Convergence of Filters on Bornological Vector Spaces and Neutrosophic Filters

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Abstract

In this research, we construct new type of convergence of bornological vector spaces called convergence of filters through using conception bounded sets. As well, we have considered several characteristics of these concepts like Fréchet filter associated with sequence, filter that has a unique limit and ultra-filter which is very useful in the study of neutrosophic topological spaces and neutrosophic filters.

Keywords: Convergence; Bornology; Filter; Fréchet filter; neutrosophic topology; neutrosophic filter.

1. Introduction

Convergence is a fundamental concept in analysis and bornology. Besides that, in bornological vector space, convergence plays an important role. The convergence of sequences is a bornological concept that was introduced and investigated by the work of [1]. In the present time, there has been renewed attention of academic researchers in bornologies in general topology considering different aspects. Considering the wide pertinence of convergence in Hausdorff metric space and the weaker Attouch–Wets convergence [2–5]. Lechicki et al. [6] were from the first who started an inclusive study of convergence of bornology for nets of sets in a metric space. In their research papers, they studied the convergence the hyperspaces that are resulted by covers of the space. Albasri [7]introduced convergence net in convex bornological vector spaces. Bornological and b-bornological Convergence properties in locally convex space introduced by D. Ayaseh and A. Ranjbari [8]. Aydemir and Albayrak [9] also considered and introduced the concept of filter bornological convergence on sequences of sets, which may be taken as a more general concept than the bornological convergence defined on topological vector spaces.

In this work, a new type of convergence in bornological vector space called “convergence of filters” has been defined. In related level for this type, many relationships are tested like separated and unique limit and bounded linear map and ultrafilter base etc. Further properties about convergence of Frechet filter, bornologically converges to a point of set, convergence of filter base of subspace is given. We denote $\mathcal{F} \overset{b}{\to} x$ to the convergence of filters F in bornological vector spaces E. Also, we use for short the term BV-spaces instead of bornological vector space.

Our study will be very applicable regarding to neutrosophic topological spaces since they are considered as generalizations of classical topological spaces. For mor detail about neutrosophic topology see [13-15].

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2. Preliminaries

In this section, fundamental concepts of bornology, bornological vector space (BV-space) and separated bornological vector space are introduced. The definitions of bornology from which the concepts of this study were built, as introduced in [10]. A vector space E on a real or complex field **F** then ß is a bornology over E which is supposed to be compatible with a vector space of E or to be a vector bornology over E, if ß is steady under vector addition, the formation of circled hulls and homothetic transformations , so it can be expressed in another way, A+B, λ A ,$\bigcup_{|\alpha|\leq 1} \alpha A$ each of them will be a member in ß whenever ß contains the sets A and B and λ ∈ **F**. Let us consider a separated BV-space (E, ß) where {0} is only taken as the only bounded vector subspace of E. The concepts of filter, ultrafilter and filter bases had been mentioned in [11,12]. A filter over a set X is a family n of subsets, $\mathcal{F}$ subcollection of P(X), such that

i. $X \in \mathcal{F}$ (that is, the set X is large sets).

ii. $\emptyset$ *does not belong to* $\mathcal{F}$ (that is, the empty set is not large set).

iii. If E is belonged to $\mathcal{F}$ and F is subset of E, then F is belonged to $\mathcal{F}$ (each set consisting a large set is large set).

If E belongs to $\mathcal{F}$ and considering F belongs to $\mathcal{F}$, then E intersects F belong to $\mathcal{F}$ (large sets containing large intersections). Now the definition of ultrafilter is going to be discuss in detail: a filter $\mathcal{F}$ on X is supposed to be an ultrafilter if for every A is subset of X, either A is belonged to $\mathcal{F}$ or X\A is belonged to $\mathcal{F}$. A filter bases are sub collection $\mathcal{F}_0$ of a filter $\mathcal{F}$ each element of $\mathcal{F}$contains some element of $\mathcal{F}_0$. Therefore, it can be easily concluded that each filter is a filter base.

