



Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, α -Cut and Applications

Muhammad Saqlain¹, Ali Hamza² and Sara Farooq²

¹ Department of Mathematics, Lahore Garrison University, Lahore, Pakistan. E-mail: msaqlain@lgu.edu.pk

² Department of Mathematics, Lahore Garrison University, Lahore, Pakistan. E-mail: alifm2909@gmail.com

² Department of Mathematics, Lahore Garrison University, Lahore, Pakistan. E-mail: sarafarooq447@gmail.com

* Correspondence: msaqlain@lgu.edu.pk

Abstract

In this paper, the primarily focus is to extend the concept of Octagonal Neutrosophic Numbers (ONN) since these numbers provide a wide range of applications while dealing with more fluctuations in the linguistic environment. Firstly, mathematical notions and definitions of Linear, Symmetric and Asymmetric types are proposed. Secondly, α – Cut is defined. Finally, a case study is done by using the TOPSIS technique of MCDM.

Keywords: Accuracy Function, Neutrosophic Numbers, Octagonal Neutrosophic Numbers (ONN), MCDM, TOPSIS.

1.Introduction

Researchers and mathematicians all over the world developed important analytical skills and problem-solving strategies to assess a broad range of issues in human resource, medicine, selection problems etc. But the most challenging issues were related to the problems which were more qualitative rather than quantitative in nature.

Thus, the need to handle uncertain situations and vagueness in practical as well as theoretical problems led the researchers to the development of theories like fuzzy, neutrosophic set theory. The neutrosophic sets (NSs) [1] reflect on the truth membership, indeterminacy membership, and falsity membership concurrently, which is more practical and adequate than FSs and IFs in selection problems, that are uncertain, incomplete, and inconsistent.

The idea of triangular, trapezoidal and pentagonal neutrosophic numbers having membership function which are dependent and independent was given by [2-4]. Single-valued neutrosophic sets are an extension of NSs which were introduced by Wang et al. [8]. Ye [9] introduced, simplified neutrosophic sets, and Peng et al. [8,9] define their novel operations and aggregation operators. Finally, there are different extensions of NSs, such as interval neutrosophic set [10], bipolar neutrosophic sets [11], and multi-valued neutrosophic sets [12].

Smarandache and many other researchers [13-20] also discussed the various extension of neutrosophic sets in TOPSIS and MCDM. Saqlain et.al. [21] proposed a new algorithm along with a new decision-making environment. Many other novel approaches were also used by many researches [22-42] in decision makings. Some Fundamental properties and applications of triangular and pentagonal neutrosophic numbers are proposed by [43-47]. With the concept of octagonal neutrosophic numbers, decision maker can deal with more fluctuations because they have more edges as compared to pentagonal numbers. In this current epoch, the neutrosophic numbers can be converted into fuzzy numbers and the ability to deal with fluctuations will be increased.

1.1 Motivation

Different researchers had already published a lot of articles on neutrosophic arena, as they applied and extended this concept in different fields such as MCDM. The conception on neutrosophic octagonal number is totally new. An important issue is that if someone wants to take Linear ONN, then what should its representation be? How should we define membership, indeterminacy and non-membership functions? From this point of view, ONN is a good choice for a decision maker in a practical scenario.

1.2 The Paper Presentation

In this paper, the concept of Octagonal Neutrosophic Numbers ONN is extended.

- Formulation of Linear, Non-Linear, Linear symmetric, Non-Linear symmetric Octagonal Neutrosophic Numbers.
- Defining the α – cut of each type.
- A case study of personal selection.

1.3 Structure of Paper

The article is structured as shown in the following Figure.

