# Fusion of Preferences with Linguistic Weighted Power Mean Operator in Complex Decision-Making Environment

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**Abstract**

This article explores the application of the linguistic 2-tuple computational model in decision-making processes, focusing on its efficiency in managing ambiguous and imprecise linguistic information, which is vital in complex decision-making environments. The main objective is to demonstrate the use of the Weighted Power Mean (WPM) operator for hierarchical aggregation, highlighting its adaptability in reflecting the priority structures of specific problems and preserving the integrity of expert opinions. The model enhances user interaction by minimizing the need for complex numerical conversions, facilitating more intuitive decision-making. The study introduces the methodology of the linguistic 2-tuples, emphasizing their practical application in various decision-making contexts through detailed case studies. It elaborates on the hierarchical aggregation model, discussing the flexibility and potential of the WPM operator to adjust the influence of individual criteria based on their importance. The article also examines potential improvements in aggregation operators to increase their effectiveness and applicability across different scenarios. This comprehensive analysis not only underscores the capabilities of linguistic computational models in modern decision-making environments but also proposes future directions for advancing these techniques to handle increasingly complex information landscapes.

**Keywords:** Linguistic 2-tuples; Decision-making; Hierarchical aggregation; Weighted Power Mean (WPM); Computational models.

## 1. Introduction

The computational model of linguistic 2-tuples [1] has been successfully applied in decision-making problems. Due to its characteristics and ease of use, the authors of this research consider this model suitable for handling linguistic information for decision making. The objective of this work is to present a method for decision making based on the use of aggregation operators in a hierarchical form for the fusion of information. For this purpose, the weighted power mean (WPM) operator is used for hierarchical aggregation [2].

The increasing complexity in decision-making within modern environments demands approaches that can handle the ambiguity and imprecision inherent in human information. The fuzzy linguistic approach, particularly through linguistic 2-tuples, offers a robust solution by allowing decisions to be modeled more naturally in terms that are intuitively understandable to end-users. This approach not



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only improves the interaction between the model and the user by reducing the need for complex numerical translations but also maintains the integrity of the original meaning expressed by experts, which is crucial in scenarios where decisions directly affect people or policies [3].

Furthermore, the implementation of aggregation operators in decision-making provides a systematic framework that helps synthesize various data sources and preferences into a cohesive and justified outcome. The flexibility of these operators, especially the Weighted Power Mean (WPM), allows for adjusting the relative influence of each criterion according to its importance, thus offering an aggregation method that truly reflects the priority structure of the problem. This adaptability is essential for developing decision models that are both versatile and aligned with the specific objectives of each situation [3].

The remainder of the article delves deeper into the methodology behind the linguistic 2-tuples and their application in decision-making processes. It will present a detailed analysis of the hierarchical aggregation model, utilizing the Weighted Power Mean operator, and provide a case study that illustrate the practical benefits and challenges of implementing this approach. Furthermore, the article will discuss potential improvements and variations of the aggregation operators, aiming to enhance their applicability and effectiveness in diverse decision-making scenarios. This exploration serves to underline the robustness of linguistic models in handling complex, real-world problems where traditional numerical methods fall short.

## 2. Background

### 3.1 Fuzzy Linguistic Approach

Computing with words is a methodology that enables computational processes and reasoning using natural language words instead of numbers [4]. This facility for using language aids in the creation and enrichment of decision-making models in which vague and imprecise information [5] can be represented by linguistic variables.

Among the proposed models, the representation of linguistic 2-tuples [6] allows for word-based computing without information loss, utilizing the concept of symbolic translation. Let $S = \{s_0, s_1, \dots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$ a value within the granularity interval of $S$. The symbolic translation of a linguistic term, $s_i$, is a number valued in the interval $[-.5, .5)$ that expresses the information difference between a quantity of information expressed by the value $\beta \in [0, g]$, obtained in a symbolic operation, and the nearest integer value, $i \in \{0, \dots, g\}$, which indicates the index of the closest linguistic label $(s_i)$in S [7].

Building on this concept, a new model for the representation of linguistic information has been developed, utilizing a pair of values (2-tuples). This representation model defines a set of functions that facilitate operations on these 2-tuples.