Example 1
Let "$B \subseteq X$". Then the family "$\mathcal{F} = \{A: B \subseteq \mathrm{A}\}$" being a filter on X.

Remark 1
Every filter $\mathcal{F}$ on X, being the intersection of all the ultrafilter finer than $\mathcal{F}$.

3. Filter Convergence in BV-Spaces

We will define convergence of filter in BV-spaces in this section to investigate intersection of filters. Also, we investigate the connection of Frechet filter with convergent filter in BV-spaces. Finally, it will be proven the convergence of filter will be unique if the space is separated BV-spaces.

Definition 1 [1]
Let E be a BV-spaces and $\mathcal{F}$ be a filter on E .Then $\mathcal{F}$ converges bornologically "fb-convergence for short "to 0 and denoted by $\mathcal{F} \xrightarrow{fb} 0$, if there exists a bounded set $B \subset E$ in such a way that $\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F}$. $\mathcal{F}$ converges bornologically to x if the filter $\mathcal{F} - x$ converges bornologically to 0. It can be assumed that "x is a bornological limit point of $\mathcal{F}$". The set of all limit points of $\mathcal{F}$ denoted by $lim(\mathcal{F})$.

Proposition 1
Let $\mathcal{F}$ converges bornologically filter to x on BV-space E then $c\mathcal{F} \xrightarrow{fb} cx$ for any number $c \in K, c \neq 0$.

Proof
Since "$\mathcal{F} \xrightarrow{fb} x$ i.e $\mathcal{F} - x \xrightarrow{fb} 0$" then there is a bounded set $B \subset E$ in such that "$\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F} - x$ then $c\{\{\lambda B; \lambda \neq 0, \lambda \in K\}\} \subseteq c(\mathcal{F} - x)$", $\{c\lambda B; \lambda, c \neq 0, \lambda, c \in K\} \subseteq c\mathcal{F} - \mathrm{c}x$, let $c\lambda = \lambda_1, \lambda_1 \neq 0, \lambda_1 \in K$ we have$\{\lambda_1 B; \lambda_1 \neq 0, \lambda_1 \in K\} \subset c\mathcal{F} - \mathrm{c}x.$ *thus* $c\mathcal{F} - \mathrm{c}x \xrightarrow{fb} 0$ $c\mathcal{F} \xrightarrow{fb} cx$.

Proposition
When a "filter $\mathcal{F}$ b-converges to x, $\mathcal{F} \xrightarrow{fb} x$ then every filter $\acute{\mathcal{F}}$ finer than $\mathcal{F}$ also b-converges to x $\acute{\mathcal{F}} \xrightarrow{fb} x$".

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Proof

Since $\mathcal{F} \overset{fb}{\rightarrow} x$ then there is a bounded set $B \subset E$ in such that $\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F} - x$. Since $\acute{\mathcal{F}}$ finer than $\mathcal{F}$,we have $\mathcal{F} \subseteq \acute{\mathcal{F}}$, then $\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \acute{\mathcal{F}} - x$, so $\acute{\mathcal{F}} \overset{fb}{\rightarrow} x$.

Proposition

Let (E, β) and (E, β') be BV- spaces, then β is finer than β' if and only if "$\mathcal{F} \overset{fb}{\rightarrow} x$" in β implies "$\mathcal{F} \overset{fb}{\rightarrow} x$" in β' for each filter $\mathcal{F}$ in E.

Proof

($\Longrightarrow$) Let β be finer than β' and let $\mathcal{F} \overset{fb}{\rightarrow} x$ in β then there is a bounded set $B \subset E$ in such a way that $\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F} - x$. Since β is finer than β'then there exists a bounded set $B_1 \subset E$ in β'such that $\{\lambda_1 B_1 : \lambda_1 \neq 0, \lambda_1 \in K\} \subset \{\lambda B: \lambda \neq 0, \lambda \in K\} \subseteq \mathcal{F} - x,$ we have $\mathcal{F} \overset{fb}{\rightarrow} x \ in \ \beta'$.

