Cosine trigonometric rules applied to average and geometric aggregating operators using extension q-rung interval-valued neutrosophic set approach

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Abstract

We introduce the concept of cosine trigonometric q-rung Diophantine neutrosophic interval-valued set (CosT q-rung DioNSIVS). The fact that CosT q-rung DioNSIVS combines q-rung neutrosophic interval-valued set, q-rung neutrosophic set and neutrosophic interval-valued set is one of its distinguishing characteristics. A new idea of CosT q-rung DioNSIVWA, CosT q-rung DioNSIVWG, GCosT q-rung DioNSIVWA and GCosT q-rung DioNSIVWG is proposed in this study. We also look at the idempotency, boundedness, commutativity, and monotonicity of the CosT q-rung DioNSIVS based on algebraic operations. We considered new kinds of two distances in the proposed models, besides Euclidean and Hamming distances. The CosT q-rung DioNSIVS method was used to analyze the cosine trigonometric aggregation procedures. The study's concluding results include several fascinating and captivating discoveries.

Keywords: Aggregating operator; CosT q-rung DioNSIVWA; CosT q-rung DioNSIVWG; GCosT q-rung DioNSIVWA; GCosT q-rung DioNSIVWG.

1 Introduction

There are many uncertainties in logic, namely the fuzzy set (FS),[1] the intuitionistic FS (IFS),[2] the interval-valued FS (IVFS),[3] the Pythagorean FS (PyFS),[4] the Pythagorean IVFS (PyIVFS),[5] the neutrosophic set (NSS),[6] the interval-valued NSS (IVNSS)[7] and the Pythagorean neutrosophic normal IVS (PyNSNIVS)[8] are put forward. There is only one grade of membership for an element in FS, such as 0 or 1. FS applications include fuzzy time series using clustering algorithms and fuzzy c-numbers.[9] Atanassov developed IFS logic, and the sum of membership grade (MG) and non-membership grade (NMG) does not exceed 1.[2] Occasionally, we use the decision-making (DM) method to solve a single problem: the sum of MG and NMG exceeds 1. Therefore, Yager[4] explained PyFS as an extension of IFS whose sum of squares of MG and NMG do not exceed 1. According to Akram et al.,[10–12] PyFS can be used for a wide range of applications. AOs in PyIVFS have been studied by Rahman et al.,[13] which we extend to group DMs. Khan[14] introduced a PyFS for the Einstein choquet integral operator application under the PyIVFS-Einstein AO. Rahman et al.[15] introduced the MAGDM approach under the PyIVFS-Einstein AO. The PyNIVF technique is based on the AOs developed

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by Yang et al.[16] Smarandache[6] introduced the NSS several years ago. Neutrosophy refers to the knowledge of neutrality, and neutrality is described as the difference between FS and IFS. It is a truth grade (TG), an indeterminacy grade (IG), and a falsehood grade (FG). It ranges from 0 to 1. The NSS is a generalization of the classical set, which includes FS, IVFS, and so on. Smarandache et al.[17] developed the PyNSIVS with several applications. Recently, Palanikumar et al. discussed many algebraic structures with applications by[18]-.[21] Medical diagnostics[22] and context analysis[23] were both the subjects of a single-valued NSS application. By enhancing distances for IFSs, such as HD and ED and applying them to MADM situations, Ejegwa[24] produced many similar features to PyFSs. Shami[25] discussed the new notion of $(2, 1)$-FS properties, weighted AOs and their applications to MCDM methods. Recently, Shami et al.[26] introduced the concept of generalized frame for orthopair FSs and (m, n)-FSs and their applications to MCDM. Yang[27] et al. discussed the concept of fuzzy c-numbers clustering procedures for fuzzy data.

In 2002, Li et al.[28] proposed generalized ordered weighted averaging operators (GOWAs). Palanikumar et al. discussed new aggregation operators with applications[29]-.[30] The paper is divided into four sections. Section 1 contains the introduction. In section 2, we describe the basic ideas of PyIVFS and IVNSS. The section **??** describes the differences between CosT-q-rung DioNSIVNs based on various distance. In Section 4 deals with the CosT-q-rung DioNSIVWA, CosT-q-rung DioNSIVWG, GCosT-q-rung DioNSIVWA and GCosT-q-rung DioNSIVWG. The conclusion can be found in Section 5.

2 Fundamental concepts

Definition 2.1. [4] Let $\mathscr{U}$ be a universal. The PyFS Υ in $\mathscr{U}$ is $\Upsilon = \Big\{\wp, \langle \Re^t_\Upsilon(\wp), \Re^f_\Upsilon(\wp)\rangle \big| \wp \in \mathscr{U}\Big\}$, where $\Re^t_\Upsilon, \Re^f_\Upsilon : \mathscr{U} \to [0,1]$ denotes MG and NMG of $\wp \in \mathscr{U}$ to Υ, respectively with $0 \leq (\Re^t_\Upsilon(\wp))^2 + (\Re^f_\Upsilon(\wp))^2 \leq 1$. For $\Upsilon = \langle \Re^t_\Upsilon, \Re^f_\Upsilon\rangle$ is called a Pythagorean fuzzy number (PyFN).

Definition 2.2. [5] The PyIVFS Υ in $\mathscr{U}$ is $\Upsilon = \Big\{\wp, \Big\langle \Re^t_\Upsilon(\wp), \Re^f_\Upsilon(\wp)\Big\rangle \Big| \wp \in \mathscr{U}\Big\}$, where $\Re^t_\Upsilon, \Re^f_\Upsilon : \mathscr{U} \to Int([0,1])$ denotes MG and NMG of $\wp \in \mathscr{U}$ to Υ, respectively with $0 \leq (\Re^{t+}_\Upsilon(\wp))^2 + (\Re^{f+}_\Upsilon(\wp))^2 \leq 1$. For $\Upsilon = \Big\langle \Big[\Re^{t-}_\Upsilon, \Re^{t+}_\Upsilon\Big], \Big[\Re^{f-}_\Upsilon, \Re^{f+}_\Upsilon\Big]\Big\rangle$ is called a Pythagorean interval-valued FN (PyIVFN).

Definition 2.3. [6] The NSS Υ in $\mathscr{U}$ is $\Upsilon = \Big\{\wp, \langle \Re^t_\Upsilon(\wp), \Re^m_\Upsilon(\wp), \Re^f_\Upsilon(\wp)\rangle \big| \wp \in \mathscr{U}\Big\}$, where $\Re^t_\Upsilon, \Re^m_\Upsilon, \Re^f_\Upsilon : \mathscr{U} \to [0,1]$ represents TG, IG and FG of $\wp \in \mathscr{U}$ to Υ, respectively with $0 \leq \Re^t_\Upsilon(\wp) + \Re^m_\Upsilon(\wp) + \Re^f_\Upsilon(\wp) \leq 3$. For $\Upsilon = \langle \Re^t_\Upsilon, \Re^m_\Upsilon, \Re^f_\Upsilon\rangle$ is called a neutrosophic number (NSN).