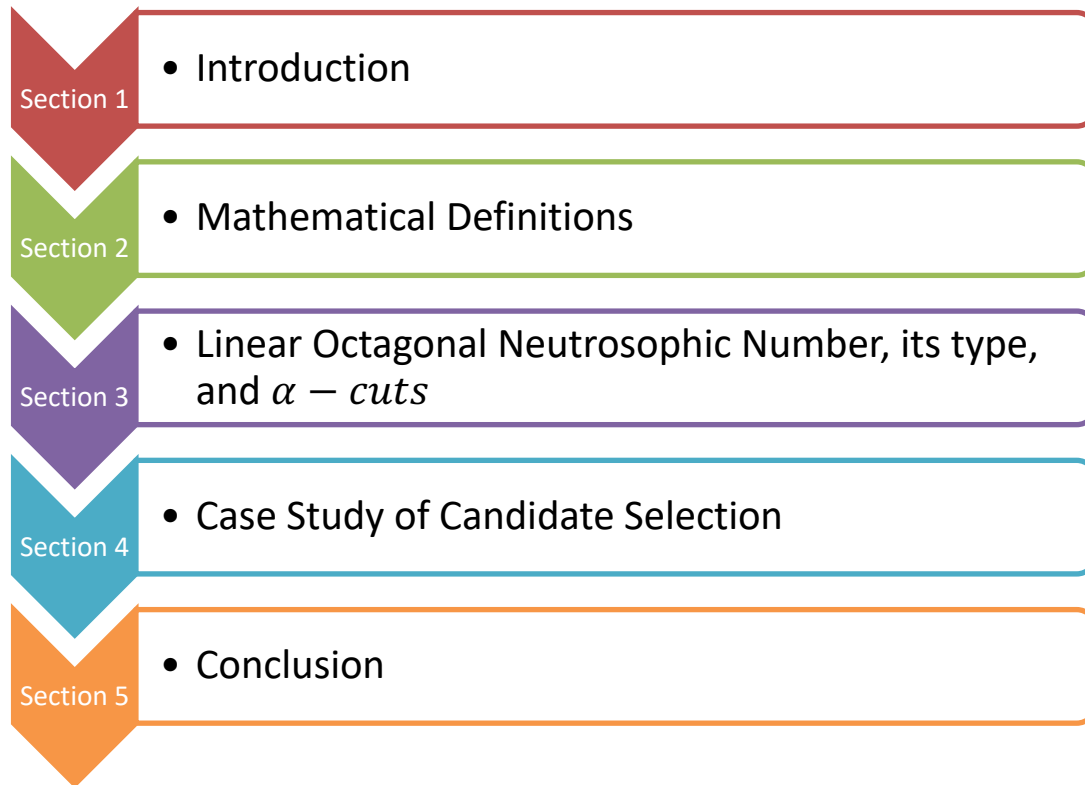


Figure 1: Pictorial view for the structure of the article

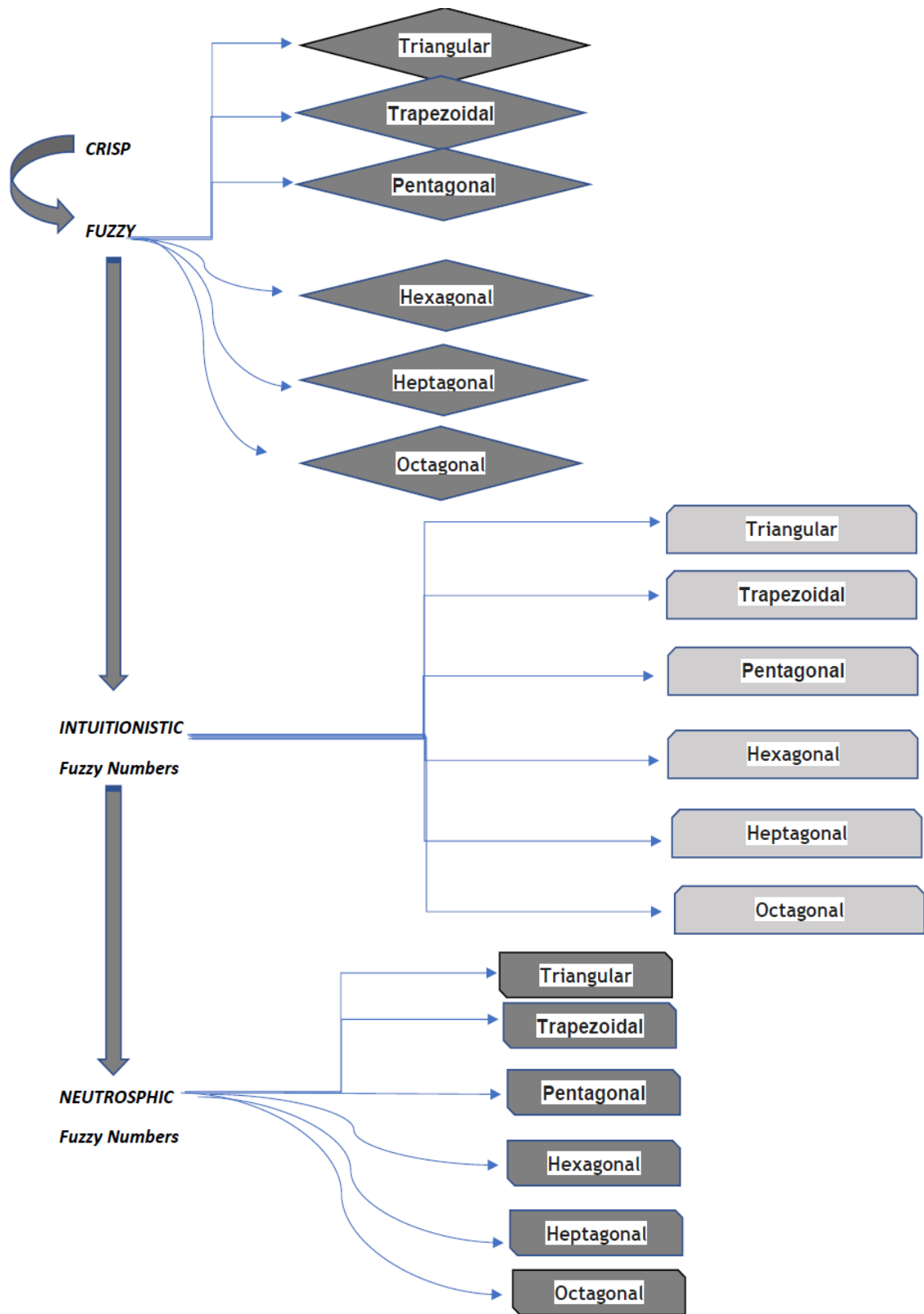


Figure 2. Flow chart of the three types, fuzzy, Intuitionistic fuzzy, and neutrosophic logic numbers

2. Mathematical Definitions

In this section, we present necessary definitions that are used throughout the paper.

Definition 2.1: Neutrosophic Set [1]: A set \widehat{nA} is neutrosophic if $\widehat{nA} = \{x; [T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x)]: x \in X\}$, where $T_{\widehat{nA}}(x): \rightarrow [0,1]$ be a truth membership function $I_{\widehat{nA}}(x)$ be a indeterminacy membership function and $F_{\widehat{nA}}(x)$ is falsity membership function $T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x)$ exhibits the following relation:

$$0^- \leq T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x) \leq 3^+$$

Definition 2.2: Triangular Neutrosophic Number [2]: Triangular single value neutrosophic number is given as $\mathcal{A}'_{\widehat{Neu}} = (p_1, p_2, p_3; r_1, r_2, r_3)$ whose truth, indeterminacy and falsity membership is given as:

$$T_{\mathcal{A}'_{\widehat{Neu}}}(x) = \begin{cases} \frac{x-p_1}{p_1-p_2} & \text{for } p_1 \leq x < p_2 \\ 1 & \text{when } x = p_2 \\ \frac{p_3-x}{p_3-p_2} & \text{for } p_2 < x \leq p_3 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\mathcal{A}'_{\widehat{Neu}}}(x) = \begin{cases} \frac{q_2-x}{q_2-q_1} & \text{for } q_1 \leq x < q_2 \\ 0 & \text{when } x = q_2 \\ \frac{x-q_2}{q_3-q_2} & \text{for } q_2 < x \leq q_3 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\mathcal{A}'_{\widehat{Neu}}}(x) = \begin{cases} \frac{x-p_1}{p_1-p_2} & \text{for } p_1 \leq x < p_2 \\ 1 & \text{when } x = p_2 \\ \frac{p_3-x}{p_3-p_2} & \text{for } p_2 < x \leq p_3 \\ 0 & \text{otherwise} \end{cases}$$

Where $0 \leq T_{\mathcal{A}'_{\widehat{Neu}}}(x) + I_{\mathcal{A}'_{\widehat{Neu}}}(x) + F_{\mathcal{A}'_{\widehat{Neu}}}(x) \leq 3, x \in \mathcal{A}'_{\widehat{Neu}}$.

