

Integration of advanced methods in decision making: plithogenic logic and neutrosophic AHP

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Abstract

In this paper, we undertake the intriguing task of uniting the advanced methods in the domain of decision making. It is about the fusion of plithogenic logic and neutrosophic HPA. It appears that in an environment where the nature of strategic decisions includes high levels of uncertainty and complexity, it is a time to seek more comprehensive methods that help in overcoming contradictions and ambiguities. This is not the first time efforts have been made to bridge the divide between decision making and artificial intelligence, but for some reason, a holistic approach to these tools is still absent. What is missing from the current literature is a framework that would tackle the two challenges of the decision-making processes, and that chaos of human judgments which is often the order of the day. To this end, we seek to articulate the missing literature, suggesting a methodology that may be useful in addressing these problems. The purpose of this model is to integrate plithogenic logic, which in its nature is a model that enables the integration of varied perspectives, and neutrosophic HPA that is theorized to be in tune with uncertainty and those fundamental contradictions in expert judgment. When this combined methodology is implemented, it seems that the outcomes achieved not only enhance the value of decision analysis, but they also allow for a more versatile and elaborate framework for evaluation of alternatives in seemingly ambiguous contexts. The truth is that the contribution of this study could be rather considerable. Theoretically, it may expand the way decision making is perceived. And more practically, it provides a way which can be more useful and comprehensible, particularly in the fields of business strategic management, policy formulation and implementation, and even project management.

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1. Introduction

Legal security implies the permanence and coherence of the norms, as well as a public, transparent and effective deliberation process for their conversion into formal law, supported by the moral consensus of the community for their promulgation, and is not limited exclusively to the legislative formalization of the laws. This concept is the fundamental pillar of the principles that structure due process, which, in the Ecuadorian legal framework, is stipulated in Article 76 of the current Constitution. This principle is based on and aims to protect the fundamental rights of people.

Essential for harmonious coexistence in society, legal certainty is related to the immutability of laws, the inclusive and effective debate for their adaptation to the formal legal sphere, with the indispensable ethical support of society for their creation, extending beyond the mere legislative legitimation of the norms. Legal certainty encompasses the non-retroactivity of laws, the principle of legality in acts of government and the delegation of powers to the judicial bodies, in short, with the fundamental norms that govern social coexistence.

The concept of legal certainty, although linked by terminological issues to the notions of order and law, its analysis exceeds the limits of this brief scrutiny. Universally recognized in the field of law, legal certainty is conceived as the practical certainty of law, ensuring the knowledge or the ability to know what is stipulated as prohibited, obligatory and permitted by the public authority in relation to the individual both in his interactions with others and from others towards him. [1]

The State, in its role as public authority regulating social interactions, not only sets guidelines and norms to be followed but, in a broader sense, has the duty to provide legal security through the exercise of its political, legal and legislative authority. This security is the guarantee that the State provides to the individual, ensuring that his person, property and rights will not be subject to violations or, if they occur, they will be subject to protection, reparation and security by society. From this principle arise the non-retroactivity of the law, the legal classification of crimes, constitutional guarantees, the power of the judge and the prescription of crimes.

On the other hand, effective judicial protection, as established in Article 75 of the Constitution of the Republic, includes the right of access to judges and courts of justice, the right to receive a judicial decision based on a right consistent with the effectiveness of judicial resolutions, among other aspects. Effective judicial protection, characterized as a right of complex configuration, encompasses multiple facets. It is recognized that developing a precise concept is challenging, since the doctrine has evolved from the right of action, whose definition is particularly complex, requiring its explanation through its various manifestations, as it is embodied in a variety of rights and procedural guarantees.

