



A Novel Fusion Method for Enhanced Multi-Criteria Decision-Making in Energy Management: Integrating Analytic Hierarchy Process

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Abstract

This paper suggests a novel fusion approach of combining Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to enhance multi-criteria decision-making for energy management. In this way, by integrating the two powerful methodologies, the fusion approach is enabled that allows for dealing with the complex and highly dynamic character of energy management decisions for which there is required the careful consideration of many conflicting criteria. The method uses A to derive weightings for each decision criterion via AHP based on expert judgments, ensuring all relevant factors are systematically considered proportionately. Subsequently, TOPSIS is applied to evaluate and rank the alternatives such that the most effective energy solutions close to the ideal solution are identified. This integration of AHP with TOPSIS leads to comprehensive analysis that draws both the strengths of these techniques and provides a powerful tool to make informed and balanced decisions in the energy sector. The effectiveness of this fusion method, when applied, could then lead to the attainment of subtler findings and dependable suggestions, making it a beneficial contribution to this field of energy management.

Keywords: fusion method; data mining methods; hierarchy analysis method; decision-making process; priority vectors; global priorities; analytical framework; method selection; evaluative criteria; comparative analysis; visualization techniques; regression methods.

1. Introduction

The development of modern society requires the search for new approaches to energy saving management processes, based on operational consideration of the fact of the occurrence of constant and rapid changes. Making effective management decisions in energy saving management processes is unthinkable without modern approaches to organizing the management process and the use of information technologies for data processing, the basis for decision support systems. In this case, the following question arises: what mechanisms are most effective for implementing decision-making in energy saving management processes? Despite the large number of studies related to the construction of solutions in the field of decision making and automatic control, this issue remains relevant [1], [24]. Decision-making is one of the regard points that are considered to entrepreneurs and managers. Engaging a suitable method in choosing the method of doing research is necessary in the aspect of decision-making. It also must be a rational process that shall result in the most appropriate alternative being chosen due to the massive impact on the part of the organization under consideration. The AHP method reaches many applications and different fields in combination with many different methods. It would be quite informative to follow the way of calculations that the AHP method applies, from complex to simpler ones. The paper refers to the procedure used when applying the Saaty method and compares it with others. This method is provided for relatively every user, as only the simplest basics of mathematics must be known to get similar results, which is very beneficial. Here, the AHP will be used for getting the weightage of the criteria of banking performance, and Grey Relational Analysis is considered for the ranking [2], [3], [15]. He has defined that this method can be applied to managerial decision-making, for instance, determining the critical technical criteria for choosing the most appropriate location of water sources and enhances the analytical hierarchical process while contributing to finding the best alternative [4] - [6].

In a modified cycle of continuous process improvement, energy saving management, the decision-making task consists of generating a set of possible options that provide a solution to the problem situation under existing restrictions, and identifying among these options one best or several preferred options that meet the requirements for them. In solving a set of decision-making problems, the results of solving all problems at the stages of the management process are used: standardization, planning, accounting, and control. There are three stages in this long process: searching for information, searching for, and finding an alternative, and choosing the best alternative [19]. At the first stage, all information available at the time of decision-making is collected: factual data, expert opinion. Where possible, mathematical models are built the views on the problem from groups of specialists influencing its solution are determined. The second stage is associated with determining what can and cannot be done in the current situation, i.e., with identification of solution options (alternatives). And the third stage includes comparison of alternatives and selection of the best solution option (or options) [7], [8], [20]. In a modified continuous process improvement cycle for energy management, the following data mining methods are used to make a decision: linear regression, neural networks, decision trees, k-nearest neighbor, and visualization, and to select the best solution option (or options) among the data mining methods, it is proposed use, as the most appropriate for the situation under consideration, a method of analyzing hierarchies, based on the judgment of experts (specialists) [9], [10].

In this paper, a new fusion [25-27] approach of SALP is presented using the Analytic Hierarchy Process (AHP) combined with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). This methodological innovation wants to offer an improvement in the decision-making capacity concerning the combined use of AHP and TOPSIS because it is ensured to measure alternatives under evaluation in a complete and precise way. AHP systematically supports the weighing of decision criteria with expert judgment, considering all relevant factors according to their relative importance. Thus, the application of TOPSIS then assists in the determination of the alternatives with the highest level of similarity and closeness to the ideal solution by way of a weight criterion. This thus brought an alignment in these two well-defined methodologies, thus resulting in a more nuanced, robust, double-layered approach to decision-making. This approach not only quantifies the subjective preferences in criterion importance but also accesses the alternative performances in a comparative manner objectively. Our method summarizes both the insights of AHP and TOPSIS into one integrative tool which provides a comprehensive framework and enhances the reliability of the process of decision-making, especially in settings where the decision will have to be made based on more criteria. The introduction of this method of fusion addresses the need for sophisticated tools of analysis, more particularly in the domain of energy management, where very often decisions are involved with intricate tradeoffs between cost, efficiency, and environmental impact. This approach provides an increase in application value, and hence the expected benefits due to the application include sound, balanced, and sustainable decision-making outcomes that would contribute to development in the theoretical and practical domains of multi-criteria decision analysis.