($\Longleftarrow$) suppose that "$\mathcal{F} \overset{fb}{\rightarrow} x$ in β implies $\mathcal{F} \overset{fb}{\rightarrow} x$ in β'" can be easily predicted.

Proposition 4

Let Ψ be the family of all filters on a BV- spaces E converges to $x \in E$ the intersection of all filters in Ψ also converges to x.

Proof

Since the intersection of all filters is filter on E, since all the filters in Ψ converge to x then the intersection of all filters is coarser than each filter in Ψ. Hence by proposition 3 the intersection of all filters is convergent to x in E.

Proposition 5

Let "(E, β) be a BV-space and let $\mathcal{F}$ be a filter on E". Then, the following statements have the same equivalent:

i. $\mathcal{F}$ approaches to a point $x \in E$.

ii. Each bounded set contains x belongs to $\mathcal{F}$.

iii. $\mathcal{F}$ is finer than the collection of all bounded sets contains x on E.

iv. For every bounded set V contains x, there is $B \in \mathcal{F}$ such that $B \subset V$.

Proof :

$(i \rightarrow ii)$ we have follows immediately from definition 1and filter base. $(ii \longrightarrow iii)$ is "an immediate consequence of the definition 1". $(iii \longrightarrow iv)$ since $\mathcal{F}$ is finer than the collection of all bounded sets contains x on E we have (iv).

Proposition 6

Let "E be a BV-space. Then a sequence $\langle x_n \rangle \subset E$ b-converges to x if and only if the Frechet filter associated with $\langle x_n \rangle$ b-converges to x".

Proof

($\Longrightarrow$) Let $\langle x_n \rangle$ in E converges bornologically to x and let $\mathcal{F}$ Frechet filter is associated with $\langle x_n \rangle$, let $x \in A \subset E$ and $A \in \mathcal{F}$ bounded set, we have A associated with $\langle x_n \rangle$ then A contains a set of the formula $\{x_n - x; n \geq n_0, n_0 \in \mathbb{N}\}$, since $x_n \overset{fb}{\rightarrow} x$ then there's found a bounded circled set $B \subset E$ and a sequence of scalars (λ_n) approaching to 0, in such a way that $x_n - x \in \lambda_n B$ for each$n \in \mathbb{N}$. Thus, there exists found bounded set $B_1 \subset E$ of x in such a way that "$\{\lambda B_1 : \lambda \neq 0, \lambda \in K\} \subseteq \mathcal{F} - x$" therefore $\mathcal{F} \overset{fb}{\rightarrow} x$.

($\Longleftarrow$) Let $\mathcal{F} \overset{fb}{\rightarrow} x$ because of $\mathcal{F}$ Frechet filter is associated with $\langle x_n \rangle$, now if a bounded $A \subset E$ belongs to the $\mathcal{F}$ Frechet filter connected with $\langle x_n \rangle$, then A contains a set of the forms $\{x_n - x; n \geq n_0, n_0 \in \mathbb{N}\}$ then if $\langle \lambda_n \rangle$ of scalars approaching to 0, we have $x_n - x \in \lambda_n B$ for every integer $n \in \mathbb{N}$ then $x_n \overset{fb}{\rightarrow} x$.

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Proposition 7

Let E be a BV- space, the filter of all bornivorous set converges bornologically to x if and only if E contains a bounded bornivorous set.

Proof:

($\Longrightarrow$) Let "$\mathcal{F}$" be a filter of all bornivorous set converges bornologically to , then we have from Definition 1 E contains a bounded subset, since $\mathcal{F}$ the filter of all bornivorous set .Hence E contains a bounded bornivorous set.

($\Longleftarrow$) Now E contains a bounded bornivorous set since the collection of all bornivorous subset of a BV-space is a filter.