And the parametric foam of this type is $(\mathcal{A}'_{\widehat{Neu}})_{\alpha, \beta, \gamma} = [T_{\widehat{Neu}1}(\alpha), T_{\widehat{Neu}2}(\alpha); J_{\widehat{Neu}1}(\beta), J_{\widehat{Neu}2}(\beta); F_{\widehat{Neu}1}(\gamma), F_{\widehat{Neu}2}(\gamma)]$, where, $T_{\widehat{Neu}1}(\alpha) = p_1 + \alpha(p_2 - p_1)$, $T_{\widehat{Neu}2}(\alpha) = p_3 - \alpha(p_3 - p_2)$, $J_{\widehat{Neu}1}(\beta) = q_2 - \beta(q_2 - q_1)$, $J_{\widehat{Neu}2}(\beta) = q_2 + \beta(q_3 - q_2)$, $F_{\widehat{Neu}1}(\gamma) = r_2 - \gamma(r_2 - r_1)$, $F_{\widehat{Neu}2}(\gamma) = r_2 + \gamma(r_3 - r_2)$, Here $0 < \alpha \leq 1, 0 < \beta \leq 1, 0 < \gamma \leq 1$ and $0 < \alpha + \beta + \gamma \leq 3$

Definition 2.3: Trapezoidal Neutrosophic Number [3]: Let \mathcal{X} be the universe of discourse, a trapezoidal neutrosophic set $\widehat{\mathcal{A}}$ in \mathcal{X} is defined by: $\widehat{\mathcal{N}} = \{(\mathcal{X}, \mathcal{T}_{\widehat{\mathcal{N}}}(\mathcal{X}), \mathcal{J}_{\widehat{\mathcal{N}}}(\mathcal{X}), \mathcal{F}_{\widehat{\mathcal{N}}}(\mathcal{X})) | x \in \mathcal{X}\}$, where $\mathcal{T}_{\widehat{\mathcal{N}}}(\mathcal{X}) \subset [0,1], \mathcal{J}_{\widehat{\mathcal{N}}}(\mathcal{X}) \subset [0,1], \mathcal{F}_{\widehat{\mathcal{N}}}(\mathcal{X}) \subset [0,1]$ are consider as three trapezoidal number, $\mathcal{T}_{\widehat{\mathcal{N}}}(\mathcal{X}) = (\mathcal{t}_{\widehat{\mathcal{N}}}^1(x), \mathcal{t}_{\widehat{\mathcal{N}}}^2(x), \mathcal{t}_{\widehat{\mathcal{N}}}^3(x), \mathcal{t}_{\widehat{\mathcal{N}}}^4(x)): \mathcal{X} \mapsto [0,1]$, $\mathcal{J}_{\widehat{\mathcal{N}}}(\mathcal{X}) = (\mathcal{j}_{\widehat{\mathcal{N}}}^1(x), \mathcal{j}_{\widehat{\mathcal{N}}}^2(x), \mathcal{j}_{\widehat{\mathcal{N}}}^3(x), \mathcal{j}_{\widehat{\mathcal{N}}}^4(x)): \mathcal{X} \mapsto [0,1]$, $\mathcal{F}_{\widehat{\mathcal{N}}}(\mathcal{X}) = (\mathcal{f}_{\widehat{\mathcal{N}}}^1(x), \mathcal{f}_{\widehat{\mathcal{N}}}^2(x), \mathcal{f}_{\widehat{\mathcal{N}}}^3(x), \mathcal{f}_{\widehat{\mathcal{N}}}^4(x)): \mathcal{X} \mapsto [0,1]$ with the condition $0 \leq \mathcal{t}_{\widehat{\mathcal{N}}}^4(x) + \mathcal{j}_{\widehat{\mathcal{N}}}^4(x) + \mathcal{f}_{\widehat{\mathcal{N}}}^4(x) \leq 3, x \in \mathcal{X}$.

Definition 2.4: Pentagonal Neutrosophic Number [4]: Pentagonal neutrosophic number ($\widehat{\mathcal{S}}$) for single valued is defined as $\widehat{\mathcal{S}} = ([m^1, n^1, o^1, p^1, q^1; \pi], [m^2, n^2, o^2, p^2, q^2; \rho], [m^3, n^3, o^3, p^3, q^3; \sigma])$ where $\pi, \rho, \sigma \in [0,1]$. The truth membership function $(\mathcal{T}_{\widehat{\mathcal{S}}}) : \mathbb{R} \mapsto [0, \mathbf{t}]$, the indeterminacy membership function $(\mathcal{I}_{\widehat{\mathcal{S}}}) : \mathbb{R} \mapsto [\mathbf{p}, 1]$ and falsity membership function $(\mathcal{F}_{\widehat{\mathcal{S}}}) : \mathbb{R} \mapsto [6, 1]$ and given as:

$$J_{\hat{s}}(x) = \begin{cases} J_{\hat{s}t1}(x) & \dot{m}^1 \leq x < \dot{n}^1 \\ J_{\hat{s}t2}(x) & \dot{n}^1 \leq x < \dot{o}^1 \\ \mu & x = \dot{o}^1 \\ J_{\hat{s}t1}(x) & \dot{o}^1 \leq x < \dot{p}^1 \\ J_{\hat{s}t1}(x) & \dot{p}^1 \leq x < \dot{q}^1 \\ 0 & \text{otherwise} \end{cases} \quad J_{\hat{s}}(x) = \begin{cases} J_{\hat{s}t1}(x) & \ddot{m}^2 \leq x < \ddot{n}^2 \\ J_{\hat{s}t2}(x) & \ddot{n}^2 \leq x < \ddot{o}^2 \\ \theta & x = \ddot{o}^2 \\ J_{\hat{s}t1}(x) & \ddot{o}^2 \leq x < \ddot{p}^2 \\ J_{\hat{s}t1}(x) & \ddot{p}^2 \leq x < \ddot{q}^2 \\ 1 & \text{otherwise} \end{cases} \quad \varepsilon_{\hat{s}}(x) = \begin{cases} \varepsilon_{\hat{s}t1}(x) & \dot{m}^3 \leq x < \dot{n}^3 \\ \varepsilon_{\hat{s}t2}(x) & \dot{n}^3 \leq x < \dot{o}^3 \\ \theta & x = \dot{o}^3 \\ \varepsilon_{\hat{s}t1}(x) & \dot{o}^3 \leq x < \dot{p}^3 \\ \varepsilon_{\hat{s}t1}(x) & \dot{p}^3 \leq x < \dot{q}^3 \\ 1 & \text{otherwise} \end{cases}$$

Where $\langle [\dot{m}^1 < \dot{n}^1 < \dot{o}^1 < \dot{p}^1 < \dot{q}^1; \pi], [\ddot{m}^2 < \ddot{n}^2 < \ddot{o}^2 < \ddot{p}^2 < \ddot{q}^2; \rho], [\dot{m}^3 < \dot{n}^3 < \dot{o}^3 < \dot{p}^3 < \dot{q}^3; \delta] \rangle$

Definition 2.5: Octagonal Neutrosophic Number [ONN] A Neutrosophic Number denoted by \hat{S} is defined as,

$$\hat{S} = \langle [(\Omega, \eta, \varepsilon_v, \varepsilon, \kappa, \acute{o}, \acute{z}): \Theta], [(\Omega^1, \eta^1, \varepsilon^1, \vnu^1, \varepsilon^1, \kappa^1, \acute{o}^1, \acute{z}^1): \Psi], [(\Omega^2, \eta^2, \varepsilon^2, \vnu^2, \varepsilon^2, \kappa^2, \acute{o}^2, \acute{z}^2): \delta] \rangle$$

Where $\Theta, \Psi, \delta \in [0, 1]$.

