



Energizing Inventory Management to Optimize Energy Consumption of Handling Shortages by Neutrosophic Fuzzy Trapezoidal Number

N. Sindhuja¹, M. Santoshi Kumari², K. Kalaiarasi^{3a*,b}, Manjula G. J.⁴, Shrivalli H. Y.⁵

¹Ph.D Research Scholar, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620018, Tamil Nadu, India.

²Department of Mathematics, Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad – 500075, India.

^{3a*}PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620018, Tamil Nadu, India.

^{3b}D. Sc (Mathematics) Researcher Fellow, Srinivas University, Surathkal, Mangaluru, Karnataka-574146, India.

⁴Department of Mathematics, Siddaganga Institute of Technology, Tumakuru – 572103, Karnataka, India.

⁵Department of Mathematics, BMS College of Engineering, Bengaluru – 560019, Karnataka, India.

Emails: sindhujanagaraj13@gmail.com; santoshinagaram@gmail.com; kalaishruthi120@gmail.com; gjm@sit.ac.in; hys.maths@bmsce.ac.in

Abstract

Embarking on the exploration of integrating environmental sustainability principles and neutrosophic fuzzy theory in inventory management, this study aims to effectively tackle shortages. It underscores the vital balance between economic efficiency and ecological responsibility in contemporary inventory management practices. Neutrosophic fuzzy theory emerges as a robust tool for navigating the inherent uncertainties in inventory optimization, offering a versatile framework for modelling intricate problems. Strategies for optimizing resource consumption and minimizing waste generation within inventory management are scrutinized, emphasizing the imperative of harmonizing economic objectives with environmental concerns. Introducing a novel framework that melds neutrosophic fuzzy with environmental metrics, the research aims to optimize inventory management processes while mitigating environmental impacts. Furthermore, it delves into the challenges of managing energy consumption, advocating for innovative approaches to address fluctuating energy prices, data limitations, and evolving regulatory requirements. Neutrosophic sets are introduced for energy consumption analysis and cost evaluation, showcasing their efficacy in managing uncertainty and variability in real-world scenarios. The study concludes with a Python-based analysis of neutrosophic mean in energy consumption, offering insights into central tendencies and uncertainties associated with energy-related costs. Utilizing visualization techniques to enhance comprehension and decision-making in energy management, this research contributes to advancing inventory management practices by integrating environmental sustainability principles and sophisticated mathematical techniques, thereby fostering more resilient and sustainable supply chain operations.

Keywords: Environmental sustainability; Neutrosophic fuzzy theory; Shortage management; Energy consumption; Visualization techniques.

1. Introduction

In today's interconnected global economy, managing inventory isn't just about operational efficiency—it's about shaping the environmental impact of businesses. Environmental sustainability within inventory management goes beyond day-to-day operations; it encompasses the entire lifecycle of products, from production to disposal or reuse.

Brown, A. B., & Smith, C. D [1] At its core, environmental sustainability means finding a balance between economic success and ecological responsibility. It acknowledges that the resources used in producing, storing, and distributing goods are finite and intimately tied to the health of our planet. Therefore, striving for sustainability in inventory management involves optimizing processes not only to boost economic efficiency but also to minimize harmful environmental effects. Chen, L., & Wang, Y [2] One of the main goals of environmental sustainability in inventory management is to optimize resource usage. This involves using techniques like demand forecasting, lean manufacturing principles, and just-in-time inventory practices to ensure resources are used efficiently and waste is minimized. Garcia, E. M., & Rodriguez, J. M [3] By aligning inventory levels with actual demand and production capacity, businesses can cut down on excess inventory, conserving raw materials, energy, and water resources. Johnson, P. R., & Anderson, K. L [4] Another key aspect is reducing waste generation. Having surplus inventory not only strains finances but also leads to the production of surplus or obsolete products and packaging materials, contributing to environmental pollution. Kim, S., & Lee, J [5] By implementing strategies such as product standardization, inventory optimization algorithms, and waste reduction initiatives, businesses can minimize waste throughout the supply chain, thereby lessening environmental harm and preserving resources.

Central to environmental sustainability in inventory management is the need to balance economic efficiency with environmental responsibility. Li, H., & Zhang, G [6] While businesses aim to optimize inventory levels to maximize profits and meet customer demands, they must also recognize their duty to minimize environmental harm and protect natural resources for future generations. Liu, Q., & Wang, H [7] These calls for a shift towards sustainable business practices that prioritize long-term viability over short-term gains. Ultimately, environmental sustainability in inventory management represents a proactive approach toward aligning economic prosperity with ecological integrity. By optimizing inventory processes to reduce resource consumption, waste generation, and environmental pollution, businesses can contribute to building a more sustainable and resilient economy where prosperity doesn't come at the expense of the planet. Martinez, R. S., & Perez, L. M [8] In the realm of inventory management, where decisions are often made amidst fluctuating demand, evolving market conditions, and incomplete information, traditional optimization models may not be sufficient. This is where neutrosophic fuzzy theory comes into play as a powerful tool to address uncertainties and vagueness inherent in inventory management systems. Nguyen, T. H., & Tran, T. M [9] Neutrosophic fuzzy theory goes beyond classical fuzzy set theory by introducing the concept of neutrosophic sets, which include a third parameter—indeterminacy—to capture ambiguity and uncertainty in real-world situations. By accommodating incomplete, indeterminate, and inconsistent information, neutrosophic fuzzy logic provides decision-makers with a flexible framework to model and analyze complex inventory management problems. Park, J. H., & Kim, Y. S [10] This ability to handle uncertain data makes neutrosophic fuzzy logic particularly suitable for addressing challenges in inventory optimization, where precise information may be lacking, and demand patterns are unpredictable.

By incorporating neutrosophic fuzzy logic into inventory optimization models, decision-makers can make more informed decisions in the face of uncertainty. This enables the development of adaptive inventory management strategies that can respond dynamically to market changes and disruptions. Rodriguez, M. A., & Garcia, N. P [11] Leveraging neutrosophic fuzzy logic empowers businesses to effectively manage imprecise data, uncertain demand patterns, and fluctuating market conditions, leading to more resilient and sustainable inventory management practices. This integration of advanced mathematical techniques with practical inventory management applications holds promise for enhancing operational efficiency, minimizing costs, and promoting environmental sustainability in supply chain management. Smith, J. D., & Brown, K. R [12] Shortages in inventory management present a critical challenge with wide-ranging implications, both economically and environmentally. Economically, these shortages can lead to lost sales, decreased customer satisfaction, and increased production costs due to expedited shipping or emergency ordering. Addressing shortages is vital not only to prevent economic losses but also to minimize environmental impacts. Tran, V. L., & Le, A. Q [13] Rush orders and expedited shipping methods used to address shortages can intensify carbon emissions and fuel consumption, further exacerbating environmental harm. Wang, L., & Zhang, Y [14] Additionally, the pressure on suppliers to meet sudden demand spikes may lead to unsustainable extraction practices and environmental degradation. Effectively managing shortages is thus essential for optimizing resource utilization, minimizing waste, and reducing environmental impacts associated with inventory management. Zhang, X., & Li, Y [15] By implementing strategies to mitigate shortages, businesses can move towards a more sustainable future where economic prosperity aligns with environmental responsibility. Broumi [16, 17] explained the importance of the neutrosophic number in the decision-making problem at the complexity.