Since every bornivorous set $B - x$ contains 0 then there is a bounded set B_1 such that $\{\lambda B_1 : \lambda \neq 0, \lambda \in K\} \subseteq \mathcal{F} - x$ therefore the filter of all bornivorous set converges bornologically to x.

Proposition 8

A BV-space "E" is separated if and only if each bornologically convergent filter in E has a unique limit.

Proof:

($\Longrightarrow$) Let $\mathcal{F}$ be a filter in E converges bornologically to x and y with $x \neq y$, now $\mathcal{F} \xrightarrow{fb} x$ then there exists a bounded set $B \subset E$ in such a way that "$\{\lambda B; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F} - x$". if $\mathcal{F} \xrightarrow{fb} y$ there is a bounded set $U \subset E$ in such a way that "$\{\lambda U; \lambda \neq 0, \lambda \in K\} \subset \mathcal{F} - y$", since $\mathcal{F}$ be a filter then $B \cap U \neq \phi$ there's found $z \in B \cap U$ in such a way that the subspace K_z spanned by z is contained in $B \cap U$ this contradicting the hypothesis that E is separated.

($\Longleftarrow$) To prove that E is a separated, suppose there's not found and element $y \neq x, and\ x, y \in E$ in such a way that the line spanned by y is bounded then there is a bounded set $x \in B \subseteq E$ and fined a bounded set $y \in U \subseteq E$ and $U \in \mathcal{F}, B \cap U \neq \phi, (\mathcal{F}$ be a filter).

Consequently, $\mathcal{F}_0 = \{B \cap U,\ x \in B \subseteq \mathcal{F}\ , y \in U \subset E\}$ is a filter base for some filter $\mathcal{F}$. The resulting filter converges at x and y. This contradiction, thus E is separated space.

4. Convergence Of Filter Base in BV- Spaces

In this section, by defining base convergence, we will complete the investigation of some properties. After that, we test convergence in product BV-space.

Definition 2

A filter base $\mathcal{F}_0$ on a BV-space E does converge bornologically to $x \in E$ if the filter whose base $\mathcal{F}_0$ converges bornologically to x and we say that x is a bornological limit point of $\mathcal{F}_0$. The set of all limit points of a filter base $\mathcal{F}_0$ is denoted by $lim(\mathcal{F}_0)$.

Proposition 9

A filter $\mathcal{F}$ on a BV-space E is b-converges to $x \in E$ iff each ultrafilter containing $\mathcal{F}$, b-converges to x.

Proof

If $\mathcal{F}$ b-converges to x, then by definition 1 we have every ultrafilter containing $\mathcal{F}$ also b-converges to x.

Conversely: let every ultrafilter containing $\mathcal{F}$ b-converges to x. We have by **Remark 1** $\mathcal{F}$ is the intersection of all ultrafilter on E finer than $\mathcal{F}$ and by proposition 4 so $\mathcal{F}$ b-converges to .

Proposition 10

Let E be a BV-space and $A \subset E$. Then A is bounded iff A belongs to each filter which b- converges to a point of A.

Proof

Let A be a bounded set and let $\mathcal{F}$ be a filter which b-converges to $x \in A$. Then we have from definition 1 $\mathcal{F}$ contains every bounded of x. Since A is bounded set contained x hence $A \in \mathcal{F}$.

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Conversely let A belong to every filter which converges bornologically to a point of A. To show that A is bounded set. Let $x \in A$ and let $\mathcal{F} \xrightarrow{fb} x$ then $A \in \mathcal{F}$ then we easily have A is a bounded set.

Proposition 11

Let G be a bornological subspace of a space E, let $y \in G$ and let $\mathcal{F}_0$ be a filter base on G if $\mathcal{F}_0 \xrightarrow{fb} y$ in G then $\mathcal{F}_0 \xrightarrow{fb} y$ in E.

