



Efficient Neutrosophic Optimization for Minimum Cost Flow Problems

Shubham Kumar Tripathi¹, Kottakkaran Sooppy Nisar^{2,3,*}, Said Broumi⁴, Ranjan Kumar^{1,*}

¹VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati, AP, India.

² Department of Mathematics, College of Science and Humanities in Alkharj, Prince Sattam Bin Abdulaziz University, Alkharj 11942, Saudi Arabia

³ Saveetha School of Engineering, SIMATS, Chennai, India.

⁴Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Morocco

Emails: shubhamt.vit22@gmail.com, *n.sooppy@psau.edu.sa, broumisaid78@gmail.com

*ranjank.nit52@gmail.com

Abstract

In the domain of optimization, linear programming (LP) is recognized as an exceptionally effective method for ensuring the most favorable outcomes. Within the context of LP, the minimum cost flow (MCF) problem is fundamental, with its primary objective being to reduce the transportation costs for a single item moving through a network, under the constraints related to capacity. This network is made up of supply nodes, directed arcs, and demand nodes and each arc has an associated cost and capacity constraint, these factors are certain. However, in practical scenarios, these factors are susceptible to variation due to causal uncertainty. The neutrosophic set theory has surfaced as a challenging approach to tackle the uncertainty that is often encountered in optimization processes. In this manuscript, our primary objective is to address the minimal cost flow (MCF) problem while accounting for the uncertainty inherent in the neutrosophic set. We specifically focus on the cost aspect as SVTN numbers and introduce a new approach based on a customized ranking function handmade for the MCF problem a pioneering endeavor within the field of neutrosophic sets. Additionally, we present numerical example to validate the effectiveness and robustness of our model.

Keywords: LPP; Minimal cost flow; Uncertainty; Neutrosophic set; SVTN numbers; Triangular neutrosophic MCF problem.

1 Introduction

In various industries and domains, decision-making and problem-solving are facilitated by Operations Research (OR), a pivotal discipline. Process optimization is permitted and elaborate decision scenarios are tackled by this powerful tool. Within the field, LP is recognized as a key optimization technique. The main aim of LP is to find optimal values for decision variables that satisfy all constraints while the objective functions are max or min. LP has a wide range of applications, including network flow problems, Job scheduling, transportation problems, resource allocation, and so on. The various researchers working in the field of optimization problems as Klein and Hannan¹ presented a technique for solving the MOILP problem that identifies some or all efficient solutions and Cavory et al.² propose an innovative method that combines a genetic algorithm with a scheduler to address job shop scheduling problems under linear constraints. Furthermore, Fisher et al.³

introduce a new approach that leverages the primal-dual ascent algorithm to address both the traveling salesman problem and the network scheduling problem. Ren and Gao⁴ established a MILP model for the integrated plan and assessment of DER systems. For solving possibilistic LP models Lai and Hwang⁵ introduced a new method to tackle an auxiliary MOLP model to address LP problems that had uncertain objectives and constraint coefficients. Nazemi and Omid⁶ have proposed an optimization approach to address the maximum flow problem arising from extensive applications for neural networks in various environments. Chen⁷ proposed a new version of the MCF problem. This version added a fixed cost to the model and aimed to reduce the average cost.

Network problems primarily encompass three distinct types of network problems: the shortest path, maximal cost flow, and minimal cost flow problems. The Minimal Cost Flow (MCF) is a significant and practical network flow model that includes demand nodes, supply nodes, and linear flow costs on the edges of a network graph. The objective of the MCF problem is to transmit flow from a collection of supply nodes, through the arcs of a network, to a set of demand nodes, all at a total minimal cost. To solve the classical MCF problem many researchers in this field such as Minoux⁸ address the challenge of solving integer MCF problems involving convex separable cost functions on the arcs, while also considering truthfulness constraints on the flows. Goldberg's⁹ scaling push-relabel method represents a significant theoretical advancement in MCF algorithms. In contrast, Goldfarb and Jin¹⁰ have proposed a novel algorithm to solve the MCF problem. During each phase, the algorithm ensures at least one flow augmentation by directly utilizing the given data, the original network, and a scaling factor. Additionally, Gopalakrishnan et al.¹¹ explore how the unique least-squares properties of node and arc incidence matrices in network flow problems can be effectively utilized in a quick primal-dual least-squares algorithm to solve the MCF problem. Furthermore, Brand et al.¹² introduce new randomized algorithms that improve runtime for solving MCF problems with polynomially bounded capacities and cost, treating them as LP problems with two-sided constraints.

