A Robust MCDM Framework for Cloud Service Selection Using Spherical Fermatean Neutrosophic Bonferroni Mean

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Abstract

This study presents an innovative approach to cloud service provider selection using the spherical Fermatean neutrosophic Bonferroni mean. As organizations increasingly rely on cloud services, selecting the optimal provider becomes critical, necessitating robust multi criteria decision making methods. Traditional approaches often fall short in capturing the diverse perspectives of decision-makers, leading to suboptimal choices. The spherical Fermatean neutrosophic Bonferroni mean addresses this gap by integrating a spherical representation that encompasses membership, non-membership and indeterminacy functions, enhanced by the Bonferroni mean. This structure effectively encapsulates the opinions of all decision makers, offering a comprehensive and balanced perspective. The study evaluates six cloud service providers based on four criteria: cost (non-beneficiary), performance, security and scalability (beneficiary). Three decision makers with distinct priorities participate in the evaluation, ensuring a thorough assessment. The proposed spherical Fermatean neutrosophic Bonferroni mean method excels in resolving ambiguity and managing risk with greater precision than conventional FNSs, providing a more accurate and effective decision-making framework. A numerical example illustrates the practical application of spherical Fermatean neutrosophic Bonferroni mean, demonstrating its utility in selecting the optimal cloud service provider for an organization.

Keywords: Spherical Fermatean neutrosophic sets; Spherical Fermatean neutrosophic Bonferroni mean; Extension of neutrosophic sets; Extension of Fermatean fuzzy sets; Multi criteria decision making

1 Introduction and Preliminaries

In the intricate domains of Multi Criteria Decision Making (MCDM) and multi group decision processes, decision theory is undergoing profound transformations. Individuals tasked with prioritizing, selecting or allocating options amidst conflicting criteria must integrate quantitative metrics with qualitative insights. They navigate a complex interplay of data streams from diverse origins, often encountering imprecision, ambiguity and uncertainty due to subjective factors, incomplete information, inconsistent measurements and intricate interactions. The selection of a Cloud Service Provider (CSP) is one such critical decision, requiring a robust framework to handle these complexities.

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The objective of this study is to explore the spherical Fermatean neutrosophic Bonferroni mean as an innovative approach to decision-making in scenarios involving diverse perspectives. By incorporating membership, non-membership and indeterminacy functions within a spherical construct, the B captures the entire spectrum of viewpoints more effectively than conventional methods. Through empirical testing and analysis, the study demonstrates that spherical Fermatean neutrosophic Bonferroni mean offers greater accuracy and effectiveness in resolving ambiguity and risk. Ultimately, the spherical Fermatean neutrosophic Bonferroni mean enhances decision making processes by providing decision makers with a comprehensive and nuanced tool to navigate complex scenarios involving diverse perspectives.

Fuzzy set theory, introduced by Zadeh in 1965, extends the classical notion of sets by allowing elements to have degrees of membership.[19] In a fuzzy set, each element is associated with a membership function that assigns it a value between 0 and 1, representing its degree of membership. This theory laid the groundwork for handling vagueness and imprecision in various domains. Building on fuzzy sets, Atanassov introduced Intuitionistic Fuzzy Sets (IFS) in 1986.[1] An IFS incorporates two functions: the membership function (MF) and the non-membership function (NMF), both ranging between 0 and 1. Additionally, the sum of these functions for any element does not exceed 1. This framework provides a more flexible approach by explicitly considering the hesitation margin, which reflects the uncertainty about the membership status of elements.

Neutrosophic sets, proposed by Smarandache, further generalize the concept of fuzzy sets by introducing three independent functions: the membership function (MF), the non-membership function (NMF) and the indeterminacy function (IF).[17] These functions map each element to values in the interval [0, 1] and the sum of these values can range from 0 to 3. This model allows for the representation of indeterminacy, offering a more comprehensive way to handle uncertainty, contradiction and incompleteness.

Fermatean fuzzy sets were introduced by Senapati and Yager as an enhancement to traditional fuzzy sets.[16] In a Fermatean fuzzy set, the degrees of membership and non-membership are raised to the power of three, reflecting a more intricate relationship between these functions. This formulation aims to provide a more detailed representation of uncertainty compared to traditional fuzzy sets. The conditions governing Fermatean Fuzzy Sets ensure that the sum of the cubed values of membership and non-membership degrees remains within the unit interval. Additionally, the degree of uncertainty for an element is defined using the difference between the maximum possible value (one) and the sum of the cubed membership and non-membership degrees. This approach provides a refined mechanism for capturing the nuances of uncertainty in various scenarios.

Fermatean neutrosophic sets merge the principles of Fermatean fuzzy sets and neutrosophic sets.[18] This advanced concept incorporates three functions to represent the degrees of membership, non-membership and indeterminacy, each raised to the power of three. The conditions for a Fermatean neutrosophic set ensure that the sum of the cubed values of membership and non-membership degrees is bounded and the cubed indeterminacy degree is also constrained within a specific range. This ensures a balanced representation of all three aspects of uncertainty. By combining the strengths of Fermatean fuzzy sets and neutrosophic sets, Fermatean neutrosophic sets offer a robust framework for handling complex and multifaceted uncertainties, making them applicable to a wide range of real-world problems. The concept of Fermatean neutrosophic has extensive applications in various fields [[3–6]]. Numerous authors have contributed to the fuzzy extension in the area of multi criteria decision making [[7–13, 15]].

Definition 1.1. [18] A set $\mathbb{F}_\epsilon = \{\langle \epsilon_\imath, \Psi(\epsilon_\imath), \Omega(\epsilon_\imath), \Lambda(\epsilon_\imath)\rangle | \epsilon_\imath \in \Upsilon\}$ is called Fermatean Neutrosophic Set (FNS)[18] in the universe of discourse Υ if $\Psi(\epsilon_\imath)$, $\Omega(\epsilon_\imath)$, $\Lambda(\epsilon_\imath) : \Upsilon \to [0,1]$ that represent the degree of MF, NMF and IF of $\epsilon_\imath \in \Upsilon$ respectively satisfy the conditions of $0 \preceq \Psi^3(\epsilon_\imath) + \Omega^3(\epsilon_\imath) \preceq 1$ and $0 \preceq \Lambda^3(\epsilon_\imath) \preceq 1$. Therefore, for FNS, $0 \preceq \Psi^3(\epsilon_\imath) + \Omega^3(\epsilon_\imath) + \Lambda^3(\epsilon_\imath) \preceq 2$ for all $\epsilon \in \Upsilon$.

