Foundations of neutrosophic convex structures

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Abstract

In this paper an idea of neutrosophic convex structures (briefly, NC-structures) is given and some of their properties are explored. Also, NC-sets, neutrosophic concave sets and neutrosophic convex hull are defined and their properties are investigated. Moreover, the notions of NC-derived operator and NC-base are studied and their relationship to NC-structures are established.

Keywords: Neutrosophic set; NC-space; neutrosophic hull operator; NC-derived operator; NC-base

1 introduction

Convexity serves as a crucial and foundational characteristic across numerous branches of mathematics. But, some specific mathematical environments, such as vector spaces, are not the most suitable for studying the basic properties of convex sets. To avoid this problem, Van de Vel[17] introduced abstract convex structures (in short, convex structures) in terms of three axioms similar to those used to define topologies. Nowadays, convexity theory has become a branch of mathematics that deals with set-theoretic structures satisfying axioms similar to the usual convex sets. In fact, convex structures have appeared in research areas such as lattices,[16] graphs,[15] and topology.[18] Concretely, a collection $\mathcal{C}$ of subsets of a set X is a convex structure over X, if $\emptyset$ and X belong to $\mathcal{C}$, moreover $\mathcal{C}$ is closed under arbitrary intersections and is closed under unions of chains. In this case, the pair $(X, \mathcal{C})$ is said to be a convex space and the members of $\mathcal{C}$ are called convex sets. A convex structure is totally determined by a special type of operator that is analogous to the closure operator in topology, and is termed the convex hull operator, which is defined as the intersection of all convex sets containing a given subset of a convex space. Convex hull operators have been investigated not only from the generalized point of view of convex structures, but also in particular cases for finite point sets, simple polygons, Brownian motion, spatial curves and epigraphs of functions. Convex hull operators have wide applications in mathematics, statistics, combinatorial optimization, economics, geometric modeling and ethology. Because convex structures are defined similarly to topologies, the natural question arises that whether they can be characterized by derivative operators. To answer this question, Chen and Shen[5] introduced the notion of derivative operators on convex spaces, termed c-derivative operators, which are suitable in the study of both convex and antimatroid spaces.

On the other hand, neutrosophic set theory was established by Smarandache[12] in 1999 as a mathematical tool that generalizes the notions of fuzzy set and intuitionistic fuzzy set, being the best choice in situations where fuzzy set and fuzzy logic cannot express false membership information and intuitionistic fuzzy set and intuitionistic fuzzy logic are not able to handle information indeterminacy. While fuzzy set theory states that

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there is a degree of membership function for each level of participation component of an uncertainty problem and intuitionistic fuzzy set theory states that in addition to the degree of membership, there is also a degree of non-membership function, in neutrosophic logic the novelty is that a proposition not only has a degree of membership and a degree of non-membership, it also has a degree of indeterminacy. Neutrosophic set theory began to receive special attention starting in 2005, when Smarandache[13] applied neutrosophic sets as an approach to solve problems involving unreliable, indeterminate and persistent data present in decision making. To achieve this, Smarandache defined the neutrosophic set operations, namely complement, union, intersection, AND and OR, and study some of their properties. Since then, research work using neutrosophic set theory and its applications in various fields has progressed rapidly because this theory is more flexible and effective because it handles, in addition to independent components, also partially independent and partially dependent components, while traditional theoretical approaches cannot address these. Thus, research related to neutrosophic sets has developed in several directions among which we can mention: medical diagnosis,[1] database,[3] image processing,[4,7,19] and decision making problem.[9] In particular, with regard to general topology and its applications, concepts and results of neutrosophic topological spaces, neutrosophic separation axioms, neutrosophic countability axioms, neutrosophic continuous functions, neutrosophic covering properties, etc. have been established, as we can see in the references.[2,6,8,10] This theoretical framework has motivated us to carry out the present research, where we use concepts, operations and properties known in neutrosophic set theory to investigate certain aspects of the theory of convex structures by introducing concepts such as neutrosophic convex structure and neutrosophic convex space. We also present the notions of neutrosophic convex hull operator, neutrosophic hull operator, neutrosophic c-derived operator and neutrosophic convex base, to establish properties and characterizations of neutrosophic convex structures that involve these notions. Based on the ideas of revolutionary topologies Smarandache's,[14] in the Conclusion section, we propose new notions of neutrosophic convex structures that can be explored by the scientific community interested in neutrosophic set theory.

2 Neutrosophic sets

Throughout this paper, let X be a nonempty set, called the *universe of discourse*.

Definition 2.1. [12] A *neutrosophic set* B on X is an object of the form

$$B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\},$$

where $\mu_B, \sigma_B, \gamma_B$ are functions from X to $[0,1]$ and $0 \le \mu_B(x) + \sigma_B(x) + \gamma_B(x) \le 3$.

We denote by $\mathcal{N}(X)$ the collection of all neutrosophic sets over X.

Definition 2.2. [8] For $A, B \in \mathcal{N}(X)$ we define the following:

1. (Inclusion) B is called a *neutrosophic subset* of A, denoted by $B \sqsubseteq A$, if $\mu_B(x) \le \mu_A(x)$, $\sigma_B(x) \ge \sigma_A(x)$ and $\gamma_B(x) \ge \gamma_A(x)$ for all $x \in X$. Also, we can say that A is a neutrosophic super set of B.
2. (Equality) B is called *neutrosophic equal* to A, denoted by $B = A$, if $B \sqsubseteq A$ and $A \sqsubseteq B$.
3. (Universal set) B is called the *neutrosophic universal set*, denoted by $\widetilde{X}$, if $\mu_B(x) = 1, \sigma_B(x) = 0$ and $\gamma_B(x) = 0$ for all $x \in X$.
4. (Empty set) B is called the *neutrosophic empty set*, denoted by $\widetilde{\emptyset}$, if $\mu_B(x) = 0, \sigma_B(x) = 1$ and $\gamma_B(x) = 1$ for all $x \in X$.
5. (Intersection) The *neutrosophic intersection* of A and B, denoted by $A \sqcap B$, is defined as

$$A \sqcap B = \{(x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}.$$

6. (Union) The *neutrosophic union* of A and B, denoted by $A \sqcup B$, is defined as

$$A \sqcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}.$$

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7. (Complement) The *neutrosophic complement* of B, denoted by B^c, is defined as

$$B^c = \{\langle x, \gamma_B(x), 1-\sigma_B(x), \mu_B(x)\rangle : x \in X\}.$$

Proposition 2.3. [8] If $A, B, C \in \mathcal{N}(X)$, then we have the following properties:

1. $A \sqcap A = A$ and $A \sqcup A = A$.
2. $A \sqcap B = B \sqcap A$ and $A \sqcup B = B \sqcup A$.
3. $A \sqcap \tilde{\emptyset} = \tilde{\emptyset}$ and $A \sqcap \tilde{X} = A$.
4. $A \sqcup \tilde{\emptyset} = A$ and $A \sqcup \tilde{X} = \tilde{X}$.
5. $A \sqcap (B \sqcap C) = (A \sqcap B) \sqcap C$ and $A \sqcup (B \sqcup C) = (A \sqcup B) \sqcup C$.
6. $(A^c)^c = A$.

Proposition 2.4. [10] Let $A, B \in \mathcal{N}(X)$. Then, $A \sqsubseteq B$ if and only if $B^c \sqsubseteq A^c$.

The union and intersection operations given in Definition 2.2 can be extended as follows.

Definition 2.5. [11] For $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ we define the following operations:

1. (Arbitrary intersection) The *arbitrary neutrosophic intersection* of the collecction $\{A_j : j \in J\}$, denoted by ${}_{j\in J} A_j$, is defined as

$${}_{j\in J} A_j = \left\{\left\langle x, \inf_{j\in J} \mu_{A_j}(x), \sup_{j\in J} \sigma_{A_j}(x), \sup_{j\in J} \gamma_{A_j}(x)\right\rangle : x \in X\right\}.$$

2. (Arbitrary union) The *arbitrary neutrosophic union* of the collecction $\{A_j : j \in J\}$, denoted by $\bigsqcup_{j\in J} A_j$, is defined as

$$\bigsqcup_{j\in J} A_j = \left\{\left\langle x, \sup_{j\in J} \mu_{A_j}(x), \inf_{j\in J} \sigma_{A_j}(x), \inf_{j\in J} \gamma_{A_j}(x)\right\rangle : x \in X\right\}.$$

Proposition 2.6. [8] If $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ and $B \in \mathcal{N}(X)$, then we have the following properties:

1. $B \sqcap \left(\bigsqcup_{j\in J} A_j\right) = \bigsqcup_{j\in J} (B \sqcap A_j)$.
2. $B \sqcup ({}_{j\in J} A_j) =_{j\in J} (B \sqcup A_j)$.
3. $({}_{j\in J} A_j)^c = \bigsqcup_{j\in J} A_j^c$.
4. $\left(\bigsqcup_{j\in J} A_j\right)^c =_{j\in J} A_j^c$.

3 NC-structures and related neutrosophic operators

3.1 NC-spaces and neutrosophic convex hull operators

Definition 3.1. A collection $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is said to be:

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1. **Neutrosophic down-directed**, if for any pair $j_1, j_2 \in J$, there exists $j_3 \in J$ such that $A_{j_3} \sqsubseteq A_{j_1}$ and $A_{j_3} \sqsubseteq A_{j_2}$.
2. **Neutrosophic up-directed**, if for any pair $j_1, j_2 \in J$, there exists $j_3 \in J$ such that $A_{j_1} \sqsubseteq A_{j_3}$ and $A_{j_2} \sqsubseteq A_{j_3}$.

Proposition 3.2. *Let $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$. Then, the following properties are equivalent:*

1. *$\{A_j : j \in J\}$ is neutrosophic up-directed.*
2. *$\left\{A_j^c : j \in J\right\}$ is neutrosophic down-directed.*

Proof. The proof is straightforward by Definition 3.1 and Proposition 2.4. □

Definition 3.3. A collection $\mho \subseteq \mathcal{N}(X)$ is called a **neutrosophic convex structure** (briefly **NC-structure**) over X if $\mho$ satisfies the following conditions:

1. $\widetilde{\emptyset}$, $\widetilde{X}$ belong to $\mho$.
2. The intersection of each collection of members of $\mho$ belongs to $\mho$.
3. The union of each neutrosophic up-directed collection of members of $\mho$ belongs to $\mho$.

The pair $(X, \mho)$ is said to be **neutrosophic convex space** (briefly NC-space).

Definition 3.4. Let $(X, \mho)$ be a NC-space. The members of $\mho$ are called **neutrosophic convex sets** (briefly NC-sets) on X. A NC-set B on X is said to be a **neutrosophic concave set** on X, if B^c belongs to $\mho$.

Proposition 3.5. *Let $(X, \mho)$ be a NC-space. Then, the following properties hold:*

1. *$\widetilde{\emptyset}$, $\widetilde{X}$ are neutrosophic concave sets on X.*
2. *The union of each collection of neutrosophic concave sets on X is a neutrosophic concave set on X.*
3. *The union of any neutrosophic down-directed collection of neutrosophic concave sets on X is a neutrosophic concave set on X.*

Proof. Follows from Definitions 3.3 and 3.4 together with Proposition 3.2. □

Example 3.6. Let X be a non-empty set.

1. Consider $\mho_{dis} = \mathcal{N}(X)$. Then, $\mho_{dis}$ is called the discrete NC-structure over X and $(X, \mho_{dis})$ is called the discrete NC-space.
2. Let $\mho_{ind} \subseteq \mathcal{N}(X)$ be defined as $\mho_{ind} = \{\widetilde{\emptyset}, \widetilde{X}\}$. Then, $\mho_{ind}$ is called the indiscrete NC-structure over X and $(X, \mho_{ind})$ is called the indiscrete NC-space.