Building upon the above literature, the classical MCF problem encompasses factors such as cost, supply, demand, and capacity. While the standard MCF formulation assumes these factors are certain and definite, but real-world scenarios introduce uncertainty, leading us to explore novel approaches. To handle these situations there are some theories like probability theory, vagueness theory, and fuzzy theory have emerged to address uncertainty in real situations. Notably, Zadeh¹³ introduced the concept of fuzzy sets in 1965 as a means to handle uncertain real-world conditions. Using fuzzy logic numerous researchers have made significant contributions to the field of fuzzy MCF problems.

Bagherian¹⁴ approaches the MCF problem by considering parameters such as arc capacities, costs, and supply or demand as interval-valued FN. Muruganandam and Srinivasan¹⁵ present an innovative algorithm for solving the uncertain transportation problem, where supply, demand, capacity, and cost are represented as trapezoidal FN. They aim to minimize transportation costs in a capacitated network. Meanwhile, Bozhenyuk et al.¹⁶ propose a novel approach that addresses both maximum flow and MCF problems in network scenarios. They take into account parameters such as capacities and transport costs of one flow unit, represented as triangular FN. Additionally, Akram et al.¹⁷ introduce an innovative approach that incorporates trapezoidal pentagonal fuzzy (TrPF) numbers for capacities and flow parameters, along with a ranking method for defuzzification. Konstantakopoulos et al.¹⁸ developed a methodology for classifying multiple vehicle routing problem variants related to goods transportation—a common challenge faced by logistics and distribution companies. Researchers have widely applied theories and their extensions, such as fuzzy theory and its extensions, to address problems involving uncertainty. Effectively addressing uncertainty in a fuzzy environment through vague reasoning becomes challenging when challenged with indeterminate and inconsistent information. Addressing indeterminacy is a crucial responsibility for fostering unambiguous data, leading to the emergence of the neutrosophic concept for explicit indeterminacy analysis, due to which F. Smarandache¹⁹ introduce the concept of the Neutrosophic Set (NS) in 1999.

Neutrosophic sets, an advanced mathematical concept, extend fuzzy logic to effectively manage data that is unclear, inconsistent, or incomplete. These sets focus on three critical measures: falsity (the degree to which information is not true), indeterminacy (when information cannot be determined), and truth. Unlike fuzzy sets, which have membership levels between 0 and 1, neutrosophic sets operate within a unique range just above 0

and just below 1. They find practical applications in modeling real-world scenarios with incomplete, inconsistent, or uncertain information. To handle these situations some researchers who work in neutrosophic environments such as Das and Edalatpanah²⁰ developed a new framework for solving ILP problems with TrNNS using an aggregate ranking function. In their research, Das²¹ investigates a transportation problem involving pentagonal Neutrosophic numbers, where parameters are uncertain. Meanwhile, Chakraborty et al.²² propose innovative methods to de-neutrosophicate trapezoidal neutrosophic numbers, aiming to convert them into crisp numbers. Additionally, Biswas and Dey²³ address MOLP problems under a neutrosophic environment, leveraging neutrosophic fuzzy approaches. Their technique considers three distinct membership degrees to provide decision-makers with potential values for optimization problems. Sinika and Ramesh²⁴ contribute to this field by proposing a novel de-neutrophication strategy using interval numbers instead of crisp numbers. Their article provides an overview of this approach, introduces a new ranking technique based on interval numbers, and explores extended neutrosophic LP. Furthermore, Das and Chakraborty²⁵ introduce a novel pentagonal neutrosophic (PN) approach for solving LP problems. This method relies on a ranking function and transforms the problem into its related to crisp LP form and Karak et al.²⁶ propose a method to order SVTN numbers based on their values and ambiguities. By applying a ratio ranking function, they transform neutrosophic LP problems into crisp LP problems, which are then solved using computational methods.

