, \{y^*\}) = (0_{\oe}, 0_y)$.

iii) for each two $\aleph EB.P\ (\oe, y)$ and $(\oe^*, y^*)$ in (X, Y) with $(\oe, y) \neq (\oe^*, y^*)$, either $(\oe, y) \notin \aleph EBKER(\{\oe^*\}, \{y^*\})$or $(\oe^*, y^*) \notin \aleph EBKER(\{\oe\}, \{y\})$.

iv) for each two $\aleph EB.P\ (\oe, y)$ and $(\oe^*, y^*)$ in (X, Y) with $(\oe, y) \neq (\oe^*, y^*)$, then $(\oe, y) \notin \aleph EBKER(\{\oe^*\}, \{y^*\})$ and $(\oe^*, y^*) \notin \aleph EBKER(\{\oe\}, \{y\})$.

Proof: It comes directly from previous results

5. Conclusions

In this present Article, we have defined neutrosophic Binary separation axioms related with the neutrosophic binary kernel, after we presented some of the basic characteristics in neutrosophic binary topological spaces and characteristics of the neutrosophic Binary nucleus. The relationships Between them were also investigated. At the inception of the thierd section, we aforementioned the definitions of the neutrosophic binary topological spaces, neutrosophic binary R_i-Spaces, $i = 0,1$, we also studied some of the relationships associated with of these separation axioms. Likewise, In the fourth section we defined the neutrosophic binary topological T_L-Space $L = 0,1,2$ and calculated the various characteristic and relationships connected with these separation axioms. We prospect that our feedbacks in this manuscript will be advantageous to the research combination and participate to the improvement of some distinct appearance of neutrosophic binary topology.

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