Example 3.7. Let $X = \{x, y, z\}$ and consider the following neutrosophic sets:

$$\begin{aligned}
A_1 &= \{\langle x, 0.1, 0, 0.9\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0, 0, 1\rangle\},\\
A_2 &= \{\langle x, 0, 0, 1\rangle, \langle y, 0.3, 0, 0.7\rangle, \langle z, 0, 0, 1\rangle\},\\
A_3 &= \{\langle x, 0, 0, 1\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0.4, 0, 0.6\rangle\},\\
A_4 &= \{\langle x, 0.6, 0, 0.4\rangle, \langle y, 0, 0, 1\rangle, \langle z, 0.8, 0, 0.2\rangle\}.
\end{aligned}$$

Then, the collection $\mho = \left\{\widetilde{\emptyset}, \widetilde{X}, A_1, A_2, A_3, A_4\right\}$ defines a NC-structure over X and hence $(X, \mho)$ is a NC-space. On the other hand, note that $\mho$ is not a neutrosophic topology over X, because $A_1, A_2 \in \mho$, but $A_1 \sqcup A_2 = \{\langle x, 0.1, 0, 0.9\rangle, \langle y, 0.3, 0, 0.7\rangle, \langle z, 0, 0, 1\rangle\} \notin \mho$.

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Definition 3.8. Let $(X, \mho)$ be a NC-space and $B \in \mathcal{N}(X)$. The **neutrosophic convex hull** of B, denoted by $Nco(B)$, is defined as

$$Nco(B) = \{G \in \mathcal{N}(X) : B \sqsubseteq G \text{ and } G \in \mho\}.$$

Note that $Nco(B)$ is the smallest element (in the sense of the neutrosophic inclusion) of $\mho$ that contains B; i.e., if $A \in \mho$ and $B \sqsubseteq A$, then $Nco(B) \sqsubseteq A$. The neutrosophic operator Nco is called the **neutrosophic convex hull operator** on $(X, \mho)$.

Proposition 3.9. *Let Nco be neutrosophic hull operator in a NC-space $(X, \mho)$ and $A, B \in \mathcal{N}(X)$. Then, the following properties hold:*

1. $Nco(\widetilde{\emptyset}) = \widetilde{\emptyset}$ ***(Neutrosophic normalization law)***,
2. $A \sqsubseteq Nco(A)$ ***(Neutrosophic extensive law)***,
3. *If* $A \sqsubseteq B$, *then* $Nco(A) \sqsubseteq Nco(B)$ ***(Neutrosophic monotone law)***.
4. $Nco(Nco(A)) = Nco(A)$ ***(Neutrosophic idempotent law)***,
5. *If* $\{A : j \in J\} \subseteq \mathcal{N}(X)$ *is a neutrosophic up-directed collection, then* $Nco\left(\bigsqcup_{j \in J} A_j\right) = \bigsqcup_{j \in J} Nco(A_j)$ ***(Neutrosophic up-directed additive law)***.
6. $A \in \mho$ *if and only if* $A = Nco(A)$.

Proof. (1) By definition, it always holds that $\widetilde{\emptyset} \sqsubseteq Nco(\emptyset)$. On the other hand, since $\widetilde{\emptyset} \sqsubseteq \widetilde{\emptyset}$ and $\widetilde{\emptyset} \in \mho$, it follows that $Nco(\widetilde{\emptyset}) \sqsubseteq \widetilde{\emptyset}$.
(2) This is an immediate consequence of the definition of $Nco(A)$.
(3) Suppose that $A \sqsubseteq B$. Then, $A \sqsubseteq B \sqsubseteq Nco(B)$ and as $Nco(B) \in \mho$, it follows that $Nco(A) \sqsubseteq Nco(B)$.
(4) By the neutrosophic extensive law, $Nco(A) \sqsubseteq Nco(Nco(A))$. On the other hand, since $Nco(A) \sqsubseteq Nco(A)$ and $Nco(A) \in \mho$, it follows that $Nco(Nco(A)) \sqsubseteq Nco(A)$.
(5) Suppose that $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is a neutrosophic up-directed collection. By the neutrosophic monotone law, $\{Nco(A_j : i \in J\}$ is also a neutrosophic up-directed collection and since $\{Nco(A_j : j \in J\} \subseteq \mho$, we have $\bigsqcup_{j \in J} Nco(A_j \in \mho$. Now, by the neutrosophic extensive law, $\bigsqcup_{j \in J} A_j \sqsubseteq \bigsqcup_{j \in J} Nco(A_j \in \mho$, which implies that $Nco\left(\bigsqcup_{j \in J} A_j\right) \sqsubseteq \bigsqcup_{j \in J} Nco(A_j$. The opposite neutrosophic inclusion is obviously obtained by the neutrosophic monotone law.
(6) By the neutrosophic extensive law, we always have $A \sqsubseteq Nco(A)$. For the opposite inclusion, suppose that $A \in \mho$. Since $A \sqsubseteq A$ and $A \in \mho$, we obtain that $Nco(A) \sqsubseteq A$. Conversely, suppose that $A = Nco(A)$. Since $Nco(A) \in \mho$, the proof is finished. □

Theorem 3.10. *Let $\gamma : \mathcal{N}(X) \to \mathcal{N}(X)$ be an application that satisfies the following properties:*

1. $\gamma(\widetilde{\emptyset}) = \widetilde{\emptyset}$ ***(Neutrosophic normalization law)***,
2. $A \sqsubseteq \gamma(A)$ ***(Neutrosophic extensive law)***,
3. *If* $A \sqsubseteq B$, *then* $\gamma(A) \sqsubseteq \gamma(B)$ ***(Neutrosophic monotone law)***,
4. $\gamma(\gamma(A)) = \gamma(A)$ ***(Neutrosophic idempotent law)***,
5. *If* $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ *is a neutrosophic upward directed collection, then* $\gamma\left(\bigsqcup_{j \in J} A_j\right) = \bigsqcup_{j \in J} \gamma(A_j)$ ***(Neutrosophic up-directed additive law)***.