Definition 1.2. [14] Let Υ be the universal set containing elements known as Spherical Fermatean Neutrosophic Sets (SFNSs). Each $\Theta_\Xi \in \Upsilon$ is defined as $\mathbb{S}_{\Theta_\Xi} = \{\langle \Theta_\Xi, \Psi_{\Theta_\Xi}, \Omega_{\Theta_\Xi}, \Lambda_{\Theta_\Xi}; \Delta_i\rangle : \Theta_\Xi \in \Upsilon\}$, where Ψ_{Θ_Ξ}, Ω_{Θ_Ξ}, Λ_{Θ_Ξ}, $\Delta : \Upsilon \to [0,1]$, represent the degrees of membership, non-membership, indeterminacy function and radius of Θ_Ξ respectively. This degrees satisfy $0 \preceq \Psi^3_{\Theta_\Xi} + \Omega^3_{\Theta_\Xi} \preceq 1$, $0 \preceq \Lambda^3_{\Theta_\Xi} \preceq 1$ and $0 \preceq \Psi^3_{\Theta_\Xi} + \Omega^3_{\Theta_\Xi} + \Lambda^3_{\Theta_\Xi} \preceq 2$ for all $\Theta_\Xi \in \Upsilon$ and $\imath = 1, 2, \ldots \Xi$. We construct the center of the sphere by

$$\langle \Psi_{\Theta_\Xi}, \Omega_{\Theta_\Xi}, \Lambda_{\Theta_\Xi}\rangle = \left\langle \frac{\sum_{\jmath=1}^{k} \Psi_{\imath,\jmath}}{k}, \frac{\sum_{\jmath=1}^{k} \Omega_{\imath,\jmath}}{k}, \frac{\sum_{\jmath=1}^{k} \Lambda_{\imath,\jmath}}{k} \right\rangle$$

and the radius is

$$\Delta_i = min\left\{ 1 \preceq \overset{max}{j} \preceq k \sqrt{(\Psi_{\Theta_\Xi} - \Psi\imath, \jmath)^2 + (\Omega_{\Theta_\Xi} - \Omega_{\imath,\jmath})^2 + (\Lambda_{\Theta_\Xi} - \Lambda_{\imath,\jmath})^2}, 1 \right\}.$$

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Let $A = \{\langle x, 0.4, 0.2, 0.3\rangle, \langle x, 0.4, 0.5, 0.3\rangle, \langle x, 0.3, 0.6, 0.4\rangle, \langle x, 0.3, 0.4, 0.3\rangle$, is the collection of FNSs. Then $\Psi_{\Theta_\Xi} = \frac{0.4+0.4+0.3+0.3}{4} = 0.35$, $\Omega_{\Theta_\Xi} = \frac{0.2+0.5+0.6+0.4}{4} = 0.43$, $\Lambda_{\Theta_\Xi} = \frac{0.3+0.3+0.4+0.3}{4} = 0.33$, The radii is

$\Delta_i = min\left\{1 \le \overset{max}{j} \le k \sqrt{(\Psi_{\Theta_\Xi} - \Psi^3_{i,j})^2 + (\Omega_{\Theta_\Xi} - \Omega^3_{i,j})^2 + (\Lambda_{\Theta_\Xi} - \Lambda^3_{i,j})^2}, 1\right\}$

$\Delta_i = min\{\max\{\sqrt{(0.35-0.4)^2 + (0.43-0.2)^2 + (0.33-0.3)^2},$

$\sqrt{(0.35-0.4)^2 + (0.43-0.5)^2 + (0.33-0.3)^2},$

$\sqrt{(0.35-0.3)^2 + (0.43-0.6)^2 + (0.33-0.4)^2},$

$\sqrt{(0.35-0.3)^2 + (0.43-0.4)^2 + (0.33-0.3)^2}\}$,1$\}$

$= \{\min\{\max\{0.23, 0.09, 0.19, 0.06\}, 1\} = 0.23\}$

Thus, the SFNS is: $\mathbb{S}_\Theta = \langle 0.35, 0.43, 0.33; 0.23\rangle$

2 Spherical Fermatean Neutrosophic Bonferroni Mean

Definition 2.1. Let $\mathbb{F}_i = \langle \Psi_{n_i}, \Omega_{n_i}, \Lambda_{n_i}; \Delta_{n_i}\rangle$ where $i = 1, 2, \ldots, \Xi$ is a collection of SFNN. For any $\mathbb{M}, \mathbb{N} > 0$, the Spherical Fermatean Neutrosophic Bonferroni Mean (B) operator is defined as

$$B^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_\Xi) = \left(\frac{1}{\Xi(\Xi-1)} \bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi} (\mathbb{F}^{\mathbb{M}}_{\mathbb{U}} \otimes \mathbb{F}^{\mathbb{N}}_{\mathbb{V}})\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}} \tag{1}$$

Theorem 2.2. *For $\mathbb{M}, \mathbb{N} > 0$ and a collection of Spherical Fermatean Neutrosophic Sets (SFNNSs) $\mathbb{F}_{\mathbb{U}} = \langle \Psi_{n_{\mathbb{U}}}, \Omega_{n_{\mathbb{U}}}, \Lambda_{n_{\mathbb{U}}}\rangle$, $\mathbb{U} = 1, 2, \ldots, \Xi$, the aggregation B is also a $SFNN$ and it is of the form*

$$B^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_\Xi) = \begin{cases} \sqrt[3]{\left(1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi} \left(1 - \Psi^{3\mathbb{M}}_{\mathbb{F}_\mathbb{U}} \Psi^{3\mathbb{N}}_{\mathbb{F}_\mathbb{V}}\right)^{\frac{1}{\Xi(\Xi-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}, \\ \sqrt[3]{1 - \left(1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi} \left(1 - (1-\Lambda^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}(1-\Lambda^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}\right)^{\frac{1}{\Xi(\Xi-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}, \\ \sqrt[3]{1 - \left(1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi} \left(1 - (1-\Omega^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}(1-\Omega^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}\right)^{\frac{1}{\Xi(\Xi-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}; \\ \sqrt[3]{\left(1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi} \left(1 - \Delta^{3\mathbb{M}}_{\mathbb{F}_\mathbb{U}} \Delta^{3\mathbb{N}}_{\mathbb{F}_\mathbb{V}}\right)^{\frac{1}{\Xi(\Xi-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}} \end{cases} \tag{2}$$

Proof:
From the basic operations, we get

$$\mathbb{F}^{\mathbb{M}}_{\mathbb{U}} = \left(\Psi^{\mathbb{M}}_{\mathbb{F}_\mathbb{U}}, \sqrt[3]{1-(1-\Lambda^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}}, \sqrt[3]{1-(1-\Omega^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}}; \Delta^{\mathbb{M}}_{\mathbb{F}_\mathbb{U}},\right)$$

and

$$\mathbb{F}^{\mathbb{N}}_{\mathbb{V}} = \left(\Psi^{\mathbb{N}}_{\mathbb{F}_\mathbb{V}}, \sqrt[3]{1-(1-\Lambda^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}}, \sqrt[3]{1-(1-\Omega^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}}; \Delta^{\mathbb{N}}_{\mathbb{F}_\mathbb{V}}\right)$$

