The Properties of Two-Fold Algebra Based on the n-standard Fuzzy Number Theoretical System

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Abstract

In this paper, we study the two-fold algebra based on the n-standard fuzzy number theoretical system as a special type of two-fold fuzzy algebras, where we study the elementary properties of the algebraic operations defined over this system. Also, we prove many results that describe the relations between two-fold substructures and sub-algebras defined by fuzzy number theoretical systems. On the other hand, we provide many different examples to explain our results.

Keywords: n-standard fuzzy number theoretical system; two-fold algebra; sub-algebra; fuzzy number theory.

1. Introduction

The concept of two-fold algebras was first defined in [4], by combining neutrosophic sets and algebraic structures. Many researchers have used these ideas in the generalization of algebraic structures and the production of new patterns of these structures, where fuzzy functions were used in the expansion of vector spaces and modules [2], and also the use of fuzzy number theory and fuzzy number theoretical systems in the construction of a new type of two-fold algebras [1].The results proved in [1] are related to one kind of fuzzy number theoretical systems, and similarly to many algebraic structures extended by fuzzy, neutrosophic, or even plithogenic logical sets [5-26], [27,28].

In this research, we generalize the study to include n-standard fuzzy number theoretical system, where we unify the concept of two-fold algebra with the concept of n-standard fuzzy number theoretical system to obtain a new algebraic structure. Then we study the basic algebraic properties of this generalization in terms of algebraic operations, partial structures, and even functions between these structures.

2. Main Discussion

Definition 2.1.
Let $\mu:\mathbb{Z}\to]0,1]$;

$$\mu(x)=\begin{cases}\frac{1}{|x|^n}\ ;\ x\neq 0\\ 1\ ;\ x=0\end{cases},$$

we define the two-fold algebra of the n-standard fuzzy number theoretical system (n-STFA) [3], $\Delta_n=\{x_{\mu(t)};\ t,x\in\mathbb{Z}\}$ with the following binary operation:
$*:\Delta_s\times\Delta_s\to\Delta_s$ such that:

$$x_{\mu(t)}*y_{\mu(s)}=(x+y)_{\mu(t)\mu(s)};\ x,y\in\mathbb{Z}.$$

Theorem 2.1.
Let Δ_n be the (n-STFA), we have:
1) * is well defined operation.
2) * is commutative.

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3) * is associative.
4) * has an identity
5) * is anti- inverse operation.

Proof.

1) Let $x_{\mu(t)} = x'_{\mu(t')}$, $y_{\mu(s)} = y'_{\mu(s')}$. Then

$$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\mu(t)\mu(s)} = (x+y)_{\frac{1}{|ts|^n}}.$$

$$x'_{\mu(t')} * y'_{\mu(s')} = (x'+y')_{\mu(t')\mu(s')} = (x'+y')_{\frac{1}{|t's'|^n}} ,$$

hence,

$$x_{\mu(t)} * y_{\mu(s)} = x'_{\mu(t')} * y'_{\mu(s')} .$$

2) $x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\frac{1}{|ts|^n}} = (y+x)_{\frac{1}{|st|^n}} = y_{\mu(s)} * x_{\mu(t)}$.

3) The proof is similar to the standard case.

4) Take $h = O_{\mu(o)} = O_1$, then

$$x_{\mu(t)} * O_{\mu(o)} = (x+o)_{\frac{1}{|t|^n}} = x_{\frac{1}{|t|^n}} = x_{\mu(t)},$$

hence

$$h = O_{\mu(o)},$$

is an identity.

5) If $y_{\mu(s)}$ is the inverse of $x_{\mu(t)}$, then

$$x_{\mu(t)} * y_{\mu(s)} = O_{\mu(o)},$$

hence,

$$(x+y)_{\frac{1}{|ts|^n}} = O_1,$$

thus ,

$$y = -x \text{ and } |s|^n = \frac{1}{|t|^n} \notin]0,1],$$

thus * is anti-inverse operation.

Definition 2.2.

Let P be an ideal $\mathbb{Z}$, we define the corresponding n-hyper-ideal of Δ_n as follows:

$$\Delta^{(P)} = \{x_{\mu(t)};\ x \in P , t \in \mathbb{Z}\}.$$

The corresponding under-ideal of Δ_n is defined as follows:

$$\Delta_{(P)} = \{x_{\mu(t)};\ t \in P , x \in \mathbb{Z}\}.$$

Theorem 2.2.

Let P be an ideal of $\mathbb{Z}$. Then

1) $\Delta^{(P)}$ is closed with respect to *.
2) $\Delta_{(P)}$ is closed with respect to *.
3) $O_{\mu(s)} \in \Delta^{(P)}$ for all $s \in \mathbb{Z}$

Proof.

1) Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$, then

$$(x,y) \in P,\ (s,t) \in \mathbb{Z},$$

thus

$$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\frac{1}{|ts|^n}} \in \Delta^{(P)},$$

that is because $x+y \in P$,

2) Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta_{(P)}$.Then

$$(x,y) \in \mathbb{Z},\ (t,s) \in P,$$

hence,

$$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\frac{1}{|ts|^n}} = (x+y)_{\mu(ts)};\ x+y \in \mathbb{Z},\ ts \in P,$$

thererfore

$$x_{\mu(t)} * y_{\mu(s)} \in \Delta_{(P)}.$$

3) It holds directly from the definition.

Theorem 2.3.

Let P,Q be two ideals of $\mathbb{Z}$. Then

1) If $P \subseteq Q$, we get $\Delta^{(P)} \subseteq \Delta^{(Q)}$

2) If $P \subseteq Q$, we get $\Delta_{(P)} \subseteq \Delta_{(Q)}$.

3) $\Delta^{(P)} \cap \Delta^{(Q)} = \Delta^{(P\cap Q)}$

4) $\Delta_{(P)} \cap \Delta_{(Q)} = \Delta_{(P\cap Q)}$.

Proof.

1) Let $x_{\mu(t)} \in \Delta^{(P)}$. Then $x \in P \subseteq Q$, and $t \in \mathbb{Z}$, thus

$$x_{\mu(t)} \in \Delta^{(Q)} \text{ and } \Delta^{(P)} \subseteq \Delta^{(Q)}.$$

2) It can be proved by a similar argument.

3) Let $x_{\mu(t)} \in \Delta^{(P\cap Q)}$. Then $x \in P \cap Q$ and $t \in \mathbb{Z}$, thus

$$x_{\mu(t)} \in \Delta^{(P)} \cap \Delta^{(Q)}.$$

Conversely, if $x_{\mu(t)} \in \Delta^{(P)} \cap \Delta^{(Q)}$, then

$$x \in P, x \in Q\ ,\ t \in \mathbb{Z},$$

hence

$$x_{\mu(t)} \in \Delta^{(P\cap Q)}.$$

4) It can be proved by a similar argument.

Definition 2.3.

Let P,Q be two ideals of $\mathbb{Z}$. Then

$$\Delta^{(P)}_{(Q)} = \{x_{\mu(t)};\ x \in P\ , t \in Q\}\ ,$$

is called a two fold hyper-under ideal (HU-ideal) of Δ_s.

If P=Q, then it is called a regular HU-ideal or (RHU-ideal).

Theorem 2.4.

Let P, Q, R, S be four ideals of $\mathbb{Z}$. Then

$\Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)} = \Delta^{(P\cap R)}_{(Q\cap S)}$.

Proof.

Let

$x_{\mu(t)} \in \Delta^{(P\cap R)}_{(Q\cap S)}$.

Then

$x \in P \cap R, t \in Q \cap S,$

thus

$x \in \Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)}$.

Conversely, if

$x_{\mu(t)} \in \Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)}$,

then

$x \in P, x \in R, t \in Q, t \in S,$

so that

$x_{\mu(t)} \in \Delta^{(P\cap R)}_{(Q\cap S)}$.

Theorem 2.5.

Let P,Q be two ideals of $\mathbb{Z}$. Then

$\Delta^{(P)} \cap \Delta_{(Q)} = \Delta^{(P)}_{(Q)}$.

Proof.

Let

$x_{\mu(t)} \in \Delta^{(P)} \cap \Delta_{(Q)}$.

Then

$$\begin{cases} x_{\mu(t)} \in \Delta^{(P)} \\ x_{\mu(t)} \in \Delta_{(Q)} \end{cases},$$

So that, $x \in P, t \in Q$ and $x_{\mu(t)} \in \Delta^{(P)}_{(Q)}$.

Conversely, if $x_{\mu(t)} \in \Delta^{(P)}_{(Q)}$, then $x \in P, t \in Q$ and

$$\begin{cases} x_{\mu(t)} \in \Delta^{(P)} \\ x_{\mu(t)} \in \Delta_{(Q)} \end{cases},$$

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hence
$x_{\mu(t)} \in \Delta^{(P)} \cap \Delta_{(Q)}$.