The truth membership function $(\Theta_{\hat{s}}): \mathbb{R} \rightarrow [0, 1]$,

the indeterminacy membership function $(\Psi_{\hat{s}}): \mathbb{R} \rightarrow [\delta, 1]$,

and the falsity membership function $(Y_{\hat{s}}): \mathbb{R} \rightarrow [\delta, 1]$ are given as follows:

$$\Theta_{\hat{s}}(x) = \begin{cases} \Theta_{\hat{s}0}(x) & \Omega \leq x < \eta \\ \Theta_{\hat{s}1}(x) & \eta \leq x < \varepsilon \\ \Theta_{\hat{s}2}(x) & \varepsilon \leq x < \vnu \\ \Theta_{\hat{s}3}(x) & \vnu \leq x < \varepsilon \\ \delta & x = \varepsilon \\ \Theta_{\hat{s}3}(x) & \varepsilon \leq x < \kappa \\ \Theta_{\hat{s}2}(x) & \kappa \leq x < \acute{o} \\ \Theta_{\hat{s}1}(x) & \acute{o} \leq x < \acute{z} \\ 0 & \text{otherwise} \end{cases} \quad \Psi_{\hat{s}}(x) = \begin{cases} \Psi_{\hat{s}0}(x) & \Omega^1 \leq x < \eta^1 \\ \Psi_{\hat{s}1}(x) & \eta^1 \leq x < \varepsilon^1 \\ \Psi_{\hat{s}2}(x) & \varepsilon^1 \leq x < \vnu^1 \\ \Psi_{\hat{s}3}(x) & \vnu^1 \leq x < \varepsilon^1 \\ \delta & x = \varepsilon^1 \\ \Psi_{\hat{s}3}(x) & \varepsilon^1 \leq x < \kappa^1 \\ \Psi_{\hat{s}2}(x) & \kappa^1 \leq x < \acute{o}^1 \\ \Psi_{\hat{s}1}(x) & \acute{o}^1 \leq x < \acute{z}^1 \\ 1 & \text{otherwise} \end{cases} \quad Y_{\hat{s}}(x) = \begin{cases} Y_{\hat{s}0}(x) & \Omega^2 \leq x < \eta^2 \\ Y_{\hat{s}1}(x) & \eta^2 \leq x < \varepsilon^2 \\ Y_{\hat{s}2}(x) & \varepsilon^2 \leq x < \vnu^2 \\ Y_{\hat{s}3}(x) & \vnu^2 \leq x < \varepsilon^2 \\ \delta & x = \varepsilon^2 \\ Y_{\hat{s}3}(x) & \varepsilon^2 \leq x < \kappa^2 \\ Y_{\hat{s}2}(x) & \kappa^2 \leq x < \acute{o}^2 \\ Y_{\hat{s}1}(x) & \acute{o}^2 \leq x < \acute{z}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where $\hat{S} = \langle [(\Omega < \eta < \varepsilon < \vnu < \varepsilon < \kappa < \acute{o} < \acute{z}): \Theta], [(\Omega^1 < \eta^1 < \varepsilon^1 < \vnu^1 < \varepsilon^1 < \kappa^1 < \acute{o}^1 < \acute{z}^1): \Psi], [(\Omega^2 < \eta^2 < \varepsilon^2 < \vnu^2 < \varepsilon^2 < \kappa^2 < \acute{o}^2 < \acute{z}^2): \delta] \rangle$

3. The definition of [LONN], Representation and Examples had been Presented

In this section, we discuss its representations, and investigate its properties.

Definition 3.1: Linear Octagonal Neutrosophic Number [LONN]

Let $\hat{A} = (\Omega, \eta, \varepsilon, \vnu; \varepsilon, \kappa, \acute{o}, \acute{z})$ be a linear octagonal neutrosophic number. It should satisfy the following conditions:

1. $\mu_{\hat{A}}(x)$ is continuous function between the interval Ω to \acute{z} for truthiness.
2. $\mu_{\hat{A}}(x)$ is a non-increasing continuous function between the interval of Ω to ε for truthiness.
3. $\mu_{\hat{A}}(x)$ is a non-decreasing continuous function between the interval ε to \acute{z} for truthiness.
4. $\mu_{\hat{A}}(x)$ is continuous function between the interval Ω^1 to \acute{z}^1 for falsity.
5. $\mu_{\hat{A}}(x)$ is a non-decreasing continuous function between the interval of Ω^1 to ε^1 for falsity.
6. $\mu_{\hat{A}}(x)$ is a non-increasing continuous function between the interval of ε^1 to \acute{z}^1 for falsity.
7. $\mu_{\hat{A}}(x)$ is continuous function between the interval Ω^2 to \acute{z}^2 for indeterminacy.
8. $\mu_{\hat{A}}(x)$ is a non-increasing continuous function between the interval of Ω^2 to ε^2 for indeterminacy.
9. $\mu_{\hat{A}}(x)$ is a non-decreasing continuous function between the interval of ε^2 to \acute{z}^2 for indeterminacy.

3.2 Linear ONN with symmetry

Let $\mathcal{A}_{LS} = (\Omega, \eta, \varepsilon, \nu, \varepsilon, \mathbb{K}, \acute{o}, \mathfrak{z})$ be a linear ONN with the following membership functions.

$$\text{Truthiness} = T_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega \\ \mathcal{h} \left(\frac{x-\Omega}{\eta-\Omega} \right) & \Omega \leq x \leq \eta \\ \mathcal{h} & \eta \leq x \leq \varepsilon \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{x-\varepsilon}{\nu-\varepsilon} \right) & \varepsilon \leq x \leq \nu \\ 1 & \nu \leq x \leq \varepsilon \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{\mathbb{K}-x}{\mathbb{K}-\varepsilon} \right) & \varepsilon \leq x \leq \mathbb{K} \\ \mathcal{h} & \mathbb{K} \leq x \leq \acute{o} \\ \mathcal{h} \left(\frac{\mathfrak{z}-x}{\mathfrak{z}-\acute{o}} \right) & \acute{o} \leq x \leq \mathfrak{z} \\ 0 & x > \mathfrak{z} \end{cases}$$