Then,

$$\mathbb{F}^{\mathbb{M}}_{\mathbb{U}} \otimes \mathbb{F}^{\mathbb{N}}_{\mathbb{V}} = \left(\Psi^{\mathbb{M}}_{\mathbb{F}_\mathbb{U}}\Psi^{\mathbb{N}}_{\mathbb{F}_\mathbb{V}}, \sqrt[3]{1-(1-\Lambda^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}(1-\Lambda^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}}, \sqrt[3]{1-(1-\Omega^3_{\mathbb{F}_\mathbb{U}})^{\mathbb{M}}(1-\Omega^3_{\mathbb{F}_\mathbb{V}})^{\mathbb{N}}}; \Delta^{\mathbb{M}}_{\mathbb{F}_\mathbb{U}}\Delta^{\mathbb{N}}_{\mathbb{F}_\mathbb{V}}\right).$$

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First let us prove

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})=\begin{cases}\sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi}(1-\Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi}(1-(1-\Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi}(1-(1-\Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})};\\ \sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\Xi}(1-\Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}\end{cases}\tag{3}$$

by mathematical induction principle on Ξ. For $\Xi=2$, we get

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{2}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})=(\mathbb{F}_{1}^{\mathbb{M}}\otimes\mathbb{F}_{2}^{\mathbb{N}})\oplus(\mathbb{F}_{2}^{\mathbb{M}}\otimes\mathbb{F}_{1}^{\mathbb{N}})\tag{4}$$

$$=\begin{cases}1-(1-\Psi_{\mathbb{F}_1}^{3\mathbb{M}}\Psi_{\mathbb{F}_2}^{3\mathbb{N}})(1-\Psi_{\mathbb{F}_2}^{3\mathbb{M}}\Psi_{\mathbb{F}_1}^{3\mathbb{N}}),\\ \sqrt[3]{(1-(1-\Lambda_{\mathbb{F}_1}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_2}^{3})^{\mathbb{N}})}\times\sqrt[3]{(1-(1-\Lambda_{\mathbb{F}_2}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_1}^{3})^{\mathbb{N}})},\\ \sqrt[3]{(1-(1-\Omega_{\mathbb{F}_1}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_2}^{3})^{\mathbb{N}})}\times\sqrt[3]{(1-(1-\Omega_{\mathbb{F}_2}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_1}^{3})^{\mathbb{N}})};\\ 1-(1-\Delta_{\mathbb{F}_1}^{3\mathbb{M}}\Delta_{\mathbb{F}_2}^{3\mathbb{N}})(1-\Delta_{\mathbb{F}_2}^{3\mathbb{M}}\Delta_{\mathbb{F}_1}^{3\mathbb{N}})\end{cases}\tag{5}$$

$$=\begin{cases}\sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{2}(1-\Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{2}(1-(1-\Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{2}(1-(1-\Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})};\\ \sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{2}(1-\Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}.\end{cases}\tag{6}$$

Assume that equation (2) holds for $\mathbb{P}=\omega$, that is,

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})=\begin{cases}\sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1-\Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1-(1-\Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})},\\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1-(1-\Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})};\\ \sqrt[3]{1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1-\Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}\end{cases}\tag{7}$$

Now let $\mathbb{P}=\omega+1$, then

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})=\left(\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})\right)\oplus\left(\bigoplus_{\mathbb{U}=1}^{\omega}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\omega+1}^{\mathbb{N}})\right)\oplus\left(\bigoplus_{\mathbb{V}=1}^{\omega}(\mathbb{F}_{\omega+1}^{\mathbb{M}}\otimes\mathbb{F}_{\mathbb{V}}^{\mathbb{N}})\right)\tag{8}$$

$$\bigoplus_{\mathbb{U}=1}^{\omega}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}}\otimes\mathbb{F}_{\omega+1}^{\mathbb{N}})=\begin{cases}\sqrt[3]{1-\prod_{\mathbb{U}=1}^{\omega}(1-\Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Psi_{\mathbb{F}_{\omega+1}}^{3\mathbb{N}})},\\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1-(1-\Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{N}})},\\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1-(1-\Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{N}})};\\ \sqrt[3]{1-\prod_{\mathbb{U}=1}^{\omega}(1-\Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}}\Delta_{\mathbb{F}_{\omega+1}}^{3\mathbb{N}})}\end{cases}\tag{9}$$

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and

$$\bigoplus_{\mathbb{V}=1}^{\omega}(\mathbb{F}_{\omega+1}^{\mathbb{M}} \otimes \mathbb{F}_{\mathbb{V}}^{\mathbb{N}}) = \begin{cases} \sqrt[3]{1 - \prod_{\mathbb{U}=1}^{\omega}(1 - \Psi_{\mathbb{F}_{\omega+1}}^{3\mathbb{M}} \Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1 - (1 - \Lambda_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1 - (1 - \Omega_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}; \\ \sqrt[3]{1 - \prod_{\mathbb{U}=1}^{\omega}(1 - \Delta_{\mathbb{F}_{\omega+1}}^{3\mathbb{M}} \Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})} \end{cases} \tag{10}$$

From equations (8-10) we get

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}} \otimes \mathbb{F}_{\mathbb{V}}^{\mathbb{N}}) = \begin{cases} \Bigg(\sqrt[3]{1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1 - \Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1 - (1 - \Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1 - (1 - \Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}; \\ \sqrt[3]{1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega}(1 - \Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})} \Bigg) \oplus \\ \Big(\sqrt[3]{1 - \prod_{\mathbb{U}=1}^{\omega}(1 - \Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Psi_{\mathbb{F}_{\omega+1}}^{3\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1 - (1 - \Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{U}=1}^{\omega}(1 - (1 - \Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{N}})}; \\ \sqrt[3]{1 - \prod_{\mathbb{U}=1}^{\omega}(1 - \Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Omega_{\mathbb{F}_{\omega+1}}^{3\mathbb{N}})} \Big) \oplus \\ \Big(\sqrt[3]{1 - \prod_{\mathbb{V}=1}^{\omega}(1 - \Psi_{\mathbb{F}_{\omega+1}}^{3\mathbb{M}} \Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{V}=1}^{\omega}(1 - (1 - \Lambda_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{\prod_{\mathbb{V}=1}^{\omega}(1 - (1 - \Omega_{\mathbb{F}_{\omega+1}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}; \\ \sqrt[3]{1 - \prod_{\mathbb{V}=1}^{\omega}(1 - \Delta_{\mathbb{F}_{\omega+1}}^{3\mathbb{M}} \Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})} \Big) \end{cases} \tag{11}$$

$$\bigoplus_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}} \otimes \mathbb{F}_{\mathbb{V}}^{\mathbb{N}}) = \begin{cases} \sqrt[3]{1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(1 - \Psi_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(1 - (1 - \Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(1 - (1 - \Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})}, \\ \sqrt[3]{1 - \prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\omega+1}(1 - \Delta_{\mathbb{F}_{\mathbb{U}}}^{3\mathbb{M}} \Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})} \end{cases} \tag{12}$$

