On Some Topological Spaces Based On Symbolic n-Plithogenic Intervals

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Abstract

In this paper, we present the topological space of intervals based symbolic m-plithogenic real numbers of orders between 2 and 5, where we clarify how m-plithogenic real intervals can be expressed according to the symbolic plithogenic partial order relation, and we use these intervals to build a topological space. On the other hand, many illustrated and related examples on open and closed sets will be provided to explain the validity of our approach.

Keywords: plithogenic number; topological space; open set, closed set; symbolic plithogenic interval.

1. Introduction

Plithogenic real numbers together form an algebraic ring [19], this ring has been studied on a wide range by many researchers, where it has been used in matrix theory, linear structures, and in number theory [13-18]. Also, symbolic n-plithogenic structures were very useful in generalizing matrix units, and eigenvalues [20-27].

Symbolic n-plithogenic real numbers are partially ordered by the following relation:

$$x_0 + \sum_{i=1}^{n} x_i p_i \le y_0 + \sum_{i=1}^{n} y_i p_i \leftrightarrow x_0 \le y_0, \sum_{i=1}^{j} x_i \le \sum_{i=1}^{j} y_i \,; j \in \{1,2..,n\}.$$

Where x_i, y_i are real numbers, and

$$p_i p_j = p_{\max(i,j).}$$

In this work, we depend on the partial order relations defined on symbolic n-pithogenic real numbers with $n \in \{2,3,4,5\}$ to build topological spaces on these intervals, and to study their elementary properties such as closed and open sets.

The study of symbolic n-plithogenic real intervals can be extended to higher orders, i.e. $n \ge 6$. For more details about neutrosophic and logical-based topological spaces, see [1-12].

2.Main Concepts and Discussion

Definition 2.1
Let $2 - sp_R$ be the symbolic 2-plithogenic ring of real numbers, intervals on $2 - sp_R$ are defined as follows:

$$\left]a_0 + \sum_{i=1}^{2} a_i p_i, \infty\right[= \left\{x_0 + \sum_{i=1}^{2} x_i p_i \in 2 - \mathrm{sp}_R \,;\, a_0 < x_0 < \infty \,, \sum_{j=0}^{1} a_j < \sum_{j=0}^{1} x_j < \infty \,, \sum_{j=0}^{2} a_j < \sum_{j=0}^{2} x_j < \infty \right\},$$

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$$\left[a_0+\sum_{i=1}^{2}a_ip_i,\infty\right[=\left\{x_0+\sum_{i=1}^{2}x_ip_i\in 2-\mathrm{sp}_R\ ;\ a_0\le x_0<\infty\,,\sum_{j=0}^{1}a_j\le\sum_{j=0}^{1}x_j<\infty\,,\sum_{j=0}^{2}a_j\le\sum_{j=0}^{2}x_j<\infty\right\},$$

$$\left]-\infty,a_0+\sum_{i=1}^{2}a_ip_i\right[=\left\{x_0+\sum_{i=1}^{2}x_ip_i\in 2-\mathrm{sp}_R;\ -\infty<x_0<a_0\,,-\infty<\sum_{j=0}^{1}x_j<\sum_{j=0}^{1}a_j\ ,-\infty<\sum_{j=0}^{2}x_j<\sum_{j=0}^{2}a_j\right\},$$

.

$$\left]-\infty,a_0+\sum_{i=1}^{2}a_ip_i\right]=\left\{x_0+\sum_{i=1}^{2}x_ip_i\in 2-\mathrm{sp}_R\ ;\ -\infty<x_0\le a_0\,,-\infty<\sum_{j=0}^{1}x_j\le\sum_{j=0}^{1}a_j\ ,-\infty<\sum_{j=0}^{2}x_j\le\sum_{j=0}^{2}a_j\right\}.$$

.