With $0 < \mathcal{h} < 1$

$$\text{Falsity} = F_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega^1 \\ \mathcal{h} \left(\frac{x-\Omega^1}{\eta^1-\Omega^1} \right) & \Omega^1 \leq x \leq \eta^1 \\ \mathcal{h} & \eta^1 \leq x \leq \varepsilon^1 \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{x-\varepsilon^1}{\nu^1-\varepsilon^1} \right) & \varepsilon^1 \leq x \leq \nu^1 \\ 1 & \nu^1 \leq x \leq \varepsilon^1 \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{\mathbb{K}^1-x}{\mathbb{K}^1-\varepsilon^1} \right) & \varepsilon^1 \leq x \leq \mathbb{K}^1 \\ \mathcal{h} & \mathbb{K}^1 \leq x \leq \acute{o}^1 \\ \mathcal{h} \left(\frac{\mathfrak{z}^1-x}{\mathfrak{z}^1-\acute{o}^1} \right) & \acute{o}^1 \leq x \leq \mathfrak{z}^1 \\ 0 & x > \mathfrak{z}^1 \end{cases}$$

$$\text{Indeterminacy} = I_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega^2 \\ \mathcal{h} \left(\frac{x-\Omega^2}{\eta^2-\Omega^2} \right) & \Omega^2 \leq x \leq \eta^2 \\ \mathcal{h} & \eta^2 \leq x \leq \varepsilon^2 \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{x-\varepsilon^2}{\nu^2-\varepsilon^2} \right) & \varepsilon^2 \leq x \leq \nu^2 \\ 1 & \nu^2 \leq x \leq \varepsilon^2 \\ \mathcal{h} + (1 - \mathcal{h}) \left(\frac{\mathbb{K}^2-x}{\mathbb{K}^2-\varepsilon^2} \right) & \varepsilon^2 \leq x \leq \mathbb{K}^2 \\ \mathcal{h} & \mathbb{K}^2 \leq x \leq \acute{o}^2 \\ \mathcal{h} \left(\frac{\mathfrak{z}^2-x}{\mathfrak{z}^2-\acute{o}^2} \right) & \acute{o}^2 \leq x \leq \mathfrak{z}^2 \\ 0 & x > \mathfrak{z}^2 \end{cases}$$

3.3 α - cut of Linear ONN with symmetry:

α - cut can be express as: $\mathcal{A}_{\alpha} = \{x \in \bar{\mathcal{X}} | T_L(\mathcal{X}), I_L(\mathcal{X}), F_L(\mathcal{X}) \geq \alpha\}$

$$\text{Truthiness} = T_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega + \frac{\alpha}{b_1}(\eta - \Omega) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta + \frac{1-\alpha}{1-b_2}(\epsilon_i - \eta) \text{ for } \alpha \in [\acute{b}_2, 1] \\ \mathcal{A}_{3L}(\alpha) = \epsilon_i + \frac{1-\alpha}{1-b_3}(\nu - \epsilon_i) \text{ for } \alpha \in [\acute{b}_3, 1] \\ \mathcal{A}_{4L}(\alpha) = \nu + \frac{1-\alpha}{1-b_4}(\epsilon - \nu) \text{ for } \alpha \in [\acute{b}_4, 1] \\ \mathcal{A}_{3R}(\alpha) = \aleph - \frac{\alpha}{b_4}(\aleph - \epsilon) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{2R}(\alpha) = \acute{o} - \frac{\alpha}{b_3}(\acute{o} - \aleph) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{1R}(\alpha) = 3 - \frac{\alpha}{b_2}(3 - \acute{o}) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \end{cases}$$

There we have $\mathcal{A}_{1L}(\alpha), \mathcal{A}_{2L}(\alpha), \mathcal{A}_{3L}(\alpha), \mathcal{A}_{4L}(\alpha)$ are increasing and $\mathcal{A}_{3R}(\alpha), \mathcal{A}_{2R}(\alpha), \mathcal{A}_{1R}(\alpha)$ are decreasing.

$$\text{Falsity} = F_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega^1 + \frac{\alpha}{b_1}(\eta^1 - \Omega^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta^1 + \frac{1-\alpha}{1-b_2}(\epsilon_i^1 - \eta^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \epsilon_i^1 + \frac{1-\alpha}{1-b_3}(\nu^1 - \epsilon_i^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \nu^1 + \frac{1-\alpha}{1-b_4}(\epsilon^1 - \nu^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \aleph^1 - \frac{\alpha}{b_4}(\aleph^1 - \epsilon^1) \text{ for } \alpha \in [\acute{b}_4, 1] \\ \mathcal{A}_{2R}(\alpha) = \acute{o}^1 - \frac{\alpha}{b_3}(\acute{o}^1 - \aleph^1) \text{ for } \alpha \in [\acute{b}_3, 1] \\ \mathcal{A}_{1R}(\alpha) = 3^1 - \frac{\alpha}{b_2}(3^1 - \acute{o}^1) \text{ for } \alpha \in [\acute{b}_2, 1] \end{cases}$$

$$\text{Indeterminacy} = I_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \eta^2 + \frac{\alpha}{b_1}(\epsilon_i^2 - \eta^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \epsilon_i^2 + \frac{1-\alpha}{1-b_2}(\nu^2 - \epsilon_i^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \nu^2 + \frac{1-\alpha}{1-b_3}(\epsilon^2 - \nu^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \epsilon^2 + \frac{1-\alpha}{1-b_4}(\aleph^2 - \epsilon^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \acute{o}^2 - \frac{\alpha}{b_3}(\acute{o}^2 - \aleph^2) \text{ for } \alpha \in [\acute{b}_4, 1] \\ \mathcal{A}_{2R}(\alpha) = 3^2 - \frac{\alpha}{b_3}(3^2 - \acute{o}^2) \text{ for } \alpha \in [\acute{b}_3, 1] \\ \mathcal{A}_{1R}(\alpha) = \overline{3^2} - \frac{\alpha}{b_2}(\overline{3^2} - \acute{o}^2) \text{ for } \alpha \in [\acute{b}_2, 1] \end{cases}$$