Therefore the result is true for $\mathbb{P} = \omega + 1$. Hence by mathematical induction the equation(2) holds for all $\mathbb{P}$. Now,

$$\frac{1}{\mathbb{P}(\mathbb{P}-1)} \bigoplus_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(\mathbb{F}_{\mathbb{U}}^{\mathbb{M}} \otimes \mathbb{F}_{\mathbb{V}}^{\mathbb{N}}) = \begin{cases} \sqrt[3]{1 - \left(\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1 - \Psi_{\mathbb{F}_{\mathbb{U}}}^{\mathbb{M}} \Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})\right)^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1 - (1 - \Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}}, \\ \sqrt[3]{\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1 - (1 - \Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1 - \Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}}; \\ \sqrt[3]{1 - \left(\prod_{\substack{\mathbb{U},\mathbb{V}=1 \\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1 - \Delta_{\mathbb{F}_{\mathbb{U}}}^{\mathbb{M}} \Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})\right)^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}} \end{cases} \tag{13}$$

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Therefore

$$FNBM^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1,\mathbb{F}_2,\ldots,\mathbb{F}_{\mathbb{P}}) = \begin{cases} \sqrt[3]{\left(1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1-\Psi_{\mathbb{F}_{\mathbb{U}}}^{\mathbb{M}}\Psi_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}, \\ \sqrt[3]{1-\left(1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1-(1-\Lambda_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Lambda_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}, \\ \sqrt[3]{1-\left(1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1-(1-\Omega_{\mathbb{F}_{\mathbb{U}}}^{3})^{\mathbb{M}}(1-\Omega_{\mathbb{F}_{\mathbb{V}}}^{3})^{\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}}; \\ \sqrt[3]{\left(1-\prod_{\substack{\mathbb{U},\mathbb{V}=1\\ \mathbb{U}\neq\mathbb{V}}}^{\mathbb{P}}(1-\Delta_{\mathbb{F}_{\mathbb{U}}}^{\mathbb{M}}\Delta_{\mathbb{F}_{\mathbb{V}}}^{3\mathbb{N}})^{\frac{1}{\mathbb{P}(\mathbb{P}-1)}}\right)^{\frac{1}{\mathbb{M}+\mathbb{N}}}} \end{cases} \quad (14)$$

Proposition 2.3 (Idempotency). *If the* $SFNNs$, $\mathbb{F}_{\mathbb{U}} = \mathbb{F} = (\Psi_{\mathbb{F}_{\mathbb{U}}}, \Omega_{\mathbb{F}_{\mathbb{U}}}, \Lambda_{\mathbb{F}_{\mathbb{U}}}; \Delta_{\mathbb{F}_{\mathbb{U}}})$ *for all* $\mathbb{U} = 1, 2, ..., \mathbb{P}$ *then* $B^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1, \mathbb{F}_2, ...\mathbb{F}_{\mathbb{P}}) = \mathbb{F}$.

Proposition 2.4 (Monotonicity). *Consider* $\mathbb{F}_{\mathbb{U}} = (\Psi_{\mathbb{F}_{\mathbb{U}}}, \Omega_{\mathbb{F}_{\mathbb{U}}}, \Lambda_{\mathbb{F}_{\mathbb{U}}}; \Delta_{\mathbb{F}_{\mathbb{U}}})$ *and* $\mathbb{M}_{\mathbb{U}} = (\Psi_{m_{\mathbb{U}}}, \Omega_{m_{\mathbb{U}}}, \Lambda_{m_{\mathbb{U}}}; \Delta_{m_{\mathbb{U}}})$ *for* $\mathbb{U} = 1, 2, 3, \ldots, \mathbb{P}$ *two SFNSs. If* $\Psi_{\mathbb{F}_{\mathbb{U}}} \leq \Psi_{m_{\mathbb{U}}}, \Omega_{\mathbb{F}_{\mathbb{U}}} \geq \Omega_{m_{\mathbb{U}}}, \Lambda_{\mathbb{F}_{\mathbb{U}}} \leq \Lambda_{m_{\mathbb{U}}}; \Delta_{\mathbb{F}_{\mathbb{U}}} \leq \Delta_{m_{\mathbb{U}}}$ *for each* $\mathbb{U}$, *then* $B^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_{\mathbb{P}}) \leq B^{\mathbb{M},\mathbb{N}}(m_1, m_2, \ldots, m_{\mathbb{P}})$.

Proposition 2.5 (Commutativity). *Consider* $\mathbb{F}_{\mathbb{U}} = (\Psi_{\mathbb{F}_{\mathbb{U}}}, \Omega_{\mathbb{F}_{\mathbb{U}}}, \Lambda_{\mathbb{F}_{\mathbb{U}}}; \Delta_{\mathbb{F}_{\mathbb{U}}})$ *as a set of SFNS. Then* $B^{\mathbb{M},\mathbb{N}}(\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_{\mathbb{P}})$ = $B^{\mathbb{M},\mathbb{N}}(\dot{\mathbb{F}}_1, \dot{\mathbb{F}}_2, \ldots, \dot{\mathbb{F}}_{\mathbb{P}})$ *where* $(\dot{\mathbb{F}}_1, \dot{\mathbb{F}}_2, \ldots, \dot{\mathbb{F}}_{\mathbb{P}})$ *is any one arrangement of* $(\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_{\mathbb{P}})$.

3 MCDM Using Spherical Fermatean Neutrosophic Sets

Let $\psi = \{\psi_1, \psi_2 \ldots \psi_n\}$ be a set of alternatives and $\lambda = \{\lambda_1, \lambda_2 \ldots \lambda_n\}$ be a set of criteria. Suppose $(\delta_{\mathbb{L}\varepsilon})_{m\times n} = \langle \Psi_{\delta_{\mathbb{L}}}, \Omega_{\delta_{\mathbb{L}}}, \Lambda_{\delta_{\mathbb{L}}} \rangle_{m\times n}$ is a Fermatean neutrosophic decision matrix, where $\Psi_{\delta_{\mathbb{L}}}$ is the degree of membership of alternatives ψ_ϵ, $\Omega_{\delta_{\mathbb{L}}}$ is the degree of neutral membership of alternatives ψ_ϵ, and $\Lambda_{\delta_{\mathbb{L}}}$ is the degree non-membership of alternatives ψ_ϵ, each alternatives Ψ_ϵ satisfy $0 \preceq \Psi^3_{\delta_{\mathbb{L}}} + \Omega^3_{\delta_{\mathbb{L}}} \preceq 1$ and $0 \preceq \Lambda^3_{\delta_{\mathbb{L}}} \preceq 1$. Therefore, $0 \preceq \Psi^3_{\delta_{\mathbb{L}}} + \Omega^3_{\delta_{\mathbb{L}}} + \Lambda^3_{\delta_{\mathbb{L}}} \preceq 2$.

We propose the following algorithm to solve MCDM problem with spherical Fermatean neutrosophic information using spherical Fermatean neutrosophic Bonferroni mean operator.

