



# Enhancing Decision-Making in Uncertain Environments: The Role of Neutrosophic Cognitive Maps in Analyzing Complex Systems

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## Abstract

Using Neutrosophic Cognitive Maps (NCM), this research tackles the very core of one of the main problems that exist in the analysis of complex structures and systems: how to represent and model decision making in situations of uncertainty, or where there is contradiction and ambiguity. This problem gets even worse in the areas of knowledge management, strategic evaluation, or design of public policies since orthodox methods do not always possess required versatility for combining partially available or contradicting information. To address this challenge, the researchers recommend Neutrosophic Cognitive Maps (NCM) as a more appropriate technique considering that the neutrosophic logic can depict and study more intricate relationships in the presence of indeterminacy. There is an iterative learning of cognitive maps which is coupled with neutrosophic analysis techniques enabling the construction of a comprehensive model capturing both certainties and the undefined and disputed areas of the evaluated systems. The findings obtained in this research demonstrate how effective NCMs are in the spatial and analytic representation of complex and multifactorial situations providing features that go beyond the conventional structure models. Apart from broadening theorization on decision-making processes in an uncertain situation, this research provides practical tools applicable in sectors such as strategic planning, complex problem solving and organizational management. In short, the study shows that neutrosophic, used as a methodological catalyst, not only expands the possibilities of analysis but also transforms how complex systems are conceptualized and managed in the academic and practical fields.

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## 1. Introduction

Neutrosophic theory, proposed by Florentin Smarandache [1], is presented as an advanced theory in the field of logic and philosophy, which provides a unique perspective to address uncertainty, ambiguity, and contradictions, critical aspects in the complex field of maritime law. This theory stands out for its influence in several key aspects.

Initially, it offers a more dynamic and detailed approach to the analysis of maritime laws and regulations, which are often in constant flux due to the international nature and rigorous regulation of the maritime sector. Thanks to its ability to address ambiguity and variability in levels of truth, Neutrosophic can improve the interpretation of laws that vary between different jurisdictions or those subject to diverse interpretations, providing fairer and more appropriate solutions that capture the complexity of maritime situations.

Furthermore, maritime law frequently faces situations where information is partial or contradictory, such as in disputes over pollution, accidents or the transit of vessels through disputed waters. In this context, neutrosophic

provides an approach to assessing statements and evidence by considering levels of certainty, falsity, and neutrality, allowing for more equitable and informed decisions by taking into account all elements of a case, including those traditionally considered ambiguous or irrelevant.

The application of Neutrosophic to maritime law analysis also promises to revolutionize the way international laws and treaties are designed. In the management of shared maritime resources, where the interests of multiple parties must be balanced, Neutrosophic logic can facilitate the creation of agreements that explicitly acknowledge and manage areas of uncertainty and conflict, promoting more durable and mutually beneficial solutions.

In terms of maritime conflict resolution, neutrosophic could lay the groundwork for developing more versatile and comprehensive arbitration and mediation methods. By accepting that the positions of the parties in a conflict may have elements of truth and error at the same time, a more collaborative and less confrontational resolution methodology is promoted, seeking solutions that come close to satisfying the interests and needs of all the parties involved.

## 2. Related Works.

### 2.1 Neutrosophic Cognitive Maps

In this study, Neutrosophic Cognitive Maps will be used, so we explain them below.

**Definition 1:** Let  $X$  be a universe of discourse. A neutrosophic set (NS) is characterized by three membership functions,  $u_A(x), r_A(x), v_A(x) : X \rightarrow ]-0,1+[$ , which satisfy the condition  $-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$  for all  $x \in X$ .  $u_A(x), r_A(x)$ , and  $v_A(x)$  are the truthiness, indeterminacy, and falsity membership functions of  $x$  in  $A$ , respectively, and their images are standard or nonstandard subsets of  $]-0,1+[$ . [15]

**Definition 2:** Let  $X$  be a universe of discourse. A single-valued neutrosophic set (SVNS)  $A$  over  $X$  is a set of the form [16] :

$$A = \{ \langle x, u_A(x), r_A(x), v_A(x) \rangle : x \in X \} \quad (1)$$

Where  $u_A, r_A, v_A : X \rightarrow [0,1]$ , satisfy the condition  $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$  for all  $x \in X$ .  $u_A(x), r_A(x)$ , and  $v_A(x)$  are the truthiness, indeterminacy, and falsity membership functions of  $x$  in  $A$ , respectively. For convenience, a Single-Valued Neutrosophic Number (SVNN) will be expressed as  $A = (a, b, c)$ , where  $a, b, c \in [0,1]$  and satisfy  $0 \leq a + b + c \leq 3$ .

