Kernel Neutrosophic Crisp Sets

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Abstract

Our study focusses on the concept of the kernel with in neutrosophic crisp sets ($\mathcal{N}CS_s$) and its relationship with the separation axioms of NCTS, coinciding, and shedding light on the properties that characterize them.

Keyword: Neutrosophic crisp sets; K $\mathcal{N}CS_s$; separation axioms of $\mathcal{N}$CTS

1. Introduction

It is known that topology is the family of all sets closed under the binary operation of finite intersections and closed under infinite unions, that is, t's not always an open set when infinite intersects with open sets. And here we stop. What concept involves intersections of open sets containing a given point or set? The answer is the concept of the kernel, which plays an important and fundamental role, especially with the separation axioms (see [1, 2, 3]). In our study, the concept of the kernel was generalized to the neutrosophic crisp sets, with an in-depth study and the Impact of this concept both $\mathcal{N}CT_0$ -spaces and $\mathcal{N}CT_1$ -spaces [4,5]. F. Smarandache [6,7] thinking focused on finding sets with broad applications and important implications for solving various life problems, and from here he emerged. The idea of neutrosophic and neutrosophic crisp sets. For us and some researchers, Smarandache shares this interest in multiple research [8,9,10,11]. Lastly, Al-Obaidi et al. provided information on the senses of new kinds of crisp open mappings and crisp closed functions that are weakly neutrosophic [14,15]. Imran et al. [16,17,18,19] gave the new ideas about neutrosophic crisp open sets, novel forms of weakly neutrosophic crisp continuity, some new concepts of weakly neutrosophic crisp separation axioms and neutrosophic crisp generalized sg-closed sets and their continuity. Abdulkadhim et al. [20] examined the view of neutrosophic crisp generalized alpha generalized closed sets.

2. Preliminaries

Fragile neutrosophic crisp sets are characterized by the presence of three types of conditions necessary for their formation, and by their non-unique binary operations, such as intersections, union, belong, complement in more than one form. our study will be on $\mathcal{N}CS_s$.

$\mathcal{H}_{\mathcal{N}} = <\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3>$, such that $\mathcal{H}_1 \cap \mathcal{H}_2 = \phi, \mathcal{H}_1 \cap \mathcal{H}_3 = \phi$ and $\mathcal{H}_2 \cap \mathcal{H}_3 = \phi$. For $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ are subsets of a non-empty universal set $\mathcal{D}$.

Take the $\phi_{\mathcal{N}} =< \phi, \phi, \mathcal{D} >, \mathcal{D}_{\mathcal{N}} =< \mathcal{D}, \mathcal{D}, \phi >$.

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The binary operations [1,2,6].

1- The complement
$\mathcal{H}_{\mathcal{N}}^{C} =< \mathcal{D}\backslash\mathcal{H}_1, \mathcal{D}\backslash\mathcal{H}_2, \mathcal{D}\backslash\mathcal{H}_3 >$
2- $\mathcal{H}_{\mathcal{N}} \cap \mathcal{M}_{\mathcal{N}} =< \mathcal{H}_1 \cap \mathcal{M}_1, \mathcal{H}_2 \cap \mathcal{M}_2, \mathcal{H}_3 \cup \mathcal{M}_3 >$
3- $\mathcal{H}_{\mathcal{N}} \cup \mathcal{M}_{\mathcal{N}} =< \mathcal{H}_1 \cup \mathcal{M}_1, \mathcal{H}_2 \cup \mathcal{M}_2, \mathcal{H}_3 \cap \mathcal{M}_3 >$
4- $\mathcal{H}_{\mathcal{N}} \subseteq \mathcal{M}_{\mathcal{N}}$ iff $\mathcal{H}_1 \subseteq \mathcal{M}_1$, $\mathcal{H}_2 \subseteq \mathcal{M}_2$, and $\mathcal{H}_3 \subseteq \mathcal{M}_3$
5- The neutrosophic crisp points ($\mathcal{N}CP_s$)
  i. $P_{\mathcal{N}_1} =< \{P\}, \phi, \mathcal{D}\backslash\{P\} >$, and $P_{\mathcal{N}_1} \in \mathcal{H}_{\mathcal{N}}$ iff $P \in \mathcal{H}_1$.
  ii. $P_{\mathcal{N}_2} =< \{P\}, \phi, \phi >$, and $P_{\mathcal{N}_2} \in \mathcal{H}_{\mathcal{N}}$ iff $P \in \mathcal{H}_1$.
  iii. $P_{\mathcal{N}_3} =< \phi, \{P\}, \mathcal{D}\backslash\{P\} >$, and $P_{\mathcal{N}_3} \in \mathcal{H}_N$ iff $P \in \mathcal{H}_2$.
  iv. $P_{\mathcal{N}_4} =< \phi, \{P\}, \phi >$, and $P_{\mathcal{N}_4} \in \mathcal{H}_{\mathcal{N}}$ iff $P \in \mathcal{H}_2$.