This legal concept is often confused with human rights. The main difference between the two concepts is that human rights have already been formalized, usually at the constitutional level, and therefore enjoy enhanced protection compared to other rights. The effective implementation of the right to effective judicial protection requires certain modifications, not only in the system of administration of justice but also in the very conception of the judicial process as a means for the adequate protection of individual rights. [2]

This leads to an extension of effective judicial protection, which requires more intensive State action than is necessary for other rights, including the sensitization of the judge, who must be considered the main person responsible for the materialization of the right. In such a case, it is necessary to recognize that, although it is proper for a constitutional State of law that all judges apply constitutional norms, principles, rules and methods, in current judicial, political and social practice this is far from being a reality, mainly because most judges are not prepared to exercise the administration of constitutional justice.

From the perspective of the radiating effect that characterizes effective judicial protection as a fundamental right, this right also influences the interpretation and application of laws by the courts. Although its impact varies according to the legal field, one of its most important manifestations is the obligation of judges and courts to interpret rights, especially constitutional rights, in the way that most favors their full effectiveness. Like any fundamental right, its essence can be discerned through its essential content. Understanding these essential contents will influence the way in which both the legislator and the judiciary configure and apply the law within the scope of their respective powers.

Violations of rights may occur in situations not explicitly provided for in legal provisions; as mentioned above, it is up to the ordinary courts to determine the specific circumstances of each particular case. In the event of such violations occurring, the legal framework must provide an appropriate mechanism for their recognition and correction.

In Ecuador, the position has been consolidated that jurisdictional decisions are subject to review in fundamental aspects such as respect for this right. It is worth highlighting the importance of the Judiciary as an essential element for rational and pluralistic dialogue in current democracies, which provides a space to debate the evolution of the norms from different critical perspectives, based on the different interpretations of reality.

The balance between the rights demanded and the duties fulfilled is articulated from the efforts mentioned, summarized in the enlightening notion that axiomatically postulates that justice as a normative principle and law as a methodology allow for achieving social peace as an effective result. For this reason, there has been and continues to be a national demand for a consistent and efficient administration of justice, that is, in accordance with ethical and legal principles, with an honest and impartial conduct that neither favors nor harms any of the parties; and with an efficient, fully effective performance, that guarantees that justice is prompt and reliable; goals

that are aspired to be achieved. Judges must follow the procedures provided to identify the presence of these causes according to the Constitution, the laws and constitutional jurisprudence, the non-observance of which results in the undermining of the mechanisms for protecting the rights and freedoms of citizens.

The proper development of due process in each case that reaches the Judiciary must be in accordance with the current legislation that regulates the procedures and guarantees respect for the rights of the parties involved. In the review of cases, certain procedural anomalies have been detected after the hearing and in the phase of summons to trial. Therefore, the general objective of this critical analysis is to strategically evaluate and prioritize measures to strengthen due process, effective judicial protection and legal security in the face of the practice of reformulation of charges by the Public Prosecutor's Office during the trial hearing.

To achieve the stated objectives, the first step was to identify the main strategies to strengthen due process, effective judicial protection and legal certainty in the face of the practice of reformulation of charges by the Public Prosecutor's Office during trial hearings. Subsequently, a Hierarchical Analytical Process (HAP) methodology enriched with the neutrosophic approach based on plithogeny was applied. It was implemented to quantify the relevance of the identified strategies [3]. The AHP, recognized as the predominant subjective weighting technique in the academic field for the evaluation of the importance of the criteria, is limited in its basic version by not adequately capturing the ambiguity inherent in human judgments [4,18], [5] .

This research overcomes the restrictions associated with the traditional AHP approach by adopting the neutrosophic set, proposed by Smarandache [6] . This set constitutes an extension of the classical, fuzzy and intuitionistic fuzzy sets (IFS), and is distinguished by incorporating degrees of truth, indeterminacy and falsity to more faithfully represent human information characterized by uncertainty and inconsistency [7] . Unlike classical fuzzy sets and IFS, which only address partial or incomplete information, the neutrosophic set can handle data that is incomplete, indeterminate and inconsistent.