1.1 Motivation and Novelties

The Neutrosophic set theory is a well-established approach for addressing uncertainty in optimization problems. The notion of the MCF problem within the context of the neutrosophic environment has been investigated by a select handful of scholars. The basic advantage of the neutrosophic set lies in its ability to aid decision-makers by taking into account the degrees of truth, falsity, and indeterminacy. Notably, the degree of indeterminacy is often regarded as an independent factor, contributing significantly to the decision-making process. Our approach aims to overcome limitations, which tend to inflate the number of constraints larger than the original problem. To address this drawback, we propose a novel solution strategy that leverages a ranking function. This approach offers improved efficiency and effectiveness in solving MCF problems with complex cost considerations. The novelties of this manuscript are as follows:

- We propose a new method that utilizes a ranking function to address the MCF problem, accounting for the uncertainty inherent in neutrosophic sets a pioneering contribution in the field of neutrosophic set literature.
- It provides a more efficient and realistic representation by taking into account all facets including falsity, indeterminacy, and truthiness degree of the decision-making process.

1.2 Objective:

The manuscript details numerous challenges with current methods for tackling minimal cost flow (MCF) problems in real-life environments, which have led to the creation of a new approach that incorporates neutrosophic logic into MCF problems. In this article, we present an innovative solution methodology for addressing the MCF problem with neutrosophic cost parameters. The main objective of this manuscript is as follows:

- To tackle the MCF problems considering the uncertainty of the neutrosophic set, focusing especially on the cost.
- To develop a new approach for solving NMFP with neutrosophic cost within the context of neutrosophic uncertainty, where the cost factor is a pivotal element.

1.3 List of abbreviations throughout the article

List of abbreviations

NS:	“neutrosophic set.”
TrNNs:	“Triangular neutrosophic numbers.”
MCF:	“Minimum cost flow.”
T:	“Initial node.”
TrpNNs:	“Trapezoidal neutrosophic numbers.”
DER:	“Distributed energy resources.”
SVTN:	“Single-valued triangular neutrosophic.”
MOLP:	“multi-objective linear programming.”
NMFP:	“Neutrosophic minimal flow problem.”
C:	“Maximum capacity of the arc.”
TrNMCF:	“Triangular neutrosophic minimum cost flow.”
SOLP:	“Single-objective linear programming.”
ILP:	“Integer linear programming.”
H:	“Terminal node.”
FN:	“Fuzzy numbers.”
MF:	“Membership function.”

1.4 Structure of the Manuscript:

The structure of this article unfolds as follows: We commence with the introduction of basic definitions in the preliminary section2. We articulate both the classical and Neutrosophic minimal cost flow problems and our proposed methodology is then presented in Section 3. Following this, in Section 4, To demonstrate its efficacy, we furnish an example. Finally, we conclude our article in the conclusion section.

2 Preliminaries

Within this portion, we introduce some basic operations and definitions that are used in the manuscript.

Definition 2.1. Neutrosophic Set:²⁷ A set $\tilde{P}_{neu}$ in the discourse universal set V , is said to be NS if $\tilde{P}_{neu} = \left\{ \left(v, \left[T_{\tilde{P}_{neu}}(v), I_{\tilde{P}_{neu}}(v), F_{\tilde{P}_{neu}}(v) \right] \right) : v \in V \right\}$ Where $F_{\tilde{P}_{neu}}(v) : V \rightarrow [0, 1]$, $I_{\tilde{P}_{neu}}(v) : V \rightarrow [0, 1]$, and $T_{\tilde{P}_{neu}}(v) : V \rightarrow [0, 1]$ are defined as the falsity-MF $F_{\tilde{P}_{neu}}(v)$, indeterminacy-MF $I_{\tilde{P}_{neu}}(v)$ and truth MF $T_{\tilde{P}_{neu}}(v)$, of an element v in $\tilde{P}_{neu}$ and respectively. $T_{\tilde{P}_{neu}}(v), I_{\tilde{P}_{neu}}(v), F_{\tilde{P}_{neu}}(v)$ satisfy the condition

$$0 \leq T_{\tilde{P}_{neu}}(v) + I_{\tilde{P}_{neu}}(v) + F_{\tilde{P}_{neu}}(v) \leq 3$$

A neutrosophic set is said to be an SVTN set,²⁸ if v is a single-valued independent variable.