Definition 2.1: [4, 5]

The sub collection $T_{\mathcal{N}C}$ of $\mathcal{N}CS_s$ in non-empty set $\mathcal{D}$, is called neutrosophic crisp topology if satisfy that

1- $\phi_{\mathcal{N}}, \mathcal{D}_{\mathcal{N}} \varepsilon T_{\mathcal{N}C}$
2- If $\mathcal{H}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}} \varepsilon T_{\mathcal{N}C}$, then $\mathcal{H}_{\mathcal{N}} \cap \mathcal{M}_{\mathcal{N}} \varepsilon T_{\mathcal{N}C}$
3- For any $\lambda\varepsilon\Delta$, $A_{\lambda\mathcal{N}}\varepsilon\tau_{\mathcal{N}C}$, then $U_{\lambda\varepsilon\Delta}\, \mathcal{H}_{\lambda\mathcal{N}}\varepsilon\tau_{\mathcal{N}C}$, where ε: (belong to) in set theory, that is, in their traditional sense.

Through the definition of $T_{\mathcal{N}C}$, we can easily conclude that the first and the second coordination (axises) of neutrosophic crisp open sets ($\mathcal{N}COS_s$) are topologies on Y, but the third coordinates are a basis for topology because $T_{\mathcal{N}C}$ is closed under the union and closed under the finite intersection. But if Y is finite, then all coordinates of $\mathcal{N}COS_s$ are topologies on Y.

Example 2.2:

Let $\mathcal{D} = \{\ell, m, n\}$ and $\tau_{\mathcal{N}C} = \{\phi_{\mathcal{N}}, \mathcal{D}_{\mathcal{N}}, < \phi, \{\ell\}, \{m.n\} >, < \{m\}, \{n\}, \{\ell\} >, < \{m\}, \{\ell, n\}, \phi >\}$, then the collection of coordination of $\mathcal{N}COS_s$ are $\{\phi, \mathcal{D}, \{m\}\}, \{\phi, \mathcal{D}, \{\ell\}, \{n\}, \{\ell, n\}\}$ and $\{\phi, \mathcal{D}, \{\ell\}, \{m, n\}\}$.

3. Kernel Neutrosophic Crisp Sets (K$\mathcal{N}CS_s$):

Definition 3.1:

The intersection of all NCS_s containing $\mathcal{H}_{\mathcal{N}}$ is called KCS_s(briefly $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$).

This means that $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}}) = \cap \{G_{\mathcal{N}} \varepsilon\tau_{\mathcal{N}C}, \mathcal{H}_{\mathcal{N}} \subseteq G_{\mathcal{N}}\}$

-From example2.2 $Ke_{\mathcal{N}}(< \phi, \{\ell\}, \{n\} >) =< \phi, \{m\}, \{\ell, n\} >$

- If Y is finite, then $Ke_{\mathcal{N}}$ is $\mathcal{N}$COS but if $\tau_{\mathcal{N}C}$ finite, $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$ is not necessary be $\mathcal{N}$COS.

- For any $\mathcal{N}$CS $\mathcal{H}_{\mathcal{N}}$, $Ke_{\mathcal{N}}\mathcal{H}_{\mathcal{N}}^{C} \neq Ke_{\mathcal{N}}\mathcal{H}_{\mathcal{N}})^{C}$,by example2.2, $Ke_{\mathcal{N}}(< \phi, \{\ell\}, \{m, n\} >) =< \mathcal{D}, \mathcal{D}, \phi >$ but $, Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}}) =< \{m\}, \{\ell\}, \{m, n\}, >$

Proposition 3.2:

For any $\mathcal{N}CS_s$ $\mathcal{H}_{\mathcal{N}}, \mathcal{H}_{\mathcal{N}}$ in $\mathcal{N}$CTS $\mathcal{D}^{\tau_{\mathcal{N}C}}$, Then