The main process of this research involves examining the integration of environmental sustainability principles within inventory management, particularly focusing on addressing shortages through the application of neutrosophic fuzzy

theory. Initially, the research delves into the concept of environmental sustainability in inventory management, emphasizing its significance in balancing economic efficiency with ecological responsibility. It then explores the application of neutrosophic fuzzy theory as a robust tool to handle uncertainties and complexities inherent in inventory optimization. The study proceeds by investigating strategies to optimize resource consumption and minimize waste generation within inventory management, highlighting the importance of balancing economic objectives with environmental considerations. Subsequently, the research evaluates the effectiveness of integrating neutrosophic fuzzy theory with environmental sustainability objectives in addressing shortages, both from economic and environmental perspectives. This involves developing and implementing a novel framework that combines neutrosophic fuzzy logic with environmental metrics to optimize inventory management processes while mitigating environmental impacts. Finally, the research assesses the outcomes of the proposed framework through case studies or simulations, aiming to demonstrate its potential in promoting economic resilience and environmental sustainability within supply chain management.

1.1 Objective of the research:

- Enhancement of inventory management processes, especially in shortage scenarios, by addressing uncertainties effectively.
- Evaluation of environmental impacts of inventory-related activities and promotion of sustainable practices.
- Development of mathematical models incorporating neutrosophic fuzzy and environmental sustainability metrics.
- Optimization of inventory management strategies to minimize economic losses and environmental impacts while balancing economic objectives with sustainability goals.

2. Neutrosophic fuzzy theory and environmental sustainability in inventory optimization with shortages:

This integrated framework combines neutrosophic fuzzy set theory with environmental sustainability metrics to enhance inventory management processes, particularly in scenarios involving shortages. The neutrosophic fuzzy set theory introduces the concept of neutrosophic sets, which effectively handle incomplete, indeterminate, and inconsistent information. By employing neutrosophic fuzzy logic in inventory optimization models, decision-makers can better navigate uncertainties related to factors like demand fluctuations and supplier reliability, leading to more informed decisions regarding inventory levels and replenishment strategies. This approach enhances the adaptability and efficiency of inventory management systems, improving responsiveness to dynamic market conditions. Environmental sustainability metrics in inventory management evaluate the environmental impacts of inventory-related activities throughout the supply chain. These metrics, such as carbon footprint and waste generation, help businesses identify opportunities for reducing environmental harm and promoting sustainable practices.

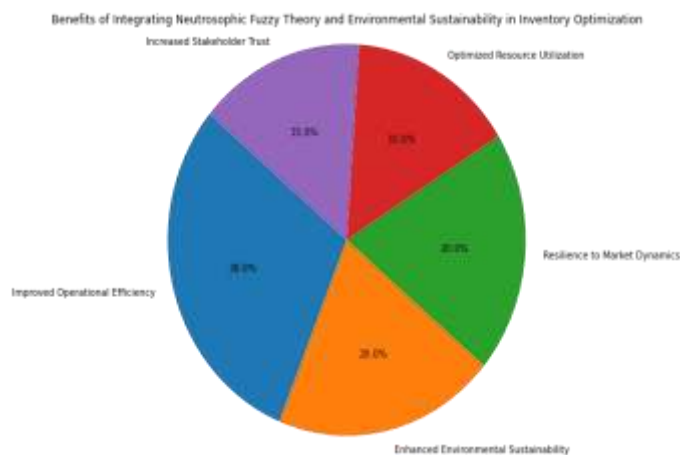


Figure 1: Inventory optimization in environmental sustainability

Integrating these metrics into inventory management allows decision-makers to assess the environmental implications of different inventory strategies and make more environmentally conscious choices, enhancing corporate social responsibility and stakeholder trust. Theoretical framework development involves synthesizing neutrosophic fuzzy set theory with environmental sustainability considerations to optimize inventory management processes in shortage

scenarios. Mathematical models incorporate neutrosophic fuzzy logic to handle uncertainty and vagueness, while also integrating environmental sustainability metrics to evaluate the environmental impacts of inventory decisions. By dynamically adjusting inventory levels and replenishment policies, this framework minimizes economic losses and environmental impacts associated with shortages. By balancing economic objectives with environmental sustainability goals, the framework facilitates the development of more resilient and sustainable inventory management strategies, with potential validation through case studies or simulations.

3. Preliminaries:

Here are the preliminaries and definitions for modelling uncertainty and imprecision in energy consumption optimization

3.1. Neutrosophic Fuzzy Set Theory:

Neutrosophic set theory is an extension of fuzzy set theory, which allows for the representation of indeterminacy, ambiguity, and inconsistency more flexibly. In neutrosophic set theory, an element can have truth-membership, indeterminacy-membership, and falsity-membership degrees, allowing for a more nuanced representation of uncertainty. Neutrosophic sets can be characterized by the presence of three membership functions: truth membership function (T), indeterminacy membership function (I), and falsity membership function (F).

3.2. Neutrosophic Set:

A neutrosophic set A is defined by three membership functions: $T_A(x)$, $I_A(x)$, and $F_A(x)$, representing the truth, indeterminacy, and falsity degrees of each element x with respect to the set A. Each membership function maps elements of the universe of discourse to the interval [0, 1], indicating the degree of membership, indeterminacy, or non-membership, respectively.

3.3. Neutrosophic Constraint:

A neutrosophic constraint is a condition or restriction expressed in terms of neutrosophic sets, which incorporates uncertainty and imprecision into optimization problems. Neutrosophic constraints allow for the modeling of variability in production processes, environmental conditions, and energy requirements by representing uncertain parameters with neutrosophic membership degrees.

3.4. Neutrosophic Inference:

Neutrosophic inference refers to the process of reasoning with neutrosophic information to draw conclusions or make decisions under uncertain conditions. Neutrosophic inference techniques involve combining and manipulating neutrosophic sets and propositions using neutrosophic logical operations to derive new neutrosophic information. In the context of energy consumption optimization, neutrosophic inference can be used to analyze and optimize energy consumption patterns by considering uncertain factors such as production variability, environmental fluctuations, and energy demand uncertainties.