Proof

Let $\mathcal{F}_0$ be a filter base on G and $\mathcal{F} \xrightarrow{fb} y$ in G then the filter whose base is $\mathcal{F}_0$ converges bornologically to y in G. Since $\mathcal{F}_0$ a sub collection of a filter $\mathcal{F}$ in G then we have $\mathcal{F}_0 \to y$ in E.

Proposition 12

Let (E_1, β_1) and (E_2, β_2) be two bornological spaces and let $x \in E_1$ let f be a mapping of E_1 into E_2 then the following statements are equivalents:

i. f is bounded linear map at x.

ii. For every filter base $\mathcal{F}_0$ on E_1 bornologically converging to x the filter base $f(\mathcal{F}_0)$ converges bornologically to $f(x)$.

iii. For every ultrafilter base $\mathcal{F}_0$ on E converging bornologically to x the ultrafilter base $f(\mathcal{F}_0)$ on E_2 bornologically converges to $f(x)$.

Proof

Let $f(\mathcal{F}_0) = \{f[B]: B \subset \mathcal{F}_0\}$

$(i) \Rightarrow (ii)$ let f be bounded linear map at x and let V be any bounded set contains x in E_1. Then $f(V)$ is bounded set contains $f(x)$ in E_2. Let $\mathcal{F}_0$ be any filter base on E_1 bornologically converges to x the filter whose base $\mathcal{F}_0$ is bornologically converges to x. Let $B \in \mathcal{F}_0$ in such a way that $B \subseteq V \Longrightarrow f(B) \subseteq f(V)$ (by f is bounded map) we have the filter whose base $f(\mathcal{F}_0)$ is bornologically converges to $f(x)$ then $f(\mathcal{F}_0)$ the filter base converges bornologically to $f(x)$.

$(ii) \to (iii)$ and $(iii) \to (i)$ can be easily proved

Proposition 13

A filter $\mathcal{F}$ on non-empty product BV-space $E = \pi\{E_\lambda: \lambda \in \Lambda\}$ bornologically converges to a point $x = \{x_\lambda: \lambda \in \Lambda\}$ of E if f the filter $\pi_\lambda[\mathcal{F}] = \{\pi_\lambda[\mathcal{F}]: F \in \mathcal{F}\}$ for every $\lambda \in \Lambda$ converges bornologically to $x_\lambda = \pi_\lambda(x)$ in E_λ.

Proof

$(\Longrightarrow)$Let $\mathcal{F}$ bornologically converges to $\in E$. Since π_λ is bounded linear for every $\lambda \in \Lambda$, we have from proposition 12 (ii) and every filter is a filter base then $\pi_\lambda[\mathcal{F}]$ bornological convergence to $\pi_\lambda(x)$

$(\Longleftarrow)$Let $\mathcal{F}$ be a filter on E in such a way that the filter $\pi_\lambda[\mathcal{F}]$ converges bornologically to a point x_λ in E_λ for every λ, then let x denote the point of E, whose $\lambda - th$ coordinate is $x_\lambda. Let\ B$ be any bounded of $x \in E$. Then there's found a base member $F = \{F_\lambda. \lambda \in \Lambda\}$ where F_λ is bounded in E_λ and $F_\lambda = E_\lambda$ for $\lambda \neq \lambda_1, \lambda_2, \ldots, \lambda_n$ in such a way that $x \in F \subset B$. now $\pi_\lambda[\mathcal{F}] = F_{\lambda i}$, which is bounded of $x_\lambda, i = 1,2,\ldots,n$. By hypothesis $\pi_{\lambda i}[\mathcal{F}]$ converges to $x_{\lambda i}$ and $\pi_{\lambda i}[\mathcal{F}]$ must contain $F_{\lambda i}$ for $i = 1,2,\ldots,n$ Hence $\pi_{\lambda i}{}^{-1}[F_{\lambda i}]$ contains a member of $\mathcal{F}$. Then by definition 1 $\pi_{\lambda i}{}^{-1}[F_{\lambda i}] \in \mathcal{F}\ for\ i = 1,2,\ldots,n$ and therefore for $\bigcap_{i=1}^{n} \pi_{\lambda i}{}^{-1}[F_{\lambda i}] \in \mathcal{F}$ but $\bigcap_{i=1}^{n} \pi_{\lambda i}{}^{-1}[F_{\lambda i}] = F$ and so $F \in \mathcal{F}$. since $F \subset B, F \in \mathcal{F}$ by definition of filter. Since B was any bounded set contained $x\ in\ E$ we have $\mathcal{F}$ converges to x.

