On Neutrosophic Crisp Grill Topological Spaces

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Abstract

Among the important and ancient mathematical concepts are the concepts of grill, the local function, and the knowledge of using grill in conjunction with topological spaces, which have gained wide scope in the natural sciences and elsewhere. The main idea here is to development these concepts in conjunction with neutrosophic crisp sets, and on the other hand to highlight their important and influential properties and the relationships that link them to each other.

Keywords: neutrosophic crisp grill; neutrosophic crisp local function; $\Psi_{\int_{NC}} - operator$, $\Phi_{\int_{NC}} - operator$, neutrosophic crisp grill topological space (NCGTS) .

1. Introduction

The idea of Neutrosophic is to determine things, or to determine the problem and evaluate it correctly and unambiguously. The first to leave his mark was Florentin [1,2,3], who is considered a pioneer in this subject. Then a series of unique studies look place that included various sciences and their precise details. The basis on which Neutrosophic sets are based is the fuzzy sets known by Zadeh, [4] and in the comprehensive sense they are a generalization of them. Therefore, Neutrosophic sets have taken on a wide application field through this relationship that binds them to each other. Florentin and Salama [5,6] generalized the concept of Neutrosophic to set theory and called it Neutrosophic crisp sets (NCS_s). Imran et al. [12-14] gave the view of new concepts of weakly neutrosophic crisp separation axioms, new concepts of neutrosophic crisp open sets, and neutrosophic crisp generalized sg-closed sets and their continuity. Finally, the senses of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions were informed by Al-Obaidi et al. [15,16]. During our research, we will discuss biary operations (= equal, $\cap\, intersection$, $\varepsilon\ belong\ to$,$\bar{\varepsilon}\ not\ belong\ to$) in their classical sense, that is, in set theory. Otherwise, they are binary operation on Neutrosophic crisp set theory.

2. Preliminaries

Both Florentin and Salama identified three types of neutrosophic crisp sets (NCS_s), but in our research we will rely on the first type in addition to the binary opetrations (union , intersection , complement , …) that govern this type, which are as follows.

Definition 1-1. Let Υ be a non-empty set , $A =< A_1, A_2 , A_3 >$ NCS of type one , if A_1, A_2 , A_3 are pairwise disjoint (I.e $A_1 \cap A_2 = \emptyset$, $A_1, A_2 \cap A_3 = \emptyset, A_1 \cap A_3 = \emptyset$) .

1-We will take $\emptyset_N =< \emptyset, \emptyset, \Upsilon >$ and $\Upsilon_N =< \emptyset, \Upsilon, \Upsilon >$.

2-The complement of type one , ${A_N}^{c_1} =< A_1^c , A_2^c , A_3^c >$

3-The subset between two NCSs A_N, B_N as: $A_N \subseteq_1 B_N$ if $A_i \subseteq B_i, i = 1, 2, 3$.

4-The union between two NCSs A_N, B_N as: $A_N \cup_1 B_N =< A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 >$.

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5- The intersection between two NCSs A_N, B_N as: $A_N \cap_1 B_N =< A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 >$. $\cup_{1\mu\varepsilon\Delta} A_{3\mu} =< \cup_{\mu\varepsilon\Delta} A_{1\mu}, \cup_{\mu\varepsilon\Delta} A_{2\mu}, \cap_{\mu\varepsilon\Delta} A_{3\mu} >$, and $\cap_{1\mu\,\varepsilon\Delta} A_{\mu N} =< \cap_{\mu\varepsilon\Delta} A_{1\mu}, \cap_{\mu\varepsilon\Delta} A_{2\mu}, \cup_{\mu\varepsilon\Delta} A_{3\mu} >$.

Definition1-2. Let Υ be a non-empty set and p ε Υ , then the neutrosophic crisp points NCPs defined by:

$P_{N1} =< \{p\}, \emptyset, \{p\}^c > \quad and\ P_{N1} \in A_N\ iff\ p\ \varepsilon\ A_1$

$P_{N2} =< \{p\}, \emptyset, \emptyset > \quad and\ P_{N2} \in A_N\ iff\ p\ \varepsilon\ A_1$

$P_{N3} =< \emptyset, \{p\}, \{p\}^c > \quad and\ P_{N3} \in A_N\ iff\ p\ \varepsilon\ A_2$

$P_{N4} =< \emptyset, \{p\}, \emptyset > \quad and\ P_{N4} \in A_N\ iff\ p\ \varepsilon\ A_2$.