Let $S = \{s_0, s_1, \dots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$a value representing the outcome of a symbolic operation, then the linguistic 2-tuple that expresses information equivalent to β is obtained using the function:

$$\Delta: [0, \mathrm{g}] \rightarrow \mathrm{S} \times [-.5, .5) \quad (1)$$

$$\Delta(\beta) = (\mathrm{s_i}, \alpha), \mathrm{con} \begin{cases} \mathrm{s_i}, & \mathrm{i} = \mathrm{round}(\beta) \\ \alpha = \beta - \mathrm{i}, & \alpha \in [-.5, .5) \end{cases}$$

Here, round is the standard rounding operator, $s_i$ is the label with the index closest to β, and α is the value of symbolic translation [7]. It is important to note that $\Delta^{-1}: \langle S \rangle \rightarrow [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. Thus, a linguistic 2-tuple ⟨S⟩ is identified with its numeric value in [0, g].

### 3.2 Linguistic Weighted Power Mean

Information aggregation is the process of combining various pieces of data to produce a single output. Aggregation operators are a type of mathematical function used for merging information. They combine $'n'$ values within a domain $'D'$ and return a value within the same domain [8].Aggregation



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operators have multiple applications across various fields [9]. In decision-making, their primary role is in the evaluation and development of alternatives [8].

Each family of operators has characteristics that allow them to model specific situations. For example, the Weighted Averaging (WA) operator allows assigning weights to information sources, which is useful for representing reliability or importance/preference. Meanwhile, the Ordered Weighted Averaging (OWA) operators [10] enable the adjustment of weights depending on the values of the data. Fuzzy integrals [11] can model redundancy, complementarity, and interactions between criteria. However, these operators are not entirely adequate for expressing properties found in human reasoning [12].

The Weighted Power Mean (WPM) aggregation operator expresses the degree of simultaneity and the relative importance of inputs (weights). It also enables the construction of hierarchical aggregation models [13]. The r-th WPM is defined as follows:

$$M_n^{[r]}(\underline{a}, \underline{w}) = (\sum_{i=1}^{n} a_i^r w_i)^{\frac{1}{r}} \quad (2)$$

where $w_i \in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$ y r can be selected to achieve desired logical properties. The Analytic Hierarchy Process (AHP) [14] can be used to determine the weights corresponding to each feature and sub-feature.

The WPM utilizes a hierarchical aggregation model known as the Logic Scoring of Preference (LSP) model [13]. One of the main strengths of the LSP model is that it can model various logical relationships between attributes and sub-features in a way that reflects the needs of different participants in the evaluation process. Table 1 displays the main operators.

Table 1: Values of Generalized Conjunction/Disjunction Functions

| **Polarization type** | **Polarization intensity** | **Symbol** | **r value** |
|---|---|---|---|
| **Disjunction** | The strongest | d | $+\infty$ |
| | Very strong | D++ | 20.63 |
| | Strong | D+ | 9.521 |
| | Medium Strong | D+- | 5.802 |
| | Medium | DA | 3.929 |
| | Medium Weak | D-+ | 2.792 |
| | Weak | D- | 2.018 |
| | Very weak | D-- | 1.449 |
| **Neutral** | | A | 1 |
| **Conjunction** | Very weak | C-- | 6.19 |
| | Weak | C- | 2.619 |
| | Medium Weak | C-+ | -0.148 |
| | Medium | AC | -0.72 |
| | Medium Strong | C+- | -1.655 |
| | Strong | C+ | -3.510 |
| | Very strong | C++ | -9.06 |
| | The strongest | C+++ | $-\infty$ |

The decision-maker can use two parameters in the aggregation process [13]:

- Degree of simultaneity (andness).
- Relative importance of the input (weights).

An interesting aspect of this operator is that it allows for the aggregation of information while taking into account which elements are mandatory and which are optional [15]. According to the author, all the elements previously addressed allow for a more realistic reflection of decision-making.