3.4 Non-Linear ONN with symmetry: Let $\acute{A}_{LS} = (\Omega, \eta, \epsilon_i, a_1, a_2, \aleph, \acute{o}, 3)_{(\acute{n}_1, \acute{n}_2, \acute{m}_1, \acute{m}_2)}$ be a non linear ONN and its membership function are:

$$\begin{aligned}
 \text{Truthiness} = T_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega \\ \ell \left(\frac{x-\Omega}{\eta-\Omega} \right)^{\dot{n}_1} & \Omega \leq x \leq \eta \\ \ell & \eta \leq x \leq \xi \\ \ell + (1-\ell) \left(\frac{x-\xi}{\nu-\xi} \right)^{\dot{n}_2} & \xi \leq x \leq \nu \\ 1 & \nu \leq x \leq \varepsilon \\ \ell + (1-\ell) \left(\frac{\mathbb{K}-x}{\mathbb{K}-\varepsilon} \right)^{\dot{m}_1} & \varepsilon \leq x \leq \mathbb{K} \\ \ell & \mathbb{K} \leq x \leq \acute{o} \\ \ell \left(\frac{\acute{o}-x}{\acute{o}-\mathbb{K}} \right)^{\dot{m}_2} & \acute{o} \leq x \leq \acute{o} \\ 0 & x > \acute{o} \end{cases} \\
 \text{Falsity} = F_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^1 \\ \ell \left(\frac{x-\Omega^1}{\eta^1-\Omega^1} \right)^{\dot{m}_1} & \Omega^1 \leq x \leq \eta^1 \\ \ell & \eta^1 \leq x \leq \xi^1 \\ \ell + (1-\ell) \left(\frac{x-\xi^1}{\nu^1-\xi^1} \right)^{\dot{m}_2} & \xi^1 \leq x \leq \nu^1 \\ 1 & \nu^1 \leq x \leq \varepsilon^1 \\ \ell + (1-\ell) \left(\frac{\mathbb{K}^1-x}{\mathbb{K}^1-\varepsilon^1} \right)^{\dot{n}_1} & \varepsilon^1 \leq x \leq \mathbb{K}^1 \\ \ell & \mathbb{K}^1 \leq x \leq \acute{o}^1 \\ \ell \left(\frac{\acute{o}^1-x}{\acute{o}^1-\mathbb{K}^1} \right)^{\dot{n}_2} & \acute{o}^1 \leq x \leq \acute{o}^1 \\ 0 & x > \acute{o}^1 \end{cases} \\
 \text{Indeterminacy} = I_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^2 \\ \ell \left(\frac{x-\Omega^2}{\eta^2-\Omega^2} \right)^{\dot{m}_1} & \Omega^2 \leq x \leq \eta^2 \\ \ell & \eta^2 \leq x \leq \xi^2 \\ \ell + (1-\ell) \left(\frac{x-\xi^2}{\nu^2-\xi^2} \right)^{\dot{m}_2} & \xi^2 \leq x \leq \nu^2 \\ 1 & \nu^2 \leq x \leq \varepsilon^2 \\ \ell + (1-\ell) \left(\frac{\mathbb{K}^2-x}{\mathbb{K}^2-\varepsilon^2} \right)^{\dot{n}_1} & \varepsilon^2 \leq x \leq \mathbb{K}^2 \\ \ell & \mathbb{K}^2 \leq x \leq \acute{o}^2 \\ \ell \left(\frac{\acute{o}^2-x}{\acute{o}^2-\mathbb{K}^2} \right)^{\dot{n}_2} & \acute{o}^2 \leq x \leq \acute{o}^2 \\ 0 & x > \acute{o}^2 \end{cases}
 \end{aligned}$$

3.5 α - cut of Non-Linear ONN with symmetry

α - cut of LONNS can be expressed by: $\mathcal{A}_{\alpha} = \{x \in \bar{\mathcal{X}} | T_{NL}(\mathcal{X}), I_{NL}(\mathcal{X}), F_{NL}(\mathcal{X}) \geq \alpha\}$

$$\text{Truthiness} = T_{NL}(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega + \left(\frac{\alpha}{b_1}\right)^{n_1} (\eta - \Omega) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta + \left(\frac{1-\alpha}{1-b_2}\right)^{n_2} (\epsilon - \eta) \text{ for } \alpha \in [\acute{b}_2, \acute{1}] \\ \mathcal{A}_{3L}(\alpha) = \epsilon + \left(\frac{1-\alpha}{1-b_3}\right)^{n_3} (\nu - \epsilon) \text{ for } \alpha \in [\acute{b}_3, \acute{1}] \\ \mathcal{A}_{4L}(\alpha) = \nu + \left(\frac{1-\alpha}{1-b_4}\right)^{n_4} (\epsilon - \nu) \text{ for } \alpha \in [\acute{b}_4, \acute{1}] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K} - \left(\frac{\alpha}{b_4}\right)^{m_1} (\mathbb{K} - \epsilon) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{2R}(\alpha) = \acute{o} - \left(\frac{\alpha}{b_3}\right)^{m_2} (\acute{o} - \mathbb{K}) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{1R}(\alpha) = 3 - \left(\frac{\alpha}{b_2}\right)^{m_3} (3 - \acute{o}) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \end{cases}$$

$$\text{Falsity} = F_{NL}(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega^1 + \frac{\alpha^{m_1}}{b_1} (\eta^1 - \Omega^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta^1 + \frac{1-\alpha^{m_2}}{1-b_2} (\epsilon^1 - \eta^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \epsilon^1 + \frac{1-\alpha^{m_3}}{1-b_3} (\nu^1 - \epsilon^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \nu^1 + \frac{1-\alpha^{m_4}}{1-b_4} (\epsilon^1 - \nu^1) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K}^1 - \frac{\alpha^{n_1}}{b_4} (\mathbb{K}^1 - \epsilon^1) \text{ for } \alpha \in [\acute{b}_4, \acute{1}] \\ \mathcal{A}_{2R}(\alpha) = \acute{o}^1 - \frac{\alpha^{n_2}}{b_3} (\acute{o}^1 - \mathbb{K}^1) \text{ for } \alpha \in [\acute{b}_3, \acute{1}] \\ \mathcal{A}_{1R}(\alpha) = 3^1 - \frac{\alpha^{n_3}}{b_2} (3^1 - \acute{o}^1) \text{ for } \alpha \in [\acute{b}_2, \acute{1}] \end{cases}$$

$$\text{Indeterminacy} = I_{NL}(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \eta^2 + \frac{\alpha^{m_1}}{b_1} (\epsilon^2 - \eta^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \epsilon^2 + \frac{1-\alpha^{m_2}}{1-b_2} (\nu^2 - \epsilon^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \nu^2 + \frac{1-\alpha^{m_3}}{1-b_3} (\epsilon^2 - \nu^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \epsilon^2 + \frac{1-\alpha^{m_4}}{1-b_4} (\mathbb{K}^2 - \epsilon^2) \text{ for } \alpha \in [\acute{o}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \acute{o}^2 - \frac{\alpha^{n_1}}{b_3} (\acute{o}^2 - \mathbb{K}^2) \text{ for } \alpha \in [\acute{b}_4, \acute{1}] \\ \mathcal{A}_{2R}(\alpha) = 3^2 - \frac{\alpha^{n_2}}{b_3} (3^2 - \acute{o}^2) \text{ for } \alpha \in [\acute{b}_3, \acute{1}] \\ \mathcal{A}_{1R}(\alpha) = \overline{3^2} - \frac{\alpha^{n_3}}{b_2} (\overline{3^2 - \acute{o}^2}) \text{ for } \alpha \in [\acute{b}_2, \acute{1}] \end{cases}$$