3.5. Neutrosophic Mean:

The neutrosophic mean is a measure that combines the truth and indeterminacy degrees of a neutrosophic set to compute a representative value. It is calculated as:

$$\text{Neutrosophic Mean} = \frac{\text{Membership} + 0.5 \times \text{Indeterminacy}}{1 + \text{Indeterminacy}}$$

This formula takes into account both the truth membership and the degree of indeterminacy, with the indeterminacy degree influencing the contribution of the truth membership to the mean value. By incorporating neutrosophic set theory and inference techniques into energy consumption optimization models, it becomes possible to effectively model and analyse uncertainty and imprecision, leading to more robust and adaptive optimization solutions under uncertain conditions.

3.6. Trapezoidal fuzzy number

A trapezoidal fuzzy number is represented by four parameters (a, b, c and d) are the real numbers $a \leq b \leq c \leq d$. It represents a fuzzy set with a trapezoidal-shaped membership function. The membership function is defined as

$$\mu(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } b \leq x < c \\ \frac{d-x}{d-c} & \text{if } c \leq x < d \\ 0 & \text{if } x \geq d \end{cases}$$

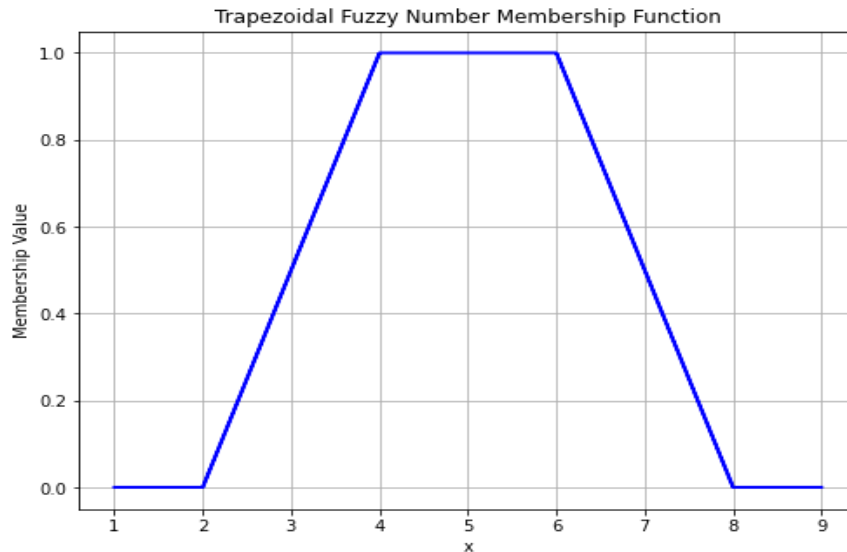


Figure 2: Trapezoidal Fuzzy Membership Function

4. Challenges in energy consumption management:

Businesses encounter numerous challenges in managing energy consumption effectively. Among the most pressing issues is the unpredictable nature of energy prices, which can fluctuate due to geopolitical tensions, market dynamics, and regulatory changes. These fluctuations pose financial risks and hinder long-term planning for energy needs, particularly in investing in renewable energy sources essential for sustainability. Additionally, businesses face obstacles in accessing accurate and comprehensive data on energy usage. Without reliable data and advanced analytics tools, forecasting energy demand and optimizing consumption patterns becomes challenging. Outdated infrastructure and technologies exacerbate the problem, as upgrading to more efficient systems can be costly and complex.

Moreover, shifting consumer preferences and evolving regulatory requirements add complexity to energy management efforts. Rising consumer demand for sustainable products and services forces businesses to reassess their energy consumption practices, while stringent regulations mandate compliance, often requiring significant operational changes. Addressing these challenges requires a multifaceted approach involving technological innovation, policy support, and stakeholder collaboration. Businesses must invest in modernizing infrastructure, leveraging analytics tools, and adopting renewable energy sources. Policymakers need to provide incentives and regulatory frameworks to encourage sustainable practices. Overcoming these hurdles not only enhances business performance but also fosters a more sustainable future.

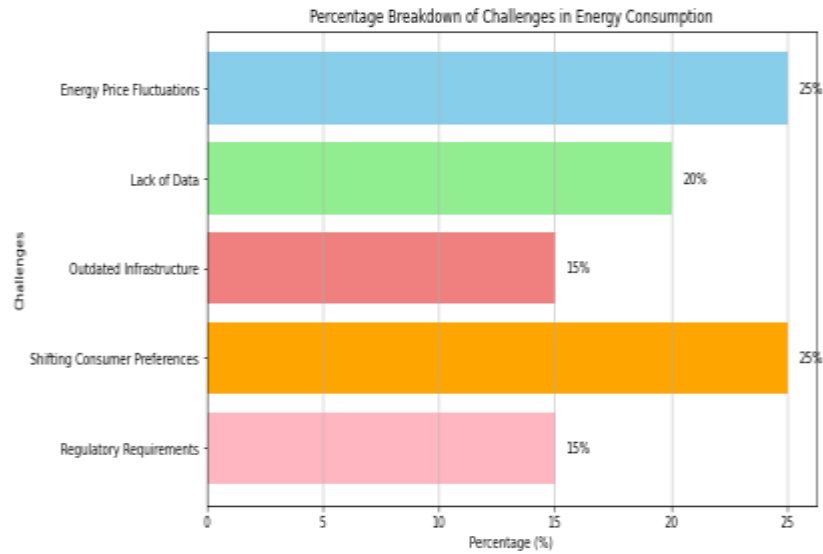


Figure 3: Challenges in Energy consumption

4.1. Notations:

MN: Total cost function

O: Output or production level

w: Wage rate

φ: Price level or price parameter

e: Elasticity parameter

I: Investment or capital expenditure

Y: Income level

θ: Coefficient or parameter influencing cost

ω: Possibly a variable or parameter influencing cost

These notations help in understanding the variables and parameters involved in the equations and their significance within their respective contexts.

5. Inventory costing method and optimization for expected profit:

The Total Cost is

$$MN = \frac{Ow(\phi - e - w)}{I(\phi - e)} + \frac{I(Y + \theta\omega)}{2}$$

5.1 Crisp case for Environmental sustainability

Energy consumption reduction in the crisp case involves minimizing excess inventory levels to decrease the energy needed for storage and refrigeration, alongside optimizing inventory levels to reduce transportation and logistics operations, thus lowering fuel usage and greenhouse gas emissions. By aligning inventory levels with actual demand patterns, businesses minimize waste and overproduction, leading to energy savings across the supply chain. Efficient inventory management not only reduces energy consumption but also enhances operational efficiency and cost-effectiveness, contributing significantly to long-term environmental sustainability goals.