Definition 2.4. [17] The PyNSS Υ is $\Upsilon = \Big\{\wp, \langle \Re^t_\Upsilon(\wp), \Re^m_\Upsilon(\wp), \Re^f_\Upsilon(\wp)\rangle \big| \wp \in \mathscr{U}\Big\}$, where $\Re^t_\Upsilon, \Re^m_\Upsilon, \Re^f_\Upsilon : \mathscr{U} \to [0,1]$ represent the TG, IG and FG of $\wp \in \mathscr{U}$ to Υ, respectively with $0 \leq (\Re^t_\Upsilon(\wp))^2 + (\Re^m_\Upsilon(\wp))^2 + (\Re^f_\Upsilon(\wp))^2 \leq 2$. For $\Upsilon = \langle \Re^t_\Upsilon, \Re^m_\Upsilon, \Re^f_\Upsilon\rangle$ is called a Pythagorean NSN (PyNSN).

Definition 2.5. [5] Let $\Upsilon = \Big\langle [\Re^{t-}, \Re^{t+}], [\Re^{f-}, \Re^{f+}]\Big\rangle$, $\Upsilon_1 = \Big\langle [\Re^{t-}_1, \Re^{t+}_1], [\Re^{f-}_1, \Re^{f+}_1]\Big\rangle$ and $\Upsilon_2 = \Big\langle [\Re^{t-}_2, \Re^{t+}_2], [\Re^{f-}_2, \Re^{f+}_2]\Big\rangle$ be any three PyIVFNs and $q > 0$. Then

1. $\Upsilon_1 \bigsqcup \Upsilon_2 = \left[\begin{array}{c}\Big[\sqrt{(\Re^{t-}_1)^2 + (\Re^{t-}_2)^2 - (\Re^{t-}_1)^2 \cdot (\Re^{t-}_2)^2}, \sqrt{(\Re^{t+}_1)^2 + (\Re^{t+}_2)^2 - (\Re^{t+}_1)^2 \cdot (\Re^{t+}_2)^2}\Big], \\ \Big[\Re^{f-}_1 \cdot \Re^{f-}_2, \Re^{f+}_1 \cdot \Re^{f+}_2\Big]\end{array}\right]$,
2. $\Upsilon_1 \bigwedge \Upsilon_2 = \left[\begin{array}{c}\Big[\Re^{t-}_1 \cdot \Re^{t-}_2, \Re^{t+}_1 \cdot \Re^{t+}_2\Big], \\ \Big[\sqrt{(\Re^{f-}_1)^2 + (\Re^{f-}_2)^2 - (\Re^{f-}_1)^2 \cdot (\Re^{f-}_2)^2}, \sqrt{(\Re^{f+}_1)^2 + (\Re^{f+}_2)^2 - (\Re^{f+}_1)^2 \cdot (\Re^{f+}_2)^2}\Big]\end{array}\right]$,
3. $q \cdot \Upsilon = \Big[\Big[\sqrt{1 - \big(1 - (\Re^{t-})^2\big)^q}, \sqrt{1 - \big(1 - (\Re^{t+})^2\big)^q}\Big], \Big[(\Re^{f-})^q, (\Re^{f+})^q\Big]\Big]$,
4. $\Upsilon^q = \Big[\Big[(\Re^{t-})^q, (\Re^{t+})^q\Big], \Big[\sqrt{1 - \big(1 - (\Re^{f-})^2\big)^q}, \sqrt{1 - \big(1 - (\Re^{f+})^2\big)^q}\Big]\Big]$.

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3 CosT q-rung DioNSIVN Operations

We discuss the concept of CosT q-rung DioNSIVS along with its basic operations. Here $\pi/2 = \theta$.

Definition 3.1. Let $\Upsilon = \left\langle [\Re^{t-}, \Re^{t+}], [\Re^{m-}, \Re^{m+}], [\Re^{f-}, \Re^{f+}], \hbar \right\rangle$ be the q-rung DioNSIVN. We define a CosT q-rung DioNSIVS is $\cos\Upsilon = \left\{ \left[\cos\left(\theta\cdot\left(\hbar\Re^{t-}_{\Upsilon}(\wp)\right)\right), \cos\left(\theta\cdot\left(\hbar\Re^{t+}_{\Upsilon}(\wp)\right)\right) \right], \left[\cos\left(\theta\cdot\left(\hbar\Re^{m-}_{\Upsilon}(\wp)\right)\right), \cos\left(\theta\cdot\left(\hbar\Re^{m+}_{\Upsilon}(\wp)\right)\right) \right], \left[1-\cos\left(\theta\cdot\left(1-\hbar\Re^{f-}_{\Upsilon}(\wp)\right)\right), 1-\cos\left(\theta\cdot\left(1-\hbar\Re^{f+}_{\Upsilon}(\wp)\right)\right) \right] \right\}$. Clearly, $\cos\Upsilon$ is a q-rung DioNSIVN, where TG, IG and FG of q-rung DioNSIVN are defined respectively, $\cos\left(\theta\cdot\Re^{t+}_{\Upsilon}(\wp)\right) : \mathscr{U} \to [0,1]$ such that $0 \leq \cos\left(\theta\cdot\Re^{t+}_{\Upsilon}(\wp)\right) \leq 1$ and $\cos\left(\theta\cdot\Re^{m+}_{\Upsilon}(\wp)\right) : \mathscr{U} \to [0,1]$ such that $0 \leq \cos\left(\theta\cdot\Re^{m+}_{\Upsilon}(\wp)\right) \leq 1$ and $1-\cos\left(\theta\cdot\left(1-\Re^{f+}_{\Upsilon}(\wp)\right)\right) : \mathscr{U} \to [0,1]$ and $\hbar \in [0,1]$ such that $0 \leq 1-\cos\left(\theta\cdot\left(1-\hbar\Re^{f+}_{\Upsilon}(\wp)\right)\right) \leq 1$. Therefore, $\cos\Upsilon = \left\{ \left[\cos\left(\theta\cdot\left(\hbar\Re^{t-}_{\Upsilon}(\wp)\right)\right), \cos\left(\theta\cdot\left(\hbar\Re^{t+}_{\Upsilon}(\wp)\right)\right) \right], \left[\cos\left(\theta\cdot\left(\hbar\Re^{m-}_{\Upsilon}(\wp)\right)\right), \cos\left(\theta\cdot\left(\hbar\Re^{m+}_{\Upsilon}(\wp)\right)\right) \right], \left[1-\cos\left(\theta\cdot\left(1-\hbar\Re^{f-}_{\Upsilon}(\wp)\right)\right), 1-\cos\left(\theta\cdot\left(1-\hbar\Re^{f+}_{\Upsilon}(\wp)\right)\right) \right] \right\}$ is a q-rung DioNSIVN.