5. Convergence Of Neutrosophic Filter Base in neutrosophic BV- Spaces

Definition

A neutrosophic filter base $\mathcal{F}_0$ on a neutrosophic BV-space E does converge bornologically to $x \in E$ if the filter whose base $\mathcal{F}_0$ converges bornologically to x and we say that x is a bornological limit point of $\mathcal{F}_0$. The set of all limit points of a filter base $\mathcal{F}_0$ is denoted by $lim(\mathcal{F}_0)$.

Proposition

A neutrosophic filter $\mathcal{F}$ on a neutrosophic BV-space E is b-converges to $x \in E$ iff each ultrafilter containing $\mathcal{F}$, b-converges to x.

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Proof
If $\mathcal{F}$ b-converges to x, then by definition 1 we have every ultrafilter containing $\mathcal{F}$ also b-converges to x.
Conversely: let every ultrafilter containing $\mathcal{F}$ b-converges to x. We have by **Remark 1** $\mathcal{F}$ is the intersection of all ultrafilter on E finer than $\mathcal{F}$ and by proposition 4 so $\mathcal{F}$ b-converges to .

Proposition
. Let E be a neutrosophic BV-space and $A \subset E$. Then A is bounded iff A belongs to each neutrosophic filter which b- converges to a point of A.

Proof
Let A be a bounded set and let $\mathcal{F}$ be a filter which b-converges to $x \in A$. Then we have from definition 1 $\mathcal{F}$ contains every bounded of x. Since A is bounded set contained x hence $A \in \mathcal{F}$.
Conversely let A belong to every filter which converges bornologically to a point of A. To show that A is bounded set. Let $x \in A$ and let $\mathcal{F} \overset{fb}{\rightarrow} x$ then $A \in \mathcal{F}$ then we easily have A is a bounded set.

Proposition
Let G be a neutrosophic bornological subspace of a space E, let $y \in G$ and let $\mathcal{F}_0$ be a neutrosophic filter base on G if $\mathcal{F}_0 \overset{fb}{\rightarrow} y$ in G then $\mathcal{F}_0 \overset{fb}{\rightarrow} y$ in E.

Proof.
Let $\mathcal{F}_0$ be a filter base on G and $\mathcal{F} \overset{fb}{\rightarrow} y$ in G then the filter whose base is $\mathcal{F}_0$ converges bornologically to y in G. Since $\mathcal{F}_0$ a sub collection of a filter $\mathcal{F}$ in G then we have $\mathcal{F}_0 \rightarrow y$ in E.

Proposition
Let (E_1, β_1) and (E_2, β_2) be two neutrosophic bornological spaces and let $x \in E_1$ let f be a mapping of E_1 into E_2 then the following statements are equivalents:

iv. f is a bounded linear map at x.

v. For every neutrosophic filter base $\mathcal{F}_0$ on E_1 bornologically converging to x the neutrosophic filter base $f(\mathcal{F}_0)$ converges bornologically to $f(x)$.

vi. For every neutrosophic ultrafilter base $\mathcal{F}_0$ on E converging bornologically to x the neutrosophic ultrafilter base $f(\mathcal{F}_0)$ on E_2 bornologically converges to $f(x)$.