Definition 2.2.
We define the spaces:

$$T_1=\left(\left]-\infty,a_0+\sum_{i=1}^{2}a_ip_i\right[,\cup,\cap,\emptyset\right),$$

$$T_2=\left(\left]a_0+\sum_{i=1}^{2}a_ip_i,\infty\right[,\cup,\cap,\emptyset\right).$$

Theorem 2.1.
T_1,T_2 are topological spaces.
Proof.
Let

$$F_k=\left]-\infty,a_0^{(k)}+\sum_{i=1}^{2}a_i^{(k)}p_i\right[\in T_1\,,$$

$$L_k=\left]a_0^{(k)}+\sum_{i=1}^{2}a_i^{(k)}p_i\,,\infty\right[\in T_2 \text{ for } K\in G\ \text{(indices set)}.$$

For

$$M=m_0+\sum_{i=1}^{2}m_ip_i\in F_k\cap F_s\ ,N=n_0+\sum_{i=1}^{2}n_ip_i\ \in\cup F_k:$$

$$\begin{cases}-\infty<m_0<a_0^{(k)}\,,-\infty<m_0<a_0^{(s)}\\ -\infty<\sum_{j=0}^{1}m_j<\sum_{j=0}^{1}a_j^{(k)}\ ,-\infty<\sum_{j=0}^{1}m_j<\sum_{j=0}^{1}a_j^{(s)}\\ -\infty<\sum_{j=0}^{2}m_j<\sum_{j=0}^{2}a_j^{(k)}\ ,-\infty<\sum_{j=0}^{2}m_j<\sum_{j=0}^{2}a_j^{(s)}\end{cases},$$

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and

$$\begin{cases} m_0 \in \left]-\infty, a_0^{(k)}\right[\cap \left]-\infty, a_0^{(s)}\right[\\ \sum_{j=0}^{1} m_j \in \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{1} a_j^{(s)}\right[\\ \sum_{j=0}^{2} m_j \in \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{2} a_j^{(s)}\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]-\infty, a_0^{(k)}\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\end{cases}.$$

Thus

$$F_k \cap F_s \in T_1 , \qquad \cup F_k \in T_1 .$$

For

$$M = m_0 + \sum_{i=1}^{2} m_i p_i \in L_k \cap L_s \ , N = n_0 + \sum_{i=1}^{2} n_i p_i \ \in \cup F_k ,$$

$$\begin{cases} a_0^{(k)} < m_0 < \infty , a_0^{(s)} < m_0 < \infty \\ \sum_{j=0}^{1} a_j^{(k)} < \sum_{j=0}^{1} m_j < \infty , \sum_{j=0}^{1} a_j^{(s)} < \sum_{j=0}^{1} m_j < \infty \\ \sum_{j=0}^{2} a_j^{(k)} < \sum_{j=0}^{2} m_j < \infty , \sum_{j=0}^{2} a_j^{(s)} < \sum_{j=0}^{2} m_j < \infty \end{cases},$$

and

$$\begin{cases} m_0 \in \left]a_0^{(k)}, \infty\right[\cap \left]a_0^{(s)}, \infty\right[\\ \sum_{j=0}^{1} m_j \in \left]\sum_{j=0}^{1} a_j^{(k)}, \infty\right[\cap \left]\sum_{j=0}^{1} a_j^{(s)}, \infty\right[\\ \sum_{j=0}^{2} m_j \in \left]\sum_{j=0}^{2} a_j^{(k)}, \infty\right[\cap \left]\sum_{j=0}^{2} a_j^{(s)}, \infty\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]a_0^{(k)}, \infty\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]\sum_{j=0}^{1} a_j^{(k)}, \infty\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]\sum_{j=0}^{2} a_j^{(k)}, \infty\right[\end{cases}.$$

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Thus

$$L_k \cap L_s \in T_2 , \qquad \cup L_k \in T_2 \ .$$

Remark 2.1.
Open sets in T_1 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{2} a_i p_i\right[\ ; \ a_i \in R ,$$

Open sets in T_2 are:

$$\left]a_0 + \sum_{i=1}^{2} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_1 are:

$$\left[a_0 + \sum_{i=1}^{2} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_2 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{2} a_i p_i\right] \ ; \ a_i \in R .$$