2. Methods

Hierarchy analysis method is a systematic procedure for hierarchically representing the elements that define the essence of any problem. The method consists of decomposing the problem into increasingly simpler component parts and further processing the sequence of judgments of the decision maker using paired comparisons. As a result, the relative degree (intensity) of interaction between elements in the hierarchy can be expressed. These judgments are then expressed numerically. The hierarchy analysis method includes procedures for synthesizing multiple judgments, obtaining priority criteria, and finding alternative solutions. It is useful to note that the values thus obtained are estimates on a ratio scale and correspond to so-called hard estimates. In hierarchical analysis methods, task elements are compared in pairs with respect to their impact (“weight” or “intensity”) on a characteristic common to them [10], [11], [14].

The main stages of the hierarchy analysis method are:

1. Developing a statement of tasks, defining the goal and important criteria for solving problems, constructing a scale of the relative importance of the criteria.
2. Construction of a hierarchy, starting from the top (goal, from a management point of view), through intermediate levels (criteria on which subsequent levels depend) to the lowest level (which is usually a list of alternatives).
3. Construction of a set of square matrices of judgments for paired comparisons for each of the lower levels - one matrix for each element adjacent to the upper level.
4. Formation of square matrices of judgments, comparing elements of any level with each other regarding their impact on the directed element.

5. Determination of consistency of judgments. Calculation and verification of the consistency index. Comparison of the consistency index with the corresponding average values for random elements. Obtaining a consistency relation.
6. Calculation of vectors of local and global priorities. Determine alignment and synthesize priorities.
7. Hierarchical synthesis to weight the eigenvectors with criterion weights and calculate the sum over all corresponding weighted components of the eigenvectors of the hierarchy level below.
8. Finding the consistency of all hierarchies by multiplying each consistency index by the priority of the corresponding criterion and summing the resulting numbers [16] - [18].

3. Results and discussion

An algorithm for solving the problem of selecting the best alternative among alternatives, described as a multicriteria optimization problem in a formulation where goals, alternatives and initial data are not clearly defined, but for which the preference relations, i.e., utility functions are clearly defined [17], [18].

Step 1: The first step is to decompose and represent the problem in a hierarchical form. At the first (highest) level there is a general goal - "Selecting the best method." At the second level there are six factors or criteria that clarify the goal, and at the third (lower) level there are five IAD methods that must be assessed in relation to the criteria of the second level. This is followed by the definition of criteria and a graphical representation of the hierarchy (Figure 1).

After the task is presented in the form of a hierarchy, the priorities of the criteria are established. To calculate weight coefficients, a group of experts is interviewed. An odd number (5, 7 or 9) of specialists with extensive experience are involved as experts. The general opinion is more accurate than the individual opinion of an individual specialist. This method is used to obtain quantitative estimates of qualitative characteristics and properties.

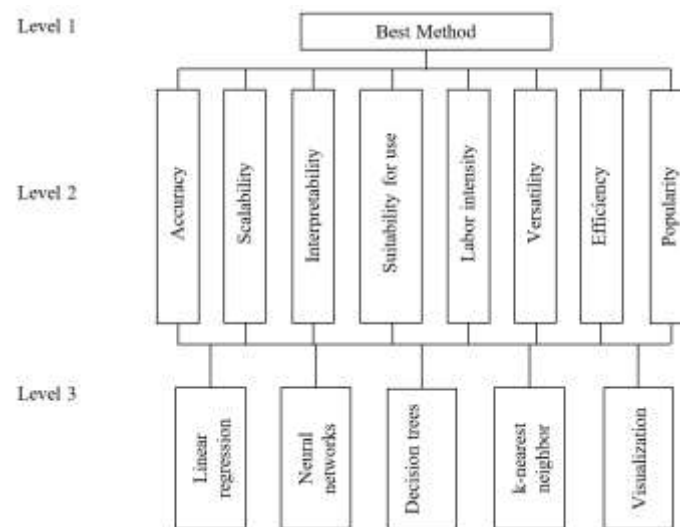


Figure 1: Scheme of task decomposition into hierarchy

When analyzing the characteristics of data mining methods based on the scale of relative importance of criteria (Table 1), experts (leading specialists in the development of decision support systems) gave assessments that are given in Tables 2 and 3.