Other important definitions are related to graphics.

**Definition 3:** A *neutrosophic graph* contains at least one indeterminate edge, represented by dotted lines [17].

**Definition 4:** A *neutrosophic directed graph* is a directed graph containing at least one indeterminate edge, which is represented by dotted lines [18].

**Definition 5:** A *neutrosophic cognitive map* (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships between the edges [19,28].

If there are  $k$  vertices  $C_1, C_2, \dots, C_k$ , each can be represented by a vector  $(x_1, x_2, \dots, x_k)$  where  $x_i \in \{0,1,I\}$  according to the state of vertex  $C_i$  at a specific time or situation:

- $x_i = 0$ : Vertex  $C_i$  is in an activated state.
- $x_i = 1$ : Vertex  $C_i$  is in a disabled state.
- $x_i = I$ : The state of vertex  $C_i$  is indeterminate.

**Definition 6 [20,21,22]:** An NCM that has edges with weights in  $\{-1, 0, 1, I\}$  is called a *Simple Neutrosophic Cognitive Map* [20].

Connections between vertices: A directed edge from  $C_m$  to  $C_n$  is called a connection and represents causality from  $C_m$  to  $C_n$ .

Associating weights to each vertex: Each vertex in the NCM is associated with a weight within the set  $\{0, 1, -1, I\}$ . The edge weight  $C_m C_n$ , denoted as  $\alpha_{mn}$ , indicates the influence of  $C_m$  on  $C_n$  and can be:

- $\alpha_{mn} = 0$ :  $C_m$  has no effect on  $C_n$ .
- $\alpha_{mn} = 1$ : An increase (decrease) in  $C_m$  results in an increase (decrease) in  $C_n$ .
- $\alpha_{mn} = -1$ : An increase (decrease) in  $C_m$  results in a decrease (increase) in  $C_n$ .
- $\alpha_{mn} = I$ : The effect of  $C_m$  on  $C_n$  is undetermined.

**Definition 7:** If  $C_1, C_2, \dots, C_k$  are the vertices of an NCM. The neutrosophic matrix  $N(E)$  is defined as  $N(E) = \alpha_{mn}$ , where  $\alpha_{mn}$  denotes the weight of the directed edge  $C_m C_n$ , with  $\alpha_{mn} \in [-1, 0, 1, I]$ .  $N(E)$  is called *the neutrosophic adjacency matrix* of the NCM.

**Definition 8:** Let  $C_1, C_2, \dots, C_k$  be the vertices of an NCM. Let  $A = (a_1, a_2, \dots, a_k)$ , where  $a_m \in \{-1, 0, 1, I\}$ .  $A$  is called *the neutrosophic instantaneous state vector* and means an on-off-indeterminate state position of the vertex at a given instant.

- $a_m = 0$  if  $C_m$  is disabled (no effect),
- $a_m = 1$  if  $C_m$  is activated (has an effect),
- $a_m = I$  if  $C_m$  is indeterminate (its effect cannot be determined).

**Definition 9:** Let  $C_1, C_2, \dots, C_k$  be the vertices of a NCM. Let be  $\overrightarrow{C_1 C_2}, \overrightarrow{C_2 C_3}, \overrightarrow{C_3 C_4}, \dots, \overrightarrow{C_m C_n}$  the edges of the NCM, then the edges constitute a *directed cycle*.

NCM is said to be *cyclic* if it has a directed cycle. It is said to be *acyclic* if you do not have any directed cycles.

**Definition 10:** An NCM containing loops is said to have *feedback*. When there is feedback in the NCM, it is said to be a *dynamical system*.

**Definition 11:** Let  $\overrightarrow{C_1 C_2}, \overrightarrow{C_2 C_3}, \overrightarrow{C_3 C_4}, \dots, \overrightarrow{C_{k-1} C_k}$  be a cycle. When  $C_m$  is activated and its causality flows through the edges of the cycle and is then the cause of  $C_m$  itself, then the dynamical system is cycling. This is true for each vertex  $C_m$  with  $m = 1, 2, \dots, k$ . The equilibrium state for this dynamical system is called the *hidden pattern*.