Algorithm 1 Multi-Criteria Decision Making (MCDM) Process

1: **Start.**
2: **Input:** To select the best alternative.
3: We employ the decision information given in matrix $(\delta_{\mathbb{L}\varepsilon})_{m\times n}$.
4: For each alternative ψ_ϵ $(\epsilon = 1, 2, \ldots, n)$, construct the spherical Fermatean neutrosophic set $\langle \Psi_\epsilon, \Omega_\epsilon, \Lambda_\epsilon; \Delta_\epsilon \rangle$ where $\langle \Psi_\epsilon, \Omega_\epsilon, \Lambda_\epsilon \rangle$ is the center and Δ_ϵ is the radius of the spherical Fermatean neutrosophic set from the decision matrix $(\delta_{\mathbb{L}\varepsilon})_{m\times n}$.
5: Operate spherical Fermatean neutrosophic Bonferroni mean operator $B\{\mathbb{L}_\imath; \imath = 1, 2, \ldots n\}$ to obtain the overall preference values ψ_ϵ $(\epsilon = 1, 2, \ldots na)$.
6: Calculate the cosine similarity measure $\cos(\Psi_\epsilon, \mathbb{I})$ $(\epsilon = 1, 2, \ldots n)$, where $\mathbb{I} = (1, 0, 0; 1)$ is the positive ideal sphere.
7: The shortest measure value of $\cos(\psi_\epsilon, \mathbb{I})$ $(\epsilon = 1, 2, \ldots n)$, is the better alternative ψ_ϵ, because it is close to the positive ideal alternative $\mathbb{I}$.
8: Rank the alternatives ψ_ϵ $(\epsilon = 1, 2, \ldots n)$ based on the spherical Fermatean neutrosophic Bonferroni mean operator $B\{\mathbb{L}_\imath; \imath = 1, 2, \ldots n\}$ evaluations and cosine similarity measure $\cos(\psi_\epsilon, \mathbb{I})$ $(\epsilon = 1, 2, \ldots n)$.
9: **Output:** Best alternative.
10: **End.**

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4 Selecting the Optimal Cloud Service Provider for an Organization

An organization seeks to identify the most suitable cloud service provider ($\mathbb{CSP}$) from a pool of six candidates ($\mathbb{CSP}$-1, $\mathbb{CSP}$-2, $\mathbb{CSP}$-3, $\mathbb{CSP}$-4, $\mathbb{CSP}$-5, $\mathbb{CSP}$-6) to enhance its operational efficiency and security while managing costs effectively. The decision-making process involves evaluating each provider based on four critical criteria: Performance, Cost, Security and Support. Performance, Security and Support are beneficial criteria, while Cost is a non-beneficial criterion.

A multidisciplinary team comprising an IT Manager (DM1), Finance Manager (DM2) and Security Officer (DM3) provides evaluations based on their expertise. The IT Manager assesses performance, the Finance Manager evaluates cost and the Security Officer examines security. The team assigns weights to each criterion: Performance (0.35), Cost (0.25), Security (0.20) and Support (0.20). The evaluations are normalized to a spherical format for consistency.

The team aggregates these evaluations using the Spherical Fermatean Neutrosophic Bonferroni Mean operator to combine inputs effectively. The cosine distance is computed for each alternative to determine proximity to the ideal solution. The CSP with the shortest distance to the ideal alternative is considered the optimal choice. This structured approach ensures a comprehensive and balanced evaluation, enabling the organization to make an informed decision that aligns with its strategic objectives.

Process Overview:

- **Define the Alternatives and Criteria:** The team identifies six cloud service providers ($\mathbb{CSP}-1$, $\mathbb{CSP}-2$, $\mathbb{CSP}-3$, $\mathbb{CSP}-4$, $\mathbb{CSP}-5$, $\mathbb{CSP}-6$) and four criteria (Performance, Cost, Security, Support). Performance, Security and Support are beneficiary criteria, while Cost is non-beneficiary criteria.
- **Gather Evaluations from Decision Makers:** Each decision maker (DM1, DM2, DM3) assesses each provider against the criteria. For instance: DM1 (IT Manager) evaluates based on performance. DM2 (Finance Manager) evaluates based on cost. DM3 (Security Officer) evaluates based on security.
- **Normalize Evaluations:** The scores from each decision maker are converted to a spherical format to ensure consistency.
- **Weighting of Criteria:** The team assigns weights to each criterion based on their importance: Performance (0.35), Cost (0.25), Security (0.20) and Support (0.20).
- **Aggregation of Evaluations using Spherical Fermatean Neutrosophic Bonferroni Mean:** The normalized evaluations from all decision makers are aggregated using the Spherical Fermatean Neutrosophic Bonferroni Mean operator to combine their inputs effectively.
- **Compute the Cosine Distance for Each Alternative:** The aggregated scores for each criterion are used to calculate the cosine distance:

  $$\cos(\delta_1, \delta_2) = 1 - \frac{\Psi_{\delta_1} \cdot \Psi_{\delta_2} + \Omega_{\delta_1} \cdot \Omega_{\delta_2} + \Lambda_{\delta_1} \cdot \Lambda_{\delta_2}}{\|\Psi_{\delta_1}\| \cdot \|\Psi_{\delta_2}\| + \|\Omega_{\delta_1}\| \cdot \|\Omega_{\delta_2}\| + \|\Lambda_{\delta_1}\| \cdot \|\Lambda_{\delta_2}\|} \times \frac{|\Delta_{\delta_1} - \Delta_{\delta_2}|}{\max(\Delta_{\delta_1}, \Delta_{\delta_2})}$$

  where $\delta_2 = \mathbb{I} = (1, 0, 0; 1)$ is the ideal sphere.
- **Rank the Alternatives:** The providers are ranked based on their overall scores. The shortest distance value of $\cos(\delta_1, \mathbb{I})$ is the better alternative Ψ_ϵ, because it is close to the ideal alternative $\mathbb{I}$.

Alternatives:

1. $\mathbb{CSP} - 1$**:** Known for high performance but comes with higher costs.
2. $\mathbb{CSP} - 2$**:** Offers balanced performance and cost but with moderate security.
3. $\mathbb{CSP} - 3$**:** High security features but at a premium cost.

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4. $\mathbb{CSP}-4$**:** Cost-effective but with lower performance and support.
5. $\mathbb{CSP}-5$**:** Excellent support services but higher in cost.
6. $\mathbb{CSP}-6$**:** Moderate performance, cost and security, with balanced features.

Criteria:

1. **Performance:** Assesses the cloud service provider's speed, reliability and overall efficiency. Critical for ensuring the smooth operation of services.
2. **Cost:** Includes initial setup costs, subscription fees and long-term financial commitments. Essential for managing the organization's budget.
3. **Security:** Evaluates the provider's security measures, including data protection, encryption and compliance with regulations. Significant for safeguarding sensitive information.
4. **Support:** Considers the quality and availability of customer support services. Important for addressing issues and ensuring continuous service.