Definition 2.3.
Let $3 - sp_R$ be the symbolic 3-plithogenic ring of real numbers, intervals on $3 - sp_R$ are defined as follows:

$$\left]a_0 + \sum_{i=1}^{3} a_i p_i, \infty\right[= \left\{x_0 + \sum_{i=1}^{3} x_i p_i \in 2 - \mathrm{sp}_R \ ; \ a_0 < x_0 < \infty , \sum_{j=0}^{k} a_j < \sum_{j=0}^{k} x_j < \infty \ ; k \in \{1,2,3\} \right\},$$

$$\left[a_0 + \sum_{i=1}^{3} a_i p_i, \infty\right[= \left\{x_0 + \sum_{i=1}^{3} x_i p_i \in 3 - \mathrm{sp}_R \ ; \ a_0 \leq x_0 < \infty , \sum_{j=0}^{k} a_j \leq \sum_{j=0}^{k} x_j < \infty \ ; \ k \in \{1,2,3\} \right\},$$

$$\left]-\infty, a_0 + \sum_{i=1}^{3} a_i p_i\right[= \left\{x_0 + \sum_{i=1}^{3} x_i p_i \in 3 - \mathrm{sp}_R \ ; \ -\infty < x_0 < a_0 , -\infty < \sum_{j=0}^{k} x_j < \sum_{j=0}^{k} a_j \ , k \in \{1,2,3\} \right\},$$

$$\left]-\infty, a_0 + \sum_{i=1}^{3} a_i p_i\right] = \left\{x_0 + \sum_{i=1}^{3} x_i p_i \in 3 - \mathrm{sp}_R \ ; \ -\infty < x_0 \leq a_0 , -\infty < \sum_{j=0}^{k} x_j \leq \sum_{j=0}^{k} a_j \ , k \in \{1,2,3\} \right\}.$$

Definition 2.4.
We define the spaces:

$$T_1 = \left(\left]-\infty, a_0 + \sum_{i=1}^{3} a_i p_i\right[, \cup, \cap, \emptyset\right) ,$$

$$T_2 = \left(\left]a_0 + \sum_{i=1}^{3} a_i p_i, \infty\right[, \cup, \cap, \emptyset\right) .$$

Theorem 2.2.
T_1, T_2 are topological spaces.
Proof.
Let

$$F_k = \left]-\infty, a_0^{(k)} + \sum_{i=1}^{3} a_i^{(k)} p_i\right[\in T_1 ,$$

$$L_k = \left]a_0^{(k)} + \sum_{i=1}^{3} a_i^{(k)} p_i , \infty\right[\in T_2 \text{ for } K \in G \text{ (indices set)}.$$

For

$$M = m_0 + \sum_{i=1}^{3} m_i p_i \in F_k \cap F_s \ , N = n_0 + \sum_{i=1}^{3} n_i p_i \ \in \cup F_k :$$

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$$\begin{cases} -\infty < m_0 < a_0^{(k)}, -\infty < m_0 < a_0^{(s)} \\ -\infty < \sum_{j=0}^{1} m_j < \sum_{j=0}^{1} a_j^{(k)}, -\infty < \sum_{j=0}^{1} m_j < \sum_{j=0}^{1} a_j^{(s)} \\ -\infty < \sum_{j=0}^{2} m_j < \sum_{j=0}^{2} a_j^{(k)}, -\infty < \sum_{j=0}^{2} m_j < \sum_{j=0}^{2} a_j^{(s)} \\ -\infty < \sum_{j=0}^{3} m_j < \sum_{j=0}^{3} a_j^{(k)}, -\infty < \sum_{j=0}^{3} m_j < \sum_{j=0}^{3} a_j^{(s)} \end{cases}.$$

and

$$\begin{cases} m_0 \in \left]-\infty, a_0^{(k)}\right[\cap \left]-\infty, a_0^{(s)}\right[\\ \sum_{j=0}^{1} m_j \in \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{1} a_j^{(s)}\right[\\ \sum_{j=0}^{2} m_j \in \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{2} a_j^{(s)}\right[\\ \sum_{j=0}^{3} m_j \in \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{3} a_j^{(s)}\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]-\infty, a_0^{(k)}\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\end{cases}.$$