Definition 2.4.
Let w_s be a non empty subset Δ_S. Then $(w_s,*)$ is called a two fold subalgebra if and if w_s is closed under *.
Definition 2.5.
Let P,Q be two ideals of $\mathbb{Z}$, we define:
1] $\Delta^{(P)} \times \Delta^{(Q)} = \{(x_{\mu(t)}, y_{\mu(s)});\ x \in P, y \in Q, (t,s) \in \mathbb{Z}\}$.
2] $\Delta_{(P)} \times \Delta_{(Q)} = \{(x_{\mu(t)}, y_{\mu(s)});\ (x,y) \in \mathbb{Z}, t \in P,\ s \in Q\}$
Definition 2.6.
Let $f: \Delta_s \to \Delta_s$ be a mapping, we say that (f) is a two fold algebra homomorphism if:
1) $f(x_{\mu(t)} * y_{\mu(s)}) = f(x_{\mu(t)}) * f(y_{\mu(s)})$.
1) If (f) is a bijection, then it is called a two fold algebra isomorphism.
Definition 2.7.
Let $g\ , h: \mathbb{Z} \to \mathbb{Z}$ such that:

$$\begin{cases} g(x+y) = g(x) + g(y) \\ h(x.y) = h(x)h(y) \end{cases}.$$

We define the regular two fold algebra homomorphism $f_{(g,h)}$ as follows:

$$f_{(g,h)}::\Delta_S \to \Delta_S.$$

With,
$f_{(g,h)}(x_{\mu(t)}) = g(x_{\mu(h(t))})$.
It is clear that

$$f_{(g,h)}\left(x_{\mu(t)} * y_{\mu(s)}\right) = f_{(g,h)}\left[(x+y)_{\frac{1}{|ts|^n}}\right] = g(x+y)_{\frac{1}{h(|ts|^n)}} = \left(g(x)+g(y)\right)_{\frac{1}{h(|t|^n)h(|s|^n)}}$$
$$= \left(g(x)\right)_{\frac{1}{h(|t|^n)}} * \left(g(y)\right)_{\frac{1}{h(|s|^n)}} = f_{(g,h)}\left(x_{\mu(t)}\right) * f_{(g,h)}\left(y_{\mu(s)}\right).$$

Theorem 2.6.
Let $f_{(g,g)}$ be a regular two-fold algebra homomorphism. Then
1) If $\Delta^{(P)}$ is a hyper ideal, then $f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{(g(P))}$
2) If $\Delta_{(P)}$ is an under ideal of Δ_S, then $f_{(g,g)}\left(\Delta_{(P)}\right) \subseteq \Delta_{(g(P))}$
3) If $\Delta^{(P)}_{(Q)}$ is a hyper-under ideal of Δ_S, then

$$f_{(g,g)}\left(\Delta^{(P)}_{(Q)}\right) \subseteq \Delta^{(g(P))}_{(g(Q))},$$

is a hyper-under ideal of Δ_S.
Proof.
1) First, we must prove that $f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{(g(P))}$.
Let $y_{\mu(s)} \in f_{(g,g)}\left(\Delta^{(P)}\right)$, then there exists $x_{\mu(t)} \in \Delta^{(P)}$ such that:

$$f_{(g,g)}\left(x_{\mu(t)}\right) = g(x)_{g(\frac{1}{|t|^n})} \in \Delta^{(g(P))}.$$

Thus,

$$f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{g(P)}.$$

2) Let $x_{\mu(t)} \in f_{(g,g)}\left(\Delta_{(P)}\right)$. Then there exists $y_{\mu(s)} \in \Delta_{(P)}$ such that:

$$x_{\mu(t)} = f_{(g,g)}\left(y_{\mu(s)}\right) = [g(y)]_{g(\frac{1}{||s|^n|})}.$$

Thus,

$$x = g(y), t = g(|s|),$$

hence

$$x_{\mu(t)} \in \Delta_{g(P)} \text{ and } f_{(g,g)}\left(\Delta_{(P)}\right) \subseteq \Delta_{g(P)}.$$

3) It holds directly from (1) and (2).

Example 2.1.
Consider $=< 2 >$, $Q =< 3 >$, then

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$\Delta^{(P)}= \{(2x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,
$\Delta^{(Q)}= \{(3x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,
$\Delta^{(P)} \cap \Delta^{(Q)}= \Delta^{(P\cap Q)}= \{(6x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$.
$\Delta_{(P)}= \{(x)_{\mu(2t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,
$\Delta_{(Q)}= \{(x)_{\mu(3t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,
$\Delta_{(P)} \cap \Delta_{(Q)}= \Delta_{(P\cap Q)}= \{(x)_{\mu(6t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$.
$\Delta^{(P)}_{(Q)}= \{(2x)_{\mu(3t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,
$\Delta_{(P)} \times \Delta_{(Q)}= \left((x)_{\mu(2t)}, (y)_{\mu(3s)}\right); x, y, t, s \in \mathbb{Z}$,
$\Delta^{(P)} \times \Delta^{(Q)}= \left((2x)_{\mu(t)}, (3y)_{\mu(s)}\right); x, y, t, s \in \mathbb{Z}$,
$\Delta^{(P)} \times \Delta_{(Q)}= \left((2x)_{\mu(t)}, (y)_{\mu(3s)}\right); x, y, t, s \in$,