Linguistic Term	Notation	(Ψ)	(Ω)	(Λ)
Extremely good	$\mathbb{L}_1$	0.95	0.05	0.3
Very good	$\mathbb{L}_2$	0.8	0.1	0.4
Good	$\mathbb{L}_3$	0.85	0.2	0.3
Medium	$\mathbb{L}_4$	0.7	0.6	0.3
Bad	$\mathbb{L}_5$	0.5	0.4	0.3
Very bad	$\mathbb{L}_6$	0.4	0.3	0.6
Extremely bad	$\mathbb{L}_7$	0.2	0.1	0.5

Table 1: SFNS Values for Linguistic Terms

DM's	Alternatives	Performance	Security	Support	Cost
ITM	$\mathbb{CSP}-1$	$\mathbb{L}_1$	$\mathbb{L}_6$	$\mathbb{L}_3$	$\mathbb{L}_4$
	$\mathbb{CSP}-2$	$\mathbb{L}_3$	$\mathbb{L}_4$	$\mathbb{L}_3$	$\mathbb{L}_6$
	$\mathbb{CSP}-3$	$\mathbb{L}_6$	$\mathbb{L}_3$	$\mathbb{L}_4$	$\mathbb{L}_4$
	$\mathbb{CSP}-4$	$\mathbb{L}_4$	$\mathbb{L}_3$	$\mathbb{L}_6$	$\mathbb{L}_3$
	$\mathbb{CSP}-5$	$\mathbb{L}_2$	$\mathbb{L}_1$	$\mathbb{L}_4$	$\mathbb{L}_5$
	$\mathbb{CSP}-6$	$\mathbb{L}_7$	$\mathbb{L}_3$	$\mathbb{L}_6$	$\mathbb{L}_3$
FM	$\mathbb{CSP}-1$	$\mathbb{L}_3$	$\mathbb{L}_7$	$\mathbb{L}_4$	$\mathbb{L}_5$
	$\mathbb{CSP}-2$	$\mathbb{L}_4$	$\mathbb{L}_5$	$\mathbb{L}_1$	$\mathbb{L}_7$
	$\mathbb{CSP}-3$	$\mathbb{L}_7$	$\mathbb{L}_4$	$\mathbb{L}_5$	$\mathbb{L}_5$
	$\mathbb{CSP}-4$	$\mathbb{L}_5$	$\mathbb{L}_1$	$\mathbb{L}_7$	$\mathbb{L}_5$
	$\mathbb{CSP}-5$	$\mathbb{L}_4$	$\mathbb{L}_3$	$\mathbb{L}_5$	$\mathbb{L}_7$
	$\mathbb{CSP}-6$	$\mathbb{L}_5$	$\mathbb{L}_4$	$\mathbb{L}_7$	$\mathbb{L}_4$
SO	$\mathbb{CSP}-1$	$\mathbb{L}_2$	$\mathbb{L}_5$	$\mathbb{L}_3$	$\mathbb{L}_3$
	$\mathbb{CSP}-2$	$\mathbb{L}_3$	$\mathbb{L}_3$	$\mathbb{L}_2$	$\mathbb{L}_5$
	$\mathbb{CSP}-3$	$\mathbb{L}_5$	$\mathbb{L}_3$	$\mathbb{L}_3$	$\mathbb{L}_3$
	$\mathbb{CSP}-4$	$\mathbb{L}_3$	$\mathbb{L}_2$	$\mathbb{L}_5$	$\mathbb{L}_4$
	$\mathbb{CSP}-5$	$\mathbb{L}_4$	$\mathbb{L}_2$	$\mathbb{L}_3$	$\mathbb{L}_6$
	$\mathbb{CSP}-6$	$\mathbb{L}_6$	$\mathbb{L}_3$	$\mathbb{L}_5$	$\mathbb{L}_3$

Table 2: Evaluations of Cloud Service Providers (CSPs) by Different Decision Makers (DMs)in linguistic terms

Step 1: Evaluate the 6 possible cloud service providers (CSPs) under 4 criteria (Performance, Cost, Security, Support). Table 2.1 presents the Fermatean Neutrosophic (FN) linguistic evaluations for these parameters across various qualitative levels, as shown in Table 2. The corresponding linguistic values are replaced in Table 3. The values are normalized as per beneficiary and non-beneficiary criteria in Table 4.

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Step 2: Using the decision matrix in Table 4, we calculate the center and radius using Equations (1) and (2). We then frame the spherical Fermatean neutrosophic set, as shown in Table 5.

Step 3: Calculate the spherical weighted normalized Fermatean neutrosophic Bonferroni mean values. The normalized values shown in Table 6.

Step 4: In Table 7, the aggregated scores for each criterion are calculated the cosine distance $\cos(\delta_1, \delta_2) =$ where $\delta_2 = \mathbb{I} = (1, 0, 0; 1)$ is the ideal sphere.

Step 5 : In Table 8, the cloud service providers are ranked based on their overall scores. The shortest distance value is the better alternative because it is close to the ideal alternative $\mathbb{I}$.

DM's	Alternatives	Performance	Security	Support	Cost
ITM	$\mathbb{CSP}-1$	$(0.95, 0.05, 0.3)$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.6, 0.5, 0.4)$
	$\mathbb{CSP}-3$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.7, 0.6, 0.3)$
	$\mathbb{CSP}-4$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$
	$\mathbb{CSP}-5$	$(0.8, 0.1, 0.4)$	$(0.95, 0.05, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$
	$\mathbb{CSP}-6$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$
FM	$\mathbb{CSP}-1$	$(0.7, 0.6, 0.3)$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.95, 0.05, 0.3)$	$(0.2, 0.1, 0.5)$
	$\mathbb{CSP}-3$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.4, 0.3, 0.6)$
	$\mathbb{CSP}-4$	$(0.4, 0.3, 0.6)$	$(0.95, 0.05, 0.3)$	$(0.2, 0.1, 0.5)$	$(0.4, 0.3, 0.6)$
	$\mathbb{CSP}-5$	$(0.7, 0.6, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.2, 0.1, 0.5)$
	$\mathbb{CSP}-6$	$(0.4, 0.3, 0.6)$	$(0.7, 0.3, 0.5)$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$
SO	$\mathbb{CSP}-1$	$(0.8, 0.1, 0.4)$	$(0.4, 0.3, 0.6)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.8, 0.1, 0.4)$	$(0.4, 0.3, 0.6)$
	$\mathbb{CSP}-3$	$(0.4, 0.3, 0.6)$	$(0.7, 0.3, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.6, 0.3)$
	$\mathbb{CSP}-4$	$(0.7, 0.6, 0.3)$	$(0.8, 0.1, 0.4)$	$(0.4, 0.3, 0.6)$	$(0.7, 0.6, 0.3)$
	$\mathbb{CSP}-5$	$(0.7, 0.6, 0.3)$	$(0.8, 0.1, 0.4)$	$(0.7, 0.3, 0.5)$	$(0.6, 0.5, 0.4)$
	$\mathbb{CSP}-6$	$(0.6, 0.5, 0.4)$	$(0.7, 0.3, 0.5)$	$(0.4, 0.3, 0.6)$	$(0.7, 0.3, 0.5)$

Table 3: Fermatean neutrosophic values of Cloud Service Providers (CSPs) by Different Decision Makers (DMs)