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Then, the collection

$$\mho_\gamma = \{A \in \mathcal{N}(X) : \gamma(A) = A\}$$

is a NC-structure over X such that $\gamma(A) = Nco(A)$ for any $A \in \mathcal{N}(X)$.

Proof. We will check that the collection $\mho_\gamma = \{A \in \mathcal{N}(X) : \gamma(A) = A\}$ is a NC-structure over X:
(1) Obviously $\widetilde{\emptyset} \in \mho_\gamma$, because $\widetilde{\emptyset} \in \mathcal{N}(X)$ and $\gamma(\widetilde{\emptyset}) = \widetilde{\emptyset}$ by the neutrosophic normalization law. Since $\gamma(\widetilde{X}) \sqsubseteq \widetilde{X}$ and $\widetilde{X} \sqsubseteq \gamma(\widetilde{X})$ by the neutrosophic extensive law, we have $\gamma(\widetilde{X}) = \widetilde{X}$. Therefore, also $\widetilde{X} \in \mho_\gamma$.
(2) Suppose that $\{A_j : j \in J\} \subseteq \mho_\gamma$ and let $B = {}_{j\in J}A_j$. Then, $A_j \in \mathcal{N}(X)$ and $\gamma(A_j) = A_j$, for any $j \in J$. Observe that $B = {}_{j\in J}A_j \sqsubseteq A_k$ for any $k \in J$. By the neutrosophic monotone law, we have $\gamma(B) \sqsubseteq \gamma(A_k) = A_k$ for any $k \in J$. Thus, $\gamma(B) \sqsubseteq {}_{j\in J}A_j = B$. By the neutrosophic extensive law, we obtain that $B \sqsubseteq \gamma(B)$.Therefore, $\gamma(B) = B$ and so, $B \in \mho_\gamma$.
(3) Suppose that $\{A_j : j \in J\} \subseteq \mho_\gamma$ is a neutrosophic up-directed collection. Then, $A_j \in \mathcal{N}(X)$ and $\gamma(A_j = A_j$ for any $j \in J$. Now, as $\bigsqcup_{j\in J} A_j \in \mathcal{N}(X)$, by the neutrosophic up-directed additive law, we get that $\gamma\left(\bigsqcup_{j\in J} A_j\right) = \bigsqcup_{j\in J} \gamma(A_j) = \bigsqcup_{j\in J} A_j$ and hence, $\bigsqcup_{j\in J} A_j \in \mho_\gamma$.

This shows that $\mho_\gamma$ is a NC-structure over X. Now, we will show that with respect to this, $\gamma(A) = Nco(A)$ for any $A \in \mathcal{N}(X)$. Observe that $\gamma(A) \in \mho_\gamma$, because by the neutrosophic idempotent law, $\gamma(\gamma(A)) = \gamma(A)$. Moreover, by the neutrosophic extensive law, $A \sqsubseteq \gamma(A)$, and since $\gamma(A) \in \mho_\gamma$, it follows that $Nco(A) \sqsubseteq \gamma(A)$. On the other hand, given that $Nco(A) \in \mho_\gamma$ and $A \sqsubseteq Nco(A)$, we have $\gamma(A) \sqsubseteq \gamma(Nco(A)) = Nco(A)$. Therefore, $\gamma(A) = Nco(A)$ whenever $A \in \mathcal{N}(X)$. □

We will call a **neutrosophic hull operator** to any application $\gamma : \mathcal{N}(X) \to \mathcal{N}(X)$ that satisfies properties (1)-(5) of Theorem 3.10.

3.2 NC-derived operators and NC-derived spaces

Definition 3.11. An application $Nd : \mathcal{N}(X) \to \mathcal{N}(X)$ is called a **neutrosophic convexly derived operator** (briefly **NC-derived operator**) over X, if the following conditions are satisfied:

1. $Nd(\widetilde{\emptyset}) = \widetilde{\emptyset}$ (**Neutrosophic normalization law**),
2. $A \sqsubseteq B$ implies $Nd(A) \sqsubseteq Nd(B)$ (**Neutrosophic monotone law**),
3. $Nd(Nd(A) \sqcup A) \sqsubseteq Nd(A) \sqcup A$ (**Neutrosophic idempotent law**),
4. If $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is a neutrosophic up-directed collection, then $Nd\left(\bigsqcup_{j\in J} A_j\right) = \bigsqcup_{j\in J} Nd(A_j)$ (**Neutrosophic up-directed additive law**).

If Nd is a NC-derived operator over X, then the pair (X, Nd) is called a **neutrosophic convexly derived space** (briefly **NC-derived space**).

Proposition 3.12. *Let Nd be a NC-derived operator over X. Then, the collection $\mho_d \subseteq \mathcal{N}(X)$ defined by*

$$\mho_d = \{A \in \mathcal{N}(X) : Nd(A) \sqsubseteq A\},$$

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*is a NC-structure over X, called the **NC-structure induced by** Nd. In addition, $\mho_d = \{Nd(A) \sqcup A : A \in \mathcal{N}(X)\}$.*

Proof. We will verify that the collection $\mho_d$ is a NC-structure over X:
(1) Obviously $\widetilde{\emptyset} \in \mho_d$, because $\widetilde{\emptyset} \in \mathcal{N}(X)$ and $Nd(\widetilde{\emptyset}) = \widetilde{\emptyset}$ by the neutrosophic normalization law. Since $\widetilde{X} \in \mathcal{N}(X)$ and $Nd(\widetilde{X}) \sqsubseteq \widetilde{X}$, also $\widetilde{X} \in \mho_d$.
(2) Suppose that $\{A_j : j \in J\} \subseteq \mho_d$ and let $B = {}_{j\in J}A_j$. Then, $A_j \in \mathcal{N}(X)$ and $Nd(A_j) \sqsubseteq A_j$ for any $j \in J$. Observe that $B \in \mathcal{N}(X)$ and $B = {}_{j\in J}A_j \sqsubseteq A_k$ for any $k \in J$. By the neutrosophic monotone law, we have $Nd(B) \sqsubseteq Nd(A_k) \sqsubseteq A_k$ for any $k \in J$. Thus, $Nd(B) \sqsubseteq {}_{j\in J}A_j = B$. Therefore, $B \in \mho_d$.
(3) Suppose that $\{A_j : j \in J\} \subseteq \mho_d$ is a neutrosophic up-directed collection and let $B = \bigsqcup_{j\in J} A_j$. Then, $A_j \in \mathcal{N}(X)$ and $Nd(A_j) \sqsubseteq A_j$ for any $j \in J$. Thus, $B = \bigsqcup_{j\in J} A_j \in \mathcal{N}(X)$ and by the neutrosophic up-directed additive law