In this study, the neutrosophic AHP is used to prioritize those measures that contribute significantly to strengthening due process. Additionally, the study seeks to increase the precision in the synthesis of the results by using the Plithogenic aggregation operator, which assumes an equidistant degree of contradiction between each criterion evaluated and the one considered most critical (dominant). Introduced by Smarandache in 2017, the Plithogenic set emerges as a generalization of the crisp, fuzzy, fuzzy intuitionistic, and neutrosophic sets, derived from the concept of Plithogenic. [9]

2. Related work.

2.1 Plithogenic set

The concept of a plithogenic set represents the genesis, formation and progression of new entities through the amalgamation of old entities, both contradictory and non-contradictory. A plithogenic set encompasses several elements, known as multidimensional attributes. $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$ where each attribute is assigned values $V = \{v_1, v_2, \dots, v_n\}$. In the framework of plithogenic sets, a fundamental aspect of each attribute's value is its membership degree and contradiction degree. For any given attribute value V , its membership degree d indicates the level of association of an element x with the set P based on specific criteria. The contradiction degree function c , on the other hand, delineates the discrepancy between the value of each attribute and that of the most significant (dominant) attribute. The contradiction degree function is essential for improving the accuracy of plithogenic aggregation operations such as intersection, union, complement, inclusion, and equality.

Definition 1: Let the two sets be plithogenic $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$

The plithogenic intersection is:

$$\begin{aligned} \tilde{a} \wedge_p \tilde{b} &= ((a_1, a_2, a_3), 1 \leq i \leq n) \wedge_p ((b_1, b_2, b_3), 1 \leq i \leq n) \\ &= \left((a_1 \wedge_p b_1), \frac{1}{2}(a_2 \wedge_F b_2) + \frac{1}{2}(a_2 \vee_F b_2), (a_3 \vee_p b_3) \right), 1 \leq i \leq n \end{aligned} \quad (1)$$

The plithogenic junction is:

$$\begin{aligned} \tilde{a} \vee_p \tilde{b} &= ((a_1, a_2, a_3), 1 \leq i \leq n) \vee_p ((b_1, b_2, b_3), 1 \leq i \leq n) \\ &= \left((a_1 \vee_p b_1), \frac{1}{2}(a_2 \wedge_F b_2) + \frac{1}{2}(a_2 \vee_F b_2), (a_3 \wedge_p b_3) \right), 1 \leq i \leq n \end{aligned} \quad (2)$$

Where

$$a_1 \wedge_p b_1 = [1 - c(v_D, v_1)] \cdot t_{norm}(v_D, v_1) + c(v_D, v_1) \cdot t_{conorm}(v_D, v_1) \quad (3)$$

$$a_1 \vee_p b_1 = [1 - c(v_D, v_1)] \cdot t_{conorm}(v_D, v_1) + c(v_D, v_1) \cdot t_{norm}(v_D, v_1) \quad (4)$$

Florentin Smarandache established a new philosophical paradigm called Neutrosophic, which delves into the genesis, essence, and broad spectrum of entities, along with their interactions within various conceptual frameworks. The Neutrosophic set, an advancement over fuzzy logic and SFI, encapsulates fuzzy data through three distinct membership functions: the truth membership function T , the indeterminacy membership function I , and the falsity membership function F [10]. These degrees of membership (truth, indeterminacy, and falsity) enable the Neutrosophic set to skillfully handle information that is inconsistent, uncertain, and imprecise [11,20]. A distinctive feature of the Neutrosophic set is the complete independence of the indeterminacy membership function from the truth and falsity membership functions, which offers a unique approach to handling ambiguity. The fundamental principles of Neutrosophic sets are described below:

Definition 2: Consider a universal set X composed of x elements, denoted by $x \in X$. Within this context, a neutrosophic set $A \subset X$ is defined by the presence of a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$, where each of these functions, $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively, are associated with subsets of $]0^-1^+[$. It is crucial to note that the set of these membership functions is not subject to any specific constraint, allowing the expression $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 3: It is defined $\tilde{a} = (a_1, a_2, a_3); \alpha, \theta, \beta$ as a single-valued triangular neutrosophic set characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsehood membership function $F_A(x)$. These functions are specified as follows:

$$T_a(x) = \begin{cases} \alpha_a \left(\frac{x-a_1}{a_2-a_1} \right) & \text{if } a_1 \leq x \leq a_2 \\ \alpha_a & \text{if } x = a_2 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$I_a(x) = \begin{cases} \frac{(a_2-x+\theta_a(x-a_1))}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \theta_a & \text{if } x = a_2 \\ \frac{(x-a_2+\theta_a(a_3-x))}{(a_3-a_2)} & \text{otherwise} \end{cases} \quad (6)$$

$$F_a(x) = \begin{cases} \frac{(a_2-x+\beta_a(x-a_1))}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \beta_a & \text{if } x = a_2 \\ \frac{(x-a_2+\beta_a(a_3-x))}{(a_3-a_2)} & \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{otherwise} \end{cases} \quad (7)$$

Where $\alpha_a, \beta_a, \theta_a \in [0,1]$ and represent the highest degree of belonging to truth, the lowest degree of belonging to indeterminacy, and the lowest degree of belonging to falsehood respectively.

2.1 Neutrosophic AHP

Introduced by Thomas L. Saaty in the early 1970s, the AHP methodology is based on quantitative multi-criteria decision-making (MCDM), which is recognized for its effectiveness in solving complex decision scenarios involving a multitude of criteria. The distinctive feature of the AHP methodology lies in its systematic decomposition of a problem into a hierarchical structure, positioning the final objective at the top level, followed by primary and secondary criteria at subsequent levels. This methodology is uniquely capable of incorporating both qualitative and quantitative factors into the decision-making process. While alternative techniques such as Analytic Network Process (ANP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), CRITIC, and SWARA have been recognized in academic works for their utility in decision making, the AHP methodology continues to be favored for its fundamental soundness, simplicity, interpretability by experts, and its adaptive and robust nature. However, the AHP methodology is not without its challenges, particularly its difficulty in capturing and processing ambiguous or imprecise data.

To overcome this limitation, fuzzy logic was merged with AHP, improving its ability to process imprecise data through membership functions defined by decision makers (DMs) based on their expertise. However, fuzzy logic does not take into account the concept of non-membership, a gap that was filled with the introduction of IFS.

The IFS system extends the scope of classical fuzzy sets to include a membership function, a non-membership function, and a degree of vacillation [14,22]. However, the IFS system does not adequately address the concept of indeterminacy, which encompasses uncertainty, vagueness, imprecision, and inconsistency – attributes frequently encountered in real-world decisions. Given the pervasive nature of indeterminacy, there has been a push

toward more refined approaches to managing imprecision, leading to the development of extensions to the IFS system, including the neutrosophic set and the Pythagorean fuzzy set. [15]

In this research, the AHP method is employed in conjunction with the neutrosophic ensemble, where the association of an element with a set is defined by a neutrosophic probability, characterized by its degrees of being true T%, indeterminate I%, and false F% within the set. Furthermore, the application of plithogenic aggregation operations takes into account the degree of uncertainty, thus improving the accuracy of the results. Therefore, the use of a neutrosophic AHP framework within a plithogenic context is justified in this research, with the aim of refining the accuracy and reliability of decision-making processes.

The methodology for employing a Plithogeny-based neutrosophic AHP framework is developed through a series of meticulously structured steps, outlined as follows:

Step 1: Segment the overall complex problem into a collection of more manageable subproblems, which facilitates a more direct approach to problem solving.

Step 2: Develop a hierarchical arrangement of the problem, placing the general objective at the top, followed by the primary criteria occupying the intermediate level and the secondary criteria arranged at the next level.