Thus

$$F_k \cap F_s \in T_1, \qquad \cup F_k \in T_1.$$

For

$$M = m_0 + \sum_{i=1}^{3} m_i p_i \in L_k \cap L_s, N = n_0 + \sum_{i=1}^{3} n_i p_i \in \cup F_k,$$

$$\begin{cases} a_0^{(k)} < m_0 < \infty, a_0^{(s)} < m_0 < \infty \\ \sum_{j=0}^{1} a_j^{(k)} < \sum_{j=0}^{1} m_j < \infty, \sum_{j=0}^{1} a_j^{(s)} < \sum_{j=0}^{1} m_j < \infty \\ \sum_{j=0}^{2} a_j^{(k)} < \sum_{j=0}^{2} m_j < \infty, \sum_{j=0}^{2} a_j^{(s)} < \sum_{j=0}^{2} m_j < \infty \\ \sum_{j=0}^{3} a_j^{(k)} < \sum_{j=0}^{3} m_j < \infty, \sum_{j=0}^{3} a_j^{(s)} < \sum_{j=0}^{3} m_j < \infty \end{cases},$$

and

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$$\begin{cases} m_0 \in \left]a_0^{(k)}, \infty\right[\cap \left]a_0^{(s)}, \infty\right[\\ \sum_{j=0}^{1} m_j \in \left]\sum_{j=0}^{1} a_j^{(k)}, \infty\right[\cap \left]\sum_{j=0}^{1} a_j^{(s)}, \infty\right[\\ \sum_{j=0}^{2} m_j \in \left]\sum_{j=0}^{2} a_j^{(k)}, \infty\right[\cap \left]\sum_{j=0}^{2} a_j^{(s)}, \infty\right[\\ \sum_{j=0}^{3} m_j \in \left]\sum_{j=0}^{3} a_j^{(k)}, \infty\right[\cap \left]\sum_{j=0}^{3} a_j^{(s)}, \infty\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]a_0^{(k)}, \infty\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]\sum_{j=0}^{1} a_j^{(k)}, \infty\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]\sum_{j=0}^{2} a_j^{(k)}, \infty\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]\sum_{j=0}^{3} a_j^{(k)}, \infty\right[\end{cases}.$$

Thus

$$L_k \cap L_s \in T_2 , \qquad \cup L_k \in T_2 \ .$$

Remark 2.2.
Open sets in T_1 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{3} a_i p_i\right[\ ; \ a_i \in R ,$$

Open sets in T_2 are:

$$\left]a_0 + \sum_{i=1}^{3} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_1 are:

$$\left[a_0 + \sum_{i=1}^{3} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_2 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{3} a_i p_i\right] \ ; \ a_i \in R .$$

Definition 2.
Let $4 - sp_R$ be the symbolic 4-plithogenic ring of real numbers, intervals on $4 - sp_R$ are defined as follows:

$$\left]a_0 + \sum_{i=1}^{4} a_i p_i , \infty\right[= \left\{x_0 + \sum_{i=1}^{4} x_i p_i \in 4 - \mathrm{sp}_R \ ; \ a_0 < x_0 < \infty , \sum_{j=0}^{k} a_j < \sum_{j=0}^{k} x_j < \infty \ ; k \in \{1,2,3,4\}\right\},$$

$$\left[a_0 + \sum_{i=1}^{4} a_i p_i , \infty\right[= \left\{x_0 + \sum_{i=1}^{4} x_i p_i \in 4 - \mathrm{sp}_R \ ; \ a_0 \le x_0 < \infty , \sum_{j=0}^{k} a_j \le \sum_{j=0}^{k} x_j < \infty \ ; \ k \in \{1,2,3,4\}\right\},$$