Let $X = \{(s_1, \alpha_1), \dots, (s_n, \alpha_n)\}$, a set of linguistic 2-tuples. The r-th Linguistic Weighted Power Mean (LWPM) is defined as follows[16] :

$$M_n^{[r]}((s_1, \alpha_1), \dots, (s_n, \alpha_n)) = (\sum_{i=1}^{n} \beta_i^r w_i)^{\frac{1}{r}} \quad (3)$$



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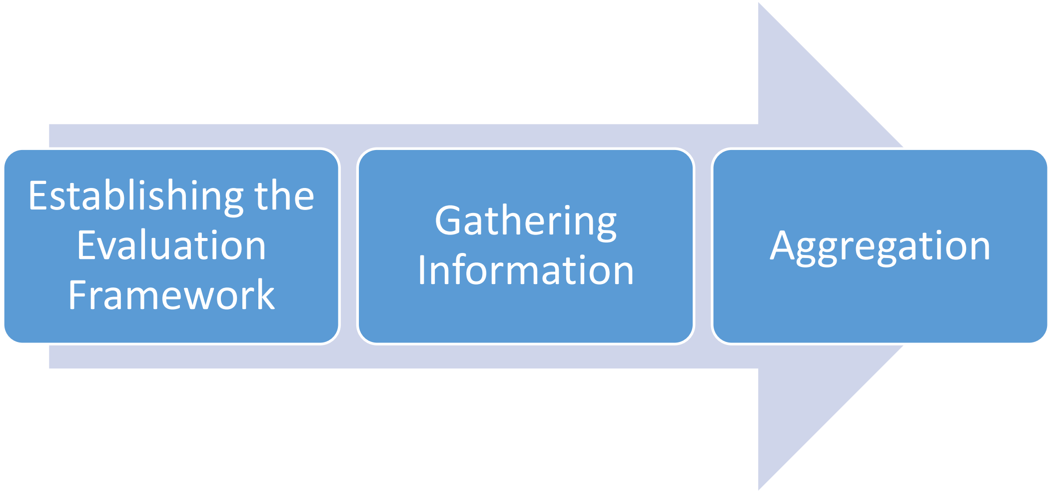
where $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$,and 'r' can be selected to achieve desired logical properties.

## 3. Methods

The proposed method includes activities such as selecting criteria and alternatives, gathering information, normalizing values, determining weight vectors, and aggregation. Below, these activities within the workflow are graphically presented (Figure 2) and described individually:




Workflow Activities for Decision-Making Prioritization:

**Establishing the Evaluation Framework:** Criteria and alternatives that will be evaluated are selected. Let $C = \{c_1, c_2, \ldots, c_k\}$ with $k \geq 2$ be the criteria to be evaluated, and $R = \{r_1, r_2, \ldots, r_j\}$ with $j \geq 2$ be the alternatives.

**Gathering Information**: Information about the preferences of the decision-makers is collected. This information represents the evaluation of each alternative according to the criteria. The utility vector [16] is represented as $V_j = \{v_{j1}, v_{j2}, \ldots, v_{jn}\}$, where $v_{jk}$ is the preference related to criterion $c_k$ of alternative $R_j$. Evaluations are given in linguistic terms S.

**Aggregation:** The aggregation function OAG: $[0,1]^n \rightarrow [0,1]$ is obtained through a hierarchical aggregation process. The Logic Scoring of Preference (LSP) model [13] is used because it realistically adjusts to the vocational guidance process. Using hierarchical aggregation operators adds flexibility to the method. The ability to directly obtain decision-maker preferences and their expression in weight vectors is another strength. For aggregation, the Linguistic Weighted Power Mean operator (LWPM) is used.

For aggregating preferences, we utilized the Logic Scoring of Preference (LSP) model coupled with the Linguistic Weighted Power Mean operator (LWPM), facilitating a flexible and realistic integration of decision-maker input through hierarchical aggregation. This methodological approach ensures a robust framework for effectively assessing alternatives in decision-making scenarios.

## 4. Case Study

In this case study Therefore, five alternatives to address the transgression of psychological integrity in minors during custody processes are proposed, along with their specific scopes and action plans to lay the groundwork for the design of policies and practices focused on protecting the rights of minors (see Table 4).