$$Nd(B) = Nd\left(\bigsqcup_{j\in J} A_j\right) = \bigsqcup_{j\in J} Nd(A_j) \sqsubseteq \bigsqcup_{j\in J} A_j = B.$$

Therefore, $B \in \mho_d$.

This shows that $\mho_d$ is a NC-structure over X. Now, we will show that $\mho_d = \{Nd(A) \sqcup A : A \in \mathcal{N}(X)\}$. For convenience we will denote $\mho_d^\diamond = \{Nd(A) \sqcup A : A \in \mathcal{N}(X)\}$. By the neutrosophic idempotent law, we have $Nd(A \sqcup Nd(A)) \sqsubseteq A \sqcup Nd(A)$ for any $A \in \mathcal{N}(X)$, which implies that $\mho_d^\diamond \subseteq \mho_d$. Also, if $A \in \mho_d$, then $Nd(A) \sqsubseteq A$ and so, $A = A \sqcup Nd(A) \in \mho_d^\diamond$. Therefore, $\mho_d = \mho_d^\diamond$. □

Proposition 3.13. *Let Nd be a NC-derived operator over X, $\mho_d \subseteq \mathcal{N}(X)$ be the NC-structure induced by Nd and Nco_d be the neutrosophic convex hull in $\mho_d$. Then, $Nco_d(A) = Nd(A) \sqcup A$ for any $A \in \mathcal{N}(X)$.*

Proof. Since $A \sqsubseteq Nd(A) \sqcup A$ and $Nd(A) \sqcup A \in \mho_d$, by the neutrosophic monotone law of Nco_d, we have $Nco_d(A) \sqsubseteq Nco_d(Nd(A) \sqcup A) = Nd(A) \sqcup A$. On the other hand, if $B \in \mho_d$ and $A \sqsubseteq B$, then $Nd(B) \sqsubseteq B$ and $Nd(A) \sqsubseteq Nd(B)$, which implies that $Nd(A) \sqcup A \sqsubseteq Nd(B) \sqcup B = B$. Now, putting $B = Nco_d(A)$, we get that $Nd(A) \sqcup A \sqsubseteq Nco_d(A)$. Therefore, $Nco_d(A) = Nd(A) \sqcup A$. □

3.3 NC-base spaces

Definition 3.14. A collection $\mathfrak{B} \subseteq \mathcal{N}(X)$ is a **neutrosophic convex base** (briefly **NC-base**) over X, if the following conditions are satisfied:

(B1) $\widetilde{X} = \bigsqcup_{j\in J} A_j$ for some neutrosophic up-directed collection $\{A_j : j \in J\} \subseteq \mathfrak{B}$.

(B2) For any collection $\{A_j : j \in J\} \subseteq \mathfrak{B}$, there exists a neutrosophic up-directed collection $\{B_k : k \in K\} \subseteq \mathfrak{B}$ such that ${}_{j\in J}A_j = \bigsqcup_{k\in K} B_k$.

(B3) If $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is a neutrosophic up-directed collection and $\{B_{jk} : k \in K_j\} \subseteq \mathfrak{B}$ is a neutrosophic up-directed collection such that $A_j = \bigsqcup_{k\in K_j} B_{jk}$ for any $j \in J$, then there exists a neutrosophic up-directed collection $\{B_i : i \in I\} \subseteq \mathfrak{B}$ such that $\bigsqcup_{j\in J} A_j = \bigsqcup_{i\in I} B_i$.

If $\mathfrak{B}$ is a NC-base over X, then the pair $(X, \mathfrak{B})$ is called a **neutrosophic convex base space** (briefly **NC-base space**).

In the following theorem, we will represent by $\mathfrak{U}(\mathfrak{B})$ the set of all neutrosophic up-directed collections contained in a NC-base $\mathfrak{B}$ over X.

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Theorem 3.15. *Let $\mathfrak{B}$ be a NC-base over X. Then, the collection $\mho_{\mathfrak{B}} \subseteq \mathcal{N}(X)$ defined by*

$$\mho_{\mathfrak{B}} = \left\{ A \in \mathcal{N}(X) : A = \bigsqcup_{j\in J} A_j \text{ for some collection } \{A_j : j \in J\} \in \mathfrak{U}(\mathfrak{B}) \right\},$$

is a NC-structure over X, called the ***NC-structure generated by*** $\mathfrak{B}$.

Proof. We will verify that the collection $\mho_{\mathfrak{B}}$ is a NC-structure over X:
(1) Clearly $\widetilde{X} \in \mho_{\mathfrak{B}}$ by (B1). Since the empty subcollection of $\mathfrak{B}$ is neutrosophic up-directed and $\widetilde{\emptyset}$ is the union of the empty subcollection of $\mathfrak{B}$, we conclude that $\widetilde{\emptyset} \in \mho_{\mathfrak{B}}$.
(2) Assume that $\{A_j : j \in J\} \subseteq \mho_{\mathfrak{B}}$. Then, $A_j \in \mathcal{N}(X)$ and there exists a neutrosophic up-directed collection $\{B_{jk} : k \in K_j\} \subseteq \mathfrak{B}$ such that $A_j = \bigsqcup_{k\in K_j} B_{jk}$ for any $j \in J$. Thus,

$${}_{j\in J}A_i = {}_{j\in J} \bigsqcup_{k\in K_j} B_{jk} = \bigsqcup_{f\in\prod_{j\in J} K_j} {}_{j\in J}B_{jf(j)}.$$