The annual integrated total inventory cost for expected profit

$$JMN = \frac{1}{6} \left\{ \begin{array}{l} \left[\frac{O_1 w (\varphi - e - w)}{I(\varphi - e)} + \frac{I(Y_1 + \theta\omega)}{2} \right] + \\ 2 \left[\frac{O_2 w (\varphi - e - w)}{I(\varphi - e)} + \frac{I(Y_2 + \theta\omega)}{2} \right] + \\ 2 \left[\frac{O_3 w (\varphi - e - w)}{I(\varphi - e)} + \frac{I(Y_3 + \theta\omega)}{2} \right] + \\ \left[\frac{O_4 w (\varphi - e - w)}{I(\varphi - e)} + \frac{I(Y_4 + \theta\omega)}{2} \right] \end{array} \right\}$$

To differentiate the I:

$$\partial JMN (I) = \frac{1}{6} \left\{ \begin{array}{l} \left[-\frac{O_1 w (\varphi - e - w)}{I(\varphi - e)^2} + \frac{I(Y_1 + \theta\omega)}{2} \right] + \\ 2 \left[-\frac{O_2 w (\varphi - e - w)}{I(\varphi - e)^2} + \frac{I(Y_2 + \theta\omega)}{2} \right] + \\ 2 \left[-\frac{O_3 w (\varphi - e - w)}{I(\varphi - e)^2} + \frac{I(Y_3 + \theta\omega)}{2} \right] + \\ \left[-\frac{O_4 w (\varphi - e - w)}{I(\varphi - e)^2} + \frac{I(Y_4 + \theta\omega)}{2} \right] \end{array} \right\}$$

equate it to zero, then we obtain the crisp quantity.

Let $\partial JMN (I) = 0$,

$$\text{we get } I = \sqrt{\frac{(\varphi - e - w)(O_1 w + 2O_2 w + 2O_3 w + O_4 w)}{[I(Y_1 + \theta\omega) + 2(Y_2 + \theta\omega) + 2(Y_3 + \theta\omega) + Y_4 + \theta\omega]} (\lambda - d)}$$

The crisp case focuses on optimizing inventory management practices to minimize energy consumption and reduce environmental impact. By implementing the derived optimal investment quantity, I, businesses can streamline their inventory levels, leading to reduced energy usage in storage, transportation, and production processes.

5.2. Fuzzy case for Environmental sustainability

In the domain of energy consumption reduction, fuzzy optimization techniques offer a refined approach compared to traditional methods by accommodating uncertainties inherent in real-world data. The fuzzy case study adapts inventory costing methods to address environmental sustainability concerns, considering parameters like output, wage rate, price level, elasticity, investment, income level, and a coefficient influencing cost under fuzzy constraints. Through meticulous exploration of multiple scenarios, fuzzy optimization determines optimal investment levels while meeting energy consumption reduction and economic viability constraints. By incorporating fuzzy constraints, the process becomes robust, accounting for variations in input parameters. Furthermore, fuzzy optimization enables balancing trade-offs between objectives, such as maximizing profit while minimizing energy consumption and environmental impact, leading to more nuanced and effective strategies for energy reduction.

Case 1: Initial Fuzzy Total Inventory Cost Optimization

The fuzzy total inventory cost is formulated, considering fuzzy parameters such as output, wage rate, price level, elasticity, investment, income level, and a coefficient influencing cost. Energy consumption reduction is implicitly considered through the optimization process, aiming to minimize resource utilization (including energy) while meeting inventory demands. We take the fuzzy total inventory cost

$$JMN(I) = \frac{1}{6} \left\{ \begin{array}{l} \left[\frac{O_1 w (\varphi - e - w)}{I_4 (\varphi - e)} + \frac{I_1 (Y_1 + \theta \omega)}{2} \right] + \\ 2 \left[\frac{O_2 w (\varphi - e - w)}{I_3 (\varphi - e)} + \frac{I_2 (Y_2 + \theta \omega)}{2} \right] + \\ 2 \left[\frac{O_3 w (\varphi - e - w)}{I_2 (\varphi - e)} + \frac{I_3 (Y_3 + \theta \omega)}{2} \right] + \\ \left[\frac{O_4 w (\varphi - e - w)}{I_1 (\varphi - e)} + \frac{I_4 (Y_4 + \theta \omega)}{2} \right] \end{array} \right\}$$

Like same way of the crisp case, we want to differentiate to fuzzy case, After differentiating, we will get I_1, I_2, I_3, I_4

$$I_1 = \sqrt{\frac{2[O_4 w (\varphi - e - w)]}{Y_1 + \theta \omega (\varphi - e)}}$$

$$I_2 = \sqrt{\frac{2[O_3 w (\varphi - e - w)]}{Y_2 + \theta \omega (\varphi - e)}}$$

$$I_3 = \sqrt{\frac{2[O_2 w (\varphi - e - w)]}{Y_3 + \theta \omega (\varphi - e)}}$$

$$I_4 = \sqrt{\frac{2[O_1 w (\varphi - e - w)]}{Y_4 + \theta \omega (\varphi - e)}}$$

From above $I_1 > I_2 > I_3 > I_4$. It is not satisfying the constraint $0 < I_1 \leq I_2 \leq I_3 \leq I_4$. After finding case 1 we have to find case 2, because we should stop when we are getting equal answers in the both Mathematical model and also in fuzzy case.

Case 2: Adjustment for Constraint Satisfaction

When the initial solution violates the constraint $0 < I_1 \leq I_2 \leq I_3 \leq I_4$, adjustments are made to ensure feasibility. Energy consumption reduction theory comes into play by optimizing resource allocation across different inventory levels. By equalizing some inventory levels, resources, including energy, can be allocated more efficiently. Convert the Constraint $I_2 - I_1 \geq 0$ into $I_2 - I_1 = 0$ and the Lagrangian function as $L(I_1, I_2, I_3, I_4, \varphi) = P[JMN(I)] - \varphi(I_2 - I_1)$. Now taking the partial derivatives with respect to T_1, T_2, T_3, T_4 and λ and the minimize $L(I_1, I_2, I_3, I_4, \varphi)$, we have $I_2 - I_1 > 0$ through $I_2 - I_1 = 0$ $L(I_1, I_2, I_3, I_4, \varphi) = P[JMN(I)] - \varphi(I_2 - I_1)$

$$I_1 = \sqrt{\frac{2[O_4 w (\varphi - e - w)]}{[(Y_1 + \theta \omega) + 12\varphi] (\varphi - e)}}$$