Definition 3.2. Let $\Upsilon = \left\langle [\Re^{t-}, \Re^{t+}], [\Re^{m-}, \Re^{m+}], [\Re^{f-}, \Re^{f+}], \hbar \right\rangle$, $\Upsilon_1 = \left\langle [\Re^{t-}_1, \Re^{t+}_1], [\Re^{m-}_1, \Re^{m+}_1], [\Re^{f-}_1, \Re^{f+}_1], \hbar \right\rangle$ and $\Upsilon_2 = \left\langle [\Re^{t-}_2, \Re^{t+}_2], [\Re^{m-}_2, \Re^{m+}_2], [\Re^{f-}_2, \Re^{f+}_2], \hbar \right\rangle$ be any three CosT q-rung DioNSIVNs and $q > 0$. Then

1. $\cos\Upsilon_1 \bigsqcup \cos\Upsilon_2 =$

$$\left[\begin{array}{c} \left[\begin{array}{c} \sqrt[2q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{t-}_1)^{2q}\right)\right)^{2q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{t-}_2)^{2q}\right)\right)^{2q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{t-}_1)^{2q}\right)\right)^{2q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{t-}_2)^{2q}\right)\right)^{2q}\end{array}}, \\ \sqrt[2q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{t+}_1)^{2q}\right)\right)^{2q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{t+}_2)^{2q}\right)\right)^{2q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{t+}_1)^{2q}\right)\right)^{2q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{t+}_2)^{2q}\right)\right)^{2q}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_1)^{q}\right)\right)^{q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_2)^{q}\right)\right)^{q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_1)^{q}\right)\right)^{q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_2)^{q}\right)\right)^{q}\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_1)^{q}\right)\right)^{q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_2)^{q}\right)\right)^{q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_1)^{q}\right)\right)^{q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_2)^{q}\right)\right)^{q}\end{array}} \end{array}\right], \\ \left[\cos^2\left(\theta\cdot\hbar\Re^{f-}_1\right)\cdot\cos^2\left(\theta\cdot\hbar\Re^{f-}_2\right), \cos^2\left(\theta\cdot\hbar\Re^{f+}_1\right)\cdot\cos^2\left(\theta\cdot\hbar\Re^{f+}_2\right)\right] \end{array}\right],$$

2. $\cos\Upsilon_1 \bigwedge \cos\Upsilon_2 =$

$$\left[\begin{array}{c} \left[\cos^2\left(\theta\cdot\hbar\Re^{t-}_1\right)\cdot\cos^2\left(\theta\cdot\hbar\Re^{t-}_2\right), \cos^2\left(\theta\cdot\hbar\Re^{t+}_1\right)\cdot\cos^2\left(\theta\cdot\Re^{t+}_2\right)\right], \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_1)^{q}\right)\right)^{q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_2)^{q}\right)\right)^{q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_1)^{q}\right)\right)^{q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{m-}_2)^{q}\right)\right)^{q}\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_1)^{q}\right)\right)^{q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_2)^{q}\right)\right)^{q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_1)^{q}\right)\right)^{q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{m+}_2)^{q}\right)\right)^{q}\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[2q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-}_1)^{2q}\right)\right)^{2q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{f-}_2)^{2q}\right)\right)^{2q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-}_1)^{2q}\right)\right)^{2q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-}_2)^{2q}\right)\right)^{2q}\end{array}}, \\ \sqrt[2q]{\begin{array}{c}\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+}_1)^{2q}\right)\right)^{2q} + \left(\cos^2\left(\theta\cdot(\hbar\Re^{f+}_2)^{2q}\right)\right)^{2q} \\ -\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+}_1)^{2q}\right)\right)^{2q}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+}_2)^{2q}\right)\right)^{2q}\end{array}} \end{array}\right] \end{array}\right],$$

3. $q\cdot\cos\Upsilon =$

$$\left[\begin{array}{c} \left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)\right)^{q}}, \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)\right)^{q}}\right], \\ \left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{q}\right)\right)^{q}}, \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{q}\right)\right)^{q}}\right], \\ \left[\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right)^{q}, \left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)^{q}\right] \end{array}\right],$$

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4. $(\cos\Upsilon)^q =$
$$\left[\begin{array}{c}\left[\left(\cos^2\left(\theta\cdot(\hbar\Re^{t-})\right)\right)^q,\left(\cos^2\left(\theta\cdot(\hbar\Re^{t+})\right)\right)^q\right],\\ \left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t-})^q\right)\right)^q},\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t+})^q\right)\right)^q}\right],\\ \left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{f-})^{2q}\right)\right)^q},\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{f+})^{2q}\right)\right)^q}\right]\end{array}\right].$$

Definition 3.3. For any two CosT q-rung DioNSIVNs $\Upsilon_1 = \left\langle[\Re_1^{t-},\Re_1^{t+}],[\Re_1^{m-},\Re_1^{m+}],[\Re_1^{f-},\Re_1^{f+}],\hbar\right\rangle$ and $\Upsilon_2 = \left\langle[\Re_2^{t-},\Re_2^{t+}],[\Re_2^{m-},\Re_2^{m+}],[\Re_2^{f-},\Re_2^{f+}],\hbar\right\rangle$. Then the ED (HD) between $\Upsilon1$ and $\Upsilon2$, respectively. Now,

$$\mathscr{D}_E\left(\Upsilon_1,\Upsilon_2\right) = \frac{1}{2}\sqrt{\begin{array}{c}\left[\frac{1+\Xi_1+1+\mho_1}{2}\chi_1-\frac{1+\Xi_2+1+\mho_2}{2}\right]^2\\ +\frac{1}{2}\left[\frac{1+\Xi_1+1+\mho_1}{2}\tau_1-\frac{1+\Xi_2+1+\mho_2}{2}\right]^2\end{array}},$$

and

$$\mathscr{D}_H\left(\Upsilon_1,\Upsilon_2\right) = \frac{1}{2}\left[\begin{array}{c}\left|\frac{1+\Xi_1+1+\mho_1}{2}-\frac{1+\Xi_2+1+\mho_2}{2}\right|\\ +\frac{1}{2}\left|\frac{1+\Xi_1+1+\mho_1}{2}-\frac{1+\Xi_2+1+\mho_2}{2}\right|\end{array}\right],$$

where

$$\Xi_1 = \cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_1^{f-})^2\right),$$

$$\mho_1 = \cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_1^{f+})^2\right),$$

$$\Xi_2 = \cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_2^{f-})^2\right),$$

$$\mho_2 = \cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^2\right)-\cos^2\left(\theta\cdot(\hbar\Re_2^{f+})^2\right).$$

4 New aggregation operators based On CosT q-rung DioNSIVNs

CosT q-rung DioNSIVWA, CosT q-rung DioNSIVWG, GCosT q-rung DioNSIVWA, and GCosT q-rung DioNSIVWG are introduced in this section.