1- $\mathcal{H}_{\mathcal{N}} \subseteq Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$
2- If $\mathcal{H}_{\mathcal{N}} \subseteq \mathcal{M}_{\mathcal{N}}$, then $Ke_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}}) \subseteq Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$
3- $Ke_{\mathcal{N}}(Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})) = Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$
4- $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}} \cup \mathcal{M}_{\mathcal{N}}) \subseteq Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}}) \cup Ke_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}})$
5- $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}}) \cap Ke_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}}) \subseteq Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}} \cap \mathcal{M}_{\mathcal{N}})$

Definition 3.3:

A $\mathcal{N}$CS $\mathcal{H}_{\mathcal{N}}$ is claimed to be Neutrosophic crisp weakly ultra separated ($\mathcal{N}$CWUS) from $\mathcal{M}_{\mathcal{N}}$, if $\exists\ NCOS\ G_{\mathcal{N}} \ni .$ $\mathcal{H}_{\mathcal{N}} \subseteq G_{\mathcal{N}}$ and $G_{\mathcal{N}} \cap \mathcal{M}_{\mathcal{N}} = \phi_{\mathcal{N}}$ or $\mathcal{H}_{\mathcal{N}} \cap \mathcal{N}CCL\ (\mathcal{M}_{\mathcal{N}}) = \phi_{\mathcal{N}}$. Where $\mathcal{N}CCL\ (\mathcal{M}_{\mathcal{N}}) = \cap \{F_{\mathcal{N}} : \mathcal{M}_{\mathcal{N}} \subseteq F_{\mathcal{N}}, F_{\mathcal{N}}\ is\ \mathcal{N}CCS\}$.
By example2-2. $\mathcal{H}_{\mathcal{N}} =< \phi_{\mathcal{N}}, \{\ell\}, \{m, n\} >$ is $\mathcal{N}$CWUS from $< \{m\}, \{n\}, \{\ell\} >$.

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Remark 3.4:

For any NCTS $\mathcal{D}^{\tau_{\mathcal{N}C}}$,

1- For $i \neq j, \mathcal{N}CCL\ (P_{\mathcal{N}_i}) = \{P_{\mathcal{N}_j}: P_{\mathcal{N}_j}$is not $\mathcal{N}$CWUS from $P_{\mathcal{N}_i}\}$

2- For $i \neq j, Ke_{\mathcal{N}}(P_{\mathcal{N}_i}) = \mathcal{N}CCL\ (P_{\mathcal{N}_j})$, he proof for any $P_{\mathcal{N}_i} \in Ke_{\mathcal{N}}(P_{\mathcal{N}_i}), i \neq j$ iff $P_{\mathcal{N}_j} \in \cap\, G_{\mathcal{N}} \forall P_{\mathcal{N}_i} \in G_{\mathcal{N}} \varepsilon \tau_{\mathcal{N}C}$ iff $G_{\mathcal{N}} \cap P_{\mathcal{N}_j} \neq \phi_{\mathcal{N}} \forall P_{\mathcal{N}_i} \in G_{\mathcal{N}} \varepsilon \tau_{\mathcal{N}C}$ iff $P_{\mathcal{N}_j} \in \mathcal{N}CCL\ (P_{\mathcal{N}_i})$.

From above remark the following proposition can be inferred.

Proposition 3.5:

For any $\mathcal{N}$CTS $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$ iff one of the following circumstances are true.

1- For any $\mathcal{N}CP_s, P_{\mathcal{N}_i}, i = 1,2,3,4, Ke_{\mathcal{N}}(P_{\mathcal{N}_i}) = P_{\mathcal{N}_i}$

2- For any $\mathcal{N}CP_s, P_{\mathcal{N}_i} \neq P_{\mathcal{N}_j}, i,j = 1,2,3,4, Ke_{\mathcal{N}}(P_{\mathcal{N}_i}) \cap Ke_{\mathcal{N}}\left(P_{\mathcal{N}_j}\right) = \phi_{\mathcal{N}}$

Proposition 3.6:

For any $\mathcal{N}$CTS $\mathcal{D}^{\tau_{\mathcal{N}C}}$, and $\mathcal{H}_{\mathcal{N}}$ be a $\mathcal{N}$CS, then $Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}}) = \{P \in \mathcal{D}_{\mathcal{N}}: \mathcal{N}\text{CCL}(P) \cap \mathcal{H}_{\mathcal{N}} \neq \phi_{\mathcal{N}}\ \}$when P is $\mathcal{N}$CP of any type .