Step 2. Based on the initial data in Tables 2 and 3, the average expert ratings for each of the criteria are determined and square matrices are compiled to compare the relative importance of the criteria in relation to the overall goal. None of the methods can be considered the only effective one, having obvious superiority over other methods [10], [13].

Let the action be evaluated according to n criterion. The weights of the criteria are given by the matrix (2-level):

Table 1: Scale of relative importance of criteria

Relative importance rating	Definition	Explanations
1	Equal importance	Equal contribution of two types of criteria to the goal
3	Moderate superiority of one over the other	Experience and judgment give slight superiority to one type of criterion over another
5	Significant or strong superiority	Experience and judgment give a strong superiority to one type of criterion over another
7	Significant superiority	One type of criterion is given such a strong superiority that it becomes practically significant
9	Very strong superiority	The evidence of the superiority of one type of criterion over another is most strongly confirmed
2,4,6,8	Intermediate solutions between two adjacent judgments	

Table 2: General assessments of experts in the development of a decision support system when analyzing the characteristics of data mining methods

Characteristics of Data Mining Methods	Expert assessments							Average rating
	I	II	III	IV	V	VI	VII	
Accuracy	9	9	9	9	9	9	9	9
Scalability	8	8	7	8	8	7	8	7,714285714
Interpretability	4	3	4	3	3	2	3	3,142857143
Suitability for use	6	6	6	5	7	6	7	6,142857143
Labor intensity	3	5	3	4	3	4	5	3,857142857
Versatility	5	4	5	4	6	4	6	4,857142857
Efficiency	9	9	9	8	9	9	8	8,714285714
Popularity	2	2	2	3	2	2	3	2,285714286

Table 3: General assessments of experts in the development of a decision support system when analyzing data mining methods by characteristics

Characteristics of Data Mining Methods (criteria)	Characteristics of Data Mining Methods	Expert assessments							Average rating
		I	II	III	IV	V	VI	VII	
		Criterion 1 (K ₁)							
Accuracy	Linear regression	6	6	7	7	6	5	6	6,142857143
	Neural networks	9	8	9	8	9	9	8	8,571428571
	Visualization	4	5	4	5	3	4	5	4,285714286
	Decision trees	4	4	3	4	3	5	4	3,857142857
	K-nearest neighbor	5	4	3	4	4	5	4	4,142857143
		Criterion 2 (K ₂)							
Scalability	Linear regression	8	7	8	9	7	8	8	7,857142857
	Neural networks	8	8	9	7	8	9	8	8,142857143
	Visualization	2	3	2	3	3	2	3	2,571428571
	Decision trees	8	7	8	7	8	7	8	7,571428571

	K-nearest neighbor	3	2	3	2	3	2	2	2,428571429
Criterion 3 (K ₃)									
Interpretability	Linear regression	6	7	8	6	7	8	6	6,857142857
	Neural networks	7	5	7	8	6	6	8	6,714285714
	Visualization	8	9	8	7	7	9	9	8,142857143
	Decision trees	8	7	9	9	8	8	8	8,142857143
	K-nearest neighbor	7	8	7	8	8	7	5	7,142857143
Criterion 4 (K ₄)									
Suitability for use	Linear regression	8	9	8	7	8	9	8	8,142857143
	Neural networks	7	8	7	8	8	7	8	7,571428571
	Visualization	9	8	7	8	8	8	9	8,142857143
	Decision trees	6	7	6	6	6	6	7	6,285714286
	K-nearest neighbor	5	6	6	5	7	6	5	5,714285714
Criterion 5 (K ₅)									
Labor intensity	Linear regression	5	4	6	5	5	6	5	5,142857143
	Neural networks	6	5	5	4	5	5	6	5,142857143
	Visualization	9	9	9	8	9	9	9	8,857142857
	Decision trees	8	9	8	7	8	8	9	8,142857143
	K-nearest neighbor	4	3	4	4	3	4	3	3,571428571
Criterion 6 (K ₆)									
Versatility	Linear regression	6	6	5	6	7	6	7	6,142857143
	Neural networks	7	7	6	7	6	7	6	6,571428571
	Visualization	5	4	5	4	5	4	4	4,428571429
	Decision trees	8	8	8	7	9	8	9	8,142857143
	K-nearest neighbor	5	4	3	5	4	3	5	4,142857143
Criterion 7 (K ₇)									
Efficiency	Linear regression	8	7	9	8	8	9	9	8,285714286
	Neural networks	9	9	8	9	8	8	8	8,428571429
	Visualization	2	3	3	2	2	3	2	2,428571429
	Decision trees	6	7	7	8	8	7	8	7,285714286
	K-nearest neighbor	8	9	9	8	8	9	9	8,571428571
Criterion 8 (K ₈)									
Popularity	Linear regression	5	5	4	5	5	4	5	4,714285714
	Neural networks	4	4	5	5	4	5	5	4,571428571
	Visualization	7	8	6	7	7	8	7	7,142857143
	Decision trees	8	8	7	7	6	6	8	7,142857143
	K-nearest neighbor	5	4	4	5	4	4	5	4,428571429