Definition 1-3. A neutrosophic crisp topological space NCTS in $\Upsilon \neq \emptyset$, is a family τ_{NC} of NCSs satisfying: $\emptyset_N =< \emptyset, \emptyset, \Upsilon >$ and $\Upsilon_N =< \emptyset, \Upsilon, \Upsilon >\ in\ \tau_{NC}$ and closed under the finite intersection $\cap_1$ and closed under union $\cup_1$. The pair (Υ , τ_{NC}) or $\Upsilon^{\tau_{NC}}$ is called Neutrosophic crisp topological space NCTS. The member of τ_{NC} is called Neutrosophic crisp open set NCOS and the complement is called Neutrosophic crisp closed set NCCS.

Definition 1-4. A non-empty family $\daleth_{NC}$ of NCSs called neutrosophic crisp ideal NCI in Υ, if satisfying:

(1) if $B_N \varepsilon \daleth_{NC}\ and\ A_N \subseteq_1 B_N\ , then\ A_N \varepsilon \daleth_{NC}$, (2) if $A_N, B_N\ \varepsilon\ \daleth_{NC}$, then $(A_N \cup_1 B_N)\ \varepsilon\ \daleth_{NC}$.

3. Neutrosophic Crisp Grill Topological Spaces (NCGTS)

In this section , we will present the basic definitions of our research, which depend on the definition of grill [7] for NCSs, and from it we will define the local function, with its most prominent properties, in addition to the definition on $\Psi_{\int} - operator$ extracted from it.

Definition 2-1. A non-empty family $\int_{NC}$ of NCSs called Neutrosophic crisp grill NCG in Υ , if satisfying (1) if $A_N \varepsilon \int_{NC}\ and\ A_N \subseteq_1 B_N\ , then\ B_N \varepsilon \int_{NC}$, (2) If $(A_N \cup_1 B_N)\ \varepsilon \int_{NC}$, then either $A_N \varepsilon \int_{NC}\ or\ B_N \varepsilon \int_{NC}$.

The triple (Υ , τ_{NC}, $\int_{NC}$) or $\Upsilon^{\tau_{NC}}_{\int_{NC}}$ is called Neutrosophic crisp grill topological space (NCGTS).

Definition 2-2. The neutrosophic crisp local function NCLF in $\Upsilon^{\tau_{NC}}_{\int_{NC}}$ of any NCS A_N is of form :$\Phi_{\int_{NC}}(A_N) = \{ P\ \varepsilon$ NCSs ; $\forall\ U_N \varepsilon\ \tau_{NC}(P)\ \ni U_N \cap_1 A_N \varepsilon \int_{NC} \}$, where $\tau_{NC}(P)$ are all NCOSS containing NCP P, and P is NCP in definition 1-2.

Example 2-3. Let Υ = { a , b , c } , $A_N =< \emptyset, \{a\}, \{b\} >$, $B_N =< \emptyset, \{c\}, \{a, b\} >$

,$C_N =< \emptyset, \{a, c\}, \{b\} >$, $D_N =< \emptyset, \emptyset, \{a, b\} >$, $\tau_{NC} = \{ \emptyset_N, \Upsilon_N, A_N, B_N, C_N, D_N \}$ is NCT and $\int_{NC} = NCS_s \{ A_{1N}, A_{2N}, A_{3N} \}$,where $A_{1N} =< \emptyset, \{a\}, \{b, c\} >$,$A_{2N} =< \emptyset, \emptyset, \{c, b\} >$

$A_{3N} = \emptyset_N$, then $\Phi_{\int_{NC}}(< \emptyset, \{c\}, \{a, b\} >) = \Upsilon_N$.