**Definition 12:** If the equilibrium state of a dynamical system is a single state, then it is called a *fixed point*. An example of a fixed point is when a dynamical system starts off being activated by  $C_1$ . If the NCM is assumed to be set at  $C_1$  and  $C_k$ , meaning the state remains as  $(1, 0, \dots, 0, 1)$ , then this neutrosophic state vector is called a fixed point.

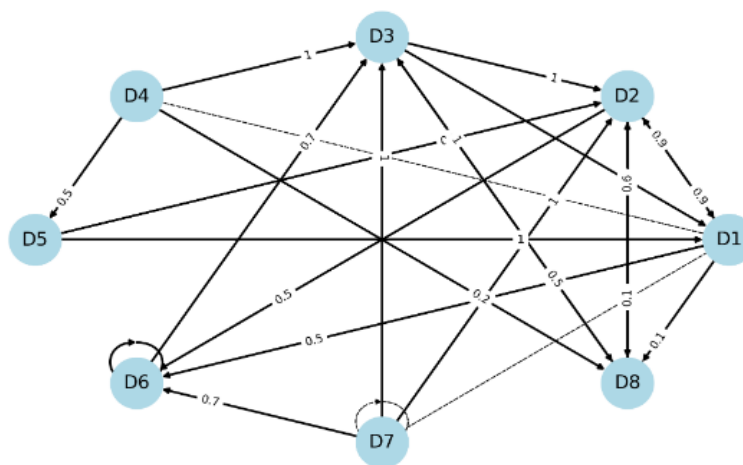
**Definition 13:** If the NCM establishes a neutrosophic state vector that repeats in the form:

$$A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_m \rightarrow A_1 \text{ LCM limit cycle [23, 24, 25].}$$

### 3. Results and discussion

Given the complexity and multifaceted nature of the relationships in the field of maritime law in Ecuador, it is suggested to adopt a methodological approach designed to delve deeper into these thematic links. As a starting point, the objective of the study was established, focusing on demonstrating how maritime legal analysis could be enriched by the application of a Neutrosophic Cognitive Map (NCM). This was followed by a detailed collection of information, which included both the examination of specialized bibliographic sources and consultations with experts in relevant fields. [ 21,22 ]

After identifying eight key areas in which Ecuadorian maritime law is deficient, as detailed in Table 2, a NCM was developed to illustrate the causal connections between these elements. This process involved defining the interactions between the various factors and their subsequent visualization through a neutrosophic cognitive map (Figure 1).



**Figure 1.** Neutrosophic cognitive map.

Source: Own elaboration.

The NCM is developed from the collection and representation of relevant knowledge. The adjacency matrix obtained, which is based on the neutrosophic values provided by the specialists, is detailed in Table 1 as an essential tool for the analysis and interpretation of causal connections within the framework of the study.

**Table 1:** Adjacency matrix.

	<b>D1</b>	<b>D2</b>	<b>D3</b>	<b>D4</b>	<b>D5</b>	<b>D6</b>	<b>D7</b>	<b>D8</b>
<b>D1</b>	0	0	0	0	Yo	0.5	0	0.1
<b>D2</b>	0.9	0	0	0	Yo	0.5	0	0.1
<b>D3</b>	1	1	0	0	0	0	Yo	0.5
<b>D4</b>	Yo	0	1	0	0.5	0	0	0.2
<b>D5</b>	1	0.1	0	0	0	0	0	0
<b>D6</b>	0	0	0.7	0	0	0	0	0
<b>D7</b>	Yo	1	1	0	0	0.7	0	0
<b>D8</b>	Yo	0.6	1	0	0	0.1	0	0

Source: Own elaboration.

Following this perspective, the calculated centrality measures are presented below (Table 2). These metrics provide a quantitative analysis of the relative relevance of nodes within the network framework, which is crucial to understand the dynamics and impact of the various components in the analyzed system.