4.1 CosT q-rung DioNSIV weighted averaging(CosT q-rung DioNSIVWA) operator

Definition 4.1. Let $\Upsilon_i = \left\langle[\Re_i^{t-},\Re_i^{t+}],[\Re_i^{m-},\Re_i^{m+}],[\Re_i^{f-},\Re_i^{f+}],\hbar\right\rangle$ be the set of CosT q-rung DioNSIVNs, $\mathscr{W} = (\kappa_1,\kappa_2,...,\kappa_n)$, and $\bigsqcup_{i=1}^{n}\kappa_{i=1}$. Prove that CosT q-rung DioNSIVWA $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n) = \bigsqcup_{i=1}^{n}\kappa_i\cos\Upsilon_i$ $(i = 1,2,...,n)$.

Theorem 4.2. *Let $\Upsilon_i = \left\langle[\Re_i^{t-},\Re_i^{t+}],[\Re_i^{m-},\Re_i^{m+}],[\Re_i^{f-},\Re_i^{f+}],\hbar\right\rangle$ be a finite collection of CosT q-rung DioNSIVNs. Prove that CosT q-rung DioNSIVWA $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)$=*

$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i},\bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\right]\end{array}\right].$$

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Proof. If $n = 2$, then CosT q-rung DioNSIVWA$(\Upsilon_1, \Upsilon_2) = \kappa_1 \cdot \cos\Upsilon_1 \bigsqcup \kappa_2 \cdot \cos\Upsilon_2$, since

$$\kappa_1 \cdot \cos\Upsilon_1 = \begin{bmatrix} \left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^{\kappa_1}},\ \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^{\kappa_1}}\right], \\ \left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^{\kappa_1}}, \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^{\kappa_1}}\right], \\ \left[\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f-})\right)\right)^{\kappa_1}, \left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f+})\right)\right)^{\kappa_1}\right] \end{bmatrix}$$

$$\kappa_2 \cdot \cos\Upsilon_2 = \begin{bmatrix} \left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^{\kappa_2}},\ \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^{\kappa_2}}\right], \\ \left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^{\kappa_2}}, \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^{\kappa_2}}\right], \\ \left[\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f-})\right)\right)^{\kappa_2}, \left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f+})\right)\right)^{\kappa_2}\right] \end{bmatrix}.$$

Now, $\kappa_1 \cdot \cos\Upsilon_1 \bigsqcup \kappa_2 \cdot \cos\Upsilon_2 =$

$$\begin{bmatrix} \left[\begin{array}{c} \sqrt[2q]{\begin{array}{c}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^{\kappa_1}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^{\kappa_2}\right)\\ -\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^{\kappa_1}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^{\kappa_2}\right)\end{array}}, \\ \sqrt[2q]{\begin{array}{c}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^{\kappa_1}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^{\kappa_2}\right)\\ -\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^{\kappa_1}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^{\kappa_2}\right)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^{\kappa_1}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^{\kappa_2}\right)\\ -\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^{\kappa_1}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^{\kappa_2}\right)\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^{\kappa_1}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^{\kappa_2}\right)\\ -\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^{\kappa_1}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^{\kappa_2}\right)\end{array}} \end{array}\right], \\ \left[\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f-})\right)\right)^{\kappa_1}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f-})\right)\right)^{\kappa_2}, \left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f+})\right)\right)^{\kappa_1}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f+})\right)\right)^{\kappa_2}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\begin{array}{c} \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^{\kappa_1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^{\kappa_2}}, \\ \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^{\kappa_1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^{\kappa_2}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^{\kappa_1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^{\kappa_2}}, \\ \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^{\kappa_1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^{\kappa_2}} \end{array}\right], \\ \left[\begin{array}{c} \left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f-})\right)\right)^{\kappa_1}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f-})\right)\right)^{\kappa_2}, \\ \left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f+})\right)\right)^{\kappa_1}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f+})\right)\right)^{\kappa_2} \end{array}\right] \end{bmatrix}$$

CosT q-rung DioNSIVWA $(\Upsilon_1, \Upsilon_2) =$

$$\begin{bmatrix} \left[\sqrt[2q]{1-\bigwedge_{i=1}^{2}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}},\ \sqrt[2q]{1-\bigwedge_{i=1}^{2}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}}\right], \\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{2}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}}, \sqrt[q]{1-\bigwedge_{i=1}^{2}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}}\right], \\ \left[\bigwedge_{i=1}^{2}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i}, \bigwedge_{i=1}^{2}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\right] \end{bmatrix}.$$

It is valid for $n \geq 3$ and hence CosT q-rung DioNSIVWA $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_l) =$

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$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{l}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i},\bigwedge_{i=1}^{l}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\right]\end{array}\right].$$

Put $n=l+1$, then CosT q-rung DioNSIVWA $(\Upsilon_1,\Upsilon_2,...,\Upsilon_l,\Upsilon_{l+1})=$

$$\left[\begin{array}{c}\left[\begin{array}{c}\sqrt[2q]{\begin{array}{c}\bigsqcup_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t-})^{2q}\right)^{\kappa_{l+1}}\right)\right)\\ -\bigwedge_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t-})^{2q}\right)^{\kappa_{l+1}}\right)\right)\end{array}},\\ \sqrt[2q]{\begin{array}{c}\bigsqcup_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t+})^{2q}\right)^{\kappa_{l+1}}\right)\right)\\ -\bigwedge_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t+})^{2q}\right)^{\kappa_{l+1}}\right)\right)\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[q]{\begin{array}{c}\bigsqcup_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m-})^{q}\right)^{\kappa_{l+1}}\right)\right)\\ -\bigwedge_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m-})^{q}\right)^{\kappa_{l+1}}\right)\right)\end{array}},\\ \sqrt[q]{\begin{array}{c}\bigsqcup_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m+})^{q}\right)^{\kappa_{l+1}}\right)\right)\\ -\bigwedge_{i=1}^{l}\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m+})^{q}\right)^{\kappa_{l+1}}\right)\right)\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\bigwedge_{i=1}^{l}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{f-})\right)\right)^{\kappa_{l+1}},\\ \bigwedge_{i=1}^{l}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\cdot\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{f+})\right)\right)^{\kappa_{l+1}}\end{array}\right]\end{array}\right]$$

$$=\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{l+1}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i},\bigwedge_{i=1}^{l+1}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\right]\end{array}\right].$$

