# Incidence Topological Spaces Generated from The Simple Undirected Graphs

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**Abstract**

In this paper, we investigate topologies produced by simple connected graphs. In particular, we associate a topology with G, called the incidence topology of G. A sub-base family to generate a incidence topology is implemented on the Vertices V set. Then we analyze some of the properties and discuss the impact topology of a few essential types of graphs. Our motivation in this section is to take a fundamental step towards the investigation of some of the characteristics of simple graphs by their corresponding incidence topology.

**Keywords:** Finite Topological Spaces; Connected Simple Graphs; Topologies; undirected graphs.

## 1. Introduction

Graph theory is one of the most important structures in discrete mathematics. It is a prominent mathematical tool in many subjects [4]. Graphs are mathematical structures consisting of vertices and edges. They are used to model dual relationships between objects in a particular collection. A graph includes vertices that serve as objects and edges by connecting these vertices [1]. Many researchers have used the relation between graph and topology theories to deduce a topology from a particular graph. Others put new specific models in the set of vertices in the graph, and others make it on the set of edges. Graphs can be divided into two types directed graphs and undirected graphs. The researchers' previous work to obtain the topology through the graph was associated with a graph of vertices adjacent to the vertex. Euler first proposed graph theory in 1736. Recently, the theory has been an achieved applied to places in various disciplines. Since the theory is according to relation combinations, it plays a crucial role in representing combinations of items and mathematical combinations. Simple set theory is also according to relational combinations. They are solved by using applications of graph theory. Because of the pervasive use of the theory, its topological structure has generated debate [3].

Using various techniques, some scientists have created topologies from a graph. Some scientists have generated topologies from graphs using various techniques. In 2013, M. Amiri et al. Developed a topology using the vertices of an undirected graph. In this paper, we outline several characteristics of a topological space by using a simple graph without isolated vertices. We describe some topological characteristics of the topology we produce with these graphs. We demonstrate that every basic graph may generate a topology. Obtain the topological space by converting the simple connected graph [10]. In this article, A synthesis between graph theory and topology has been made. A topology with the set of vertices for any simple graphs has been associated, called incidence topology. The study of some properties of this new model of topology has been presented on a certain few important type of graphs.



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## 2. Preliminaries

In this section, some fundamental definitions and theorems related to the graph theory, approximation spaces and topological spaces used in the work are presented.

**Definition 2.1.[1]** A graph $G$ is a pair$(V, E)$ or$(V(G), E(G))$ where is a non-empty set called vertices or nodes and$E$ is element subsets of $V$ called edges or links.

**Definition 2.3.[4]** A simple graph is called complete graph, if any two distinct vertices are joined by an edge denoted by $K_n$ . A graph $G$ is regular if the vertices of $G$ has the same degree.

**Definition 2.4.[4]** A complete biparitite graph is a bipartite graph in which each vertex in the first set is joined to each vertex in the second set by exactly one edge. The complete bipartite graph with n vertice s of first set and m vertices of second set is dented by $K_{n,m}$.

**Definition 2.5. [2]**

A tree is an undirected connected graph with no cycles

**Definition 2.6.[3]** A spanning tree of a graph $G$ is a sub graph of $G$ that is a tree that contain all the vertices of $G$ with no cycles.

**Definition 2.7.[12]** If X is a non-empty set, a collection $\mathcal{T} \subseteq$ (X) is said to be a topology on X if the following condition holds: I. X and $\emptyset$ belongs to $\mathcal{T}$ . II. The intersection of a finite numbers of elements in , is in $\mathcal{T}$. III. The union of any number of sets in $\mathcal{T}$ belongs to . Then is called a topological space.

**Definition 2.8.[12]** A sub base S for a topology on X is a collection of subsets of X whose union equals X . The topology generated by the sub basis S is defined to be the collection of all unions of finite intersections of elements of S .

**Definition 2.9.[6]** In a space (X,$\mathcal{T}$) , a collection β of open subset of is called a basis for $\mathcal{T}$ if every open is a union of element of β .

## 3. Incidence Topology on Simple Connected Graphs

Let G = (V, E) be a simple connected graph. In this section, we associate a topology with G, called the incidence topology of G. A sub-base family to generate a incidence topology is implemented on the Vertices V set. Then we analyze some of the properties and discuss the impact topology of a few essential types of graphs. Our motivation in this section is to take a fundamental step towards the investigation of some of the characteristics of simple graphs by their corresponding incidence topology.