The increasing functions are $\mathcal{A}_{1L}(\alpha), \mathcal{A}_{2L}(\alpha), \mathcal{A}_{3L}(\alpha), \mathcal{A}_{4L}(\alpha)$ with respect to α and the decreasing functions are $\mathcal{A}_{1R}(\alpha), \mathcal{A}_{2R}(\alpha), \mathcal{A}_{3R}(\alpha)$ with respect to α . $\mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha)$ with respect to α .

$$\begin{aligned}
 \text{Truthiness} = T_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega \\ \mathcal{P} \left(\frac{x-\Omega}{\eta-\Omega} \right)^{\dot{n}_1} & \Omega \leq x \leq \eta \\ \mathcal{P} & \eta \leq x \leq \xi \\ \mathcal{P} - (\mathcal{P} - \mathcal{P}) \left(\frac{x-\xi}{a_1-\xi} \right)^{\dot{n}_2} & \xi \leq x \leq a_1 \\ 1 & a_1 \leq x \leq a_2 \\ \mathcal{P} - (\mathcal{P} - \mathcal{P}) \left(\frac{\mathcal{K}-x}{\mathcal{K}-a_2} \right)^{\dot{m}_1} & a_2 \leq x \leq \mathcal{K} \\ \mathcal{P} & \mathcal{K} \leq x \leq \acute{o} \\ \mathcal{P} \left(\frac{\acute{o}-x}{\acute{o}-\acute{o}} \right)^{\dot{m}_2} & \acute{o} \leq x \leq \mathfrak{z} \\ 0 & x > \mathfrak{z} \end{cases} \\
 \text{Falsity} = F_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^1 \\ \mathcal{Q} \left(\frac{x-\Omega^1}{\eta^1-\Omega^1} \right)^{\dot{n}_1} & \Omega^1 \leq x \leq \eta^1 \\ \mathcal{W} & \eta^1 \leq x \leq \xi^1 \\ \mathcal{W} - (\mathcal{W} - \mathcal{Q}) \left(\frac{x-\xi^1}{b_1^1-\xi^1} \right)^{\dot{n}_2} & \xi^1 \leq x \leq a_1^1 \\ 1 & b_1^1 \leq x \leq a_2^1 \\ \mathcal{W} - (\mathcal{W} - \mathcal{S}) \left(\frac{\mathcal{K}^1-x}{\mathcal{K}^1-b_2^1} \right)^{\dot{m}_1} & b_2^1 \leq x \leq \mathcal{K}^1 \\ \mathcal{W} & \mathcal{K}^1 \leq x \leq \acute{o}^1 \\ \mathcal{S} \left(\frac{\acute{o}^1-x}{\acute{o}^1-\acute{o}^1} \right)^{\dot{m}_2} & \acute{o}^1 \leq x \leq \mathfrak{z}^1 \\ 0 & x > \mathfrak{z}^1 \end{cases} \\
 \text{Indeterminacy} = I_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^2 \\ \mathcal{Y} \left(\frac{x-\Omega^2}{\eta^2-\Omega^2} \right)^{n_1} & \Omega^2 \leq x \leq \eta^2 \\ \mathcal{X} & \eta^2 \leq x \leq \xi^2 \\ \mathcal{X} - (\mathcal{X} - \mathcal{Y}) \left(\frac{x-\xi^2}{c_1^2-\xi^2} \right)^{n_2} & \xi^2 \leq x \leq a_1^2 \\ 1 & c_1^2 \leq x \leq a_1^2 \\ \mathcal{X} - (\mathcal{X} - \mathcal{Z}) \left(\frac{\mathcal{K}^2-x}{\mathcal{K}^2-c_2^2} \right)^{m_1} & c_2^2 \leq x \leq \mathcal{K}^2 \\ \mathcal{Z} & \mathcal{K}^2 \leq x \leq \acute{o}^2 \\ \mathcal{Z} \left(\frac{\acute{o}^2-x}{\acute{o}^2-\acute{o}^2} \right)^{m_2} & \acute{o}^2 \leq x \leq \mathfrak{z}^2 \\ 0 & x > \mathfrak{z}^2 \end{cases}
 \end{aligned}$$

4. Case Study

To demonstrate the feasibility and productiveness of the proposed method, here is the most useful real-life problem. Suppose we have three different persons which have different degree, experience and number of publications. How can we select the person who has more potential to deal with situations?

Numerical Example: Suppose that U is the universe. Suppose that the HR which is responsible for recruiting, interviewing and placing workers, wants to hire a new person in company. Three different persons (A, B, C) apply for this opportunity, which have different degrees, experiences and publications. On the base of choice parameters $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$ we have to select the best one.

A	B	C
{C ₁ (0.72,0.35,0.71,0.77,0.41,0.73,0.77,0.81) (0.93,0.83,0.63,0.88,0.94,0.99,0.96,0.60) (0.86,0.95,0.99,0.97,0.94,0.93,0.95,0.91)}	{C ₁ (0.33,0.73,0.34,0.25,0.26,0.74,0.45,0.29) (0.33,0.46,0.59,0.79,0.85,0.79,0.74,0.86) (0.88,0.83,0.55,0.75,0.98,0.64,0.96,0.90)}	{C ₁ (0.23,0.33,0.63,0.56,0.45,0.35,0.73,0.28) (0.76,0.55,0.69,0.34,0.24,0.63,0.95,0.91) (0.94,0.73,0.95,0.95,0.48,0.94,0.96,0.74)}
{C ₂ (0.35,0.65,0.36,0.54,0.33,0.65,0.43,0.56) (0.75,0.45,0.95,0.38,0.68,0.79,0.57,0.13) (0.96,0.99,0.78,0.79,0.97,0.36,0.97,0.95)}	{C ₂ (0.25,0.55,0.36,0.54,0.33,0.65,0.43,0.56) (0.93,0.83,0.83,0.58,0.84,0.69,0.76,0.80) (0.96,0.99,0.98,0.99,0.97,0.76,0.87,0.95)}	{C ₂ (0.23,0.65,0.26,0.54,0.63,0.65,0.41,0.59) (0.75,0.45,0.85,0.38,0.78,0.79,0.67,0.13) (0.98,0.89,0.88,0.79,0.97,0.96,0.87,0.85)}
{C ₃ (0.24,0.33,0.44,0.55,0.56,0.34,0.45,0.89) (0.35,0.46,0.58,0.79,0.85,0.71,0.64,0.96) (0.84,0.73,0.85,0.95,0.98,0.84,0.96,0.94)}	{C ₃ (0.24,0.33,0.74,0.35,0.46,0.54,0.85,0.19) (0.88,0.86,0.58,0.85,0.85,0.61,0.64,0.86) (0.98,0.93,0.95,0.95,0.98,0.84,0.66,0.84)}	{C ₃ (0.24,0.23,0.44,0.25,0.26,0.34,0.85,0.89) (0.35,0.44,0.78,0.79,0.75,0.71,0.94,0.96) (0.74,0.63,0.95,0.95,0.98,0.94,0.98,0.94)}