Step 3: Constitute a panel of decision makers (DM)

Step 4: Instruct DMs to formulate pairwise comparison matrices using a triangular neutrosophic scale. To do so, they are instructed to apply the values described, which assign the linguistic variables to their corresponding triangular neutrosophic scales, as follows:

- Very weakly important (VWI): (0.10, 0.30, 0.35), (0.10, 0.20, 0.15)
- Weakly important (WI): (0.15, 0.25, 0.10), (0.60, 0.20, 0.30)
- Partially important (PI): (0.40, 0.35, 0.50), (0.60, 0.10, 0.20)
- Equally important (EI): (0.65, 0.60, 0.70), (0.80, 0.10, 0.10)
- Very important (VI): (0.70, 0.65, 0.80), (0.90, 0.20, 0.10)
- Very strongly important (VSI): (0.90, 0.85, 0.90), (0.70, 0.20, 0.20)
- Absolutely important (AI): (0.95, 0.90, 0.95), (0.90, 0.10, 0.10)

This methodology enables DMs to accurately assess and quantify the importance of criteria and sub criteria, incorporating the nuances of uncertainty, imprecision and inconsistency inherent in human judgment and decision-making processes.

Step 5: Implement the plithogenic aggregation operation to consolidate the matrices created by the DMs, improving the accuracy by considering the equidistant degrees of contradiction between the established criteria. By employing plithogenic aggregation operators, the matrices generated by the DMs are synthesized into a unified matrix, thus streamlining the decision-making process.

Step 6: Calculate the precise (crisp) values of the unified decision matrix using equation (8):

$$S(a) = \frac{1}{8}(a_1 + b_1 + c_1) \times (2 + \alpha - \theta - \beta) \quad (8)$$

Step 7: Determine the local weight for each criterion by initially summing the values in each row and subsequently performing a normalization process to achieve proportional weights.

3. Materials and methods

This study focuses on the strategic evaluation and prioritization of measures aimed at strengthening due process, effective judicial protection and legal certainty, in the context of the reformulation of charges by the Public Prosecutor's Office during oral trial hearings. The methodology adopted incorporates a multicriteria approach, specifically through the use of the AHP neutrosophic method complemented by the plithogenic aggregation operator, to address the complexity and multifaceted nature of the issue.

The phases or steps to follow to carry out the research initially consider carrying out an exhaustive analysis with the collaboration of experts in the area and a detailed review of the relevant literature, to identify and select pertinent measures that can effectively contribute to strengthening due process, effective judicial protection and legal certainty.

A panel of decision makers (DMs) is formed, which is critical to the implementation of the proposed neutrosophic AHP framework. The identified measures are discussed with the DMs to assess their relevance. Based on the contributions made by the DMs, the measures will be categorized under different classifications and subsequently organized hierarchically. In this way, a better organization of the information and its subsequent application, if necessary, is expected. Pairwise comparison matrices are created to evaluate the main measures and the derived sub-measures, using neutrosophic scales. This facilitates the inclusion of subjective judgments and ambiguity in the evaluation.

By using the plithogenic aggregation operator, the DM matrices are combined into a consolidated matrix, allowing for the effective integration of multiple perspectives. From the aggregated matrix, the crisp values are determined and the local weights of each measure and sub-measure are calculated by normalization. In addition, the overall weight intensity of each sub-measure is estimated. Finally, the sub-measures will be ranked according to the intensity of their overall weight, thus establishing a priority order among them.

The research is framed within a non-experimental, cross-sectional and descriptive design, where the analysis is based on the perception of experts and the review of documentary sources, without manipulation of variables. The choice of an AHP neutrosophic approach allows for a comprehensive assessment that considers the uncertainty, ambiguity, and subjectivity inherent in the evaluation of the proposed measures. This methodology strengthens the decision-making process, allowing for a more precise interpretation and a more holistic evaluation of the measures proposed to address the challenges identified in the field of justice.