Theorem 4.3. *(idempotency property) If all* $\Upsilon_i=\Big\langle[\cos(\theta\cdot\hbar\Re_i^{t-}),\cos(\theta\cdot\hbar\Re_i^{t+})],[\cos(\theta\cdot\hbar\Re_i^{m-}),\cos(\theta\cdot\hbar\Re_i^{m+})],[\cos(\theta\cdot\hbar\Re_i^{f-}),\cos(\theta\cdot\hbar\Re_i^{f+})],\hbar\Big\rangle(i=1,2,...,n)$ *are equal and* $[\cos^2(\theta\cdot(\hbar\Re_i^{t-})^{2q}),\cos^2(\theta\cdot(\hbar\Re_i^{t+})^{2q})]=[(\cos(\theta\cdot(\hbar\Re_i^{t-})))^{2q},(\cos(\theta\cdot(\hbar\Re_i^{t+})))^{2q}]$ *and* $[\cos^2(\theta\cdot(\hbar\Re_i^{m-})^{q}),\cos^2(\theta\cdot(\hbar\Re_i^{m+})^{q})]=[(\cos(\theta\cdot(\hbar\Re_i^{m-})))^{q},(\cos(\theta\cdot(\hbar\Re_i^{m+})))^{q}]$*, then CosT q-rung DioNSIVWA* $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)=\Upsilon$.

Proof. Since $[\cos(\theta\cdot\hbar\Re_i^{t-}),\cos(\theta\cdot\hbar\Re_i^{t+})]=[\cos(\theta\cdot\hbar\Re^{t-}),\cos(\theta\cdot\hbar\Re^{t+})]$, $[\cos(\theta\cdot\hbar\Re_i^{m-}),\cos(\theta\cdot\hbar\Re_i^{m+})]=[\cos(\theta\cdot\hbar\Re^{m-}),\cos(\theta\cdot\hbar\Re^{m+})]$ and $[\cos(\theta\cdot\hbar\Re_i^{f-}),\cos(\theta\cdot\hbar\Re_i^{f+})]=[\cos(\theta\cdot\hbar\Re^{f-}),\cos(\theta\cdot\hbar\Re^{f+})]$, for

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$i = 1, 2, ..., n$ and $\bigsqcup_{i=1}^{n} \kappa_i = 1$. Now, CosT q-rung DioNSIVWA $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$

$$
\begin{aligned}
&= \left[\begin{array}{c}
\left[\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^{\kappa_i}}, \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^{\kappa_i}}\right], \\
\left[\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^{\kappa_i}}, \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^{\kappa_i}}\right], \\
\left[\bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{\kappa_i}, \bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{\kappa_i}\right]
\end{array}\right] \\
&= \left[\begin{array}{c}
\left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}}, \sqrt[2q]{1\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}}\right], \\
\left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}}, \sqrt[q]{1\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}}\right], \\
\left[\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}, \left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)^{\bigsqcup_{i=1}^{n}\kappa_i}\right]
\end{array}\right] \\
&= \left[\begin{array}{c}
\left[\sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)\right)}, \sqrt[2q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)\right)}\right], \\
\left[\sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)\right)}, \sqrt[q]{1-\left(1-\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)\right)}\right], \\
\left[\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right), \left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)\right]
\end{array}\right] \\
&= \cos\Upsilon.
\end{aligned}
$$

Theorem 4.4. *(Boundedness property) Let* $\Upsilon_i = \left\langle [\Re_{ij}^{t-}, \Re_{ij}^{t+}], [\Re_{ij}^{m-}, \Re_{ij}^{m+}][\Re_{ij}^{f-}, \Re_{ij}^{f+}], \hbar \right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the CosT q-rung DioNSIVWAs. Since* $\underbrace{\Re^{t-}} = \inf \Re_{ij}^{t^-}$, $\overbrace{\Re^{t-}} = \sup \Re_{ij}^{t^-}$, $\underbrace{\Re^{t+}} = \inf \Re_{ij}^{t^+}$, $\overbrace{\Re^{t+}} = \sup \Re_{ij}^{t^+}$, $\underbrace{\Re^{m-}} = \inf \Re_{ij}^{m^-}$, $\overbrace{\Re^{m-}} = \sup \Re_{ij}^{m^-}$, $\underbrace{\Re^{m+}} = \inf \Re_{ij}^{m^+}$, $\overbrace{\Re^{m+}} = \sup \Re_{ij}^{m^+}$, $\underbrace{\Re^{f-}} = \inf \Re_{ij}^{f^-}$, $\overbrace{\Re^{f-}} = \sup \Re_{ij}^{f^-}$, $\underbrace{\Re^{f+}} = \inf \Re_{ij}^{f^+}$, $\overbrace{\Re^{f+}} = \sup \Re_{ij}^{f^+}$, $j = 1, 2, ..., i_j$ *and* $1 \le i \le n$. *Prove that,*

$$
\begin{aligned}
\left\langle [\underbrace{\Re^{t-}}, \underbrace{\Re^{t+}}], [\underbrace{\Re^{m-}}, \underbrace{\Re^{m+}}], [\overbrace{\Re^{f-}}, \overbrace{\Re^{f+}}], \hbar \right\rangle &\le CosTq-rungDioNSIVWA(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) \\
&\le \left\langle [\overbrace{\Re^{t-}}, \overbrace{\Re^{t+}}], [\overbrace{\Re^{m-}}, \overbrace{\Re^{m+}}], [\underbrace{\Re^{f-}}, \underbrace{\Re^{f+}}], \hbar \right\rangle.
\end{aligned}
$$

.

Proof. Since, $\underbrace{\Re^{t-}} = \inf \Re_{ij}^{t^-}$, $\overbrace{\Re^{t-}} = \sup \Re_{ij}^{t^-}$, $\underbrace{\Re^{t+}} = \inf \Re_{ij}^{t^+}$, $\overbrace{\Re^{t+}} = \sup \Re_{ij}^{t^+}$ and $\underbrace{\Re^{t-}} \le \Re_{ij}^{t^-} \le \overbrace{\Re^{t-}}$ and

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$\underbrace{\Re^{t+}} \leq \Re_{ij}^{t^+} \leq \overbrace{\Re^{t+}}$. We have, $\underbrace{\Re^{t-}} + \underbrace{\Re^{t+}}$

$$\begin{aligned}
&= \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)}\right)^{\kappa_i}} + \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)}\right)^{\kappa_i}} \\
&\leq \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{t^-})^{2q}\right)\right)^{\kappa_i}} + \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{t^+})^{2q}\right)\right)^{\kappa_i}} \\
&\leq \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)}\right)^{\kappa_i}} + \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)}\right)^{\kappa_i}} \\
&= \overbrace{\Re^{t-}} + \overbrace{\Re^{t+}}.
\end{aligned}$$