Definition 2.2. Single-valued triangular neutrosophic (SVTN) number:²⁹

A set $\tilde{P}\tilde{N}_{neu} = \langle (p_n^1, p_n^2, p_n^3), (\alpha_n^1, \alpha_n^2, \alpha_n^3), (d_n^1, d_n^2, d_n^3) \rangle$ is called SVTN number if it defines a truth MF $T_{\tilde{P}\tilde{N}_{neu}}(v)$, indeterminacy MF, $I_{\tilde{P}\tilde{N}_{neu}}(v)$ and the falsity-MF $F_{\tilde{P}\tilde{N}_{neu}}(v)$ of an element v in $\tilde{P}\tilde{N}_{neu}$ is defined as,

$$T_{\widetilde{PN}_{neu}}(v) = \begin{cases} \left(\frac{v-p_n^1}{p_n^2-p_n^1} \right) & p_n^1 \leq v < p_n^2 \\ 1 & p_n^2 = v \\ \left(\frac{p_n^3-v}{p_n^3-p_n^2} \right) & p_n^2 < v \leq p_n^3 \\ 0 & \text{otherwise} \end{cases},$$

$$I_{\widetilde{PN}_{neu}}(v) = \begin{cases} \left(\frac{\alpha_n^2-v}{\alpha_n^2-\alpha_n^1} \right) & \alpha_n^1 \leq v < \alpha_n^2 \\ 0 & \alpha_n^2 = v \\ \left(\frac{v-\alpha_n^2}{\alpha_n^3-\alpha_n^2} \right) & \alpha_n^2 < v \leq \alpha_n^3 \\ 1 & \text{otherwise} \end{cases} \text{ and}$$

$$F_{\widetilde{PN}_{neu}}(v) = \begin{cases} \left(\frac{d_n^2-v}{d_n^2-d_n^1} \right) & d_n^1 \leq v < d_n^2 \\ 0 & d_n^2 = v \\ \left(\frac{v-d_n^2}{d_n^3-d_n^2} \right) & d_n^2 < v \leq d_n^3 \\ 1 & \text{otherwise} \end{cases}$$

with the condition $0 \leq T_{\widetilde{PN}_{neu}}(v) + I_{\widetilde{PN}_{neu}}(v) + F_{\widetilde{PN}_{neu}}(v) \leq 3$

Definition 2.3. Arithmetic Operations for SVTN number:³⁰

Let $\widetilde{PN}_{neu} = \langle (p_n^1, p_n^2, p_n^3), (\alpha_n^1, \alpha_n^2, \alpha_n^3), (d_n^1, d_n^2, d_n^3) \rangle$ and $\widetilde{QN}_{neu} = \langle (q_n^1, q_n^2, q_n^3), (\beta_n^1, \beta_n^2, \beta_n^3), (f_n^1, f_n^2, f_n^3) \rangle$ be two SVTN numbers and $\Gamma > 0$ then-

- (i) $\widetilde{PN}_{neu} \oplus \widetilde{QN}_{neu} = \left\langle (p_n^1 + q_n^1, p_n^2 + q_n^2, p_n^3 + q_n^3), (\alpha_n^1 + \beta_n^1, \alpha_n^2 + \beta_n^2, \alpha_n^3 + \beta_n^3), (d_n^1 + f_n^1, d_n^2 + f_n^2, d_n^3 + f_n^3) \right\rangle$
- (ii) $\widetilde{PN}_{neu} \otimes \widetilde{QN}_{neu} = \left\langle (p_n^1 \cdot q_n^1, p_n^2 \cdot q_n^2, p_n^3 \cdot q_n^3), (\alpha_n^1 \cdot \beta_n^1, \alpha_n^2 \cdot \beta_n^2, \alpha_n^3 \cdot \beta_n^3), (d_n^1 \cdot f_n^1, d_n^2 \cdot f_n^2, d_n^3 \cdot f_n^3) \right\rangle$
- (iii) $\Gamma \odot \widetilde{PN}_{neu} = \langle (\Gamma \cdot p_n^1, \Gamma \cdot p_n^2, \Gamma \cdot p_n^3), (\Gamma \cdot \alpha_n^1, \Gamma \cdot \alpha_n^2, \Gamma \cdot \alpha_n^3), (\Gamma \cdot d_n^1, \Gamma \cdot d_n^2, \Gamma \cdot d_n^3) \rangle$

Definition 2.4. ²⁹ Let $\widetilde{PN}_{neu} = \langle (p_n^1, p_n^2, p_n^3), (\alpha_n^1, \alpha_n^2, \alpha_n^3), (d_n^1, d_n^2, d_n^3) \rangle$ be an SVTN number and define a ranking function $De_{neu} : N(\mathbb{R}) \rightarrow \mathbb{R}$ such as

$$De_{neu}(\widetilde{PN}_{neu}) = \frac{1}{12} [(p_n^1 + 2 \cdot p_n^2 + p_n^3) + (\alpha_n^1 + 2 \cdot \alpha_n^2 + \alpha_n^3) + (d_n^1 + 2 \cdot d_n^2 + d_n^3)]$$

where $N(\mathbb{R})$ denotes a set of SVTN numbers characterized by a set of the real number $\mathbb{R}$.