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$$\left]-\infty, a_0+\sum_{i=1}^{4} a_i p_i\right[=\left\{x_0+\sum_{i=1}^{4} x_i p_i \in 4-\mathrm{sp}_R\ ;\ -\infty<x_0<a_0, -\infty<\sum_{j=0}^{k} x_j<\sum_{j=0}^{k} a_j\ , k \in\{1,2,3,4\}\right\},$$

$$\left]-\infty, a_0+\sum_{i=1}^{4} a_i p_i\right]=\left\{x_0+\sum_{i=1}^{4} x_i p_i \in 4-\mathrm{sp}_R\ ;\ -\infty<x_0 \leq a_0, -\infty<\sum_{j=0}^{k} x_j \leq \sum_{j=0}^{k} a_j\ , k \in\{1,2,3,4\}\right\}.$$

Definition 2.6.
We define the spaces:

$$T_1=\left(\left]-\infty, a_0+\sum_{i=1}^{4} a_i p_i\right[, \cup, \cap, \emptyset\right),$$

$$T_2=\left(\left]a_0+\sum_{i=1}^{4} a_i p_i, \infty\right[, \cup, \cap, \emptyset\right).$$

Theorem 2.3.
T_1, T_2 are topological spaces.
Proof.
Let

$$F_k=\left]-\infty, a_0^{(k)}+\sum_{i=1}^{3} a_i^{(k)} p_i\right[\in T_1,$$

$$L_k=\left]a_0^{(k)}+\sum_{i=1}^{4} a_i^{(k)} p_i, \infty\right[\in T_2 \text{ for } K \in G \text{ (indices set).}$$

For

$$M=m_0+\sum_{i=1}^{4} m_i p_i \in F_k \cap F_s\ , N=n_0+\sum_{i=1}^{4} n_i p_i\ \in \cup F_k:$$

$$\begin{cases} -\infty<m_0<a_0^{(k)}, -\infty<m_0<a_0^{(s)} \\ -\infty<\sum_{j=0}^{1} m_j<\sum_{j=0}^{1} a_j^{(k)}, -\infty<\sum_{j=0}^{1} m_j<\sum_{j=0}^{1} a_j^{(s)} \\ -\infty<\sum_{j=0}^{2} m_j<\sum_{j=0}^{2} a_j^{(k)}, -\infty<\sum_{j=0}^{2} m_j<\sum_{j=0}^{2} a_j^{(s)} \\ -\infty<\sum_{j=0}^{3} m_j<\sum_{j=0}^{3} a_j^{(k)}, -\infty<\sum_{j=0}^{3} m_j<\sum_{j=0}^{3} a_j^{(s)} \\ \infty<\sum_{j=0}^{4} m_j<\sum_{j=0}^{4} a_j^{(k)}, -\infty<\sum_{j=0}^{4} m_j<\sum_{j=0}^{4} a_j^{(s)} \end{cases},$$

and

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$$\begin{cases} m_0 \in \left]-\infty, a_0^{(k)}\right[\cap \left]-\infty, a_0^{(s)}\right[\\ \sum_{j=0}^{1} m_j \in \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{1} a_j^{(s)}\right[\\ \sum_{j=0}^{2} m_j \in \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{2} a_j^{(s)}\right[\\ \sum_{j=0}^{3} m_j \in \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{3} a_j^{(s)}\right[\\ \sum_{j=0}^{4} m_j \in \left]-\infty, \sum_{j=0}^{4} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{4} a_j^{(s)}\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]-\infty, a_0^{(k)}\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\\ \sum_{j=0}^{4} n_j \in \cup \left]-\infty, \sum_{j=0}^{4} a_j^{(k)}\right[\end{cases}.$$