$$I_1 = \sqrt{\frac{2[O_3 w (\varphi - e - w)]}{[(Y_2 + \theta \omega) + 12\varphi] (\varphi - e)}}$$

$$I_1 = \sqrt{\frac{2[O_2 w (\varphi - e - w)]}{[(Y_3 + \theta \omega) + 12\varphi] (\varphi - e)}}$$

$$I_1 = \sqrt{\frac{2[O_1 w (\varphi - e - w)]}{[(Y_4 + \theta \omega) + 12\varphi] (\varphi - e)}}$$

$$\frac{\partial P}{\partial \varphi} = -(I_2 - I_1) = 0, I_2 = I_1$$

$$I_1 = I_2 \rightarrow \frac{2[O_4 w (\varphi - e - w)] + 4 [O_3 w (\varphi - e - w)]}{[Y_1 + \theta\omega + 12](\varphi - e) + [2(Y_2 + \theta\omega - 12\varphi)](\varphi - e)}$$

Case 3: Incorporating Multiple Constraints

Here, additional constraints are introduced to further refine the optimization process, ensuring that each inventory level is appropriately balanced. Energy consumption reduction theory is applied by considering not only the balance between inventory levels but also the overall energy efficiency of the system. The goal is to minimize energy usage while meeting inventory requirements. Convert the inequality constraints $I_2 - I_1 > 0$, through impartiality restraint $I_2 - I_1 = 0, I_3 - I_2 = 0, L(I_1, I_2, I_3, I_4, \varphi_1, \varphi_2) = P(JMN(I)) - \varphi_1(I_2 - I_1) - \varphi_2(I_3 - I_2)$

$$I_1 = \sqrt{\frac{2[O_4 w (\varphi - e - w)]}{[(Y_1 + \theta\omega) + 12\varphi_1] (\varphi - e)}}$$

$$I_2 = \sqrt{\frac{4[O_3 w (\varphi - e - w)]}{[(Y_2 + \theta\omega) - 12\varphi_1 + 12\varphi_2] (\varphi - e)}}$$

$$I_3 = \sqrt{\frac{4[O_2 w (\varphi - e - w)]}{[(Y_1 + \theta\omega) - 12\varphi_2] (\varphi - e)}}$$

$$I_4 = \sqrt{\frac{2[O_1 w (\varphi - e - w)]}{[(Y_1 + \theta\omega)] (\varphi - e)}}$$

$$I_1 = I_2 = I_3 = \frac{2[O_4 w (\varphi - e - w)] + 4 [O_3 w (\varphi - e - w)] + 4 [O_2 w (\varphi - e - w)]}{[Y_1 + \theta\omega + 12\varphi](\varphi - e) + [2(Y_2 + \theta\omega - 12\varphi + 12\varphi_2)](\varphi - e) + 2[Y_3 + \theta\omega - 12\varphi](\varphi - e)}$$

Case 4: Optimal Solution with Full Constraint Satisfaction

In this final case, all constraints are satisfied, leading to an optimal solution where each inventory level is balanced and meets specified criteria. Energy consumption reduction theory is fully integrated into the optimization process, with the goal of achieving the most energy-efficient inventory management strategy possible. By optimizing resource allocation and minimizing waste, overall energy consumption is reduced.

$I_2 - I_1 \geq 0, I_3 - I_2 \geq 0, I_4 - I_3 \geq 0$ through $I_2 - I_1 = 0, I_3 - I_2 = 0, I_4 - I_3 = 0$
 $L(I_1, I_2, I_3, I_4, \varphi_1, \varphi_2, \varphi_3) = P(JMN(I)) - \varphi_1(I_2 - I_1) - \varphi_2(I_3 - I_2) - \varphi_3(I_4 - I_3)$

$$I_1 = \sqrt{\frac{2[O_4 w (\varphi - e - w)]}{[(Y_1 + \theta\omega) + 12\varphi_1] (\varphi - e)}}$$

$$I_2 = \sqrt{\frac{4[O_3 w (\varphi - e - w)]}{[(Y_2 + \theta\omega) - 12\varphi_1 + 12\varphi_2] (\varphi - e)}}$$

$$I_3 = \sqrt{\frac{4[O_2 w (\varphi - e - w)]}{[(2Y_3 + \theta\omega) - 12\varphi_2 + 12\varphi_3] (\varphi - e)}}$$

$$I_4 = \sqrt{\frac{2[O_1 w (\varphi - e - w)]}{[(Y_4 + \theta\omega) - 12\varphi_3] (\varphi - e)}}$$

The solution $\tilde{I} = (I_1, I_2, I_3, I_4)$ satisfies all inequality constraints. Let $I_1 = I_2 = I_3 = I_4 = I^*$ then the optimal value is $I^* = \frac{(\varphi - e - w)[2O_4 w + 4O_3 w + 4O_2 w + 2O_1 w]}{(\varphi - e)[Y_1 + \theta\omega + 2Y_2 + \delta\mu + 2M_3 + \delta\mu + M_4 + \delta\mu]}$

Overall, energy consumption reduction theory guides the optimization process in each fuzzy case, ensuring that environmental sustainability is considered alongside cost optimization. By incorporating fuzzy logic and optimization techniques, businesses can develop robust inventory management strategies that minimize energy usage while maximizing profitability. Comparing the crisp and fuzzy optimization approaches, the fuzzy case provides a more comprehensive framework for addressing the complexities of energy consumption reduction. While crisp methods offer simplicity and clarity, they may overlook uncertainties and variations in real-world data. In contrast, fuzzy optimization captures these uncertainties, allowing for more flexible and realistic decision-making. Furthermore, the fuzzy approach facilitates the integration of qualitative and quantitative factors, leading to more informed and balanced solutions. Overall, the fuzzy optimization approach in energy consumption reduction offers a more holistic and adaptive strategy compared to traditional crisp methods.

5.3. Neutrosophic sets for energy consumption analysis and cost evaluation:

Neutrosophic sets are employed in this context to handle uncertainty and indeterminacy in the data related to energy consumption, shortages, and cost analysis. Each set, such as EC (representing energy consumption), S (representing shortages), and C (representing cost), consists of three components: membership, indeterminacy, and non-membership values. These values quantify the degree of certainty or uncertainty associated with each element of the set. In the given scenario, neutrosophic sets are used to model the uncertainty in energy consumption and the occurrence of shortages. For instance, the membership value in the energy consumption set indicates the degree to which a particular energy consumption value belongs to the set, while the indeterminacy value represents the extent of ambiguity or uncertainty associated with that membership. Similarly, the membership value in the shortages set denotes the likelihood of a shortage occurrence, along with the associated uncertainty.