Proof
Let $f(\mathcal{F}_0) = \{f[B]: B \subset \mathcal{F}_0\}$
$(i) \Rightarrow (ii)$ let f be bounded linear map at x and let V be any bounded set contains x in E_1. Then $f(V)$ is bounded set contains $f(x)$ in E_2. Let $\mathcal{F}_0$ be any neutrosophic filter base on E_1 bornologically converges to x the filter whose base $\mathcal{F}_0$ is bornologically converges to x. Let $B \in \mathcal{F}_0$ in such a way that $B \subseteq V \Longrightarrow f(B) \subseteq f(V)$ (by f is bounded map) we have the filter whose base $f(\mathcal{F}_0)$ is bornologically converges to $f(x)$ then $f(\mathcal{F}_0)$ the filter base converges bornologically to $f(x)$.
$(ii) \rightarrow (iii)$ and $(iii) \rightarrow (i)$ can be easily proved

Proposition
A neutrosophic filter $\mathcal{F}$ on non-empty product neutrosophic BV-space $E = \pi\{E_\lambda : \lambda \in \Lambda\}$ bornologically converges to a point $x = \{x_\lambda : \lambda \in \Lambda\}$ of E if f the filter $\pi_\lambda[\mathcal{F}] = \{\pi_\lambda[\mathcal{F}]: F \in \mathcal{F}\}$ for every $\lambda \in \Lambda$ converges bornologically to $x_\lambda = \pi_\lambda(x)$ in E_λ.

Proof
$(\Longrightarrow)$Let $\mathcal{F}$ bornologically converges to $\in E$. Since π_λ is bounded linear for every $\lambda \in \Lambda$, we have from proposition 12 (ii) and every filter is a filter base then $\pi_\lambda[\mathcal{F}]$ bornological convergence to $\pi_\lambda(x)$
$(\Longleftarrow)$Let $\mathcal{F}$ be a neutrosophic filter on E in such a way that the filter $\pi_\lambda[\mathcal{F}]$ converges bornologically to a point x_λ in E_λ for every λ, then let x denote the point of E, whose $\lambda - th$ coordinate is $x_\lambda. Let\ B$ be any bounded of $x \in E$. Then there's found a base member $F = \{F_\lambda . \lambda \in \Lambda\}$ where F_λ is bounded in E_λ and $F_\lambda = E_\lambda$ for $\lambda \neq \lambda_1, \lambda_2, \ldots, \lambda_n$ in such a way that $x \in F \subset B$. now $\pi_\lambda[\mathcal{F}] = F_{\lambda i}$, which is bounded of $x_\lambda, i = 1,2, \ldots, n$. By hypothesis $\pi_{\lambda i}[\mathcal{F}]$ converges to $x_{\lambda i}$ and $\pi_{\lambda i}[\mathcal{F}]$ must

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contain $F_{\lambda i}$ for $i = 1,2,\dots,n$ Hence $\pi_{\lambda i}{}^{-1}[F_{\lambda i}]$ contains a member of $\mathcal{F}$. Then by definition 1 $\pi_{\lambda i}{}^{-1}[F_{\lambda i}] \in \mathcal{F}\ for\ i = 1,2,\dots,n$ and therefore for $\bigcap_{i=1}^{n} \pi_{\lambda i}{}^{-1}[F_{\lambda i}] \in \mathcal{F}$ but $\bigcap_{i=1}^{n} \pi_{\lambda i}{}^{-1}[F_{\lambda i}] = F$ and so $F \in \mathcal{F}$. since $F \subset B, F \in \mathcal{F}$ by definition of filter. Since B was any bounded set contained $x\ in\ E$ we have $\mathcal{F}$ converges to x.

6. Conclusion

In this paper, we have studied a very important concept with its relations to neutrosophic topologies, where we constructed new type of convergence of bornological vector spaces called convergence of filters through using conception bounded sets. As well, we have considered several characteristics of these concepts like Fréchet filter associated with sequence, filter that has a unique limit and ultra-filter.

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