**Table 2:** Centrality analysis.

<b>Node</b>	<i>of(I saw)</i>	<i>identification(vi)</i>	<i>td(vi)</i>
<b>D1</b>	0.6+I	2.9+3I	3.5+4I
<b>D2</b>	1.5+yo	2.7	4.2+I
<b>D3</b>	2.5+yo	3.7	6.2+I
<b>D4</b>	1.7+I	0	1.7+I
<b>D5</b>	1.1	0.5+2I	1.6+2I
<b>D6</b>	0.7	1.8	2.5
<b>D7</b>	2.7+yo	0+me	2.7+2I
<b>D8</b>	1.7+I	0.9	2.6+I

Source: Own elaboration.

In the context of static analysis in the NCM, initial results are obtained that incorporate the element of indeterminacy “I” within their neutrosophic values. To refine these results, it is essential to carry out a process known as deneutrosophication, recommended by [23]. This process involves replacing the indeterminacy parameter I, which ranges between 0 and 1, considering in this case “I” as 0.5. The importance of this method lies in its ability to produce more defined and precise results, which significantly simplifies the understanding of the interconnections present in the analysis in question (Table 3).

**Table 3:** Deneutrosophized centrality.

<b>Node</b>	<i>td(vi)</i>
<b>D1</b>	5.5
<b>D2</b>	4.7
<b>D3</b>	6.7
<b>D4</b>	2.2
<b>D5</b>	2.6
<b>D6</b>	2.5
<b>D7</b>	3.7
<b>D8</b>	3.1

Source: Own elaboration.

These values reflect the strength of the outgoing and incoming relationships, as well as the overall centrality of each node within the cognitive map, after considering indeterminacy as a medium-degree Neutrosophic influence. Neutrosophic analysis, applied to the deficiencies of maritime law in Ecuador, provides a unique tool to assess the complexity and interrelations between various factors that affect the effectiveness of maritime regulation in the country. The adoption of the NCM allows to visualize and quantify how the different elements interact with each other, including legal, regulatory, infrastructure and environmental protection aspects, among others. Table 1, by

presenting the neutrosophic adjacency matrix, facilitates the understanding of the causal relationships between the various elements identified [26, 27, 28]. Serve to quantify the relative importance of each element within the network of maritime law deficiencies. The application of deneutrosophication (Table 3) clarifies and simplifies the analysis by considering indeterminacy with a neutrosophic value, which is essential for a better understanding of the complex and multifaceted dynamics of maritime law in Ecuador.

From Table 3, it can be observed that D1 (Outdated legal framework) and D3 (Infrastructure and logistics) are the nodes with the highest total centrality (5.5 and 6.7 respectively), indicating that they are the areas of greatest impact within the maritime law system in Ecuador. This suggests that updates to the legal framework and improvements in infrastructure and logistics could have the most significant effects on improving the system.

The adjacency matrix and causal relationships illustrated show how certain deficiencies have a direct impact on others. For example, the deficiency in international cooperation (D8) directly influences marine resource management (D3) and training and education (D7), reflecting the importance of international cooperation in these areas.

Deneutrosophication process in the analysis lies in its ability to provide a more accurate and detailed interpretation of the results. The assignment of a mean value to the indeterminacy “I” improves the comparison and analysis regarding the relevance and impact of various deficiencies. On the other hand, the application of the NCM together with the deneutrosophication procedure proves to be an effective tool to examine and understand the complex interactions between deficiencies in the framework of maritime law in Ecuador.

#### 4. Conclusion

The present study aimed to explore and analyze the deficiencies in the field of maritime law in Ecuador, applying the neutrosophic theory and the use of Neutrosophic Cognitive Maps (NCM). The neutrosophic methodology allowed not only to identify but also to quantify the causal relationships between various deficiencies detected in the legal and practical framework of the maritime sector. Deneutrosophication analysis, have revealed which aspects of maritime law require primary attention. Among them, the outdated legal framework and challenges in infrastructure and logistics emerged as nodes of greatest impact, pointing out clear guidelines for future reforms and improvements. Furthermore, the research highlighted the importance of international cooperation and effective management of marine resources, underlining how global interconnection decisively influences the effectiveness of national maritime law. The application of NCM and neutrosophic theory has not only allowed for a richer and more nuanced understanding of the shortcomings of Ecuadorian maritime law but has also provided a replicable methodology for analyzing complex legal systems in other contexts. This methodological approach opens new avenues for legal analysis, offering the possibility of addressing uncertainty and indeterminacy in a structured and meaningful way.

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