DM's	Alternatives	Performance	Security	Support	Cost
ITM	$\mathbb{CSP}-1$	$(0.95, 0.05, 0.3)$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.3, 0.7, 0.5)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.5, 0.6, 0.4)$
	$\mathbb{CSP}-3$	$(0.6, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.6, 0.7, 0.3)$
	$\mathbb{CSP}-4$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.6, 0.5, 0.4)$	$(0.6, 0.7, 0.3)$
	$\mathbb{CSP}-5$	$(0.8, 0.1, 0.4)$	$(0.95, 0.05, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.3, 0.4, 0.6)$
	$\mathbb{CSP}-6$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.6, 0.5, 0.4)$	$(0.6, 0.7, 0.3)$
FM	$\mathbb{CSP}-1$	$(0.7, 0.6, 0.3)$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.3, 0.4, 0.6)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.95, 0.05, 0.3)$	$(0.1, 0.2, 0.5)$
	$\mathbb{CSP}-3$	$(0.2, 0.1, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.3, 0.4, 0.6)$
	$\mathbb{CSP}-4$	$(0.4, 0.3, 0.6)$	$(0.95, 0.05, 0.3)$	$(0.2, 0.1, 0.5)$	$(0.3, 0.4, 0.6)$
	$\mathbb{CSP}-5$	$(0.7, 0.6, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.3, 0.6)$	$(0.1, 0.2, 0.5)$
	$\mathbb{CSP}-6$	$(0.4, 0.3, 0.6)$	$(0.7, 0.3, 0.5)$	$(0.2, 0.1, 0.5)$	$(0.6, 0.7, 0.3)$
SO	$\mathbb{CSP}-1$	$(0.8, 0.1, 0.4)$	$(0.4, 0.3, 0.6)$	$(0.7, 0.6, 0.3)$	$(0.3, 0.7, 0.5)$
	$\mathbb{CSP}-2$	$(0.7, 0.6, 0.3)$	$(0.7, 0.3, 0.5)$	$(0.8, 0.1, 0.4)$	$(0.3, 0.4, 0.6)$
	$\mathbb{CSP}-3$	$(0.4, 0.3, 0.6)$	$(0.7, 0.3, 0.5)$	$(0.7, 0.6, 0.3)$	$(0.6, 0.7, 0.3)$
	$\mathbb{CSP}-4$	$(0.7, 0.6, 0.3)$	$(0.8, 0.1, 0.4)$	$(0.4, 0.3, 0.6)$	$(0.6, 0.7, 0.3)$
	$\mathbb{CSP}-5$	$(0.7, 0.6, 0.3)$	$(0.8, 0.1, 0.4)$	$(0.7, 0.3, 0.5)$	$(0.5, 0.6, 0.4)$
	$\mathbb{CSP}-6$	$(0.6, 0.5, 0.4)$	$(0.7, 0.3, 0.5)$	$(0.4, 0.3, 0.6)$	$(0.3, 0.7, 0.5)$

Table 4: Normalized Fermatean neutrosophic values of Cloud Service Providers(CSPs)

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Alternatives	Performance	Security
$\mathbb{CSP}-1$	(0.82, 0.25, 0.33; 0.37)	(0.40, 0.30, 0.50; 0.30)
$\mathbb{CSP}-2$	(0.70, 0.60, 0.30; 0.00)	(0.60, 0.30, 0.53; 0.21)
$\mathbb{CSP}-3$	(0.40, 0.30, 0.50; 0.30)	(0.70, 0.50, 0.37; 0.24)
$\mathbb{CSP}-4$	(0.60, 0.50, 0.40; 0.35)	(0.82, 0.15, 0.40; 0.21)
$\mathbb{CSP}-5$	(0.73, 0.43, 0.33; 0.35)	(0.82, 0.25, 0.33; 0.37)
$\mathbb{CSP}-6$	(0.40, 0.30, 0.50; 0.30)	(0.70, 0.40, 0.43; 0.24)
Alternatives	Support	Cost
$\mathbb{CSP}-1$	(0.70, 0.60, 0.30; 0.00)	(0.30, 0.60, 0.53; 0.21)
$\mathbb{CSP}-2$	(0.82, 0.25, 0.33; 0.37)	(0.30, 0.40, 0.50; 0.30)
$\mathbb{CSP}-3$	(0.60, 0.40, 0.47; 0.28)	(0.50, 0.60, 0.40; 0.35)
$\mathbb{CSP}-4$	(0.40, 0.30, 0.50; 0.30)	(0.50, 0.60, 0.40; 0.35)
$\mathbb{CSP}-5$	(0.60, 0.40, 0.47; 0.28)	(0.30, 0.40, 0.50; 0.30)
$\mathbb{CSP}-6$	(0.40, 0.30, 0.50; 0.30)	(0.50, 0.70, 0.37; 0.24)

Table 5: Spherical Fermatean neutrosophic set representation across various $\mathbb{CSP}$ instances.

Method	$\mathbf{B_{(P=1,Q=2)}}$	$\mathbf{B_{(P=3,Q=2)}}$
$\mathbb{CSP}-1$	(0.2565, 0.6099, 0.5129; 0.0199)	(0.2365, 0.2753, 0.2172; 0.1069)
$\mathbb{CSP}-2$	(0.3442, 0.4422, 0.5129; 0.0254)	(0.2229, 0.1755, 0.2126; 0.1050)
$\mathbb{CSP}-3$	(0.1995, 0.5837, 0.5036; 0.0318)	(0.1234, 0.2766, 0.2428; 0.1077)
$\mathbb{CSP}-4$	(0.2458, 0.5079, 0.5015; 0.0350)	(0.1584, 0.2017, 0.2215; 0.1416)
$\mathbb{CSP}-5$	(0.3120, 0.4333, 0.5130; 0.0364)	(0.2529, 0.1435, 0.1938; 0.1133)
$\mathbb{CSP}-6$	(0.1514, 0.5537, 0.5189; 0.0221)	(0.0846, 0.2392, 0.2816; 0.0877)
Method	$\mathbf{B_{(P=2,Q=2)}}$	$\mathbf{B_{(P=2,Q=1)}}$
$\mathbb{CSP}-1$	(0.1839, 0.7090, 0.6497; 0.0066)	(0.3102, 0.5207, 0.4872; 0.0269)
$\mathbb{CSP}-2$	(0.2080, 0.6047, 0.6476; 0.0074)	(0.3021, 0.4629, 0.4831; 0.0267)
$\mathbb{CSP}-3$	(0.1146, 0.6997, 0.6570; 0.0082)	(0.2251, 0.5275, 0.5160; 0.0274)
$\mathbb{CSP}-4$	(0.1441, 0.6426, 0.6462; 0.0102)	(0.2718, 0.4726, 0.4952; 0.0343)
$\mathbb{CSP}-5$	(0.2117, 0.5799, 0.6371; 0.0116)	(0.3413, 0.4221, 0.4627; 0.0408)
$\mathbb{CSP}-6$	(0.0800, 0.6741, 0.6786; 0.0057)	(0.1809, 0.5037, 0.5478; 0.0235)