$$K = \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{pmatrix},$$

where k_{ij} is the weight of the i -criterion in relation to the j -criterion, and $k_{ii} = 1$ and $k_{ij} \cdot k_{ji} = 1, i, j = 1, \dots, n$.

In these matrices, called pairwise comparison matrices, an element of the *i*-row and *j*-column is defined as the ratio of the average expert assessment of the *i*-criterion to the average expert assessment of the *j*-criterion. It means the relative importance of the *i*-criterion in relation to the *j*-criterion (Table 4).

Table 4: Comparison matrix of criteria in relation to the goal

Overall satisfaction of alternatives	<i>K</i> ₁	<i>K</i> ₂	<i>K</i> ₃	<i>K</i> ₄	<i>K</i> ₅	<i>K</i> ₆	<i>K</i> ₇	<i>K</i> ₈
<i>K</i> ₁	1	1,16667	2,86364	1,46512	2,33333	1,85294	1,03279	3,93750
<i>K</i> ₂	0,85714	1	2,45455	1,25581	2,00000	1,58824	0,88525	3,37500
<i>K</i> ₃	0,34921	0,40741	1	0,51163	0,81481	0,64706	0,36066	1,37500
<i>K</i> ₄	0,68254	0,79630	1,95455	1	1,59259	1,26471	0,70492	2,68750
<i>K</i> ₅	0,42857	0,50000	1,22727	0,62791	1	0,79412	0,44262	1,68750
<i>K</i> ₆	0,53968	0,62963	1,54545	0,79070	1,25926	1	0,55738	2,12500
<i>K</i> ₇	0,96825	1,12963	2,77273	1,41860	2,25926	1,79412	1	3,81250
<i>K</i> ₈	0,25397	0,29630	0,72727	0,37209	0,59259	0,47059	0,26230	1

Step 3. The evaluation of the eigenvector components by rows is calculated, the elements of the matrix columns are summed, and the results are normalized to obtain an estimate of the priority vector:

At the 2nd level, the elements of the priority vector

$$\vec{X} = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$$

are determined by the formula

$$x_i = \frac{e^{\frac{1}{n} \ln \prod_{j=1}^n k_{ij}}}{\sum_{i=1}^n e^{\frac{1}{n} \ln \prod_{j=1}^n k_{ij}}}$$

where *i*=1,2,3, ..., *n* is the number of criteria. In our case *x*₁=0,197, *x*₂=0,169, *x*₃=0,069, *x*₄=0,134, *x*₅=0,084, *x*₆=0,106, *x*₇=0,191, *x*₈=0,050.

Step 4. The consistency index in each matrix is calculated. First, each column of judgments is summed, then the sum of the first column is multiplied by the value of the first component of the normalized priority vector, the sum of the second column is multiplied by the second component:

$$\lambda_{max} = \sum_{i=1}^n x_i \sum_{j=1}^n k_{ji}$$

where *λ*_{max} is the maximum own value. In our case

$$\lambda_{max} = 0,197 \times 5,079 + 0,169 \times 5,926 + 0,069 \times 14,545 + 0,134 \times 7,442 + 0,084 \times 11,852 + 0,106 \times 9,412 + 0,191 \times 5,246 + 0,05 \times 20 = 8.$$

Step 5. The consistency index is determined by the following formula

$$CI = \frac{\lambda_{max}}{n - 1}$$

where *CI* is the consistency index, *n* is the number of elements being compared.

For an inversely symmetric matrix, always *λ*_{ma} ≥ *n*; 8 ≥ 8:

$$CI = \frac{8 - 8}{8 - 1} = 0,000 = 0\%$$

To determine how accurately the *CI* reflects the consistency of judgments, it must be compared with a random consistency index (*CI*), which corresponds to a matrix with random values selected from a scale of relative importance, assuming an equal probability of choosing any of the given numbers.

Step 6. In this step, the consistency relation is calculated:

$$CR = \frac{CI}{slind(n)},$$

where CR is the consistency relation; $slind(n)$ - random index for n criterion.