**Definition 3.1.**

Let $G(V, E)$ be connected a simple graph consist of a non-empty set $V(G)$of vertices and a set $E(G)$ of edges. If $v_1$ and $v_2$ are vertices and e is an edge such that $e = v_1v_2$ , then e is said to join $v_1$ and $v_2$ ,each vertex ($v_1$and $v_2$ ) is incident with e.Bear in mind the $GN(e_n)$ is a set of all vertices that are incident to the edge, define $N_G(E)$ as follows $N_G(E)=\{N(e) \in E(G)\}$,we have $V =\cup_{e\in E} N_G(E)$. Hence $N_G(E)$from a sub basis for a topology $\mathcal{T}_G$ on $V$,called incidence topology of G.

**Example 3.2.**

Consider the simple connected graph$G(V, E)$ in figure (3.7) where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E == \{ e_1, e_2, e_3, e_4, e_5\}$.



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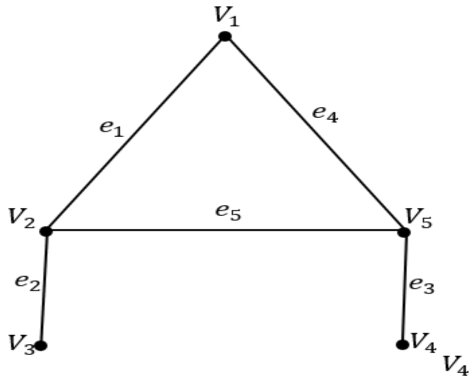


Figure (3.7): Simple graph

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows: $N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_2, v_3\}$ , $N(e_3) = \{v_4, v_5\}$ , $N(e_4) = \{v_1, v_5\}$ , $N(e_5) = \{v_2, v_5\}$
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_3\}, \{v_2, v_3\}\{v_4, v_5\}, \{v_1, v_5\}, \{v_2, v_5\}, \{v_1\}, \{v_2\}, \{v_5\}\}$
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_3\}, \{v_2, v_3\}\{v_4, v_5\}, \{v_1, v_5\}, \{v_2, v_5\}, \{v_1\}, \{v_2\}, \{v_5\}, \{v_1, v_2, v_3\},$
$\{v_1, v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_3, v_5\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_5\},$
$\{v_2, v_3, v_5\}, \{v_1, v_4, v_5\}, \{v_2, v_4, v_5\}\{v_1, v_4, v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_5\}, \{v_2, v_5\},$
$\{v_1, v_2, v_4, v_5\}\}$.

**Example 3.3.**

Consider the graph$G(V, E)$ in figure 1 where:
$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.


Figure 1: Connected graph

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows:
$N(e_1) = \{v_1, v_3\}$ , $N(e_2) = \{v_2, v_3\}$ , $N(e_3) = \{v_3, v_4\}$ , $N(e_4) = \{v_4, v_5\}$ , $N(e_5) = \{v_4, v_6\}$
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_3\}, \{v_2, v_3\}\{v_4, v_5\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_3\}, \{v_4\}\}$
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_3\}, \{v_2, v_3\}\{v_4, v_5\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_3\}, \{v_4\},$
$\{v_1, v_2, v_3\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_3, v_4, v_6\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_4, v_5\},$
$\{v_2, v_3, v_4, v_6\}, \{v_3, v_4, v_5\}, \{v_3, v_4, v_6\}, \{v_4, v_5, v_6\}\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_4, v_5\},$
$\{v_1, v_2, v_3, v_4, v_6\}, \{v_3, v_4, v_5, v_6\}, \{v_1, v_3, v_4, v_5, v_6\}\}$.

**Remark 3.4**.
Incidence topology of $P_n$ where $n \geq 3$ is not discrete because the set contains just a few vertices are open.

**Remark 3.5.**
The path graph $P_n$ ,wheren=2 is not incidence topology.

**Example 3.6.**
Consider the path graph $G(V, E)$ in figure where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3\}$.



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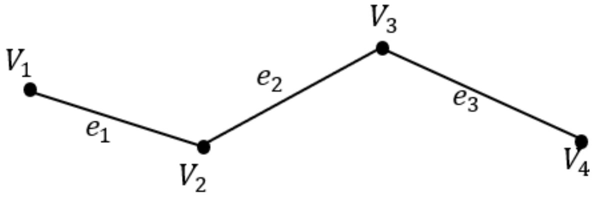


Figure 2: Path graph $p_4$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows $N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_2, v_3\}$ , $N(e_3) = \{v_3, v_4\}$ .
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_2\}, \{v_3\}\}$.
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_2\}, \{v_3\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}$

**Example 3.7.**
Consider the path graph $G(V, E)$ in figure 3 where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{ e_1, e_2, e_3, e_4, e_5\}$.