4. Results and discussion

The exploration of the specialized literature, complemented by consultative interactions with experts in the field, allowed us to identify an initial spectrum of twenty-one measures, which were meticulously categorized into five main domains and, subsequently, through a process of synthesis and critical evaluation, consolidated into fifteen key initiatives. These initiatives are structured around five fundamental strategies, thus outlining the framework for action derived from this rigorous analysis. The set of results obtained from this procedure is detailed below.

E1. Ensure transparency in the procedure for reformulating positions

- Clarity in reformulation criteria: Establish clear parameters that justify the need to reformulate the positions.
- Pre-Notification to Defense: Ensuring that the defense receives proper notice before the restatement of charges.
- Documentation of Reasons: Obligation to document in detail the reasons that support the decision to reformulate the charges.
- Right to reply: Guarantee a mechanism for the defense to challenge the reformulation of the charges.

E2. Improve training and education

- Judicial Training in Fundamental Rights: Continuing education programs for judges and prosecutors on respect for fundamental rights during the process.
- Raising awareness on the impact of job restatements: Workshops on the ethical and legal implications of job restatements.
- Updates on Procedures and Regulations: Courses on the latest legislative updates and best judicial practices.

E3. Improvements to the legal framework

- Review of Current Legislation: Analysis and proposal of modifications to current legislation to limit the arbitrary reformulation of charges.
- Creation of Legal Safeguards: Implementation of protection mechanisms to prevent abuses in the reformulation of positions.
- Clarification of Roles and Responsibilities: Precise definition of the responsibilities of the Prosecution and the defense in the process.

E4. Strengthening the role of defense

- Access to defense resources: Improve access to legal resources and legal assistance for defendants.

- Protection against unwarranted changes: Establish procedures to protect defendants from unwarranted changes in charges.

E5. Promotion of restorative justice

- Alternatives to re-charging: exploring the use of restorative justice as an alternative to re-charging.
- Victim Inclusion in the Process: Promote an approach that includes the victim in decision-making regarding the reformulation of charges.
- Remedial Measures: Implementation of remedial measures that may be considered before proceeding to reformulation of charges.

Experts selected to participate in the research were asked to conduct peer comparison evaluations of the previously identified measures. Each DM recorded their ratings on pairwise comparison matrices, using the triangular neutrosophic linguistic scale provided specifically for this study. Table 1 shows the pairwise comparison responses provided by DMs for the five main strategies.

Table 1: Pairwise comparison matrix for the main selected strategies.

	E1	E2	E3	E4	E5
E1	VI	PI	EI	VSI	VSI
E2	VSI	EI	VSI	VSI	AI
E3	VI	WI	EI	VSI	AI
E4	PI	WI	PI	EI	VSI
E5	VSI	VWI	WI	PI	EI
DM2					
E1	VI	PI	EI	VSI	VSI
E2	PI	EI	VSI	VSI	VSI
E3	VI	WI	EI	VSI	AI
E4	PI	WI	PI	EI	VSI
E5	VSI	VWI	WI	PI	EI
DM3					
E1	PI	EI	VSI	EI	VI
E2	AI	EI	VI	VSI	VSI
E3	VI	PI	EI	VSI	VSI
E4	WI	WI	PI	EI	VI
E5	PI	WI	WI	PI	EI

From the collected data, the plithogenic aggregation operator, which is based on the equidistant degree of contradiction between each criterion and the dominant criterion, is used to construct the aggregate matrix. It is stipulated that the degrees of contradiction between the criteria and the dominant criterion are established as follows: $c_1 = 0, c_2 = 0.2, c_3 = 0.4, c_4 = 0.6, c_5 = 0.8$. This plithogenic aggregation methodology contributes to increasing the precision in the synthesis of the data. The aggregate matrix is calculated by applying Equation 1. This matrix is then transformed into a matrix of crisp values using Equation 8.