Since, $\underbrace{\Re^{m-}} = \inf \Re_{ij}^{m^-}$, $\overbrace{\Re^{m-}} = \sup \Re_{ij}^{m^-}$, $\underbrace{\Re^{m+}} = \inf \Re_{ij}^{m^+}$, $\overbrace{\Re^{m+}} = \sup \Re_{ij}^{m^+}$ and $\underbrace{\Re^{m-}} \leq \Re_{ij}^{m^-} \leq \overbrace{\Re^{m-}}$ and $\underbrace{\Re^{m+}} \leq \Re_{ij}^{m^+} \leq \overbrace{\Re^{m+}}$. We have, $\underbrace{\Re^{m-}} + \underbrace{\Re^{m+}}$

$$\begin{aligned}
&= \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)}\right)^{\kappa_i}} + \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)}\right)^{\kappa_i}} \\
&\leq \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{m^-})^{q}\right)\right)^{\kappa_i}} + \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{m^+})^{q}\right)\right)^{\kappa_i}} \\
&\leq \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)}\right)^{\kappa_i}} + \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)}\right)^{\kappa_i}} \\
&= \overbrace{\Re^{m-}} + \overbrace{\Re^{m+}}.
\end{aligned}$$

Since, $\underbrace{\Re^{f-}} = \inf \Re_{ij}^{f^-}$, $\overbrace{\Re^{f-}} = \sup \Re_{ij}^{f^-}$, $\underbrace{\Re^{f+}} = \inf \Re_{ij}^{f^+}$, $\overbrace{\Re^{f+}} = \sup \Re_{ij}^{f^+}$ and $\underbrace{\Re^{f-}} \leq \Re_{ij}^{f^-} \leq \overbrace{\Re^{f-}}$ and $\underbrace{\Re^{f+}} \leq \Re_{ij}^{f^+} \leq \overbrace{\Re^{f+}}$. We have,

$$\begin{aligned}
\underbrace{\Re^{f-}} + \underbrace{\Re^{f+}} &= \bigwedge_{i=1}^{n}\underbrace{\left(\cos^2\left(\theta\cdot\overset{\kappa_i}{(}\hbar\Re^{f-})\right)\right)} + \bigwedge_{i=1}^{n}\underbrace{\left(\cos^2\left(\theta\cdot\overset{\kappa_i}{(}\hbar\Re^{f+})\right)\right)} \\
&\leq \bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{f^-})\right)\right)^{\kappa_i} + \bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{f^+})\right)\right)^{\kappa_i} \\
&\leq \bigwedge_{i=1}^{n}\overbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right)^{\kappa_i}} + \bigwedge_{i=1}^{n}\overbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)^{\kappa_i}} \\
&= \overbrace{\Re^{f-}} + \overbrace{\Re^{f+}}.
\end{aligned}$$

Therefore,

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$$\frac{\bigsqcup_{i=1}^{n}\kappa_i}{2}\times\left[\frac{\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)}\right)^{\kappa_i}}\right)^2+\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)}\right)^{\kappa_i}}\right)^2}{2}-\frac{\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)}\right)^{\kappa_i}}\right)^2+\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\underbrace{\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)}\right)^{\kappa_i}}\right)^2}{2}+2-\frac{\left(\bigwedge_{i=1}^{n}\overbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right)^{\kappa_i}}\right)^2+\left(\bigwedge_{i=1}^{n}\overbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)^{\kappa_i}}\right)^2}{2}\right]$$

$$\leq\frac{\bigsqcup_{i=1}^{n}\kappa_i}{2}\times\left[\frac{\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{t^-})^{2q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{t^+})^{2q}\right)\right)^{\kappa_i}}\right)^2}{2}-\frac{\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{m^-})^{q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{m^+})^{q}\right)\right)^{\kappa_i}}\right)^2}{2}+2-\frac{\left(\bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{f^-})\right)\right)^{\kappa_i}\right)^2+\left(\bigwedge_{i=1}^{n}\left(\cos^2\left(\theta\cdot(\hbar\Re_{ij}^{f^+})\right)\right)^{\kappa_i}\right)^2}{2}\right]$$

$$\leq\frac{\bigsqcup_{i=1}^{n}\kappa_i}{2}\times\left[\frac{\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{(\cos^2\left(\theta\cdot(\hbar\Re^{t-})^{2q}\right)}\right)^{\kappa_i}}\right)^2+\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{(\cos^2\left(\theta\cdot(\hbar\Re^{t+})^{2q}\right)}\right)^{\kappa_i}}\right)^2}{2}-\frac{\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{(\cos^2\left(\theta\cdot(\hbar\Re^{m-})^{q}\right)}\right)^{\kappa_i}}\right)^2+\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\overbrace{(\cos^2\left(\theta\cdot(\hbar\Re^{m+})^{q}\right)}\right)^{\kappa_i}}\right)^2}{2}+2-\frac{\left(\bigwedge_{i=1}^{n}\underbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f-})\right)\right)^{\kappa_i}}\right)^2+\left(\bigwedge_{i=1}^{n}\underbrace{\left(\cos^2\left(\theta\cdot(\hbar\Re^{f+})\right)\right)^{\kappa_i}}\right)^2}{2}\right].$$

Hence,

$$\begin{aligned}\left\langle[\underbrace{\Re^{t-}},\underbrace{\Re^{t+}}],[\underbrace{\Re^{m-}},\underbrace{\Re^{m+}}],[\overbrace{\Re^{f-}},\overbrace{\Re^{f+}}],\hbar\right\rangle &\leq CosTq-rungDioNSIVWA(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)\\ &\leq \left\langle[\overbrace{\Re^{t-}},\overbrace{\Re^{t+}}],[\overbrace{\Re^{m-}},\overbrace{\Re^{m+}}],[\underbrace{\Re^{f-}},\underbrace{\Re^{f+}}],\hbar\right\rangle.\end{aligned}$$

Theorem 4.5. *(Monotonicity property) Let* $\Upsilon_i = \left\langle[\Re_{t_{ij}}^{t-},\Re_{t_{ij}}^{t+}],[\Re_{t_{ij}}^{m-},\Re_{t_{ij}}^{m+}],[\Re_{t_{ij}}^{f-},\Re_{t_{ij}}^{f+}],\hbar\right\rangle$ *and* $\mathscr{W}_i = \left\langle[\Re_{h_{ij}}^{t-},\Re_{h_{ij}}^{t+}],[\Re_{h_{ij}}^{m-},\Re_{h_{ij}}^{m+}],[\Re_{h_{ij}}^{f-},\Re_{h_{ij}}^{f+}],\hbar\right\rangle (i = 1,2,...,n); (j = 1,2,...,i_j)$ *be the CosT q-rung DioNSIVWAs. For any i, if* $\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{t-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{t+}\right)^2\right)\leq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{t-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{t+}\right)^2\right)$ *and* $\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{m-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{m+}\right)^2\right)\leq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{m-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{m+}\right)^2\right)$ $\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f+}\right)^2\right)\geq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f+}\right)^2\right)$ *or* $\Upsilon_i\leq\mathscr{W}_i$*. Prove that CosT q-rung DioNSIVWA* $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)\leq$ *CosT q-rung DioNSIVWA* $(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$*.*

Proof. For any i, $\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{t-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{t+}\right)^2\right)\leq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{t-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{t+}\right)^2\right)$. Therefore, $1-\cos^2\left(\theta\cdot(\hbar\Re_{t_i}^{t-})^2\right)+1-\cos^2\left(\theta\cdot(\hbar\Re_{t_i}^{t+})^2\right)\geq1-\cos^2\left(\theta\cdot(\hbar\Re_{h_i}^{t-})^2\right)+1-\cos^2\left(\theta\cdot(\hbar\Re_{h_i}^{t+})^2\right)$.