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Table 2: Alternatives to promote comprehensive development and protection of minors' rights in custody contexts.

| No. | Alternative | Description |
|---|---|---|
| **A1** | Improvement of communication between parents and professionals. | Implementation of workshops and training sessions for parents and professionals on effective communication techniques. |
| **A2** | Increase in emotional and psychological support for minors. | Development of emotional and psychological support programs in educational and community centers. |
| **A3** | Mediation programs to reduce parental conflict. | Establishment of free or subsidized family mediation services during custody processes. |
| **A4** | Streamlining of custody judicial procedures. | Reform of legal procedures to simplify and expedite custody cases, with an emphasis on the best interests of the child. |
| **A5** | Training and awareness for the involved professionals. | Mandatory courses and update programs for judges, lawyers, and social workers on child psychology and minors' rights. |

These proposed alternatives aim to effectively address the factors impacting the psychological integrity of minors during custody processes, promoting a safer and more supportive environment for their development and well-being. Criteria are selected (Table 3

Table 3: Criteria description.

| No | Criteria | Description |
|---|---|---|
| C1 | Effectiveness | Measures the degree to which a project achieves its intended outcomes. Effectiveness assesses the direct impact of the project's outputs on its ultimate goals. |
| C2 | Cost-benefit | Evaluates the economic efficiency of the project, comparing the total expected costs to the benefits that will be derived. This criterion helps determine whether a project provides good value for the investment made. |
| C3 | Viability | Assesses the project's likelihood of successful implementation considering current resources, capabilities, and environmental factors. It looks at the feasibility and practicality of the project. |
| C4 | Social impact | Examines the effects of the project on the community and society at large. This includes looking at who benefits from the project and how it affects various social groups. |
| C5 | Sustainability | Focuses on whether the project can maintain its output or benefits over time without exhausting the natural, social, or economic resources it relies on. It often includes environmental sustainability as well. |
| C6 | Time | Considers the duration required to complete the project and whether it fits within the necessary or available timelines. Time efficiency is crucial for projects with tight deadlines or those aligned with specific events or seasons. |

The attributes will be assessed using the following linguistic scale (Table 4). (Figure 2)

Table 4: Linguistic Scale

| No. | Labels | Membership Functions |
|---|---|---|
| $s_0$ | Very Low (VL) | (0.0, 0.0, 0.25) |
| $s_1$ | Low (L) | (0.0, 0.25, 0.50) |
| $s_2$ | Medium (M) | (0.25, 0.50, 0.75) |
| $s_3$ | High (H) | (0.50, 0.75, 1.0) |



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| | | |
|---|---|---|
| $s_4$ | Very High (VH) | (0.75, 1.0, 1.0) |

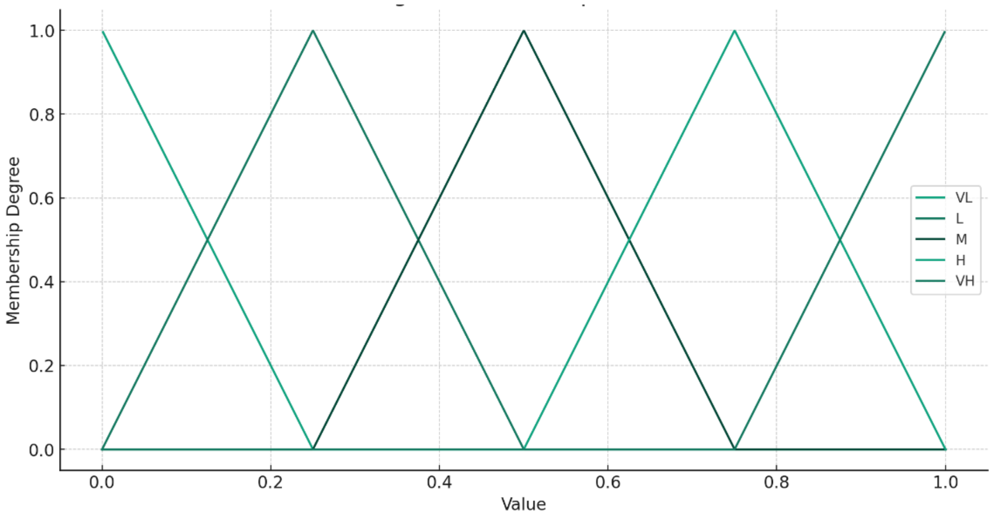


Figure 2: Set of labels used.

The weights of individual criteria are determined using the Analytic Hierarchy Process (AHP) [17] method. Among these criteria, effectiveness and cost-benefit are considered key factors in evaluating the alternatives.