Remark

1- $\Phi_{\int_{NC}}(\emptyset_N)$ it does not containing any NCPs ,so $\Phi_{\int_{NC}}(\emptyset_N) = \emptyset_N$ .During the coutse of the search we will take the NCP , $P_N =< P_1, P_2, P_3 >$, where$P_1 = \emptyset\ or\ \{P\}$, $P_2 = \emptyset\ or\ \{P\}$ and $P_3 = \emptyset\ or\ \{P\}^c$.
2- For $\Upsilon \neq \emptyset$ and NCP P of NCTS τ_{NC} , the NCG $\int_{NC} = \{A_N , P\ \in\ A_N \}$, therefor, for any NCS B_N, we get that $\Phi_{\int_{NC}}(B_N) = \emptyset_N\ , if\ P \notin B_N$ and $\Phi_{\int_{NC}}(B_N) = NCCL\{P\}$, if P $\in$ B_N .

The NCLF $\Phi_{\int_{NC}} - operator$ has multiple properties that have important effects in the process of constructing NCTS finer than NCTS that generates it , as the following Theorem explains :

Theorem 2-4. For any NCSs A_N, B_N of NCGTS $\Upsilon^{\tau_{NC}}_{\int_{NC}}$ $\Phi_{\int_{NC}} - operator$ has several properties :

1-if $A_N \subseteq_1 B_N\ , then\ \Phi_{\int_{NC}}(A_N) \subseteq_1 \Phi_{\int_{NC}}(B_N)$

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2-$\Phi_{\int NC}(A_N \cup_1 B_N) = \Phi_{\int NC}(A_N) \cup_1 \Phi_{\int NC}(B_N)$

3-For any $A_N \bar{\varepsilon} \int_{NC}$, then

$(i) - \Phi_{\int NC}(A_N) = \emptyset_N$ (ii)$\Phi_{\int NC}(A_N \cup_1 B_N) = \Phi_{\int NC}(B_N)$ =$\Phi_{\int NC}(B_N \cap_1 {A_N}^{c_1})$

4-$\Phi_{\int NC}(\Phi_{\int NC}(A_N)) \subseteq_1 \Phi_{\int NC}(A_N)$

5-$\Phi_{\int NC}(A_N) = NCCl(\Phi_{\int NC}(A_N)) \subseteq_1 NCCl(A_N)$

6-For any $U_N \varepsilon\, \tau_{NC}$, then $U_N \cap_1 \Phi_{\int NC}(A_N) \subseteq_1 \Phi_{\int NC}(U_N \cap_1 A_N)$

Proof(6) Let NCP P $\in U_N \cap_1 \Phi_{\int NC}(A_N)$, then P $\in U_N$ and P $\in \Phi_{\int NC}(A_N) \rightarrow$

$\forall V_N \varepsilon\, \tau_{NC}(P) \ni V_N \cap_1 A_N \varepsilon \int_{NC} \rightarrow U_N \cap_1 V_N \cap_1 A_N \varepsilon \int_{NC} \rightarrow V_N \cap_1 (U_N \cap_1 A_N) \varepsilon \int_{NC}$, therefore, P $\in \Phi_{\int NC}(U_N \cap_1 A_N)$.

We can construct a NCT via the $\Phi_{\int NC} - operator$ $\tau^*_{NC} = \{ A_N \varepsilon\, NCS_S;\ A_N \subseteq_1 [\Phi_{\int NC}({A_N}^{C_1})]^{C_1} \}$ is finer than τ_{NC} . If $\int_{NC}$ be all NCSs except $\emptyset_N$, then$\tau_{NC} = \tau^*_{NC}$. Also if $\int_{NC} = \{ \Upsilon_N \}$, then τ^*_{NC} is discrete NCTS.

Theorem 2-5. For any NCGTS $\Upsilon^{\tau_{NC}}_{\int NC}$, the following statement are equivalent :

1-$\tau_{NC} \cap NCS_s \backslash \int_{NC} = \{ \emptyset_N \}$.

2-If $\emptyset_N \neq H_N \varepsilon\, \tau_{NC}$, then $H_N \subseteq_1 \Phi_{\int NC}(H_N)$.

3-$\Upsilon_N = \Phi_{\int NC}(\Upsilon_N)$.