The value of $slind(n)$ is taken from Table 5 depending on the size of the matrix.

Table 5: Random index table

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}
Matrix size	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Random consistency	0	0	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49	1,51	1,48	1,56	1,57	1,59

The ratio of the CI to the average CI value for a matrix of the same order is called the consistency ratio (CR). And if we divide the IS by a number corresponding to the random consistency of a matrix of the same order, we obtain a consistency ratio (CR), which should be of the order of less than 10%, but not more than 20%:

$$CR = \frac{0,000}{1,41} = 0,000 = 0\%$$

Step 7. Let the number of decision alternatives be equal to m . The priorities of the options are specified by matrices (3rd level):

$$L_d = \begin{pmatrix} l_{d11} & l_{d12} & \dots & l_{d1m} \\ l_{d21} & l_{d22} & \dots & l_{d2m} \\ \dots & \dots & \dots & \dots \\ l_{dm1} & l_{dm2} & \dots & l_{dmm} \end{pmatrix}, \quad d = \overline{1, m}$$

where L_d is the priority matrix of d - criterion options; l_{dij} - priority of the i - option in relation to the j - option of the d - criterion, and $l_{dij} = 1$ and $l_{dij} \cdot l_{dji} = 1, i, j = 1, \dots, m$.

Step 8. The evaluation of the eigenvector components by rows is calculated, the elements of the matrix columns are summed, and the results are normalized to obtain an estimate of the priority vector.

At the 3rd level, elements of priority vectors

$$\vec{Y}_d = \begin{pmatrix} y_{1d} \\ y_{2d} \\ \dots \\ y_{md} \end{pmatrix}, \quad d = \overline{1, n}$$

are determined by the formula

$$y_{id} = \frac{e^{\frac{1}{m} \ln \prod_{j=1}^m l_{dij}}}{\sum_{i=1}^m e^{\frac{1}{m} \ln \prod_{j=1}^m l_{dij}}}, \quad d = \overline{1, n}.$$

Step 9. The consistency index in each matrix is calculated. First, each column of judgments is summed, then the sum of the first column is multiplied by the value of the first component of the normalized priority vector, the sum of the second column is multiplied by the second component:

$$\lambda_{dmax} = \sum_{i=1}^m y_{di} \sum_{j=1}^m l_{dji}$$

Step 6. In this step, the consistency relation is calculated: where λ_{dmax} is the maximum eigenvalue for d - criterion.

Step 10. The consistency index is determined by the following formula

$$CI_d = \frac{\lambda_{dmax}}{m - 1}$$

where CI_d is the consistency index for d - criterion; m is the number of elements being compared.

For an inversely symmetric matrix, always $\lambda_{max} \geq m$.

Step 11. In this step, the consistency relation is calculated:

$$CR_d = \frac{CI_d}{slind(m)}, \quad d = \overline{1, n}$$

where CR_d is the consistency ratio for d - criterion; $slind(m)$ - random index for matrix size m . The value of $slind(m)$ is taken from Table 5 depending on the size of the matrix.

The results of calculating priority vectors, calculating the consistency index and consistency ratio for the third level of the hierarchy are given in Tables 6-13:

Table 6: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_1

K_1	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	0,717	1,433	1,593	1,483	0,228
Neural networks	1,395	1	2,000	2,222	2,069	0,317
Visualization	0,698	0,500	1	1,111	1,034	0,159
Decision trees	0,628	0,450	0,900	1	0,931	0,143
K-nearest neighbor	0,674	0,483	0,967	1,074	1	0,153
					λ_{max}	5,000
					CI	0,000
					CR	0,000

Table 7: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_2

K_2	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	0,965	3,056	1,038	3,235	0,275
Neural networks	1,036	1	3,167	1,075	3,353	0,285
Visualization	0,327	0,316	1	0,340	1,059	0,090
Decision trees	0,964	0,930	2,944	1	3,118	0,265
K-nearest neighbor	0,309	0,298	0,944	0,321	1	0,085
					λ_{max}	5,000
					CI	0,000
					CR	0,000

Table 8: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_3

K_3	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	1,021	0,842	0,842	0,960	0,185
Neural networks	0,979	1	0,825	0,825	0,940	0,181

Visualization	1,188	1,213	1	1	1,140	0,220
Decision trees	1,188	1,213	1	1	1,140	0,220
K-nearest neighbor	1,042	1,064	0,877	0,877	1	0,193
					λ_{max}	5,000
					<i>CI</i>	0,000
					<i>CR</i>	0,000