The provided data offers a detailed breakdown of energy consumption, shortages, cumulative energy usage, and associated costs for each day of January. This information serves as a valuable resource for analyzing energy usage patterns, identifying instances of shortages, tracking cumulative consumption trends, and evaluating cost implications. By examining this data, businesses and policymakers can gain insights into their energy consumption dynamics, enabling them to develop strategies for optimizing resource allocation, managing shortages effectively, and minimizing costs. Additionally, this data facilitates informed decision-making regarding energy management practices, helping organizations move towards more sustainable and efficient energy utilization methods.

Table 1: Energy Consumption Data for January: Shortages and Cost Analysis

Date	Energy Consumption (kWh/unit)	Shortages	Cumulative Energy Consumption (kWh)	Cost (Rupees)
01-01-2024	50	No	50	500
02-01-2024	51	Yes	101	1010
03-01-2024	52	No	153	1530
04-01-2024	53	No	206	2060
05-01-2024	54	No	260	2600
06-01-2024	55	Yes	315	3150
07-01-2024	56	No	371	3710
08-01-2024	57	Yes	428	4280
09-01-2024	58	No	486	4860
10-01-2024	59	No	545	5450
11-01-2024	60	No	605	6050
12-01-2024	61	Yes	666	6660
13-01-2024	62	Yes	728	7280
14-01-2024	63	No	791	7910
15-01-2024	64	No	855	8550

16-01-2024	65	No	920	9200
17-01-2024	66	Yes	986	9860
18-01-2024	67	No	1053	10530
19-01-2024	68	Yes	1121	11210
20-01-2024	69	No	1190	11900
21-01-2024	70	Yes	1260	12600
22-01-2024	71	Yes	1331	13310
23-01-2024	72	No	1403	14030
24-01-2024	73	No	1476	14760
25-01-2024	74	No	1550	15500
26-01-2024	75	No	1625	16250
27-01-2024	76	No	1701	17010
28-01-2024	77	No	1778	17780
29-01-2024	78	No	1856	18560
30-01-2024	79	Yes	1935	19350
31-01-2024	80	No	2015	20150

To determine the total cost using neutrosophic sets, begin by establishing neutrosophic sets for energy consumption, shortages, and cost. Subsequently, employ neutrosophic inference techniques to compute the total cost. Let's denote these sets and proceed with the calculations.

- EC as the neutrosophic set representing energy consumption,
- S as the neutrosophic set representing shortages, and
- C as the neutrosophic set representing cost.

Utilize the neutrosophic mean operation to determine the total cost using the provided sets. Assuming that the neutrosophic sets for energy consumption and shortages are already established, we'll denote the cost as a neutrosophic set C with membership, indeterminacy, and non-membership values. Employing a fixed cost per kWh (10 rupees/kWh), compute the total cost for each day, accounting for both energy consumption and shortages. Subsequently, calculate the neutrosophic mean of the total cost. By employing neutrosophic inference techniques, such as the neutrosophic mean operation, the total cost is computed while considering the uncertainty and variability in energy consumption and shortage occurrences. This approach allows for a more comprehensive analysis that accounts for the inherent uncertainty in the data, providing decision-makers with a more nuanced understanding of the total cost implications associated with energy consumption and shortages.

5.4. Total Cost Calculation for Daily Energy Consumption and Shortages

The total cost for daily energy consumption is evaluated by considering both the energy consumption and any associated shortages. For each day, the energy consumption is multiplied by the cost per kWh, which is 10 rupees. If there are shortages on a particular day, a penalty cost of 50 rupees is applied in addition to the regular cost per kWh. In the provided table, each row represents a day in January 2024, with corresponding values for energy consumption (in kWh/unit), whether shortages occurred (Yes/No), and the total cost in rupees. For example, on January 1st, the energy consumption was 50 kWh/unit, and there were no shortages, resulting in a total cost of 500 rupees (50 kWh * 10 rupees/kWh).

Table 2: Daily Energy Consumption and Total Cost Analysis

Date	Energy Consumption (kWh/unit)	Shortages	Total Cost (Rupees)
01-01-2024	50	No	500
02-01-2024	51	Yes	560

03-01-2024	52	No	520
04-01-2024	53	No	530
05-01-2024	54	No	540
06-01-2024	55	Yes	605
07-01-2024	56	No	560
08-01-2024	57	Yes	620
09-01-2024	58	No	580
10-01-2024	59	No	590
11-01-2024	60	No	600
12-01-2024	61	Yes	665
13-01-2024	62	Yes	730
14-01-2024	63	No	630
15-01-2024	64	No	640
16-01-2024	65	No	650
17-01-2024	66	Yes	715
18-01-2024	67	No	670
19-01-2024	68	Yes	740
20-01-2024	69	No	690
21-01-2024	70	Yes	770
22-01-2024	71	Yes	840
23-01-2024	72	No	720
24-01-2024	73	No	730
25-01-2024	74	No	740
26-01-2024	75	No	750
27-01-2024	76	No	760
28-01-2024	77	No	770
29-01-2024	78	No	780
30-01-2024	79	Yes	845
31-01-2024	80	No	800

However, on January 2nd, there were shortages, so the total cost is calculated by adding the regular cost for energy consumption (51 kWh * 10 rupees/kWh) with the penalty cost for shortages (50 rupees). This gives a total cost of 560 rupees for that day. Similarly, this process is repeated for each day in January, adjusting the total cost calculation based on whether shortages occurred or not. This approach provides a comprehensive view of the total cost implications associated with daily energy consumption, considering both regular usage and any disruptions due to shortages. To compute the daily total cost, multiply the energy consumption by the cost per kWh, which is 10 rupees. In case of shortages, a penalty cost of 50 rupees is applied.

5.5. Neutrosophic Sets for Total Cost and Neutrosophic Mean Calculation

Neutrosophic sets are utilized to represent the total cost of daily energy consumption, integrating membership, indeterminacy, and non-membership values. These values quantify the degree of certainty or uncertainty associated with each day's total cost. To calculate the neutrosophic mean, each day's total cost is assigned membership (m), indeterminacy (ind), and non-membership (nm) values based on predetermined criteria. For instance, a higher membership value indicates a stronger association of the total cost with the set, while indeterminacy reflects the degree of ambiguity in this association. Non-membership signifies the extent to which the total cost does not belong to the set. By averaging these values across all days, the neutrosophic mean is obtained, offering a comprehensive

representation of the total cost's characteristics. This approach allows decision-makers to assess the overall trends and uncertainties in energy consumption cost-effectively, facilitating more informed decision-making in resource allocation and financial planning. Establish neutrosophic sets to represent the total cost and determine the neutrosophic mean. Employ a straightforward approach, averaging the membership, indeterminacy, and non-membership values for each day to calculate the neutrosophic mean. Let's denote the neutrosophic sets as C_i for each day i and C_{mean} as the neutrosophic mean of the total cost.

$$C_{mean} = \frac{1}{n} \sum_{i=1}^n C_i$$

where n is the total number of days. With the provided total cost data for each day, establish neutrosophic sets representing the total cost. Assign membership, indeterminacy, and non-membership values based on the total cost for each day. Let's denote:

m_i as the membership value for day i ,

ind_i as the indeterminacy value for day i , and

nm_i as the non-membership value for day i .