By (B2), it follows that for any $f \in \prod_{j\in J} K_j$, there exists a neutrosophic up-directed collection $\{B_{jk} : k \in K_j\} \subseteq \mathfrak{B}$ such that ${}_{j\in J}B_{jf(j)} = \bigsqcup_{k\in K_j} B_{jk}$. We will show that $\left\{ {}_{j\in J}B_{jf(j)} : f \in \prod_{j\in J} K_j \right\}$ is a neutrosophic up-directed collection. Indeed, if $f, g \in \prod_{j\in J} K_j$, then $f(j)$ and $g(j)$ belong to K_j, $B_{jf(j)}$ and $B_{jg(j)}$ belong to $\{B_{jk} : k \in K_j\}$ for any $j \in J$. Since $\{B_{jk} : k \in K_j\}$ is a neutrosophic up-directed collection, there exists $B_{jk_j} \in \{B_{jk} : k \in K_j\}$ such that $B_{jf(j)} \sqsubseteq B_{jk_j}$ and $B_{jg(j)} \sqsubseteq B_{jk_j}$. Let us define the function $h : J \to \bigcup_{j\in J} K_j$ by $h(j) = jk_j$ for any $j \in J$. Thus, $h \in \prod_{j\in J} K_j$, $\bigsqcup_{j\in J} B_{jf(j)} \sqsubseteq \bigsqcup_{j\in J} B_{jh(j)}$ and $\bigsqcup_{j\in J} B_{jg(j)} \sqsubseteq \bigsqcup_{j\in J} B_{jh(j)}$. This shows that $\left\{ {}_{j\in J}B_{jf(j)} : f \in \prod_{j\in J} K_j \right\}$ is a neutrosophic up-directed collection. Now, by (B3), there exists a neutrosophic up-directed collection $\mathfrak{B}' \subseteq \mathfrak{B}$ such that

$${}_{j\in J}A_j = \bigsqcup_{f\in\prod_{j\in J} K_j} {}_{j\in J}B_{jf(j)} = \bigsqcup_{B\in\mathfrak{B}'} B,$$

which implies that ${}_{j\in J}A_j \in \mho_{\mathfrak{B}}$.
(3) Assume that $\{A_j : j \in J\} \subseteq \mho_{\mathfrak{B}}$ is a neutrosophic up-directed collection. Then, for all $j \in J$, $A_j \in \mathcal{N}(X)$ and there exists a neutrosophic up-directed collection $\{B_{jk} : k \in K_j\} \subseteq \mathfrak{B}$ such that $A_j = \bigsqcup_{k\in K_j} B_{jk}$. Thus, by (B3), there exists a neutrosophic up-directed collection $\{B_i : i \in I\} \subseteq \mathfrak{B}$ such that

$$\bigsqcup_{j\in J} A_i = \bigsqcup_{i\in I} B_i.$$

Therefore, $\bigsqcup_{j\in J} A_i \in \mho_{\mathfrak{B}}$.

From the above, we get that $\mho_{\mathfrak{B}}$ is a NC-structure over X. □

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4 Preservation under neutrosophic functions

Definition 4.1. Let $(X, \mho_X)$ and $(Y, \mho_Y)$ be two NC-spaces. A function $f : X \to Y$ is called:

1. **Neutrosophic convex preserving** (briefly **NCP**), if $f^{-1}(B) \in \mho_X$ for any $B \in \mho_Y$.
2. **Neutrosophic convex to convex** (briefly **NCC**), if $f(A) \in \mho_Y$ for any $A \in \mho_X$.

We will use the notation $f : (X, \mho_X) \to (Y, \mho_Y)$ to represent a neutrosophic function between two NC-spaces $(X, \mho_X)$ and $(Y, \mho_Y)$.

Henceforth, given $A \in \mathcal{N}(X)$, put $\mathcal{P}(A) = \{B \in \mathcal{N}(X) : B \sqsubseteq A\}$.

Theorem 4.2. *Let $f : (X, \mho_X \to (Y, \mho_Y)$ be a neutrosophic function between two NC-spaces. Then, the following statements are equivalent:*

1. *f is NCP.*
2. *If $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is a neutrosophic up-directed collection, then* $f\left(Nco_X\left(\bigsqcup_{j\in J} A_j\right)\right) \sqsubseteq \bigsqcup_{j\in J} Nco_Y(f(A_j)$.
3. *$f(Nco_X(A) \sqsubseteq Nco_Y(f(A))$ for any $A \in \mathcal{N}(X)$.*

Proof. (1) $\Longrightarrow$ (3). Assume that $A \in \mathcal{N}(X)$ and let $B \in \mho_Y$ such that $f(A) \sqsubseteq B$. Since f is NCP, we have $f^{-1}(B) \in \mho_X$ and as $A \sqsubseteq f^{-1}(f(A)) \sqsubseteq f^{-1}(B)$, it follows that $Nco_X(A) \sqsubseteq f^{-1}(B)$. Thus, $f(Nco_X(A)) \sqsubseteq f(f^{-1}(B)) \sqsubseteq B$. Since B is an arbitrary neutrosophic $\mho_Y$-convex superset of $f(Nco_X(A))$, we conclude that $f(Nco_X(A)) \sqsubseteq \{B \in \mho_Y : f(A) \sqsubseteq B\} = Nco_Y(f(A))$.

(3) $\Longrightarrow$ (1). Let $B \in \mho_Y$. Then, $f^{-1}(B) \in \mathcal{N}(X)$. By the hypothesis and the neutrosophic monotone law, we have $f(Nco_X(f^{-1}(B))) \sqsubseteq Nco_Y(f(f^{-1}(B))) \sqsubseteq Nco_Y(B) = B$ and so, $Nco_X(f^{-1}(B)) \sqsubseteq f^{-1}(f(Nco_X(f^{-1}(B)))) \sqsubseteq f^{-1}(B)$. On the other hand, since the neutrosophic inclusion $f^{-1}(B) \sqsubseteq Nco_X(f^{-1}(B))$ is always true, we conclude that $Nco_X(f^{-1}(B)) = f^{-1}(B)$ and hence, $f^{-1}(B) \in \mho_X$, which proves that f is NCP.