3 Our Proposed model

Before presenting our proposed algorithm, we introduce a sub-section that discusses the existing crisp model in the minimum cost flow (MCF) problem and the approach involving the neutrosophic environment with the NMFP with neutrosophic cost.

3.1 Existing crisp model in MCF problem

Let a directed graph $G^\eta = (N_\eta, \aleph_\eta)$ is considered, where $N_\eta = \{1, 2, 3, \dots, t\}$ is the set of finite nodes and $\aleph_\eta$ represents the set of arcs.

τ_{qt} : Per-unit flow from the arc q to t .

Ck_{qt} : Cost-per-unit flow from arc q to t .

ℓ_{qt} : Lower capacity from the arc q to t .

p_{qt} : Upper capacity from the arc q to t .

Υ_q : Represents the supply, demand, or transshipment node.

The node q is identified as a transshipment node if Υ_q equals 0. If Υ_q is less than 0, then the node q is identified as a demand node. If Υ_q is greater than 0, then the node q is identified as a supply node.

The general formulation of the mathematical model for the classical MCF problem is to be considered as follows:

$$\text{Mini } Z = \sum_{(q,t) \in \aleph_\eta} Ck_{qt} \tau_{qt}$$

$$\text{Subject to } \sum_{t:(q,t) \in \aleph_\eta} \tau_{qt} - \sum_{t:(t,q) \in \aleph_\eta} \tau_{tq} = \Upsilon_q, \quad \forall q \in N_\eta \quad (\text{Flow conservation equations})$$

$$0 \leq \ell_{qt} \leq \tau_{qt} \leq p_{qt} \quad \forall (q, t) \in \aleph_\eta \quad (\text{Flow capacity constraints})$$

3.2 Transformation of the crisp model MCF problem into NMFP with neutrosophic cost

In this section, consider the scenario where we substitute the cost Ck_{qt} convert into a neutrosophic cost per unit flow $\widetilde{Ck_{qt}^\eta}$ from arc q to t . Then the general formulation of the mathematical model for the NMFP with neutrosophic cost is to be considered as follows:

$$\text{Mini } \widetilde{O}^\eta \approx \sum_{(q,t) \in \aleph_\eta} \widetilde{Ck_{qt}^\eta} \tau_{qt} \quad (1)$$

Subject to constraints

$$\sum_{t:(q,t) \in \aleph_\eta} \tau_{qt} - \sum_{t:(t,q) \in \aleph_\eta} \tau_{tq} = \Upsilon_q, \quad \forall q \in N_\eta \quad (2)$$

$$0 \leq \ell_{qt} \leq \tau_{qt} \leq p_{qt} \quad \forall (q, t) \in \aleph_\eta \quad (3)$$

3.3 Algorithm: A novel approach for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters

Let's consider a directed graph whose arcs denote the SVTN cost per unit flow $\widetilde{Ck_{qt}^\eta}$ from arc q to t . In this section, our proposed algorithm by using a new ranking function tends to provide a novel methodology for finding the NMFP with neutrosophic cost considering as SVTN number for cost parameters.

The steps of the algorithm are as follows table 1.

Table 1: The steps of the algorithm

Steps	Algorithm
Step 01:	Formulate the mathematical model of the problem by the model of equation 1 , 2 , and 3 .
Step 02:	Transformation of the NMFP with neutrosophic cost obtained in step 01 into crisp model MCF problem by using the definition 2.4
Step 03:	Solve this MCF problem obtained in step 02 by using the existing methods or software (LINGO 18.0) and obtain the NMFP with neutrosophic cost problem.
Step 04:	Substitute all values τ_{qt} in the objective equation 1 to get the optimal value in the form of an SVTN number is $\langle (p_n^{1*}, p_n^{2*}, p_n^{3*}), (\alpha_n^{1*}, \alpha_n^{2*}, \alpha_n^{3*}), (d_n^{1*}, d_n^{2*}, d_n^{3*}) \rangle$
Step 05:	The End

To illustrate our proposed algorithm, we consider an example, a flow network shown in Figure 1.