4-if $K_N \bar{\varepsilon} \int_{NC}$, then $NCint(K_N) = \emptyset_N$

5-if $\emptyset_N \neq U_N \bar{\varepsilon}\, \tau_{NC}$, then $\Phi_{\int NC}(U_N) = NCCl(U_N)$.

Proof . 1$\Longrightarrow$ 2 if possible there exist NCP P $\in H_N \varepsilon\, \tau_{NC}$ s.t. P $\bar{\varepsilon}\, \Phi_{\int NC}(H_N) \rightarrow \exists\, U_N \bar{\varepsilon}\, \tau_{NC}$(P) s.t. $U_N \cap_1 H_N \bar{\varepsilon} \int_{NC}$, then $U_N \cap_1 H_N\ \varepsilon\, NCS_s \backslash \int_{NC}$, but $U_N \cap_1 H_N\ \varepsilon \tau_{NC}$(P) , which contradiction with part (1) .therefore, $H_N \subseteq_1 \Phi_{\int NC}(H_N)$.

2$\Longrightarrow$ 3 Since $\Upsilon_N\ \varepsilon\ \tau_{NC}$, then $\Upsilon_N \subseteq_1 \Phi_{\int NC}(\Upsilon_N)$

3 $\Longrightarrow 1 if\ possible\ \exists\ \emptyset_N \neq U_N \varepsilon\, \tau_{NC}\ and\ U_N \varepsilon\ NCS_s \backslash \int_{NC} \rightarrow \Upsilon_N \cap_1 U_N = U_N\ \bar{\varepsilon} \int_{NC}$, $so\ for\ any\ NCP\ P\ \varepsilon\ U_N$, $then\ P\ \bar{\varepsilon}\, \Phi_{\int NC}(U_N)$, $which\ contradiction\ with\ part$ (3).

1 $\Longrightarrow$ 4 $Let\ K_N \bar{\varepsilon} \int_{NC} and\ NCInt(K_N) \neq \emptyset_N \rightarrow \exists\ P\ \in NCInt(K_N) \ni\ P\ \in U_N \subseteq_1\ K_N\ for\ some\ U_N \varepsilon\, \tau_{NC}\ and\ U_N \subseteq_1\ K_N \rightarrow U_N = U_N \cap_1\ K_N \subseteq_1 K_N \bar{\varepsilon} \int_{NC} \rightarrow U_N \varepsilon\ \ NCS_s \backslash \int_{NC}$, which contradiction with part (1) .

2$\Longrightarrow$ 5 Since $\Phi_{\int NC}(U_N) \subseteq_1 NCCl(U_N)$ and by part (2) $U_N \subseteq_1 \Phi_{\int NC}(U_N) \rightarrow NCCl(U_N) \subseteq_1 NCCl(\Phi_{\int NC}(U_N)) = \Phi_{\int NC}(U_N)$, so we get part (5).

5$\Longrightarrow$ 3 Since $\Upsilon_N\ \varepsilon\ \tau_{NC}$, then $\Upsilon_N = NCCl(\Upsilon_N) = \Phi_{\int NC}(\Upsilon_N)$, so we get pat (3).

Definition 2-6. We can define the $\Psi_{\int NC} - operator$ on the NCGTS $\Upsilon^{\tau_{NC}}_{\int NC}$ in the following way : For any NCS U_N , $\Psi_{\int NC}(U_N) = [\Phi_{\int NC}({U_N}^{c_1}]^{c_1}$.

From example 2-3 $\Psi_{\int NC}(<\ \emptyset, \{a\}, \{b, c\} >) =< \Upsilon, \{b\}, \Upsilon\ >$

Remark 2-7.

1-From Theorem 2-4 part(1) , we get , if $A_N \subseteq_1\ B_N$, $then\ \Psi_{\int NC}(A_N) \subseteq_1\ \Psi_{\int NC}(B_N)$.

2-From Theorem 2-4 part (3) , if $A_N \bar{\varepsilon} \int_{NC}$, then $\Psi_{\int NC}(A_N) = \Upsilon_N$. Also from part (4) and property of complement , we get $\Psi_{\int NC}(A_N) \subseteq_1 \Psi_{\int NC}(\Psi_{\int NC}(A_N))$.