Thus

$$F_k \cap F_s \in T_1 ,$$

For

$$M = m_0 + \sum_{i=1}^{4} m_i p_i \in L_k \cap L_s \ , N = n_0 + \sum_{i=1}^{4} n_i p_i \ \in \cup F_k ,$$

$$\begin{cases} a_0^{(k)} < m_0 < \infty \ , a_0^{(s)} < m_0 < \infty \\ \sum_{j=0}^{1} a_j^{(k)} < \sum_{j=0}^{1} m_j < \infty \ , \sum_{j=0}^{1} a_j^{(s)} < \sum_{j=0}^{1} m_j < \infty \\ \sum_{j=0}^{2} a_j^{(k)} < \sum_{j=0}^{2} m_j < \infty \ , \sum_{j=0}^{2} a_j^{(s)} < \sum_{j=0}^{2} m_j < \infty \\ \sum_{j=0}^{3} a_j^{(k)} < \sum_{j=0}^{3} m_j < \infty \ , \sum_{j=0}^{3} a_j^{(s)} < \sum_{j=0}^{3} m_j < \infty \\ \sum_{j=0}^{4} a_j^{(k)} < \sum_{j=0}^{4} m_j < \infty \ , \sum_{j=0}^{4} a_j^{(s)} < \sum_{j=0}^{4} m_j < \infty \end{cases},$$

and

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$$\begin{cases} m_0 \in \left]a_0^{(k)},\infty\right[\cap \left]a_0^{(s)},\infty\right[\\ \sum_{j=0}^{1} m_j \in \left]\sum_{j=0}^{1} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{1} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{2} m_j \in \left]\sum_{j=0}^{2} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{2} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{3} m_j \in \left]\sum_{j=0}^{3} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{3} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{4} m_j \in \left]\sum_{j=0}^{4} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{4} a_j^{(s)},\infty\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]a_0^{(k)},\infty\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]\sum_{j=0}^{1} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]\sum_{j=0}^{2} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]\sum_{j=0}^{3} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{4} n_j \in \cup \left]\sum_{j=0}^{4} a_j^{(k)},\infty\right[\end{cases}.$$

Thus

$$L_k \cap L_s \in T_2 , \qquad \cup L_k \in T_2 \ .$$

Remark 2.3.

Open sets in T_1 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{4} a_i p_i\right[\ ; \ a_i \in R ,$$

Open sets in T_2 are:

$$\left]a_0 + \sum_{i=1}^{4} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_1 are:

$$\left[a_0 + \sum_{i=1}^{4} a_i p_i , \infty\right[\ ; \ a_i \in R ,$$

Closed sets in T_2 are:

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$$\left]-\infty, a_0 + \sum_{i=1}^{4} a_i p_i\right] \; ; \; a_i \in R \, .$$

Definition 2.7.

Let $4 - sp_R$be the symbolic 5-plithogenic ring of real numbers, intervals on $4 - sp_R$ are defined as follows:

$$\left]a_0 + \sum_{i=1}^{5} a_i p_i, \infty\right[= \left\{x_0 + \sum_{i=1}^{5} x_i p_i \in 5 - \mathrm{sp}_R \; ; \; a_0 < x_0 < \infty \, , \sum_{j=0}^{k} a_j < \sum_{j=0}^{k} x_j < \infty \, ; k \in \{1,2,3,4,5\} \right\},$$

$$\left[a_0 + \sum_{i=1}^{5} a_i p_i, \infty\right[= \left\{x_0 + \sum_{i=1}^{5} x_i p_i \in 5 - \mathrm{sp}_R \; ; \; a_0 \le x_0 < \infty \, , \sum_{j=0}^{k} a_j \le \sum_{j=0}^{k} x_j < \infty \, ; \; k \in \{1,2,3,4,5\} \right\},$$

$$\left]-\infty, a_0 + \sum_{i=1}^{5} a_i p_i\right[= \left\{x_0 + \sum_{i=1}^{5} x_i p_i \in 5 - \mathrm{sp}_R \; ; \; -\infty < x_0 < a_0 \, , -\infty < \sum_{j=0}^{k} x_j < \sum_{j=0}^{k} a_j \; , k \in \{1,2,3,4,5\} \right\},$$

$$\left]-\infty, a_0 + \sum_{i=1}^{5} a_i p_i\right] = \left\{x_0 + \sum_{i=1}^{5} x_i p_i \in 5 - \mathrm{sp}_R \; ; \; -\infty < x_0 \le a_0 \, , -\infty < \sum_{j=0}^{k} x_j \le \sum_{j=0}^{k} a_j \; , k \in \{1,2,3,4,5\} \right\}.$$

Definition 2.8.