4 Numerical Example

Let's investigate an example of a network diagram, as presented below. This network consists of seven nodes and fourteen arcs. Each arc's cost is expressed using an SVTN number for the cost parameters. The objective is to determine the flow within the network while minimizing the overall cost. See Figure 1 for the visual representation of this flow network and using the data of table 2 .

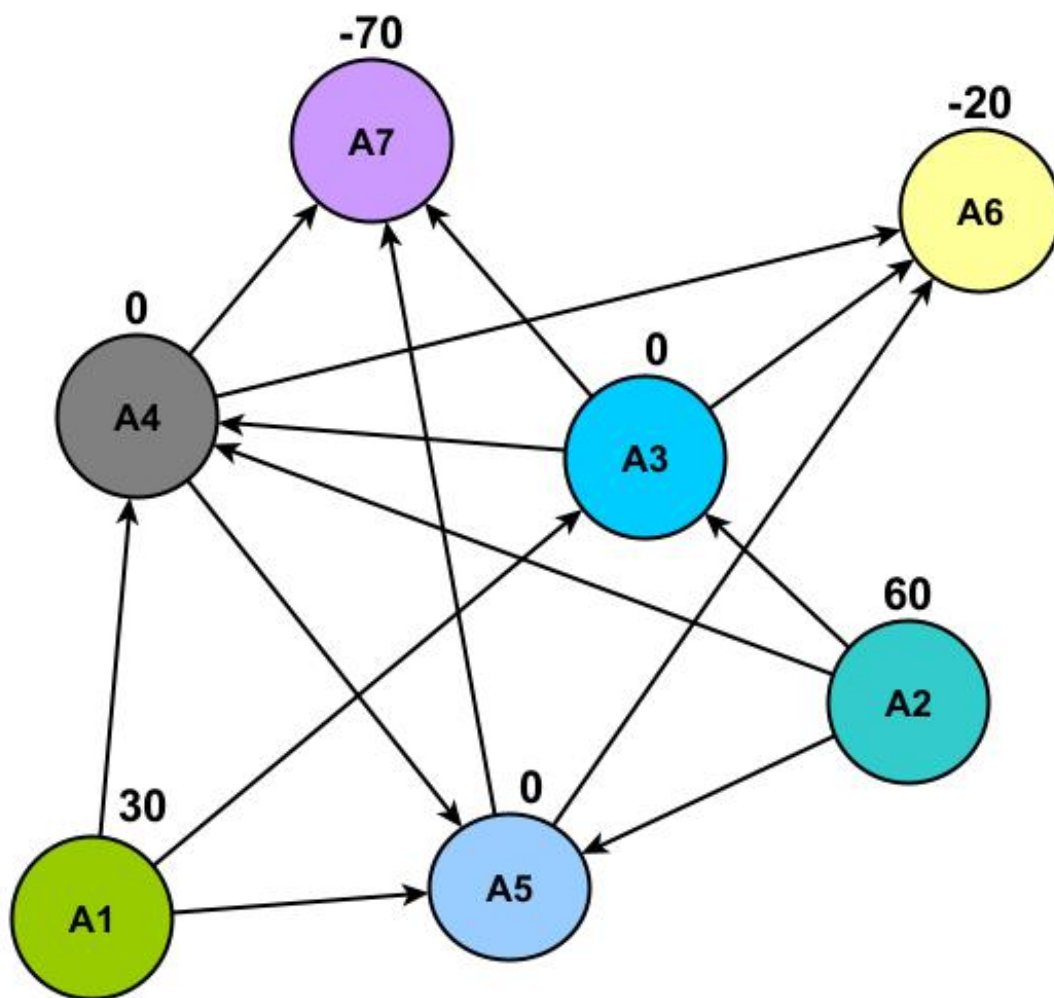


Figure 1: The network consider for Solving MCF problem under neutrosophic environment