Table 9: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_4

K_4	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	1,075	1	1,295	1,425	0,227
Neural networks	0,930	1	0,930	1,205	1,325	0,211
Visualization	1	1,075	1	1,295	1,425	0,227
Decision trees	0,772	0,830	0,772	1	1,100	0,175
K-nearest neighbor	0,702	0,755	0,702	0,909	1	0,159
					λ_{max}	5,000
					<i>CI</i>	0,000
					<i>CR</i>	0,000

Table 10: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_5

K_5	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	1	0,581	0,632	1,440	0,167
Neural networks	1	1	0,581	0,632	1,440	0,167
Visualization	1,722	1,722	1	1,088	2,480	0,287
Decision trees	1,583	1,583	0,919	1	2,280	0,264
K-nearest neighbor	0,694	0,694	0,403	0,439	1	0,116
					λ_{max}	5,000
					<i>CI</i>	0,000
					<i>CR</i>	0,000

Table 11: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_6

K_6	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	0,935	1,387	0,754	1,483	0,209
Neural networks	1,070	1	1,484	0,807	1,586	0,223
Visualization	0,721	0,674	1	0,544	1,069	0,150
Decision trees	1,326	1,239	1,839	1	1,966	0,277
K-nearest neighbor	0,674	0,630	0,935	0,509	1	0,141

	λ_{max}	5,000
	CI	0,000
	CR	0,000

Table 12: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_7

K_7	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	0,983	3,412	1,137	0,967	0,237
Neural networks	1,017	1	3,471	1,157	0,983	0,241
Visualization	0,293	0,288	1	0,333	0,283	0,069
Decision trees	0,879	0,864	3,000	1	0,850	0,208
K-nearest neighbor	1,034	1,017	3,529	1,176	1	0,245
					λ_{max}	5,000
					CI	0,000
					CR	0,000

Table 13: Results of calculating priority vectors, calculating the consistency index and consistency relations for the third level of the hierarchy according to K_8

K_8	Linear regression	Neural networks	Visualization	Decision trees	K-nearest neighbor	Priority vector
Linear regression	1	1,031	0,660	0,660	1,065	0,168
Neural networks	0,970	1	0,640	0,640	1,032	0,163
Visualization	1,515	1,563	1	1	1,613	0,255
Decision trees	1,515	1,563	1	1	1,613	0,255
K-nearest neighbor	0,939	0,969	0,620	0,620	1	0,158
					λ_{max}	5,000
					CI	0,000
					CR	0,000

The Radar (Spider) Chart above is relative and serves to compare the performance profiles of any five (Linear regression, Neural networks, Visualization, Decision trees, k-nearest neighbor) data mining methods to another (Linear regression, Neural networks, Visualization, Decision trees, k-nearest neighbor) based on any two different evaluation criteria (K_7 , K_8) (figure 3). The roles of each of these axes emanate from the center of the chart and correspond to one of the methods, while the distances along them represent the priority vector of the method under the given criterion. Such multidimensional representation allows making an assessment of the relative importance or the performance of the overall set of approaches [21]. The chart is drawn by plotting the priority vectors of each method on criteria K_7 (in red) and K_8 (in blue), then joining them to have a closed loop. The area inside each loop represents the graphical view of the overall priority with respect to the considered criteria of the methods and provides a way for one to compare the performances of the methods through visual means [22].

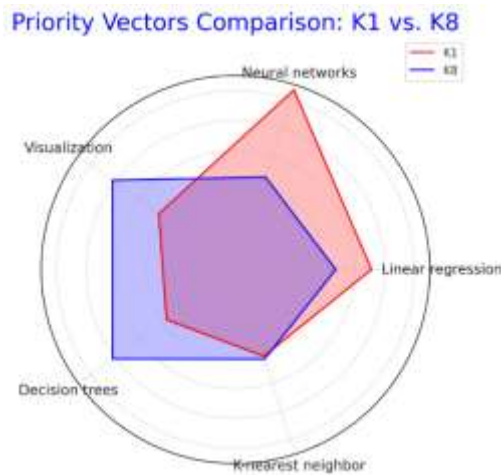


Figure 2: Priority Vectors Comparison: K_1 vs K_8

Comparatively, the Radar Chart provides an easy comparison since its viewers are able to make patterns and trade-offs between methods. For example, one can take a method with a higher enclosed area under a criterion as more favorable in that particular evaluative context. In case the area for one method on another criterion overlaps to a great extent and is more than the first, then it carries implications for the shift in relative importance/performance and, hence, shows that the selection of an appropriate data mining technique is multidimensional in nature.

Step 12. The next step is to apply the principle of synthesis. To identify composite, or global, method priorities in the matrix, local priorities are arranged relative to each criterion, each column of vectors is multiplied by the priority of the corresponding criterion, and the result is added along each row.