(2) $\Longrightarrow$ (3). Let $A \in \mathcal{N}(X)$. Using the fact that $\{f(B) : B \in \mathcal{P}(A)\}$ is a neutrosophic up-directed collection, we have

$$\begin{aligned} f(Nco_X(A)) &= f\left(Nco_X\left(\bigsqcup\{B : B \in \mathcal{P}(A)\}\right)\right) \\ &\sqsubseteq \bigsqcup\{Nco_Y(f(B)) : B \in \mathcal{P}(A)\} \\ &= Nco_Y\left(\bigsqcup\{f(B) : B \in \mathcal{P}(A)\}\right) \\ &= Nco_Y\left(f\left(\bigsqcup\{B : B \in \mathcal{P}(A)\}\right)\right) \\ &= Nco_Y(f(A)). \end{aligned}$$

(3) $\Longrightarrow$ (2). Let $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ be a neutrosophic up-directed collection. Then,

$$\begin{aligned} f\left(Nco_X\left(\bigsqcup_{j\in J} A_j\right)\right) &= f\left(\bigsqcup_{j\in J} Nco_X(A_j)\right) \\ &= \bigsqcup_{j\in J} f(Nco_X(A_j)) \\ &\sqsubseteq \bigsqcup_{j\in J} Nco_Y(f(A_j)). \end{aligned}$$

□

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Proposition 4.3. *Let $f : (X, \mho_X) \to (Y, \mho_Y)$ and $g : (Y, \mho_Y) \to (Z, \mho_Z)$ be two neutrosophic functions between NC-spaces. If f and g are NCP, then $g \circ f$ is also NCP.*

Theorem 4.4. *Let $f : (X, \mho_X) \to (Y, \mho_Y)$ be a neutrosophic function between two NC-spaces. Then, the following properties are equivalent:*

1. *f is NCC.*
2. *If $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ is a neutrosophic up-directed collection, then* $\bigsqcup_{j\in J} Nco_Y(f(A_j)) \sqsubseteq f\left(Nco_X\left(\bigsqcup_{j\in J} A_j\right)\right)$.
3. *$Nco_Y(f(A)) \sqsubseteq f(Nco_X(A))$ for any $A \in \mathcal{N}(X)$.*

Proof. (1) $\Longrightarrow$ (3). Let $A \in \mathcal{N}(X)$. Then, $Nco_X(A) \in \mho_X$ and as f is NCC, we have $f(Nco_X(A)) \in \mho_Y$, which implies that $Nco_Y(f(Nco_X(A))) \sqsubseteq f(Nco_X(A))$. By the neutrosophic extensive and neutrosophic monotone laws, we get that $Nco_Y(f(A)) \sqsubseteq Nco_Y(f(Nco_X(A))) \sqsubseteq f(Nco_X(A))$.

(3) $\Longrightarrow$ (1). Let $A \in \mho_X$. Then, $A \in \mathcal{N}(X)$ and $Nco_X(A) = A$. By hypothesis, we have $Nco_Y(f(A)) \sqsubseteq f(Nco_X(A)) = f(A)$ and so, $f(A) \in \mho_Y$, which shows that f is NCC.

(2) $\Longrightarrow$ (3). Let $A \in \mathcal{N}(X)$. Since $\{f(B) : B \in \mathcal{P}(A)\}$ is a neutrosophic up-directed collection, we have

$$\begin{aligned} Nco_X(f(A)) &= Nco_X\left(f\left(\bigsqcup\{B : B \in \mathcal{P}(A)\}\right)\right) \\ &= Nco_X\left(\bigsqcup\{f(B) : B \in \mathcal{P}(A)\}\right) \\ &= \bigsqcup\{Nco_X(f(B)) : B \in \mathcal{P}(A)\} \\ &\sqsubseteq f\left(Nco_Y\left(\bigsqcup\{B : B \in \mathcal{P}(A)\}\right)\right) \\ &= f(Nco_Y(A)). \end{aligned}$$

(3) $\Longrightarrow$ (2). Let $\{A_j : j \in J\} \subseteq \mathcal{N}(X)$ be a neutrosophic up-directed collection. Then,

$$\begin{aligned} \bigsqcup_{j\in J} Nco_Y(f(A_j)) &\sqsubseteq \bigsqcup_{j\in J} f(Nco_X(A_j)) \\ &= f\left(\bigsqcup_{j\in J} Nco_X(A_j)\right) \\ &= f\left(Nco_X\left(\bigsqcup_{j\in J} A_j\right)\right). \end{aligned}$$

□

Proposition 4.5. *Let $f : (X, \mho_X) \to (Y, \mho_Y)$ and $g : (Y, \mho_Y) \to (Z, \mho_Z)$ be two neutrosophic functions between NC-spaces. If f and g are NCC, then $g \circ f$ is also NCC.*

Definition 4.6. Let (X, Nd_X) and (Y, Nd_Y) be two NC-derived spaces. A function $f : X \to Y$ is called **neutrosophic c-derived preserving** (briefly **NDP**), if

$$f(Nd_X(A)) \sqsubseteq f(A) \sqcup Nd_Y(f(A)) \text{ for all } A \in \mathcal{N}(X).$$

We will use the notation $f : (X, Nd_X) \to (Y, Nd_Y)$ to represent a neutrosophic function between two NC-derived spaces (X, Nd_X) and (Y, Nd_Y).