Table 2: Consider the below data for Solving NMFP under neutrosophic environment

T	H	Neutrosophic arc cost	C
1	4	<(0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5)>	10
3	7	<(1,5,8),(1.5,3,6.5),(4,7,9)>	90
1	3	<(1,4,7),(1,3,5),(3.5,6,7.5)>	40
3	4	<(2,4,6),(1.5,2.5,4.5),(3,5,7)>	20
1	5	<(1,3,5),(0.5,1.5,3.5),(3,4,6)>	15
4	7	<(15,20,25),(19,21,27),(17,20,24)>	20
2	5	<(1.5,2.5,3.5),(1,1.5,3),(2,3,4)>	40
4	6	<(20,25,30),(24,26,32),(22,25,29)>	30
2	4	<(1,1.5,4),(0.5,1,2.5),(2.25,3,4.25)>	80
4	5	<(10,15,20),(14,16,22),(12,15,19)>	15
2	3	<(1,2,3),(0.5,1.5,2.5),(1.5,2.5,3.5)>	50
5	7	<(15,20,25),(19,21,27),(17,20,24)>	100
3	6	<(1,5,8),(1.5,4,5,7.5),(4,6,5,9)>	100
5	6	<(13,18,23),(17,19,25),(15,18,22)>	15

Solution:

Step 01: To solve the problem, first of all, transform the above network into a mathematical model named equation 4

$$\begin{aligned}
 Min(\widetilde{O}^n) = & \langle (0.5, 2.5, 4.5), (1, 2, 3), (1.5, 3.5, 5.5) \rangle \tau_{14} + \langle (1, 4, 7), (1, 3, 5), (3.5, 6, 7.5) \rangle \tau_{13} + \\
 & \langle (1, 3, 5), (0.5, 1.5, 3.5), (3, 4, 6) \rangle \tau_{15} + \langle (1.5, 2.5, 3.5), (1, 1.5, 3), (2, 3, 4) \rangle \tau_{25} + \\
 & \langle (1, 1.5, 4), (0.5, 1, 2.5), (2.25, 3, 4.25) \rangle \tau_{24} + \langle (1, 2, 3), (0.5, 1.5, 2.5), (1.5, 2.5, 3.5) \rangle \tau_{23} + \\
 & \langle (1, 5, 8), (1.5, 4.5, 7.5), (4, 6.5, 9) \rangle \tau_{36} + \langle (1, 5, 8), (1.5, 3, 6.5), (4, 7, 9) \rangle \tau_{37} + \\
 & \langle (2, 4, 6), (1.5, 2.5, 4.5), (3, 5, 7) \rangle \tau_{34} + \langle (15, 20, 25), (19, 21, 27), (17, 20, 24) \rangle \tau_{47} + \\
 & \langle (20, 25, 30), (24, 26, 32), (22, 25, 29) \rangle \tau_{46} + \langle (10, 15, 20), (14, 16, 22), (12, 15, 19) \rangle \tau_{45} + \\
 & \langle (15, 20, 25), (19, 21, 27), (17, 20, 24) \rangle \tau_{57} + \langle (13, 18, 23), (17, 19, 25), (15, 18, 22) \rangle \tau_{56}.
 \end{aligned} \quad (4)$$

Subject to constraints:

$$\left. \begin{aligned}
 & \tau_{57} + \tau_{56} - \tau_{45} - \tau_{25} - \tau_{15} = 0; \quad 0 \leq \tau_{47} \leq 20; \quad 0 \leq \tau_{57} \leq 100 \\
 & \tau_{15} + \tau_{14} + \tau_{13} = 30; \quad 0 \leq \tau_{14} \leq 10; \quad \tau_{25} + \tau_{24} + \tau_{23} = 60; \quad 0 \leq \tau_{15} \leq 15 \\
 & 0 \leq \tau_{23} \leq 50, \quad -\tau_{36} - \tau_{46} - \tau_{56} = -20; \quad 0 \leq \tau_{25} \leq 40; \quad 0 \leq \tau_{24} \leq 80; \\
 & 0 \leq \tau_{37} \leq 90; \quad \tau_{54} + \tau_{46} + \tau_{47} - \tau_{14} - \tau_{42} - \tau_{34} = 0; \quad 0 \leq \tau_{56} \leq 15; \\
 & 0 \leq \tau_{13} \leq 40; \quad 0 \leq \tau_{34} \leq 20; \quad \tau_{37} + \tau_{36} + \tau_{34} - \tau_{23} - \tau_{13} = 0; \quad 0 \leq \tau_{36} \leq 100; \\
 & 0 \leq \tau_{45} \leq 15; \quad 0 \leq \tau_{46} \leq 30; \quad -\tau_{37} - \tau_{47} - \tau_{57} = -70;
 \end{aligned} \right\} \quad (5)$$