In the final stage, the normalization process is implemented to calculate and rank the local weight of the criteria, which facilitates their ranking according to their relevance. This procedure ensures a detailed and precise evaluation of the measures, thus allowing a well-founded classification of the strategies to be implemented to strengthen due process, effective judicial protection and legal certainty in the event of the reformulation of charges by the Public Prosecutor's Office during trial hearings. See Table 2.

Table 2: Normalized matrix with weight vector for each strategy.

	E1	E2	E3	E4	E5	Weight
E1	0.19	0.2	0.19	0.2	0.2	0.198
E2	0.24	0.2	0.2	0.19	0.2	0.206
E3	0.24	0.2	0.2	0.19	0.19	0.207
E4	0.11	0.2	0.2	0.2	0.2	0.183
E5	0.22	0.2	0.2	0.21	0.2	0.206

In order to establish the precise values, local weight and hierarchical structure of the proposed measures, a consistent procedure is adopted. This methodological approach allows, from the data collected, to generate a set of matrices that are subsequently consolidated into a comprehensive matrix. This matrix efficiently compiles the weights or degrees of influence of each measure, both in the global context and within their respective categories or strategies. Table 3 provides a summary of the findings derived from this process.

Table 3: Local and global weights derived from the analysis performed.

Strategies	Weight	Measures	Local weight	Overall weight
Ensure transparency in the process of reformulating positions	0.20	Clarity in reformulation criteria	0.116	0.023
		Pre-defense notification	0.144	0.029
		Documentation of reasons	0.468	0.093
		Right to reply	0.272	0.054
Improving training and education	0.21	Judicial Training in Fundamental Rights	0.590	0.122
		Awareness of the impact of job reformulation	0.325	0.067
		Updates on procedures and regulations	0.085	0.018
Improvements in the legal framework	0.21	Review of current legislation	0.590	0.122
		Creation of legal guarantees	0.325	0.067
		Clarification of roles and responsibilities	0.085	0.018
Strengthening the role of defense	0.18	Access to defense resources	0.550	0.101
		Protection against unjustified changes	0.450	0.083
Promoting restorative justice	0.21	Alternatives to the reformulation of charges	0.091	0.019
		Inclusion of the victim in the process	0.391	0.080
		Remedial measures	0.518	0.107

The findings of this study indicate that, at a strategic level, there is a relatively even distribution of weights between the different strategies proposed, with the strategies focused on “Improving Training and Education”, “Improvements to the Legal Framework” and “Promoting Restorative Justice” being the most preferred according to the assessment of the experts consulted. These results emphasize the importance of establishing transparent and reasoned procedures, underlining the need for ongoing training and pointing out the urgency of updating and improving existing legislation.

At the global level, measures aimed at improving judicial training and education were given a prominent place, especially those related to “Judicial Training in Fundamental Rights”. This aspect reflects a unanimous consensus on the urgent need to provide justice operators with comprehensive knowledge of fundamental rights, an element considered vital to strengthening due process. This convergence of opinions highlights the recognition of judicial training as a fundamental pillar for promoting a more just and effective justice system.

Furthermore, the “Review of Current Legislation” was identified as a high-priority measure, showing an imminent demand for legislative reforms that ensure fair and efficient judicial procedures. The allocation of the greatest local weight to this measure underlines the perception of a critical need to adapt and modernize the legal framework to adequately respond to contemporary challenges of the justice system. [16, 17, 18]

The assessment of the measure related to “Access to Defense Resources” demonstrated its critical relevance to ensure the full exercise of the right to defense, an indispensable pillar for the integrity of the judicial system. The assignment of significant weight to this measure reflects the collective awareness of the importance of an effective defense as an essential component of a fair and transparent judicial process. For their part, the “Reparation Measures,” which receive special attention within this strategy, highlight the tendency to seek solutions that prioritize the repair of damage and reconciliation between the parties involved over the imposition of sanctions. This approach not only represents a step towards more humanized judicial systems and effective conflict resolution, but also underlines the evolution of legal thought towards paradigms that emphasize restoration and reparation as preferred mechanisms of justice.