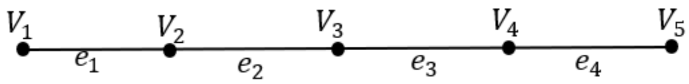


Figure 3: Path graph $P_5$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows$N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_2, v_3\}$ , $N(e_3) = \{v_3, v_4\}$ , $N(e_4) = \{v_4, v_5\}$ .
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_2\}, \{v_3\}, \{v_4\}\}$
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2, v_3\},$
$\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_4, v_5\},$
$\{v_2, v_3, v_4, v_5\}, \{v_3, v_4, v_5\}, \{v_2, v_4, v_5\}\{v_2, v_3\}, \{v_3, v_4\}, \{v_2, v_4\}$.

**Remark 3.8.**
Suppose $G(V, E)$ be a complete graph and complete bipartite graph verify the incidence topology is discrete topology when $n \geq 3$ .

**Example 3.3.9.**
Consider the complete graph $G(V, E)$ in figure 4 where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{ e_1, e_2, e_3, e_4\}$.

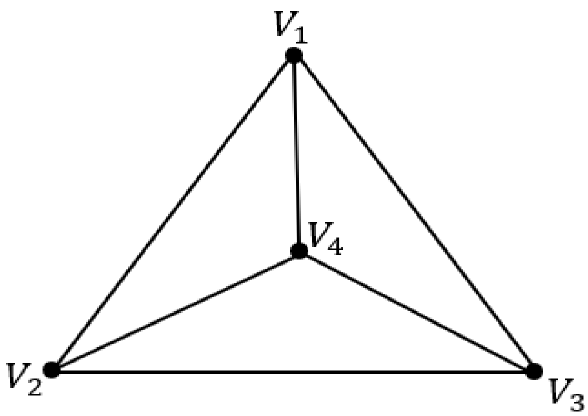


Figure 4: A complete graph $K_4$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows:
$N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_2, v_3\}$ , $N(e_3) = \{v_1, v_3\}$ , $N(e_4) = \{v_1, v_4\}$, $N(e_5) = \{v_2, v_4\}$, $N(e_6) = \{v_3, v_4\}$.



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By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}$
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$

**Example 3.10.**

Consider the complete bipartite graph $G(V, E)$ in figure 5 where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$.

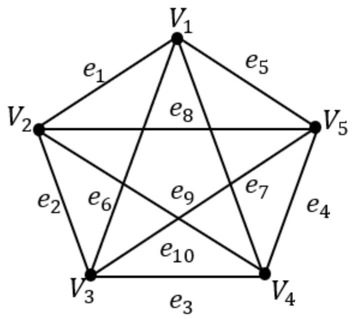


Figure 5: A complete graph $K_5$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows:
$N(e_1) = \{v_1, v_2\}$, $N(e_2) = \{v_2, v_3\}$, $N(e_3) = \{v_3, v_4\}$, $N(e_4) = \{v_4, v_5\}$, $N(e_5) = \{v_1, v_5\}$, $N(e_6) = \{v_1, v_3\}$, $N(e_7) = \{v_1, v_4\}$, $N(e_8) = \{v_2, v_5\}$, $N(e_9) = \{v_2, v_4\}$, $N(e_{10}) = \{v_3, v_5\}$.
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_5\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}\}$.
$T_G = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_4, v_5\}, \{v_1, v_5\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_5\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_4, v_5\}, \{v_2, v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3, v_5\}\}$.

**Example 3.11.**

Consider the complete graph $G(V, E)$ in figure where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$.

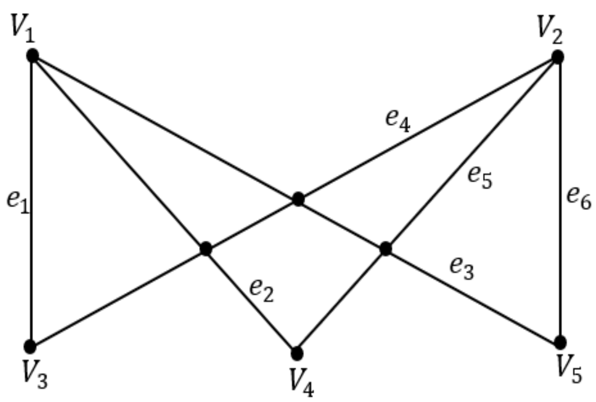


Figure 6: A complete bipartite graph $K_{2,3}$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows:
$N(e_1) = \{v_1, v_3\}$, $N(e_2) = \{v_1, v_4\}$, $N(e_3) = \{v_1, v_5\}$, $N(e_4) = \{v_2, v_3\}$, $N(e_5) = \{v_2, v_4\}$,



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$N(e_6) = \{v_2, v_5\}$.
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}$.
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_5\}, \{v_2, v_3, v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_2\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_4, v_5\}, \{v_2, v_3, v_4, v_5\}$.