After solving **step: 02 and step 03**

We obtain the basic variables $\tau_{13} = 30, \tau_{14} = 0, \tau_{15} = 0, \tau_{23} = 50, \tau_{24} = 0, \tau_{25} = 10, \tau_{34} = 0, \tau_{36} = 10, \tau_{37} = 70, \tau_{45} = 0, \tau_{46} = 0, \tau_{47} = 0, \tau_{56} = 10, \tau_{57} = 0$

Step: 04 and Step 05

Substitute all basic variables τ_{qt} in the objective equation 4 to get optimal value is in the form of SVTN numbers as $\langle (305, 825, 1265), (355, 625, 1085), (670, 1070, 1380) \rangle$ and $\tau_{13} = 30, \tau_{14} = 0, \tau_{15} = 0, \tau_{23} = 50, \tau_{24} = 0, \tau_{25} = 10, \tau_{34} = 0, \tau_{36} = 10, \tau_{37} = 70, \tau_{45} = 0, \tau_{46} = 0, \tau_{47} = 0, \tau_{56} = 10, \tau_{57} = 0$

End

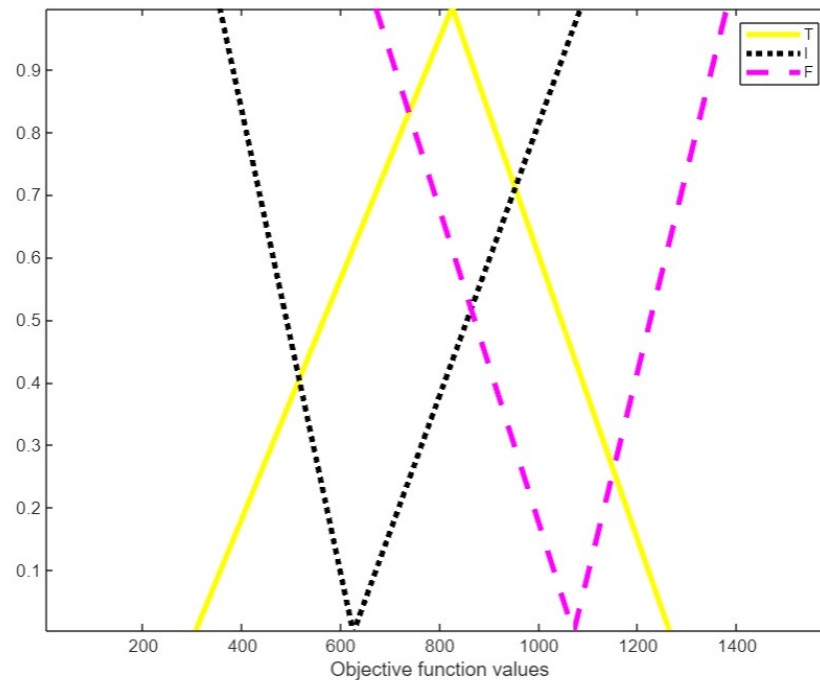


Figure 2: The graphical representation of $\langle (305, 825, 1265), (355, 625, 1085), (670, 1070, 1380) \rangle$

In Figure 2, the graphical representation of the optimal solution of a given numerical example with SVTN numbers $\langle (305, 825, 1265), (355, 625, 1085), (670, 1070, 1380) \rangle$ is shown, with the yellow line depicting truth-MF, the black dot depicting indeterminacy-MF, and the magenta dot depicting falsity-MF.

Conclusion

The minimum cost flow (MCF) problem aims to find the least cost way to move flow through a network, considering factors like cost, supply, demand, and capacity. And also discusses the neutrosophic set theory has surfaced as a challenging approach to tackle the uncertainty that is often encountered in optimization processes. In this manuscript, we introduce a new algorithm using SVTN numbers to solve the NMFP with neutrosophic cost, which also predicts crisp MCF problems. By applying a ratio ranking function, they transform neutrosophic LP problems into crisp LP problems, which are then solved using computational methods. For this purpose, we used a numerical example to solve the NMFP with neutrosophic cost with SVTN numbers into crisp LP problems, by using the existing methods or software (LINGO 18.0) and obtain the NMFP with neutrosophic cost problem. The new algorithms can be used for real-world issues like assignments, job scheduling, and transportation in the future.

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