Table 6: Comparison of CSPs based on different methods

Method	COS($\mathbb{CSP}$-1, $\mathbb{I}$)	COS($\mathbb{CSP}$-2, $\mathbb{I}$)	COS($\mathbb{CSP}$-3, $\mathbb{I}$)
$B_{(P=1,Q=2)}$	0.6997	0.5584	0.7575
$B_{(P=3,Q=2)}$	0.5006	0.4373	0.7164
$B_{(P=2,Q=2)}$	0.8134	0.7732	0.8824
$B_{(P=2,Q=1)}$	0.6118	0.5995	0.7162
Method	COS($\mathbb{CSP}$-4, $\mathbb{I}$)	COS($\mathbb{CSP}$-5, $\mathbb{I}$)	COS($\mathbb{CSP}$-6, $\mathbb{I}$)
$B_{(P=1,Q=2)}$	0.6858	0.5940	0.8087
$B_{(P=3,Q=2)}$	0.5987	0.3583	0.7963
$B_{(P=2,Q=2)}$	0.8454	0.7642	0.9171
$B_{(P=2,Q=1)}$	0.6437	0.5411	0.7693

Table 7: Cosine distance values for different $B_{(P,Q)}$ configurations across various $\mathbb{CSP}$ instances.

Method	Ranking	Best Service Provider
$B_{(P=1,Q=2)}$	$\mathbb{CSP}-2 < \mathbb{CSP}-5 < \mathbb{CSP}-4 < \mathbb{CSP}-1 < \mathbb{CSP}-3 < \mathbb{CSP}-6$	$\mathbb{CSP}-2$
$B_{(P=3,Q=2)}$	$\mathbb{CSP}-5 < \mathbb{CSP}-2 < \mathbb{CSP}-1 < \mathbb{CSP}-4 < \mathbb{CSP}-3 < \mathbb{CSP}-6$	$\mathbb{CSP}-5$
$B_{(P=2,Q=2)}$	$\mathbb{CSP}-5 < \mathbb{CSP}-2 < \mathbb{CSP}-1 < \mathbb{CSP}-4 < \mathbb{CSP}-3 < \mathbb{CSP}-6$	$\mathbb{CSP}-5$
$B_{(P=2,Q=1)}$	$\mathbb{CSP}-5 < \mathbb{CSP}-2 < \mathbb{CSP}-1 < \mathbb{CSP}-4 < \mathbb{CSP}-3 < \mathbb{CSP}-6$	$\mathbb{CSP}-5$

Table 8: Ranking and Best Service Provider for different B configurations.

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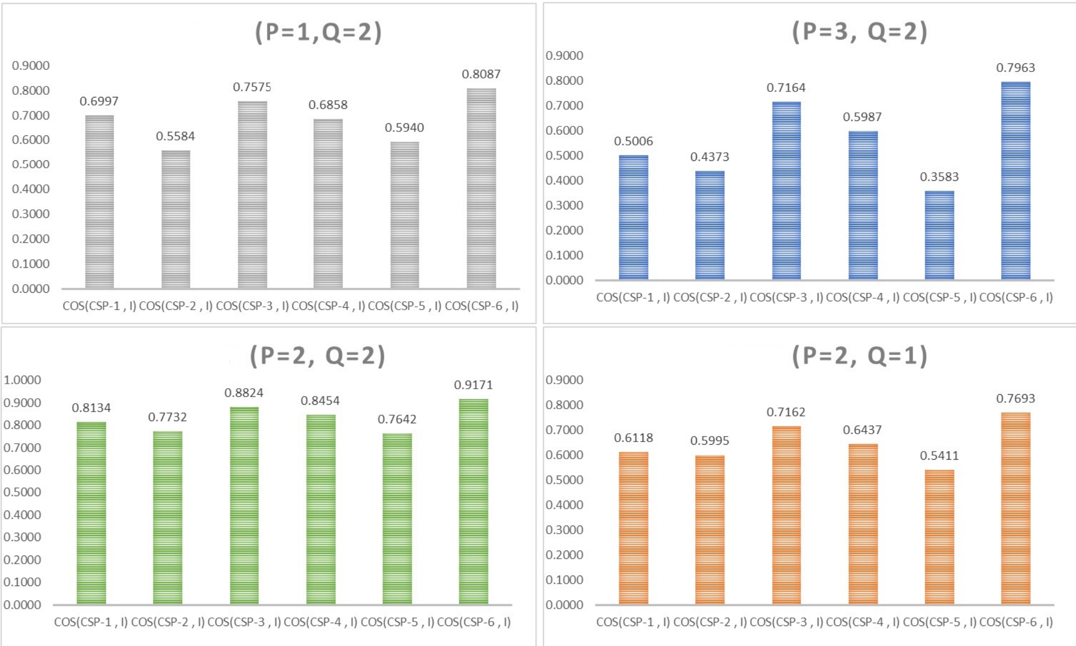

Figure 1: Comparison of $B(P, Q)$ for different cloud service provider

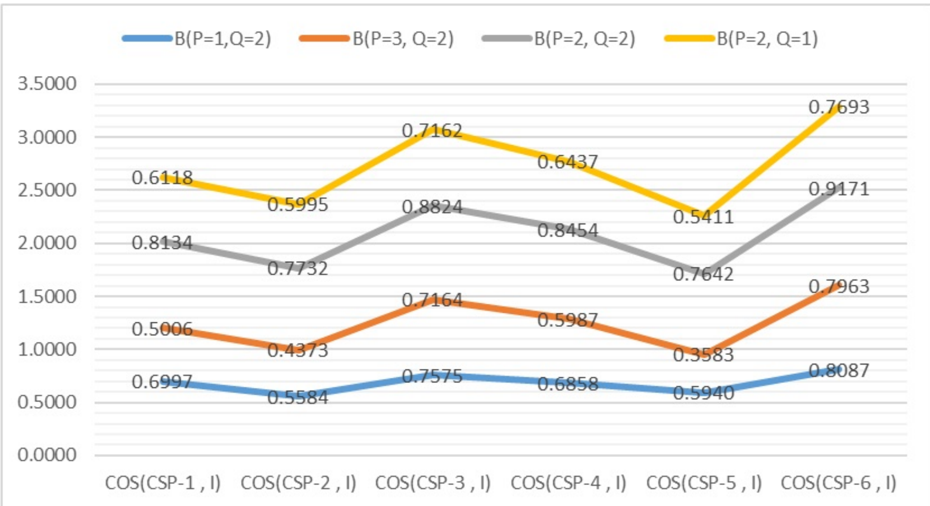

Figure 2: Comparison of different cloud service provider rank based on their overall score

5 Conclusion

The proposed SFNS-based MCDM approach provides a robust framework for evaluating and selecting the best alternative among multiple options. By integrating the evaluations of multiple decision makers and accounting for degrees of membership, neutral membership and non-membership, this approach offers a comprehensive and accurate decision-making process.

In the case study of selecting the best cloud service provider, the approach identified the provider that best met the organization's performance, financial and security requirements. This demonstrates the practical applicability and effectiveness of the SFNS-based MCDM method in real-world decision-making scenarios.

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