Next, from the \vec{Y}_d ($d = \overline{1, n}$) priority vectors, we will create a matrix

$$Y = (\vec{Y}_1, \vec{Y}_2, \dots, \vec{Y}_n) = \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{pmatrix}.$$

Let's define global priorities for independence:

$$\vec{GN} = \begin{pmatrix} GN_1 \\ GN_2 \\ \dots \\ GN_m \end{pmatrix} = Y \cdot \vec{X} = \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix},$$

where GN_i ($i = \overline{1, m}$) global priority of the i -option under independence.

In our case

$$\vec{GN} = \begin{pmatrix} 0,228 & 0,275 & 0,185 & 0,227 & 0,167 & 0,209 & 0,237 & 0,168 \\ 0,317 & 0,285 & 0,181 & 0,211 & 0,167 & 0,223 & 0,241 & 0,163 \\ 0,159 & 0,090 & 0,220 & 0,227 & 0,287 & 0,150 & 0,069 & 0,225 \\ 0,143 & 0,265 & 0,220 & 0,175 & 0,264 & 0,277 & 0,208 & 0,255 \\ 0,153 & 0,085 & 0,193 & 0,159 & 0,116 & 0,141 & 0,245 & 0,158 \end{pmatrix} \times \begin{pmatrix} 0,197 \\ 0,169 \\ 0,069 \\ 0,134 \\ 0,084 \\ 0,106 \\ 0,191 \\ 0,050 \end{pmatrix} = \begin{pmatrix} 0,224 \\ 0,243 \\ 0,158 \\ 0,216 \\ 0,159 \end{pmatrix}$$

As a result of the calculations, we obtain the following values (Table 14).

In this analysis, we apply the principle of synthesis to derive composite or global priorities of various data mining methods across multiple criteria. This approach involves aggregating local priorities, which are determined relative to each criterion, by multiplying each column of priority vectors by the priority of the corresponding criterion and then summing the results across each row. This methodological step facilitates a holistic evaluation of each data mining method's performance, reflecting a comprehensive integration of multiple evaluative dimensions.

The mathematical formulation for calculating global priorities is expressed as follows: $\overline{GN} = Y \cdot \vec{X}$, where Y is a matrix composed of priority vectors Y_d (for $d=1$ to n), and \vec{X} is a vector of criterion priorities. The element GN_i within the resulting vector \overline{GN} represents the global priority of the i^{th} option under the assumption of independence among criteria.

In the context of our case, the matrix Y encapsulates priority vectors across different criteria for each data mining method - Linear regression, Neural networks, Visualization, Decision trees, and K-nearest neighbor. The vector \vec{X} reflects the relative importance of each criterion. Upon performing the matrix multiplication, we derive a vector of global priorities \overline{GN} , signifying a synthesized measure of each method's overall suitability or effectiveness across all evaluated criteria.

The computed global priorities are as follows:

- Linear regression: 0.224
- Neural networks: 0.243
- Visualization: 0.158
- Decision trees: 0.216
- K-nearest neighbor: 0.159

These findings indicate that Neural networks exhibit the highest global priority, suggesting an overarching suitability across the evaluated criteria. This analysis underscores the utility of the synthesis principle in facilitating multidimensional decision-making processes, allowing for an integrated assessment of options across a spectrum of evaluative criteria. This integrative approach enables a nuanced understanding of the relative strengths and weaknesses of each data mining method, guiding informed and balanced decision-making in the selection of the most appropriate method for specific data analysis tasks.

Table 14: Results of global priorities

Criteria	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	Generalized or global priorities
Vector of priorities for each criterion	0,197	0,169	0,069	0,134	0,084	0,106	0,191	0,050	
Name of data mining methods	Priority vector of each data mining method								
Linear regression	0,228	0,275	0,185	0,227	0,167	0,209	0,237	0,168	0,224
Neural networks	0,317	0,285	0,181	0,211	0,167	0,223	0,241	0,163	0,243
Visualization	0,159	0,090	0,220	0,227	0,287	0,150	0,069	0,255	0,158
Decision trees	0,143	0,265	0,220	0,175	0,264	0,277	0,208	0,255	0,216
K-nearest neighbor	0,153	0,085	0,193	0,159	0,116	0,141	0,245	0,158	0,159

Let's define the maximum global priority under independence:

When analyzing the results, you can be convinced that the neural network method (0,243) is superior to all other data mining methods. This means using the neural network method to ensure more accurate decisions are made.