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Hence, $\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{t-}\right)^2\right)\right)^{\kappa_i}+\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{t+}\right)^2\right)\right)^{\kappa_i}\geq\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{t-}\right)^2\right)\right)^{\kappa_i}+\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{t+}\right)^2\right)\right)^{\kappa_i}$ implies that

$$\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{t-}\right)^{2q}\right)\right)^{\kappa_i}}+\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{t+}\right)^{2q}\right)\right)^{\kappa_i}}$$
$$\leq\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{t-}\right)^{2q}\right)\right)^{\kappa_i}}+\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{t+}\right)^{2q}\right)\right)^{\kappa_i}}.$$

For any i, $\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{m-}\right)^q\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{m+}\right)^q\right)\leq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{m-}\right)^q\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{m+}\right)^q\right)$. Therefore, $1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m-}\right)^q\right)+1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m+}\right)^q\right)\geq1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m-}\right)^q\right)+1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m+}\right)^q\right)$.

Hence, $\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m-}\right)^q\right)\right)^{\kappa_i}+\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m+}\right)^q\right)\right)^{\kappa_i}\geq\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m-}\right)^q\right)\right)^{\kappa_i}+\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m+}\right)^q\right)\right)^{\kappa_i}$.

Hence, $\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m-}\right)^q\right)\right)^{\kappa_i}}+\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{t_i}^{m+}\right)^q\right)\right)^{\kappa_i}}$
$\leq\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m-}\right)^q\right)\right)^{\kappa_i}}+\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot\left(\hbar\Re_{h_i}^{m+}\right)^q\right)\right)^{\kappa_i}}$.

For any i,$\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f+}\right)^2\right)\geq\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f-}\right)^2\right)+\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f+}\right)^2\right)$.

Therefore, $2-\frac{\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f-}\right)\right)\right)^2+\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot\left(\hbar\Re_{t_{ij}}^{f+}\right)\right)\right)^2}{2}\leq 2-\frac{\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f-}\right)\right)\right)^2+\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot\left(\hbar\Re_{h_{ij}}^{f+}\right)\right)\right)^2}{2}$.

Now,

$$\frac{1}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ti}^{t^-})^{2q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ti}^{t^+})^{2q}\right)\right)^{\kappa_i}}\right)^2}{2}\\ -\frac{\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ti}^{m^-})^{q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{ti}^{m^+})^{q}\right)\right)^{\kappa_i}}\right)^2}{2}\\ +2-\frac{\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot(\hbar\Re_{t_{ij}}^{f-})\right)\right)^2+\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot(\hbar\Re_{t_{ij}}^{f+})\right)\right)^2}{2}\end{array}\right]$$

$$\leq\frac{1}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{hi}^{t^-})^{2q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{hi}^{t^+})^{2q}\right)\right)^{\kappa_i}}\right)^2}{2}\\ -\frac{\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{hi}^{m^-})^{q}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_{hi}^{m^+})^{q}\right)\right)^{\kappa_i}}\right)^2}{2}\\ +2-\frac{\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot(\hbar\Re_{h_{ij}}^{f-})\right)\right)^2+\left(\bigwedge_{i=1}^{n}\cos^2\left(\theta\cdot(\hbar\Re_{h_{ij}}^{f+})\right)\right)^2}{2}\end{array}\right].$$

Hence, CosT q-rung DioNSIVWA $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)\leq$ CosT q-rung DioNSIVWA $(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$.

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4.2 CosT q-rung DioNSIV Weighted Geometric(CosT q-rung DioNSIVWG) Operator

Definition 4.6. Let $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{m-}, \Re_i^{m+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ be the collection of CosT q-rung DioNSIVNs. Then CosT q-rung DioNSIVWG $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \bigwedge_{i=1}^{n} (\cos \Upsilon_i)^{\kappa_i}$ $(i = 1, 2, ..., n)$.

Theorem 4.7. *Let $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ be the set of CosT q-rung DioNSIVNs. Prove that CosT q-rung DioNSIVWG $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) =$*

$$\left[\begin{array}{c} \left[\bigwedge_{i=1}^{n} \left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})\right)\right)^{\kappa_i}, \bigwedge_{i=1}^{n} \left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})\right)\right)^{\kappa_i}\right], \\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^q\right)\right)^{\kappa_i}}, \sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^q\right)\right)^{\kappa_i}}\right], \\ \left[\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})^{2q}\right)\right)^{\kappa_i}}, \sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})^{2q}\right)\right)^{\kappa_i}}\right] \end{array}\right].$$

Corollary 4.8. *Idempotency, boundedness and monotonicity are satisfied in the case of CosT q-rung DioNSIVWG operator.*

4.3 Generalized CosT q-rung DioNSIVWA (GCosT q-rung DioNSIVWA) Operator

Definition 4.9. Let $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{m-}, \Re_i^{m+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ be the collection of CosT q-rung DioNSIVN. The GCosT q-rung DioNSIVWA operator is GCosT q-rung DioNSIVWA $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \left(\bigsqcup_{i=1}^{n} \kappa_i (\cos \Upsilon_i)^q\right)^{1/q}$.

Theorem 4.10. *Let $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{m-}, \Re_i^{m+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ be the set of CosT q-rung DioNSIVNs. Prove that GCosT q-rung DioNSIVWA $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) =$*

$$\left[\begin{array}{c} \left[\begin{array}{c} \left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}, \\ \left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}, \\ \left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^{2q}\right)^{1/q}}, \\ \sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^{2q}\right)^{1/q}} \end{array}\right] \end{array}\right].$$

Proof. It is necessary to demonstrate that, $\bigsqcup_{i=1}^{n} \kappa_i (\cos \Upsilon_i)^q =$

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$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^q\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i},\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right]\end{array}\right].$$

Put $n=2$, $\kappa_1(\cos\Upsilon_1)^q\bigsqcup\kappa_2(\cos\Upsilon_2)^q=$

$$\left[\begin{array}{c}\left[\begin{array}{c}\sqrt[2q]{\begin{array}{c}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}+\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\\ -\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\end{array}},\\ \sqrt[2q]{\begin{array}{c}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}+\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\\ -\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\\ -\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\end{array}},\\ \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\\ -\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{m+})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_1}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_1},\\ \left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_1}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_2^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_1}\end{array}\right]\end{array}\right]$$

implies that,

$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{2}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{2}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{2}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^q\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{2}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{2}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i},\bigwedge_{i=1}^{2}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right]\end{array}\right].$$

In general, $\bigsqcup_{i=1}^{l}\kappa_i(\cos\Upsilon_i)^q=$

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$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^q\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{l}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i},\bigwedge_{i=1}^{l}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right]\end{array}\right].$$

If $n = l+1$, then $\bigsqcup_{i=1}^{l}\kappa_i(\cos\Upsilon_i)^q + \kappa_{l+1}(\cos\Upsilon_{l+1})^q = \bigsqcup_{i=1}^{l+1}\kappa_i(\cos\Upsilon_i)^q$.