**Example 3.12.**
Consider the complete bipartite graph $G(V, E)$ in figure 7 where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

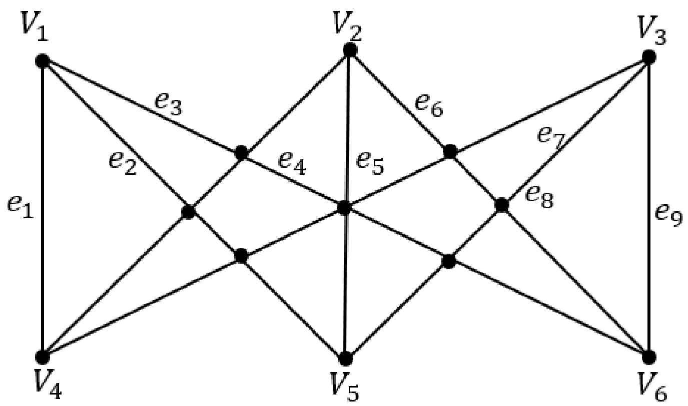


Figure 7: A complete bipartite graph $K_{3,3}$

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows:
$N(e_1) = \{v_1, v_4\}$ , $N(e_2) = \{v_1, v_5\}$ , $N(e_3) = \{v_1, v_6\}$ , $N(e_4) = \{v_2, v_4\}$, $N(e_5) = \{v_2, v_5\}$, $N(e_6) = \{v_2, v_6\}$, $N(e_7) = \{v_3, v_4\}$, $N(e_8) = \{v_3, v_5\}$, $N(e_9) = \{v_3, v_6\}$.
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_6\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_3, v_6\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}$.
$T_G=\{\emptyset, V, \{v_1,v_4\}, \{v_1,v_5\}, \{v_1,v_6\}, \{v_2,v_4\}, \{v_2,v_5\}, \{v_2,v_6\}, \{v_3,v_4\}, \{v_3,v_5\}, \{v_3,v_6\}, \{v_1,v_2,v_3,v_4,v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_1,v_2,v_3\}, \{v_1,v_2,v_3,v_4\}, \{v_1,v_2,v_4,v_5\}, \{v_1,v_2,v_4\}, \{v_1,v_2,v_5\}, \{v_2,v_3,v_4\}, \{v_2,v_3,v_4,v_5\}, \{v_2,v_3,v_5\}, \{v_3,v_4,v_5\}, \{v_1,v_3, v_4,v_5\}, \{v_1,v_3,v_4\}, \{v_1,v_4,v_5\}, \{v_2,v_4,v_5\}, \{v_1,v_3,v_5\}, \{v_1,v_2,v_3,v_5\}, \{v_1,v_4,v_6\}, \{v_1,v_2,v_6\}, \{v_1,v_3,v_4,v_6\}, \{v_1,v_5,v_6\}, \{v_1,v_2,v_4,v_6\}, \{v_1,v_2,v_5,v_6\}, \{v_1,v_3,v_6\}, \{v_2,v_4,v_6\}, \{v_1,v_3,v_5,v_6\}, \{v_3,v_5,v_6\}, \{v_2,v_3,v_6\}, \{v_3,v_4,v_6\}, \{v_1,v_2,v_3,v_6\}, \{v_1,v_2,v_3,v_4,v_6\}, \{v_1,v_2,v_4,v_5,v_6\}, \{v_2,v_3,v_4,v_6\}, \{v_2,v_3,v_5,v_6\}, \{v_3,v_4,v_5,v_6\}, \{v_1,v_3,v_4,v_5,v_6\}, \{v_1,v_3,v_4,v_6\}, \{v_1,v_4,v_5,v_6\}, \{v_2,v_4,v_5,v_6\}, \{v_1,v_3,v_5,v_6\}, \{v_1,v_2,v_3,v_5,v_6\}, \{v_1,v_3,v_4,v_5,v_6\}, \{v_2,v_3,v_4,v_5,v_6\}, \{v_1,v_2\}, \{v_1,v_3\}, \{v_2,v_3\}, \{v_4,v_5\}, \{v_5,v_6\}\}$.