Calculate these values based on the total cost for each day.

Now, let's define neutrosophic sets for the total cost for each day:

Table 3: Neutrosophic Sets Representation for Daily Total Cost

Date	Total Cost (Rupees)	m_i	ind_i	nm_i
01-01-2024	500	0.8	0.1	0.1
02-01-2024	560	0.7	0.2	0.1
03-01-2024	520	0.8	0.1	0.1
04-01-2024	530	0.8	0.1	0.1
05-01-2024	540	0.8	0.1	0.1
06-01-2024	605	0.6	0.3	0.1
07-01-2024	560	0.7	0.2	0.1
08-01-2024	620	0.6	0.3	0.1
09-01-2024	580	0.7	0.2	0.1
10-01-2024	590	0.7	0.2	0.1
11-01-2024	600	0.7	0.2	0.1
12-01-2024	665	0.6	0.3	0.1
13-01-2024	730	0.5	0.4	0.1
14-01-2024	630	0.7	0.2	0.1
15-01-2024	640	0.7	0.2	0.1
16-01-2024	650	0.7	0.2	0.1
17-01-2024	715	0.6	0.3	0.1
18-01-2024	670	0.7	0.2	0.1
19-01-2024	740	0.5	0.4	0.1
20-01-2024	690	0.7	0.2	0.1
21-01-2024	770	0.5	0.4	0.1

22-01-2024	840	0.4	0.5	0.1
23-01-2024	720	0.6	0.3	0.1
24-01-2024	730	0.6	0.3	0.1
25-01-2024	740	0.6	0.3	0.1
26-01-2024	750	0.6	0.3	0.1
27-01-2024	760	0.6	0.3	0.1
28-01-2024	770	0.6	0.3	0.1
29-01-2024	780	0.6	0.3	0.1
30-01-2024	845	0.5	0.4	0.1
31-01-2024	800	0.6	0.3	0.1

Calculating the total cost and neutrosophic mean based on neutrosophic sets for energy consumption, shortages, and cost offers several benefits in inventory management and decision-making. By incorporating neutrosophic sets and inference techniques, decision-makers can effectively handle uncertainties and complexities inherent in inventory optimization, particularly in scenarios involving shortages. This approach enables more informed decisions regarding inventory levels, replenishment strategies, and resource allocation, leading to improved operational efficiency and cost-effectiveness. Additionally, the neutrosophic mean provides a nuanced representation of the total cost, considering both the membership and indeterminacy degrees, which allows decision-makers to balance economic objectives with environmental sustainability considerations. Overall, leveraging neutrosophic sets and inference techniques enhances the adaptability and resilience of inventory management systems, promoting more sustainable and robust supply chain practices.

This 3D scatter plot visualizes the relationship between the membership value (m_i), indeterminacy (ind_i), and total cost for each day. Each point in the plot represents a day from January 1 to January 31, 2024. The x-axis denotes the membership value, ranging from 0.4 to 0.8, indicating the degree to which the total cost belongs to the defined neutrosophic set. The y-axis represents the indeterminacy, varying from 0.1 to 0.5, signifying the uncertainty associated with the total cost estimation. The z-axis displays the total cost in rupees, ranging from 500 to 845. The color intensity of the points corresponds to the total cost value, with lighter shades indicating higher costs. This visualization offers insights into the distribution of total costs concerning membership, indeterminacy, and their associated values.

By identifying the relationship between membership value, indeterminacy, and total cost through this visualization, we can derive several benefits related to energy consumption. Firstly, it provides a clearer understanding of the variability and uncertainty associated with energy-related costs. This insight enables decision-makers to anticipate and adapt to fluctuations in energy expenses more effectively, leading to improved budgeting and resource allocation. Additionally, by identifying patterns or trends in the data, stakeholders can implement proactive measures to optimize energy consumption, reduce costs, and enhance overall efficiency. Furthermore, this analysis facilitates the development of targeted strategies for mitigating financial risks associated with energy consumption, thereby promoting sustainability and long-term economic viability. Overall, by leveraging insights from this visualization, organizations can make informed decisions to optimize energy consumption practices and achieve cost savings while minimizing environmental impact.

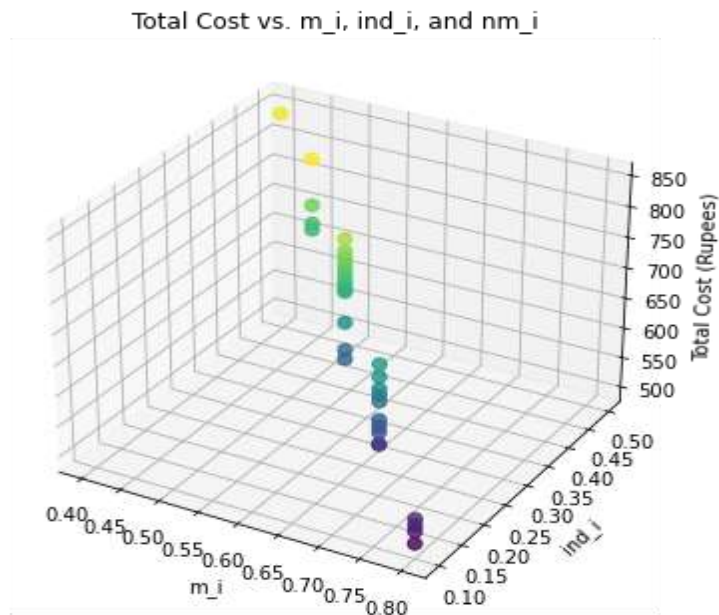


Figure 4: Variation of Total cost and Neutrosophic mean

5.6. Neutrosophic mean analysis of energy consumption costs through Python:

By implementing a method to calculate neutrosophic sets and find the neutrosophic mean based on total cost data for each day in a Python code. Initially, we define the total cost data, including the date and the corresponding total cost in rupees. Then, we compute the membership, indeterminacy, and non-membership values for each day using predefined formulas. These values represent the degree of truth, indeterminacy, and falsity of the total cost data, respectively, within a specified range. Next, we calculate the neutrosophic mean by averaging the membership, indeterminacy, and non-membership values across all days. Finally, we display the resulting neutrosophic mean, providing insights into the overall trend of the total cost data and its associated uncertainty. This approach enables us to analyze and interpret the total cost data using neutrosophic logic, which captures the inherent uncertainties and complexities present in real-world scenarios.