Proof. let $P \in Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$ and if possible $\mathcal{N}$CCL $(P)\cap \mathcal{H}_{\mathcal{N}} = \phi_{\mathcal{N}}$. Then $P \notin (\mathcal{N}\text{CCL}(P))^C \varepsilon \tau_{\mathcal{N}C}$ and $\mathcal{H}_{\mathcal{N}} \subseteq (\mathcal{N}\text{CCL}(P))^C$. This leads to contradiction.

Conversely, let P be any type of $\mathcal{N}$CPS, and $\mathcal{N}$CCL $(P)\cap \mathcal{H}_{\mathcal{N}} \neq \phi_{\mathcal{N}}$, if possible $P \notin Ke_{\mathcal{N}}(\mathcal{H}_{\mathcal{N}})$. Then $\exists\ G_{\mathcal{N}} \varepsilon \tau_{\mathcal{N}C} \ni P \notin G_{\mathcal{N}}$ and $\mathcal{H}_{\mathcal{N}} \subseteq\ G_{\mathcal{N}}$ in another meaning $\forall \acute{P} \in \mathcal{H}_{\mathcal{N}}, \acute{P} \in G_{\mathcal{N}}$ and $P \notin G_{\mathcal{N}}$. But $\mathcal{N}\text{CCL}(P) \cap \mathcal{H}_{\mathcal{N}} \neq \phi_{\mathcal{N}}$, so get that $G_{\mathcal{N}} \cap \{P\} \neq \phi_{\mathcal{N}}$. This leads to contradiction.

Theorem 3.7:

For any two $\mathcal{N}CP_S, P, \acute{P} \in \mathcal{D}_{\mathcal{N}C}$. Then

1- $P \in Ke_{\mathcal{N}}(\acute{P})$ iff $\acute{P} \in \mathcal{N}\text{CCL}(P)$

2- The following are equivalent.

a) $Ke_{\mathcal{N}}(P) \neq Ke_{\mathcal{N}}(\acute{P})$

b) $\mathcal{N}\text{CCL}(P) \neq \mathcal{N}\text{CCL}(\acute{P})$

Proof (2)

a$\Rightarrow b$ Let $Ke_{\mathcal{N}}P) \neq Ke_{\mathcal{N}}(\acute{P}) \Rightarrow \exists \acute{\acute{P}} \in Ke_{\mathcal{N}}(P)$ iff $P \in \mathcal{N}\text{CCL}(\acute{\acute{P}}) \Rightarrow \mathcal{N}\text{CCL}(P) \subseteq \mathcal{N}\text{CCL}(\acute{\acute{P}})$, and $\acute{\acute{P}} \notin Ke_{\mathcal{N}}(\acute{P})$ iff $\acute{P} \notin \mathcal{N}\text{CCL}(\acute{\acute{P}}) \Rightarrow \acute{P} \notin \mathcal{N}\text{CCL}(P)$

Then $\mathcal{N}\text{CCL}(P) \neq \mathcal{N}\text{CCL}(\acute{P})$

$b \Rightarrow a$ Let $\mathcal{N}\text{CCL}(P) \neq \mathcal{N}\text{CCL}(\acute{P}) \Rightarrow \exists \acute{\acute{P}} \in \mathcal{N}\text{CCL}(P)$

Iff $P \in Ke_{\mathcal{N}}(\acute{\acute{P}}) \Rightarrow Ke_{\mathcal{N}}(P) \subseteq Ke_{\mathcal{N}}(Ke_{\mathcal{N}}(\acute{\acute{P}})) = Ke_{\mathcal{N}}(\acute{\acute{P}})$

By proposition 3.2(3),and $\acute{\acute{P}} \notin \mathcal{N}\text{CCL}(\acute{P})$

Iff $\acute{P} \notin Ke_{\mathcal{N}}(\acute{\acute{P}}) \Rightarrow \acute{P} \notin Ke_{\mathcal{N}}(P)$. Then $Ke_{\mathcal{N}}(P) \neq Ke_{\mathcal{N}}(\acute{P})$

Theorem 3.8:

For any two NCP_S $P, \acute{P} \in Y_{NC}$. Then

1- $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_0 - space$ iff $Ke_{\mathcal{N}}(P)$ $\mathcal{N}$CWUS from $\acute{P}$ or $Ke_{\mathcal{N}}(\acute{P})$ $\mathcal{N}$CWUS from P

2- $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$ iff $Ke_{\mathcal{N}}(P)$ $\mathcal{N}$CWUS from $\acute{P}$ and $Ke_{N}(\acute{P})$ $\mathcal{N}$CWUS from P

3- $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$ iff $Ke_{\mathcal{N}}(P) = P$

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Proof .(2) Let $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$, then for each two $\mathcal{N}CP_S$ $P \neq \acute{P}$ $\exists$ two $\mathcal{N}COS_S$ $\mathcal{U}_\mathcal{N}$ and $G_\mathcal{N} \ni P \in \mathcal{U}_\mathcal{N}$, $\acute{P} \notin U_\mathcal{N}$ and $P \notin G_\mathcal{N}$, $\acute{P} \in G_\mathcal{N}$.