The following visual heatmap illustrates a multidimensional dataset measuring the performance level under different datamining methods with respect to evaluative criteria. In each row of a heatmap, it gives the type of datamining method employed, such as Linear regression, Neural networks, Visualization, Decision trees, K-nearest neighbor see in Figure 3.

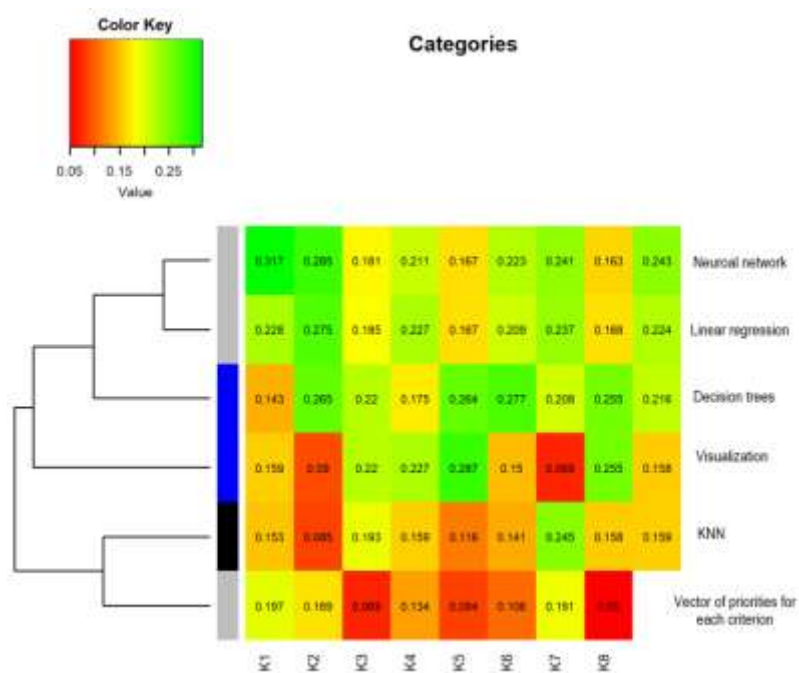


Figure 3: Heatmap of Priority Vectors for Data Mining Methods

Columns correspond to individual evaluative criteria (K1 through K8), respectively, with an additional column corresponding to generalized or global priorities synthesizing the performance across all criteria into a single composite metric. The color in the heatmap thus visualizes an encoding of the values of the priority vector with darker colors showing higher-priority items and lighter colors showing items with lower priority. This encoding makes at-a-glance comparisons of how each method ranks against the criteria and overall easy and intuitive, hence enabling stakeholders to quickly identify patterns, strengths, and weaknesses in the performance of the methods. That is, the map indicated the priority and superiority of the best method value when based on Neural networks. Accounting for the general effect that all criteria have, the Neural network prevails for all methods, proving its robustness and suitability to assure, among the evaluated scenarios, the occurrence of a more accurate decision-making process. This is an excellent example of the powerful power of heatmaps, summarizing complex multidimensional datasets in a comprehensible way that allows assessing the different options comprehensively in comparison to others against the set of criteria. That just underlines the point to which such analytic tools are useful for this domain in supporting informed, data-driven decision processes in this domain of method selection in data mining [23].

4. Conclusions

From this perspective, the new combination of the Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) discussed in the subsequent sections offers a sound theoretical framework in multi-criteria decision-making area to energy management. This fusion methodology of AHP combines in a logical way its strengths in qualitative assessment with the quantitative evaluation capabilities of TOPSIS, making it feasible for a comprehensive and wholesome approach to decision-making. The integration of these methodologies has allowed us to obtain an excellent and very flexible tool, which greatly supports the process of decision-making, especially within scenarios so complex and well-defined by many criteria, often conflicting. This fusion approach has already proved to have a clear evaluation and ranking mechanism of the energy management solutions, supporting the best choice, the most efficient and sustainable one. Further, the method synthesizes the evaluative dimensions into one coherent analytic framework, guaranteeing that, through this kind of analysis, decision-makers are able to exercise well-considered choices underpinned by a rigorous examination balancing many criteria. This further contributes to the theoretical and practical advances in decision-making research by presenting a model that would be helpful in adapting to diverse decision contexts other than energy management. These possibilities are due chiefly to the flexibility and scalability of this approach, which, as such, appears to hold great promise for researchers and practitioners looking to develop highly capable decision-support tools in their respective domains. In one word, fusion with AHP of TOPSIS has added more value to the toolkit of multi-criteria decision analysis and even provided a more dependable and efficient way of navigating the complexities involved in dealing with the modern energy management challenges. Thus, future research may seek further integration with other decision models or even this approach in diversified contexts in such a manner that its effectiveness is proved and adopted into other industries.

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