Neutrosophic Mean:

Membership: 0.4992987377279102

Indeterminacy: 0.10000000000000003

Non-membership: 0.4007012622720899

The Neutrosophic Mean is a measure that combines the membership, indeterminacy, and non-membership values of a neutrosophic set to calculate a representative value. In this context, the membership value of approximately 0.499 indicates the degree to which the total cost data for energy consumption falls within the specified range or condition. The indeterminacy value of approximately 0.1 reflects the ambiguity or uncertainty associated with the total cost data, suggesting that there is a certain level of unpredictability or variability in the cost calculations. Lastly, the non-membership value of approximately 0.401 represents the degree to which the total cost data does not meet the specified range or condition. Together, these values provide a nuanced understanding of the energy consumption costs, accounting for both the certainty and uncertainty inherent in the data.

The analysis of energy consumption data is paramount in this endeavour as it directly impacts the overall cost associated with energy resource utilization. Through the utilization of neutrosophic sets and the Neutrosophic Mean to assess the associated costs, decision-makers can gain valuable insights into the variability, uncertainty, and imprecision inherent in energy-related expenses. This enhanced understanding facilitates more informed decision-making, empowering the development of adaptive energy management strategies capable of addressing fluctuating costs, optimizing resource allocation, and mitigating financial risks effectively. Furthermore, the integration of neutrosophic logic and inference techniques into energy consumption analysis allows decision-makers to effectively

accommodate the uncertainties and complexities intrinsic to energy-related data, resulting in more resilient and dependable cost assessments and management practices.

5.7. Visualizing neutrosophic mean in energy consumption analysis:

In the provided visualization, the red point represents the neutrosophic mean, which is a statistical measure used to assess the central tendency of a dataset characterized by uncertainty and imprecision. The axes represent membership, indeterminacy, and non-membership values, respectively. The position of the mean point in this 3D space reflects its membership, indeterminacy, and non-membership values. This visualization allows decision-makers to understand the overall trend of the dataset in terms of its central tendency and the degree of uncertainty associated with it.

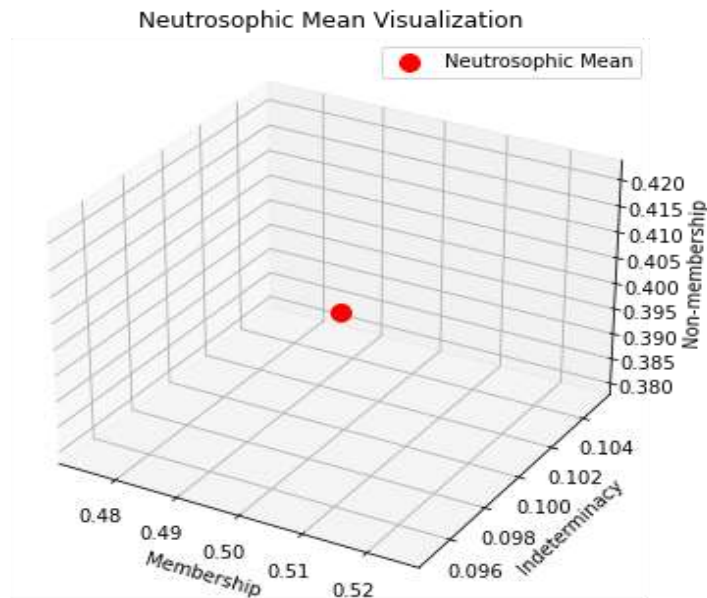


Figure 5: Neutrosophic mean of membership functions

Identifying the neutrosophic mean in the context of energy consumption provides valuable insights for decision-making in energy management. By analyzing the neutrosophic mean, stakeholders can gain a comprehensive understanding of the average cost pattern, taking into account uncertainty and imprecision factors. This insight enables them to develop more robust energy management strategies tailored to address fluctuations in costs effectively, optimize resource allocation, and minimize financial risks. Moreover, by incorporating neutrosophic logic into energy consumption analysis, decision-makers can make more informed decisions, leading to improved efficiency and sustainability in energy usage.

5.8. Advantages of neutrosophic fuzzy set theory over alternative approaches:

Benefits	Neutrosophic Fuzzy Set Theory	Other Methods
Handling Uncertainty	Effective handling of incomplete, indeterminate, and inconsistent information.	Limited capability to address uncertainty adequately.
Flexibility	Offers flexibility in representing uncertainty with truth, indeterminacy, and falsity degrees.	Often rigid in representing uncertainty, leading to oversimplification.

Adaptability	Enables adaptation to dynamic and complex situations with varying levels of certainty.	Struggles to adapt to changing conditions and uncertainties effectively.
Decision-Making Support	Provides more informed decision-making through nuanced representation of uncertainty.	May lead to suboptimal decisions due to incomplete or inaccurate information representation.
Robustness	Offers robustness against data variability and imprecision, leading to more reliable results.	Vulnerable to errors and biases arising from limited consideration of uncertainty.

6. Result and Discussion:

The results and discussion highlight the significant advantages of integrating neutrosophic fuzzy theory with environmental sustainability principles in inventory management. This fusion enables better management of shortages and energy consumption, leading to optimized inventory levels and replenishment strategies. Decision-makers benefit from a more flexible framework that enhances adaptability to uncertainties, resulting in more informed and effective decision-making processes. Overall, this approach promotes resilience, sustainability, and efficiency within supply chain operations, offering valuable insights for improving inventory management practices.

7. Conclusion:

The integration of neutrosophic fuzzy theory and environmental sustainability principles in inventory management, with a focus on shortages, offers a comprehensive approach to handling uncertainties and enhancing decision-making. By combining neutrosophic fuzzy logic with sustainability metrics, this framework improves adaptability and efficiency while promoting environmentally conscious practices. Challenges in energy consumption management are effectively tackled through neutrosophic sets, enabling more accurate cost evaluations and strategic energy management. This research enhances our understanding of energy-related costs, facilitating the development of adaptive strategies and fostering resilient and sustainable supply chain operations. Ultimately, this integrated approach has the potential to reduce costs compared to traditional methods by optimizing inventory levels, minimizing waste, and improving resource allocation.

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