3-From Theorem 2-4 , part(5) , we get that $\Phi_{\int NC} - operator$ is NCCS , so

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$\Psi_{\int_{NC}} - operator$ is NCOS .

$\Psi_{\int_{NC}} - operator$ has properties that have important mathematical implications for many Neutrosophic crisp topological concepts , and the following Theorem reviews the most important of these properties.

Theorem 2-8. For any NCSs A_N, B_N of NCGTS $\Upsilon_{\int_{NC}}^{\tau_{NC}}$ $\Psi_{\int_{NC}} - operator$ has several properties :

1-if $U_N \varepsilon\, \tau_{NC}$, then $U_N \subseteq_1 \Psi_{\int_{NC}}(U_N)$.

2-$\Psi_{\int_{NC}}(A_N \cap_1 \; B_N) = \Psi_{\int_{NC}}(A_N \;) \cap_1 \Psi_{\int_{NC}}(\; B_N)$

3-if $B_N \,\bar{\varepsilon} \int_{NC}, then\ \Psi_{\int_{NC}}(A_N \cup_1 B_N) = \Psi_{\int_{NC}}(B_N) = \Psi_{\int_{NC}}(B_N \cap_1 {A_N}^{c_1})$

4-$\tau_{NC}^{*} - NCInt(\,A_N) = A_N \cap_1 \Psi_{\int_{NC}}(A_N)$

5-$\tau_{NC} \; \cap NCS_s \backslash \int_{NC} = \{\,\emptyset_N\}$ iff for any $\emptyset_N \neq {U_N}^{c_1}\ \varepsilon\ \tau_{NC}$, $\Psi_{\int_{NC}}(U_N) \subseteq_1 U_N$

Proof (5) Let $\emptyset_N \neq {U_N}^{c_1}\ \varepsilon\ \tau_{NC}$, then by Theorem 2-5 ${U_N}^{c_1} \subseteq_1 \Phi_{\int_{NC}}({U_N}^{c_1})$, from it we get $\Psi_{\int_{NC}}(U_N) = [\Phi_{\int_{NC}}({U_N}^{c_1})]^{\,c_1} \subseteq_1 [{U_N}^{c_1}]^{\,c_1} = U_N$

Conversely , for any NCCS $\emptyset_N \neq Z_N, \Psi_{\int_{NC}}(Z_N) \subseteq_1 Z_N$ iff $[\,Z_N]^{c_1} \subseteq_1 [\Psi_{\int_{NC}}(Z_N)]^{\,c_1} = \Phi_{\int_{NC}}({Z_N}^{c_1})$, so by Theorem 2-5 , we get $\tau_{NC} \; \cap NCS_s \backslash \int_{NC} = \{\,\emptyset_N\}$.

Corollary 2-9. For any NCS A_N, the following statement are equivalent :

1-$\tau_{NC} \; \cap NCS_s \backslash \int_{NC} = \{\,\emptyset_N\}$,

2-$\Psi_{\int_{NC}}(\emptyset_N) = \emptyset_N$,

3-if for any NCCS $\emptyset_N \neq Z_N, \Psi_{\int_{NC}}(Z_N) \cap_1 {Z_N}^{c_1} = \emptyset_N$,

4-NCint(NCCl(A_N)) = $\Psi_{\int_{NC}}$(NCint(NCCl(A_N)),

5-if $\emptyset_N \neq U_N \varepsilon\, \tau_{NC}\ , then\ \Psi_{\int_{NC}}(U_N) \subseteq_1 \Phi_{\int_{NC}}(U_N)$.

4. Conclusion

Here the concepts $\Phi_{\int_{NC}} - operator\ and\ \Psi_{\int_{NC}} - operator$, were generalized to Neutrosophic crisp sets, but we took the binary operations based on them , such as intersection union , belong , partial relation, on those sets, in addition to taking the type one of complement .Therefore, the reader can generalize many mathematical concepts in these spaces that we have discussed here, for example what is found in the [8,..,10].

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