**Definition 3.13.**
Let $G = (V, E)$be a graph, we call H is a subgraph from G if $V(H) \subseteq V(G), E(H) \subseteq E(G)$, in this case we would write $H \subseteq G$ The spanning sub graph from a graph G is a sub graph acquired by edge deletions only. A deduced sub graph of a graph Gis a sub graph acquired by vertex deletions along with the incident edges

**Remark 3.14.**
Aspin tree of a graph G is a sub graph of G that is tree that contain all the vertices of G with no cycles. The incidence topology of spin tree is not discrete.

**Example 3.15.**
Let G be a graph (as it is shown in figure 8). Then;



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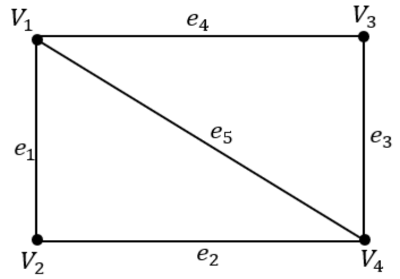


Figure 8: graph G

1. Let $G = ST_1$(spin tree as it is shown in figure 9). Then;

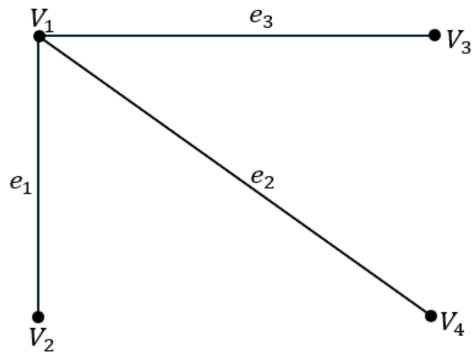


Figure 9: spin tree ( $T_1$) of G

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows: $N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_1, v_4\}$ , $N(e_3) = \{v_1, v_3\}$ .
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}, \{v_1, v_4\}, \{v_1, v_3\}, \{v_1\}$.
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_1, v_4\}, \{v_1, v_3\}, \{v_1\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}\}$

2. Let $G = ST_2$(spin tree as it is shown in figure 10). Then;
3.


Figure 10: spin tree ($T_2$) of G

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows: $N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_1, v_3\}$ , $N(e_3) = \{v_2, v_4\}$ .
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_1\}, \{v_2\}\}$.
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_1\}, \{v_2\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}\}$

4. Let $G = ST_3$(spin tree as it is shown in figure 11). Then;



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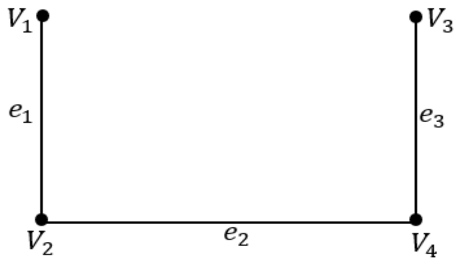


Figure 11: spin tree ($T_3$)of G

We will find the incidence topology $N\mathcal{T}_G$ of the graph $G$ as follows: $N(e_1) = \{v_1, v_2\}$ , $N(e_2) = \{v_2, v_4\}$ , $N(e_3) = \{v_3, v_4\}$ .
By taking finitely intersection of basis obtained
$NB_G(E) = \{\emptyset, \{v_1, v_2\}$ ,$\{v_2, v_4\}$ ,$\{v_3, v_4\}, \{v_2\}, \{v_4\}\}$.
$\mathcal{T}_G = \{\emptyset, V, \{v_1, v_2\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_2\}, \{v_4\}, \{v_1, v_2, v_4\}\}$.

**Proposition 3.16**. Suppose that $\mathcal{T}_G$is the incidence topology of the graph G . If $d(v) \geq 2$, then $\{v\} \in \mathcal{T}_G$ for every $\in V$ .
Proof. Since G is a simple graph and for any degree of $v$, we have $\bigcap_{i=2}^{\infty} N_{e_i} = \{v\}$ such that $vN_{e_i}$ for all $i = 1,2,3, ....$ Now by the definition of $\mathcal{T}_G, \{v\}$ is an element in the basis of $\mathcal{T}_G$. Hence $\{v\} \in \mathcal{T}_G$
The following corollary is a trivial result for the previous proposition.

**Corollary 3.17.** Let $G$ be a graph. If $d(v) \geq 2$ for all $v \in V$, then $\mathcal{T}_G$ is a discrete topology.

## 4. Conclusion

In this paper it is shown that topologies can be generated by simple connected graphs. It is studied topologies generated by certain graphs. Therefore, it is seen that there is a topology generated by every undirected graph. Properties proved by these generated topologies are presented. The study of some properties of this new model of topology has been presented.

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