[In this above, matrix (C₁, C₂, C₃) is mentioned in the row and persons (A,B,C) are mentioned in the column]

STEP 1: Defuzzify the Octagonal Neutrosophic number by using Accuracy Function [21]

$$D^{TNO_N} = \left(\frac{\Omega + \eta + \epsilon + \nu + \epsilon + \kappa + \delta + \alpha}{8} \right), D^{INO_N} = \left(\frac{\Omega^1 + \eta^1 + \epsilon^1 + \nu^1 + \epsilon^1 + \kappa^1 + \delta^1 + \alpha^1}{8} \right), D^{FNO_N} = \left(\frac{\Omega^2 + \eta^2 + \epsilon^2 + \nu^2 + \epsilon^2 + \kappa^2 + \delta^2 + \alpha^2}{8} \right)$$

Then the neutrosophic soft matrix is

Criteria	A ₁	A ₂	A ₃
C ₁	(0.6,0.8,0.9)	(0.4,0.6,0.8)	(0.4,0.5,0.9)
C ₂	(0.4,0.5,0.7)	(0.4,0.7,0.9)	(0.4,0.6,0.9)
C ₃	(0.4,0.6,0.9)	(0.4,0.7,0.9)	(0.4,0.7,0.9)

STEP 2: For normalized aggregate fuzzy decision matrix.

$$\bar{r}_{ij} = \left(\frac{\bar{a}_{ij}}{c_{ij}}, \frac{\bar{b}_{ij}}{c_{ij}}, \frac{\bar{c}_{ij}}{c_{ij}} \right)$$

Criteria	A ₁	A ₂	A ₃
C ₁	(0.6,0.8,1.0)	(0.5,0.7,1.0)	(0.4,0.5,1.0)
C ₂	(0.5,0.7,1.0)	(0.4,0.7,1.0)	(0.4,0.6,1.0)
C ₃	(0.4,0.6,1.0)	(0.4,0.7,1.0)	(0.4,0.7,1.0)

Aggregate decision matrix for criteria weighting

$$\bar{W}_1 = (0.3, 0.4, 0.5), \bar{W}_2 = (0.5, 0.6, 0.7), \bar{W}_3 = (0.1, 0.2, 0.3)$$

STEP 3. Weighted normalized fuzzy decision matrix. $\bar{P}_{ij} = \bar{r}_{ij}$ multiply-by w_j

Criteria	A ₁	A ₂	A ₃
C ₁	(0.1,0.32,0.5)	(0.1,0.28,0.5)	(0.1,0.2,0.5)
C ₂	(0.2,0.42,0.7)	(0.2,0.42,0.7)	(0.2,0.36,0.7)
C ₃	(0.04,0.12,0.3)	(0.04,0.14,0.3)	(0.04,0.14,0.3)

STEP 4. Find FNIS AND FPIS

$$A^+ = (P_1^+, P_2^+, P_3^+ \dots P_n^+)$$

$$P_j^+ = \max (P_{ij3}) \quad i=1,2,\dots,m, \quad j=1,2,3,\dots,n$$

$$A^- = (P_1^-, P_2^-, P_3^- \dots P_n^-)$$

$$P_j^- = \min (P_{ij3}) \quad i=1,2,\dots,m, \quad j=1,2,3,\dots,n$$

$$A^+ = (P_1^+(0.5,0.5,0.5), P_2^+(0.7,0.7,0.7), P_3^+(0.3,0.3,0.3))$$

$$A^- = (P_1^-(0.1,0.1,0.1), P_2^-(0.2,0.2,0.2), P_3^-(0.04,0.04,0.04))$$

Now by $d(\bar{x}, \bar{y}) = \sqrt{\frac{1}{3}(\mathbf{a}_1 - \hat{\mathbf{a}}_2)^2 + (\mathbf{b}_1 - \hat{\mathbf{b}}_2)^2 + (\mathbf{c}_1 - \hat{\mathbf{c}}_2)^2}$

Positive Ideal Solution

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	0.253	0.263	0.288
\mathcal{C}_2	0.330	0.330	0.349
\mathcal{C}_3	0.182	0.176	0.176

Negative Ideal Solution

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	0.050	0.041	0.023
\mathcal{C}_2	0.063	0.063	0.046
\mathcal{C}_3	0.012	0.015	0.015

STEP 5. Now calculate distance between each weighted alternative

$\bar{d}_i^* = \sum_{j=1}^n \check{d}(v_{ij}, v_j^*), d_i^- = \sum_{j=1}^n \check{d}(v_{ij}, v_j^-)$

$\bar{d}_1^* = 0.765 \quad \hat{d}_1^- = 0.125$

$\bar{d}_2^* = 0.769 \quad \hat{d}_2^- = 0.119$

$\bar{d}_3^* = 0.813 \quad \hat{d}_3^- = 0.084$

STEP 6. Closeness coefficient

$\bar{C}C_i = \frac{d_i^-}{d_i^- + d_i^*}$

$\bar{C}C_1 = \frac{0.125}{0.125 + 0.765} = 0.140$

$\bar{C}C_2 = \frac{0.119}{0.119 + 0.769} = 0.134$

$\bar{C}C_3 = \frac{0.084}{0.084 + 0.813} = 0.093$

Strategy	Result value	Rank
$\bar{C}C_1$	0.140	1
$\bar{C}C_2$	0.134	2
$\bar{C}C_3$	0.093	3

Clearly

$\mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_3$, The person deserves this post is \mathcal{A}_1 .

5. Conclusions

In this article, types of octagonal neutrosophic number (Linear, Non-Linear, Symmetric, Asymmetric) are proposed and their α – cuts were also derived. Octagonal Neutrosophic Numbers are very useful in solving multi criteria decision making MCDM problems from daily life since this number can deal with more fluctuations. To discuss the applicability and productiveness in daily life issues a case study was done using TOPSIS technique of MCDM. In which firstly numbers were converted from octagonal to fuzzy using accuracy function and then used in

the existing method. In forthcoming work, we'll propose the aggregate operators of Octagonal Neutrosophic Numbers and their matrix notions with operations.

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