We define the spaces:

$$T_1 = \left(\left]-\infty, a_0 + \sum_{i=1}^{5} a_i p_i\right[, \cup, \cap, \emptyset\right),$$

$$T_2 = \left(\left]a_0 + \sum_{i=1}^{5} a_i p_i, \infty\right[, \cup, \cap, \emptyset\right).$$

Theorem 2.4.

T_1, T_2 are topological spaces.

Proof.

Let

$$F_k = \left]-\infty, a_0^{(k)} + \sum_{i=1}^{3} a_i^{(k)} p_i\right[\in T_1 \, ,$$

$$L_k = \left]a_0^{(k)} + \sum_{i=1}^{5} a_i^{(k)} p_i \, , \infty\right[\in T_2 \text{ for } K \in G \text{ (indices set)}.$$

For

$$M = m_0 + \sum_{i=1}^{5} m_i p_i \in F_k \cap F_s \; , N = n_0 + \sum_{i=1}^{5} n_i p_i \; \in \cup \, F_k \; :$$

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$$\begin{cases} -\infty < m_0 < a_0^{(k)}, -\infty < m_0 < a_0^{(s)} \\ -\infty < \sum_{j=0}^{1} m_j < \sum_{j=0}^{1} a_j^{(k)}, -\infty < \sum_{j=0}^{1} m_j < \sum_{j=0}^{1} a_j^{(s)} \\ -\infty < \sum_{j=0}^{2} m_j < \sum_{j=0}^{2} a_j^{(k)}, -\infty < \sum_{j=0}^{2} m_j < \sum_{j=0}^{2} a_j^{(s)} \\ -\infty < \sum_{j=0}^{3} m_j < \sum_{j=0}^{3} a_j^{(k)}, -\infty < \sum_{j=0}^{3} m_j < \sum_{j=0}^{3} a_j^{(s)} \\ -\infty < \sum_{j=0}^{4} m_j < \sum_{j=0}^{4} a_j^{(k)}, -\infty < \sum_{j=0}^{4} m_j < \sum_{j=0}^{4} a_j^{(s)} \\ -\infty < \sum_{j=0}^{5} m_j < \sum_{j=0}^{5} a_j^{(k)}, -\infty < \sum_{j=0}^{5} m_j < \sum_{j=0}^{5} a_j^{(s)} \end{cases},$$

and

$$\begin{cases} m_0 \in \left]-\infty, a_0^{(k)}\right[\cap \left]-\infty, a_0^{(s)}\right[\\ \sum_{j=0}^{1} m_j \in \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{1} a_j^{(s)}\right[\\ \sum_{j=0}^{2} m_j \in \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{2} a_j^{(s)}\right[\\ \sum_{j=0}^{3} m_j \in \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{3} a_j^{(s)}\right[\\ \sum_{j=0}^{4} m_j \in \left]-\infty, \sum_{j=0}^{4} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{4} a_j^{(s)}\right[\\ \sum_{j=0}^{5} m_j \in \left]-\infty, \sum_{j=0}^{5} a_j^{(k)}\right[\cap \left]-\infty, \sum_{j=0}^{5} a_j^{(s)}\right[\end{cases},$$

also

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$$\begin{cases} n_0 \in \cup \left]-\infty, a_0^{(k)}\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]-\infty, \sum_{j=0}^{1} a_j^{(k)}\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]-\infty, \sum_{j=0}^{2} a_j^{(k)}\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]-\infty, \sum_{j=0}^{3} a_j^{(k)}\right[\\ \sum_{j=0}^{4} n_j \in \cup \left]-\infty, \sum_{j=0}^{4} a_j^{(k)}\right[\\ \sum_{j=0}^{5} n_j \in \cup \left]-\infty, \sum_{j=0}^{5} a_j^{(k)}\right[\end{cases}.$$