Example 2.2.
Consider: $g, h: \mathbb{Z} \to \mathbb{Z}$ such that:

$$g(x) = 5x,\ h(y) = y^2,$$

we have:

$$\begin{cases} g(x+y) = g(x) + g(y) \\ h(xy) = h(x)\,h(y) \end{cases}.$$

Define $f_{(g,h)}: \Delta_s \to \Delta_s$ such that:

$$f_{(g,h)}\left(x_{\mu(t)}\right) = \left(g(x)_{\mu(h(t))}\right) = (5x)_{\frac{1}{|t^{2n}|}} = (5x)_{\frac{1}{t^{2n}}}.$$

The mapping $f_{(g,h)}$ is a two fold homomorphism.
Also, $ker\ (f_{(g,h)}) = \{x_{\mu(t)} \in \Delta_s\ such\ that\ f_{(g,h)}(x_{\mu(t)}) = O_{\mu(0)}\}$, so that

$$\begin{cases} 5x = 0\ , \\ t^2 = 1 \Rightarrow t = 1\ , \end{cases}.$$

hence,

$$ker\ (f_{(g,h)}) = \{O_{\mu(0)}, O_{\mu(1)}, O_{\mu(-1)}\}.$$

Definition 2.8.
Let $f_{(g_1,h_1)}, f_{(g_2,h_2)}: \Delta_s \to \Delta_s$ be two fold homomorphisms. Then
We define. $f_{(g_1,h_1)} \circ f_{(g_2,h_2)}: \Delta_s \to \Delta_s$ such that:

$$f_{(g_1,h_1)} \circ f_{(g_2,h_2)}\left(x_{\mu(t)}\right) = f_{(g_1\circ g_2, h_1\circ h_2)}\left(x_{\mu(t)}\right) = \left(g_1 \circ g_2(x)_{\mu(h_1\circ h_2(t))}\right).$$

We denote to the set of all two fold algebra homomorphisms by F_S.

Remark 2.1.
Since (o) is associative, and non-commutative, then
$O: F_S \times F_S \to F_S$ is associative and non-commutative operation, with $F_{(I,I)}$ as an identity, where
$I: \mathbb{Z} \to \mathbb{Z}$ such that

$$T(x) = x.$$

Theorem 2.7.
Let $\Delta^{(P)}, \Delta_{(Q)}$ be two hyper/under two fold ideals of Δ_s. Then
1) $\Delta^{(P)}$ is two fold subalgebra of Δ_s.
2) $\Delta_{(Q)}$ is two fold subalgebra of Δ_s.
3) $\Delta^{(P)}_{(Q)}$ is two fold subalgebra of Δ_s

Proof.
1) Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$. Then $x, y \in P, t, s \in \mathbb{Z}$, thus,

$$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\mu(ts)} \in \Delta^{(P)},$$

that is because $x, y \in P$,
hence $(\Delta^{(P)}.*)$ is closed u.nder (*), and $\Delta^{(P)}$ is two fold subalgebra of Δ_s.
2) Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$ Then $x, y \in \mathbb{Z}, t, s \in Q$, thus,

$$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\mu(ts)} \in \Delta_{(Q)},$$

that is because $x + y \in \mathbb{Z}$, $ts \in Q$ and $(\Delta_{(Q)}, *)$ is closed and then it is a two fold subalgebra.
3) If holds directly from (1) and (2).

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Definition 2.9.

Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta_s$, we say that $x_{\mu(t)} \sim y_{\mu(s)}$ if there exists $z_{\mu(c)} \in \Delta_s$ such that: $x_{\mu(t)} * z_{\mu(c)} = y_{\mu(s)}$.

3. Conclusion

In this paper, we studied the two-fold algebra based on the n-standard fuzzy number theoretical system as a special type of two-fold fuzzy algebras, where we proved the elementary properties of the algebraic operations defined over this system. Also, we proved many results that describe the relations between two-fold substructures and sub-algebras defined by fuzzy number theoretical systems. On the other hand, we provided many different examples to explain our results.

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