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Now, $\bigsqcup_{i=1}^{l}\kappa_i(\cos\Upsilon_i)^q+\kappa_{l+1}(\cos\Upsilon_{l+1})^q=\kappa_1(\cos\Upsilon_1)^q\bigsqcup\kappa_2(\cos\Upsilon_2)^q\bigsqcup...\bigsqcup\kappa_l(\cos\Upsilon_l)^q\bigsqcup\kappa_{l+1}(\cos\Upsilon_{l+1})^q$

$$
=\begin{bmatrix}
\begin{bmatrix}
\sqrt[2q]{\begin{matrix}\left(\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)\right)^q\right)^{\kappa_i}}\right)^{2q}+\\ \left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q},\\ -\left(\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)\right)^q\right)^{\kappa_i}}\right)^{2q}\cdot\\ \left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t-})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{2q}\end{matrix}},\\
\sqrt[2q]{\begin{matrix}\left(\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{2q}+\\ \left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\\ -\left(\sqrt[2q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{2q}\cdot\\ \left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{t+})^{2q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\end{matrix}}
\end{bmatrix},\\
\begin{bmatrix}
\sqrt[q]{\begin{matrix}\left(\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{q}+\\ \left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q},\\ -\left(\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{q}\cdot\\ \left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m-})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\end{matrix}}\\
\sqrt[q]{\begin{matrix}\left(\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{q}+\\ \left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m+})^{q}\right)\right)^q\right)^{\kappa_1}}\right)^{q}\\ -\left(\sqrt[q]{1-\bigwedge_{i=1}^{l}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{q}\cdot\\ \left(\sqrt[q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{m+})^{q}\right)\right)^{2q}\right)^{\kappa_1}}\right)^{q}\end{matrix}}
\end{bmatrix},\\
\begin{bmatrix}
\bigwedge_{i=1}^{l}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_1},\\
\bigwedge_{i=1}^{l}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\cdot\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_{l+1}^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_1}
\end{bmatrix}
\end{bmatrix}
$$

Thus, $\bigsqcup_{i=1}^{l+1}\kappa_i(\cos\Upsilon_i)^q=$

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$$\left[\begin{array}{c}\left[\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}},\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m-})^{q}\right)\right)^q\right)^{\kappa_i}},\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_1^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right],\\ \left[\bigwedge_{i=1}^{l+1}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i},\bigwedge_{i=1}^{l+1}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right]\end{array}\right].$$

Hence, $\left(\bigsqcup_{i=1}^{l+1}\kappa_i(\cos\Upsilon_i)^q\right)^{1/q} =$

$$\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q},\\ \left(\sqrt[2q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q},\\ \left(\sqrt[q]{1-\bigwedge_{i=1}^{l+1}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^{q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{l+1}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^2\right)^{1/q}},\\ \sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{l+1}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^2\right)^{1/q}}\end{array}\right]\end{array}\right].$$

It is true for any l.

Remark 4.11. A CosT q-rung DioNSIVWG operator is substituted for GCosT q-rung DioNSIVWA when $q = 1$.

Corollary 4.12. *Idempotent, bounded, and monotonic properties are fulfilled by the GCosT q-rung DioNSIVWA operator.*

4.4 Generalized CosT q-rung DioNSIVWG (GCosT q-rung DioNSIVWG) Operator

Definition 4.13. Let $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{m-}, \Re_i^{m+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ be a finite collection of CosT q-rung DioNSIVNs. Then GCosT q-rung DioNSIVWG $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n) = \frac{1}{q}\left(\bigwedge_{i=1}^{n}(q\cdot\cos\Upsilon_i)^{\kappa_i}\right)$ $(i = 1, 2, ..., n)$.

Theorem 4.14. *Let* $\Upsilon_i = \left\langle [\Re_i^{t-}, \Re_i^{t+}], [\Re_i^{m-}, \Re_i^{m+}], [\Re_i^{f-}, \Re_i^{f+}], \hbar \right\rangle$ *be the set of CosT q-rung DioNSIVNs. Prove that GCosT q-rung DioNSIVWG* $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$=

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$$\left[\begin{array}{c}\left[\begin{array}{c}\sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t-})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^{2q}\right)^{1/q}},\\ \sqrt[2q]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[2q]{1-\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{t+})\right)\right)^{2q}\right)^q}\right)^{\kappa_i}\right)^{2q}\right)^{1/q}}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m-})^q\right)\right)^q\right)^{\kappa_i}}\right)^{1/q},\\ \left(\sqrt[q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{m+})^q\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f-})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q},\\ \left(\sqrt[2q]{1-\bigwedge_{i=1}^{n}\left(1-\left(\cos^2\left(\theta\cdot(\hbar\Re_i^{f+})^{2q}\right)\right)^q\right)^{\kappa_i}}\right)^{1/q}\end{array}\right]\end{array}\right].$$

Remark 4.15. The CosT q-rung DioNSIVWG operator is substituted for the GCosT q-rung DioNSIVWG operator when $q = 1$.

Corollary 4.16. *Idempotent, boundedness, and monotonicity properties must be fulfilled by the GCosT q-rung DioNSIVWG operator.*

5 Conclusion:

We introdueced the new distance such as EDs and HDs for CosT q-rung DioNSIVS. The concept of CosT q-rung DioNSIVWA, CosT q-rung DioNSIVWG, GCosT q-rung DioNSIVWA and GCosT q-rung DioNSIVWG. We present a new idea for cosine trigonometric rules. Examined as well is the CosT q-rung DioNSIVS based on algebraic operations such as idempotency, boundedness, commutativity and monotonicity. We discussed additional kinds of two distances in the proposed models, in addition to ED and HD distances. This concept is applicable to both ongoing research projects and real time applications. Given the widespread knowledge of the hybridization of fuzzy soft sets with FSs, his work could benefit from the new generalization of fuzzy soft sets, (a, b)-fuzzy soft sets, cubic FSs and cubic fuzzy soft sets.

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