The application of plithogenic logic and neutrosophic AHP, enriched with the contributions of plithogenic logic, to address legal issues, particularly in the evaluation and prioritization of strategies aimed at strengthening due process, effective judicial protection and legal certainty, has revealed significant implications for both legal theory and practice. The adoption of these advanced methodological approaches represents a paradigmatic advance in the way of analyzing and resolving legal challenges, offering a more nuanced and in-depth view of the complexities inherent to the judicial system [19, 20, 21].

From a theoretical perspective, the application of plithogenic logic and neutrosophic AHP underlines the importance of considering ambiguity, uncertainty and contradiction as intrinsic elements of legal decision-making. Traditionally, legal analysis has sought clarity and certainty, undeniably valuable principles. However, legal reality often faces situations where these ideals are not fully achievable due to the intrinsically complex nature of legal issues. Plithogenic logic, by allowing for a richer and more detailed representation of judgments and preferences, together with neutrosophic HPA, which facilitates the structuring and evaluation of these complexities, provides a solid framework for addressing such multidimensionality [22, 23, 24].

In practical terms, the use of these approaches provides legal practitioners with powerful tools for critical evaluation and prioritization of legal and judicial measures. The ability to quantitatively and systematically analyze the degrees of importance, contradiction and ambiguity of expert opinions allows for more informed and thoughtful deliberation. This is especially relevant in the context of legal reforms and judicial policies, where well-informed decisions can have significant effects on the effectiveness and fairness of the justice system.

Furthermore, the integration of plithogenic logic and neutrosophic AHP in legal problem solving promotes greater inclusion and participation in the decision-making process. By capturing a broader range of perspectives and assessments, these advanced methods move towards more democratic and transparent decision-making in the legal field, recognizing the diversity of opinions and experiences among different actors in the judicial system.

The application of these innovative approaches in legal analysis not only improves the quality and accuracy of decisions made, but also lays the groundwork for future research at the intersection of logic, decision theory, and law. As the field of law continues to evolve in response to complex and dynamic challenges, the adoption of advanced analytical tools such as plithogenic logic and neutrosophic AHP may become essential components to address these complexities with greater effectiveness and insight.

5. Conclusion

The present study focused on assessing and prioritizing strategies and measures to strengthen due process, effective judicial protection, and legal certainty, specifically in the context of the reformulation of charges by the Public Prosecutor's Office during oral trial hearings. Plithogenic logic and the neutrosophic AHP were identified as the central methodological tools for this study. The former provided a framework to manage the uncertainty,

ambiguity, and contradictions inherent in legal analysis. The use of the neutrosophic AHP provided a systematic structure to decompose and assess the complexity of the problem into hierarchical levels, taking into account the elements of indeterminacy characteristic of real life. The integration of these methods facilitated the quantitative and qualitative assessment of strategies and measures, thus allowing for a deeper understanding and more rigorous analysis of the options available to improve the judicial system.

The results obtained indicate a critical need to implement strategies that promote transparency in the judicial process, strengthen the training and capacity building of justice operators, update and refine the legal framework, strengthen the role of the defense and encourage alternative approaches such as restorative justice. These areas, identified as priorities by the consulted experts, reflect a consensus on the fundamental aspects that require attention in order to move towards a more just, equitable and effective justice system. The findings provide legislators, judges, prosecutors, other justice operators and academics with a solid basis on which to design and implement interventions aimed at improving the justice system. The study illustrates the value of applying advanced decision-making methodologies in the analysis of legal problems, highlighting how these tools can facilitate more rigorous deliberation and more informed decision-making in the legal field.

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