Table 5: Criteria Weight Distribution Table

| | **C1** | **C2** | **C3** | **C4** | **C5** | **C6** |
|---|---|---|---|---|---|---|
| w | 0.29 | 0.40 | 0.10 | 0.05 | 0.06 | 0.10 |

Subsequently, the assessment is carried out for each requirement with respect to the selected criteria (Table 6).

Table 6: Valuation of Alternatives

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| A1 | Medium (M) | High (H) | Medium (M) | Low (L) | Low (L) | Medium (M) |
| A2 | High (H) | High (H) | Medium (M) | Low (L) | Low (L) | Medium (M) |
| A3 | High (H) | Very High (VH) | Medium (M) | Medium (M) | Medium (M) | Medium (M) |
| A4 | High (H) | Very High (VH) | Medium (M) | Low (L) | Medium (M) | High (H) |
| A5 | Medium (M) | Very High (VH) | Medium (M) | Low (L) | Medium (M) | High (H) |

The hierarchical aggregation structure obtained is shown below. Aggregation operators that reflect simultaneity as established by LSP [18, 19] were used.

Table 7: Aggregation Structure

| **Initial Entries** | **Operator** | | **Block ID** | **Operator** | | **Block ID** |
|---|---|---|---|---|---|---|
| C1 | 0.420 | C- | Cost-Effectiveness | 0,69 | C-- | Global Priority |
| C2 | 0.580 | | | | | |
| C3 | | | | 0.1 | | |



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| | | | | | | |
|---|---|---|---|---|---|---|
| C4 | 0.454 | C- | Sustainable Impact: | 0.11 | | |
| C5 | 0.546 | | | | | |
| C6 | | | | 0.1 | | |

Weight values of criteria are normalized during intermediate aggregation steps to ensure their sum equals one, meeting the requirements of the Weighted Power Mean (WPM) operator.

The results of the aggregation of the criteria allow for the ordering of the alternatives. In this case, the order of priority is as follows: A4 > A3 > A5 > A2 > A1

Table 8: Results of the aggregation.

| Proyecto | AG |
|---|---|
| A1 | ($s_3$,- 0.48) |
| A2 | ($s_3$,- 0.17) |
| A3 | ($s_3$, 0.43) |
| A4 | ($s_3$, 0.45) |
| A5 | ($s_3$, 0.29) |

Among the advantages pointed out by specialists are the relative ease of the technique and the high flexibility provided using the aggregation model employed. The results also show the applicability of decision support models based on information aggregation. Additionally, the linguistic output allows for greater interpretability of the model.

## 6. Conclusion

The article discusses the advanced use of computational models, specifically linguistic 2-tuples, in decision-making. These models allow for efficient handling of ambiguous and imprecise linguistic information, which is crucial in modern decision-making environments. The use of the Weighted Power Mean (WPM) operator for hierarchical aggregation is highlighted as particularly effective, adapting to the relative importance of each criterion to reflect the priority structure of specific problems. This approach not only improves the interaction between the model and the user by reducing the need for complex numerical translations but also preserves the integrity of the original meaning expressed by experts, which is fundamental in scenarios where decisions directly affect people or policies.

For future work, it is suggested to explore the expansion of these models into other complex decision domains where linguistic information plays a critical role, such as crisis management or urban planning. Additionally, investigating the integration of machine learning techniques to further refine the selection and weighting of criteria in aggregation models could result in even more accurate and adaptive decision support systems. Evaluating the efficacy of different aggregation operators across various contexts could also provide valuable insights into optimizing these models for specific needs.

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## References

[1] Malhotra, T., & Gupta, A. (2023). A systematic review of developments in the 2-tuple linguistic model and its applications in decision analysis. Soft Computing, 27(4), 1871-1905.Cordón, L.G.P., *Modelos de recomendación con falta de información. Aplicaciones al sector turístico*. 2008, Universidad de Jaén.

[2] Leyva Vazquez, M. Y., Quiroz Martinez, M. A., Diaz Sanchez, J. H., & Aguilera Balseca, J. L. (2021). Decision model for qos in networking based on hierarchical aggregation of information. In Advances in Artificial Intelligence, Software and Systems Engineering: Proceedings of the AHFE 2020 Virtual Conferences on Software and Systems Engineering, and Artificial Intelligence and Social Computing, July 16-20, 2020, USA (pp. 361-368). Springer International Publishing.