Thus

$$F_k \cap F_s \in T_1 , \qquad \cup F_k \in T_1 .$$

For

$$M = m_0 + \sum_{i=1}^{5} m_i p_i \in L_k \cap L_s \ , N = n_0 + \sum_{i=1}^{5} n_i p_i \ \in \cup F_k ,$$

$$\begin{cases} a_0^{(k)} < m_0 < \infty , a_0^{(s)} < m_0 < \infty \\ \sum_{j=0}^{1} a_j^{(k)} < \sum_{j=0}^{1} m_j < \infty , \sum_{j=0}^{1} a_j^{(s)} < \sum_{j=0}^{1} m_j < \infty \\ \sum_{j=0}^{2} a_j^{(k)} < \sum_{j=0}^{2} m_j < \infty , \sum_{j=0}^{2} a_j^{(s)} < \sum_{j=0}^{2} m_j < \infty \\ \sum_{j=0}^{3} a_j^{(k)} < \sum_{j=0}^{3} m_j < \infty , \sum_{j=0}^{3} a_j^{(s)} < \sum_{j=0}^{3} m_j < \infty \\ \sum_{j=0}^{4} a_j^{(k)} < \sum_{j=0}^{4} m_j < \infty , \sum_{j=0}^{4} a_j^{(s)} < \sum_{j=0}^{4} m_j < \infty \\ \sum_{j=0}^{5} a_j^{(k)} < \sum_{j=0}^{5} m_j < \infty , \sum_{j=0}^{5} a_j^{(s)} < \sum_{j=0}^{5} m_j < \infty \end{cases},$$

and

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$$\begin{cases} m_0 \in \left]a_0^{(k)},\infty\right[\cap \left]a_0^{(s)},\infty\right[\\ \sum_{j=0}^{1} m_j \in \left]\sum_{j=0}^{1} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{1} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{2} m_j \in \left]\sum_{j=0}^{2} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{2} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{3} m_j \in \left]\sum_{j=0}^{3} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{3} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{4} m_j \in \left]\sum_{j=0}^{4} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{4} a_j^{(s)},\infty\right[\\ \sum_{j=0}^{5} m_j \in \left]\sum_{j=0}^{5} a_j^{(k)},\infty\right[\cap \left]\sum_{j=0}^{5} a_j^{(s)},\infty\right[\end{cases},$$

also

$$\begin{cases} n_0 \in \cup \left]a_0^{(k)},\infty\right[\\ \sum_{j=0}^{1} n_j \in \cup \left]\sum_{j=0}^{1} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{2} n_j \in \cup \left]\sum_{j=0}^{2} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{3} n_j \in \cup \left]\sum_{j=0}^{3} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{4} n_j \in \cup \left]\sum_{j=0}^{4} a_j^{(k)},\infty\right[\\ \sum_{j=0}^{5} n_j \in \cup \left]\sum_{j=0}^{5} a_j^{(k)},\infty\right[\end{cases}.$$

Thus

$$L_k \cap L_s \in T_2 , \qquad \cup L_k \in T_2 .$$

Remark 2.4.
Open sets in T_1 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{5} a_i p_i\right[\;\; ; \; a_i \in R ,$$

Open sets in T_2 are:

$$\left]a_0 + \sum_{i=1}^{5} a_i p_i , \infty\right[\;\; ; \; a_i \in R ,$$

Closed sets in T_1 are:

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$$\left[a_0 + \sum_{i=1}^{5} a_i p_i , \infty\right[\;\; ; \; a_i \in R \, ,$$

Closed sets in T_2 are:

$$\left]-\infty, a_0 + \sum_{i=1}^{5} a_i p_i\right] \;\; ; \; a_i \in R \, .$$

Conclusion

In this paper, we presented the topological space of intervals based symbolic m-plithogenic real numbers of orders between 2 and 5, where we clarified how m-plithogenic real intervals can be expressed according to the symbolic plithogenic partial order relation, and we use these intervals to build a topological space. On the other hand, many illustrated and related examples on open and closed sets are provided to explain the validity of our approach.

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