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Proposition 4.7. *Let $f : (X, Nd_X) \to (Y, Nd_Y)$ and $g : (Y, Nd_Y) \to (Z, Nd_Z)$ be two neutrosophic functions between NC-derived spaces. If f and g are NDP, then $g \circ f$ is also NDP.*

Proof. Let $A \in \mathcal{N}(X)$. Since $f : (X, Nd_X) \to (Y, Nd_Y)$ is NDP, we have

$$f(Nd_X(A)) \sqsubseteq f(A) \sqcup Nd_Y(f(A))$$

and so,

$$\begin{aligned}(g \circ f)(Nd_X(A)) &= g(f(Nd_X(A))) \\ &\sqsubseteq g(f(A) \sqcup Nd_Y(f(A))) \\ &= g(f(A)) \sqcup g(Nd_Y(f(A))) \\ &= (g \circ f)(A) \sqcup g(Nd_Y(f(A))).\end{aligned}$$

Since $g : (Y, Nd_Y) \to (Z, Nd_Z)$ is NDP and $f(A) \in \mathcal{N}(Y)$, we have

$$g(Nd_Y(f(A)) \sqsubseteq g(f(A)) \sqcup Nd_Z(g(f(A))) = (g \circ f)(A) \sqcup Nd_Z((g \circ f)(A)),$$

which implies that

$$\begin{aligned}(g \circ f)(Nd_X(A)) &\sqsubseteq (g \circ f)(A) \sqcup g(Nd_Y(f(A))) \\ &\sqsubseteq (g \circ f)(A) \sqcup Nd_Z((g \circ f)(A)).\end{aligned}$$

This shows that $g \circ f$ is NDP. □

Proposition 4.8. *Let (X, Nd_X) and (Y, Nd_Y) be two NC-derived spaces. If $f : (X, Nd_X) \to (Y, Nd_Y)$ is NDP, then $f : (X, \mho_{d_X}) \to (Y, \mho_{d_Y})$ is NCP.*

Proof. Suppose that $f : (X, Nd_X) \to (Y, Nd_Y)$ is NDP and let $A \in \mathcal{N}(X)$. Then, $f(Nd_X(A)) \sqsubseteq f(A) \sqcup Nd_Y(f(A))$ and by Proposition 3.13, we have $Nco_{d_X}(A) = Nd_X(A) \sqcup A$ and $Nco_{d_Y}(f(A)) = Nd_Y(f(A)) \sqcup f(A)$. Thus, $f(Nco_{d_X}(A)) = f(Nd_X(A) \sqcup A) = f(Nd_X(A)) \sqcup f(A) \sqsubseteq f(A) \sqcup Nd_Y(f(A)) = Nco_{d_Y}(f(A))$ and by Theorem 4.2, we get that $f : (X, \mho_{d_X}) \to (Y, \mho_{d_Y})$ is NCP. □

Definition 4.9. Let $(X, \mathfrak{B}_X)$ and $(Y, \mathfrak{B}_Y)$ be two NC-base spaces. A function $f : X \to Y$ is called **neutrosophic c-base preserving** (briefly **NBP**), if $f^{-1}(A) \in \mathfrak{B}_X$ for any $A \in \mathfrak{B}_Y$.

We will use the notation $f : (X, \mathfrak{B}_X) \to (Y, \mathfrak{B}_Y)$ to represent a neutrosophic function between two NC-base spaces $(X, \mathfrak{B}_X)$ and $(Y, \mathfrak{B}_Y)$.

Proposition 4.10. *Let $f : (X, \mathfrak{B}_X) \to (Y, \mathfrak{B}_Y)$ and $g : (Y, \mathfrak{B}_Y) \to (Z, \mathfrak{B}_Z)$ be two neutrosophic functions between NC-base spaces. If f and g are NBP, then $g \circ f$ is also NBP.*

Proposition 4.11. *Let $(X, \mathfrak{B}_X)$ and $(Y, \mathfrak{B}_Y)$ be two NC-base spaces. If $f : (X, \mathfrak{B}_X) \to (Y, \mathfrak{B}_Y)$ is NBP, then $f : (X, \mho_{\mathfrak{B}_X}) \to (Y, \mho_{\mathfrak{B}_Y})$ is NCP.*

Proof. Let $A \in \mho_{\mathfrak{B}_Y}$. Then, there exists a neutrosophic up-directed collection $\{A_j : j \in J\} \subseteq \mathfrak{B}_Y$ such that $A = \bigsqcup_{j \in J} A_j$. Thus, $\{f^{-1}(A_j) : j \in J\} \subseteq \mathfrak{B}_X$ is a neutrosophic up-directed collection and $f^{-1}(A) = f^{-1}\left(\bigsqcup_{j \in J} A_j\right) = \bigsqcup_{j \in J} f^{-1}(A_j)$, which implies that $f^{-1}(A) \in \mho_{\mathfrak{B}_X}$. Therefore, $f : (X, \mho_{\mathfrak{B}_X}) \to (Y, \mho_{\mathfrak{B}_Y})$ is NCP. □

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Conclusion

We have expanded the literature related to neutrosophic set theory by establishing the most relevant properties of neutrosophic convex structures and neutrosophic convex spaces. We have introduced the concepts of neutrosophic convex hull operator, neutrosophic convexly derived operator and neutrosophic convex base to investigate their relationship with neutrosophic convex structures. We have analyzed the behavior of such notions under some special neutrosophic functions. Due to the fact that various modifications of the notion of neutrosophic topology have been proposed by Smarandache in,[14] we consider that the notions and results provided in this article can be explored following the idea of revolutionary topologies, proposing the following avantgarde convex structures: NonStandard Neutrosophic Convex Structure, NeutroConvex Structure, AntiConvex Structure, Refined Neutrosophic Convex Structure, Refined Neutrosophic Crisp Convex Structure, SuperHyperConvex Structure, Neutrosophic SuperHyperConvex Structure, NonStandard Convex Structure, Largest Extended NonStandard Real Convex Structure, Neutrosophic Triplet Weak/Strong Convex Structures, Neutrosophic Extended Triplet Weak/Strong Convex Structures, Neutrosophic Duplet Convex Structure, Neutrosophic Extended Duplet Convex Structure, Neutrosophic MultiSet Convex Structure.

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