[3] Castro Arias, S. T., Quiroz Martinez, M. A., Leyva Vazquez, M. Y., & Flores Ortiz, A. C. (2021, June). Architectural Design for the Implementation of Learning Analytics: Case Study at Salesian Polytechnic University. In International Conference on Applied Human Factors and Ergonomics (pp. 260-268). Cham: Springer International Publishing.



Doi: https://doi.org/10.54216/FPA.160106
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[4] Teruel, K. P., Vázquez, M. Y. L., & Sentí, V. E. (2014). Consensus process in mental models and its application to agile software development in bioinformatics. Revista Cubana de Información en Ciencias de la Salud (ACIMED), 25(3), 317-331.

[5] Dutta, B., D. Guha, and R. Mesiar, *A model based on linguistic 2-tuples for dealing with heterogeneous relationship among attributes in multi-expert decision making.* Fuzzy Systems, IEEE Transactions on, 2015. **23**(5): p. 1817-1831.

[6] Herrera, F. and L. Martínez, *A 2-tuple fuzzy linguistic representation model for computing with words.* Fuzzy Systems, IEEE Transactions on, 2000. **8**(6): p. 746-752.

[7] Torra, V. and Y. Narukawa, *Modeling decisions: information fusion and aggregation operators*. 2007: Springer.

[8] Beliakov, G., A. Pradera, and T. Calvo, *Aggregation functions: a guide for practitioners*. 2007: Springer.

[9] Yager, R.R., *On ordered weighted averaging aggregation operators in multicriteria decisionmaking.* Systems, Man and Cybernetics, IEEE Transactions on, 1988. **18**(1): p. 183-190.

[10] Chigane, K., & Ouassaid, M. (2024). Experimental power control of a single-phase DC/AC converter using fuzzy integral sliding mode approach for photovoltaic systems. International Journal of Dynamics and Control, 12(2), 494-513.

[11] Dujmović, J.J., *Continuous preference logic for system evaluation.* Fuzzy Systems, IEEE Transactions on, 2007. **15**(6): p. 1082-1099.

[12] Dujmović, J.J. and H. Nagashima, *LSP method and its use for evaluation of Java IDEs.* International Journal of Approximate Reasoning, 2006. **41**(1): p. 3-22.

[13] Saaty, T.L., *What is the analytic hierarchy process?*, in *Mathematical models for decision support*. 1988, Springer. p. 109-121.

[14] Islam, M. R., Aziz, M. T., Alauddin, M., Kader, Z., & Islam, M. R. (2024). Site suitability assessment for solar power plants in Bangladesh: A GIS-based analytical hierarchy process (AHP) and multi-criteria decision analysis (MCDA) approach. Renewable Energy, 220, 119595.

[15] Al-Subhi, S. H. S., Mahdi, G. S. S., Alava, M. V., Pérez, M. P. Y. P., & Vázquez, M. L. (2017). Linguistic weighted power means and its application for decision-making. International Journal of Innovation and Applied Studies, 22(1), 38-43.

[16] Long, G., Hu, Y., Liu, G., Rui, Z., & Zhang, W. (2024, February). Research on Weight Determination Method of Cable Damage Index Based on Game Theory. In 2023 5th International Conference on Civil Architecture and Urban Engineering (ICCAUE 2023) (pp. 263-274). Atlantis Press.

[17] Castro, J., Laranjeiro, N., & Vieira, M. (2023, July). Exploring Logic Scoring of Preference for DoS Attack Detection in Microservice Applications. In 2023 IEEE International Conference on Web Services (ICWS) (pp. 573-584). IEEE.

[18] Leyva Vazquez, M. Y., Quiroz Martinez, M. A., Diaz Sanchez, J. H., & Aguilera Balseca, J. L. (2021). Decision model for qos in networking based on hierarchical aggregation of information. In Advances in Artificial Intelligence, Software and Systems Engineering: Proceedings of the AHFE 2020 Virtual Conferences on Software and Systems Engineering, and Artificial Intelligence and Social Computing, July 16-20, 2020, USA (pp. 361-368). Springer International Publishing.




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