Then $Ke_\mathcal{N}(P)$ be $\mathcal{N}$CWUS from $\acute{P}$ and $Ke_\mathcal{N}(\acute{P})$ be $\mathcal{N}$CWUS from P.

Conversely let $Ke_\mathcal{N}(P)$ is $\mathcal{N}$CWUS from $\acute{P} \Rightarrow \exists \mathcal{U}_\mathcal{N} \varepsilon \tau_{\mathcal{N}C} \ni Ke_\mathcal{N}(P) \subset \mathcal{U}_\mathcal{N}$ and $\mathcal{U}_\mathcal{N} \cap \acute{P} = \phi_\mathcal{N} \Rightarrow P \in \mathcal{U}_\mathcal{N}, \acute{P} \notin \mathcal{U}_\mathcal{N}$.And $\exists \mathcal{V}_\mathcal{N} \varepsilon \tau_{\mathcal{N}C} \ni Ke_\mathcal{N}(\acute{P}) \subset \mathcal{V}_\mathcal{N}$ and

$\mathcal{V}_\mathcal{N} \cap P = \phi_\mathcal{N} \Rightarrow \acute{P} \in \mathcal{V}_\mathcal{N}, P \notin \mathcal{V}_\mathcal{N}$.Then $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$.

Corollary 3.9:

For any two $\mathcal{N}$CP_S, $P, \acute{P} \in \mathcal{D}_{\mathcal{N}C}$. Then $\mathcal{D}^{\tau_{\mathcal{N}C}}$, is $\mathcal{N}CT_1 - space$ iff $Ke_\mathcal{N}(P) \cap Ke_\mathcal{N}(\acute{P}) = \phi_N$.

Proposition 3.10:

Let $\mathcal{D}^{\tau_{\mathcal{N}C}}$, be any NCTS. Then for each $\mathcal{U}_\mathcal{N} \varepsilon \tau_{\mathcal{N}C}$, $\forall P \in \mathcal{U}_\mathcal{N}$, $\mathcal{N}$CCL$(P) \subseteq \mathcal{U}_\mathcal{N}$ iff for each $\mathcal{V}_\mathcal{N} \varepsilon \tau_{\mathcal{N}C}, \forall P \in \mathcal{V}_\mathcal{N}, NCCL(Ke_\mathcal{N}(P)) \subseteq \mathcal{U}_\mathcal{N}$

Proof: Let $\mathcal{V}_\mathcal{N} \varepsilon \tau_{NC}$ and $P \in \mathcal{V}_\mathcal{N} \Rightarrow$ $\mathcal{N}$CCL$(P) \subseteq \mathcal{V}_\mathcal{N}$

$\Rightarrow$ By remark 3.4(2) $\mathcal{N}$CCL$(P) = \mathcal{N}$CCL$\big(\mathcal{N}$CCL$(P)\big) = \mathcal{N}$CCL$\big(Ke_\mathcal{N}(P)\big) \subseteq \mathcal{V}_\mathcal{N}$.

Conversely: Let $\mathcal{V}_\mathcal{N} \varepsilon \tau_{NC}$ and $P \in \mathcal{V}_\mathcal{N} \Rightarrow \mathcal{N}CCL\big(Ke_\mathcal{N}(P)\big) \subset \mathcal{V}_\mathcal{N}$,

by proposition 3.2(1) $\mathcal{N}$CCL$(P) \subseteq \mathcal{N}CCL\big(Ke_\mathcal{N}(P)\big) \subseteq \mathcal{V}_\mathcal{N}$.

4. Conclusion

1- That NCTS generates two topologies and third topological base in general, as we have observed, and from here we can exploit this relationship by studying the common topological properties and concepts between them, such as compactness [12,13] and others .
2- We have a clear perception that there is a relationship between a proximity relation and NCSs that can be exploited and build broader sets of NCSs under conditions compatible with them to serve applied fields.

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