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FULL LENGTH ARTICLE

Stochastic Diffusion Process Models for Driving Innovation in Market-Driven Product Development

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ABSTRACT

In order to analyse the diffusion of new technological products in rapidly changing market environments, this paper presents two new stochastic diffusion models: SDM1 and SDM2. The two models also utilize a stochastic market-size function in capturing the rather random growth of potential users inherent in most real-world markets. SDM1 applies the exponential distribution to model the market growth rate for cases characterized by high increase, while SDM2 adapts the Erlang distribution to reflect the S-shape associated with long-term adoption. The presented models rely on stochastic differential equations with recourse to calculus, and they adopt stochastic geometric Brownian motion and a logistic growth function for adoption rates. This makes it possible to capture effects of learning as well as the non-regularity of adoption over time. Empirical results from benchmark models using Apple iPhone and Samsung Galaxy smartphone sales data show the better performance of SDM1 and SDM2. Performance is measured using model parameters, goodness-of-fit tests, and forecast accuracy, all of which show that the proposed methods are very efficient. These models have a rich theoretical background that provides a foundation for explaining adoption patterns and facilitates managers' and policymakers' understanding of consumers, inventory control, and the design of significant marketing strategies for technology products in a stochastic world. Both suggested algorithms outperform state-of-the-art techniques in terms of accuracy. SDM1 outperforms the other models with an accuracy of 95.32 percent, while SDM2 achieves the greater forecasting accuracy of 97.3 percent.

Keywords: SDM2 ▪ SDM1 ▪ MAPE ▪ SDE ▪ RMSE ▪ MSE ▪ MAE

1. INTRODUCTION

Adoption of technical breakthroughs is a significant factor that plays an important role in changing the structure of the market, including consumer behaviour. On the other hand, projecting the development of new goods in real-world settings is a difficult undertaking because of the inherent unpredictability present in customer tastes, market changes, and external variables [1, 2]. Despite their widespread use, conventional probabilistic diffusion models are not capable of effectively capturing the unpredictable and ever-changing character of market expansion. This work presents two sophisticated stochastic diffusion models, SDM1 and SDM2, which combine dynamic market shares and spurious changes to offer an accurate illustration of the adoption method. The purpose of this investigation is to address these shortcomings. The frameworks are intended to improve our knowledge of diffusion processes, especially in markets characterised by rapid technical improvements and unpredictable customer behaviour.

It is crucial for organisations to have a solid understanding of the diffusion process in order to optimise advertising approaches, inventory management, and manufacturing scheduling. The vast majority of traditional models assume that mar-

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ket size is fixed and that adoption rates are predetermined. This restricts their application in real-world settings, which are characterised by revenue potential and adoption behaviour affected by externalities [3, 4]. Through the incorporation of stochastic components into the modelling framework, SDM1 and SDM2 are able to tackle these deficiencies. The incorporation of Brownian motion and changing development parameters results in a strong technique for projecting the widespread use of breakthroughs over time.

Perhaps the most important aspect of SDM1 is its use of an exponential distribution to depict market expansion. This hypothesis depicts the rapid initial growth often seen in the early phases of technological acceptance [5, 6], when prospective adopters rise rapidly because of vigorous advertising and growing consumer awareness. SDM2, on the other hand, uses an Erlang distribution that represents market expansion with an S-shaped curve. This curve reflects a more gradual diffusion procedure that takes both early adopters and late-stage customers into consideration. Together, the models accommodate a wide variety of market conditions and are therefore flexible across goods and businesses.

The use of Itô calculus and stochastic differential equations (SDEs) allows the stochastic aspect of SDM1 and SDM2 to be realised. A realistic picture of the diffusion process may be obtained through these computational methods, which allow unplanned changes to be included in penetration rates and increasing market size [7]. The use of geometric Brownian equations ensures that estimates take account of quirks and ambiguities in market functioning, including shifts in customer preferences, variations in financial situations, and technological developments. This method greatly improves the forecasting capacity and precision of the algorithms, making them suitable for unpredictable and complicated market situations.

Scientific verification of SDM1 and SDM2 reveals that both are preferable to conventional diffusion models. Using practical sales information from products such as Apple iPhones and Samsung Galaxy mobile devices, these projections provide a more accurate representation of observed usage trends and exceed benchmark simulations in forecast accuracy [8]. Key measures illustrating their usefulness include root mean squared error (RMSE), mean absolute percentage error (MAPE), and the coefficient of determination (R^2). Beyond their mathematical soundness, the models enable businesses to construct focused marketing efforts, optimise manufacturing schedules, and establish efficient stock-control methods. They are useful to executives and lawmakers because their stochastic elements offer an achievable and complete structure for understanding innovation diffusion in a chaotic market context.

A comprehensive examination of the approach behind SDM1 and SDM2 is presented in the subsequent sections [9]. This examination covers their mathematical structure, essential assumptions, and implementation situations. Statistical evaluation and outcome assessment are also explored to demonstrate their practical importance and usefulness in forecasting the acceptance of technical breakthroughs. The investigation intends to advance diffusion modelling and offer a solid basis for future studies and applications in innovation management.

Consumers may embrace an invention or service through a series of steps collectively referred to as the implementation procedure. Fundamental behavioural theory suggests that a discernible period elapses between a potential purchaser becoming aware of an invention and deciding to accept it. The adoption method is thus a time-varying sequential operation. A prospective purchaser undergoes several phases before finally deciding to embrace the innovation. These phases are arranged hierarchically as information understanding, persuasion, decision making, execution, and verification [10]. The outcome of each phase may affect the pattern of invention dissemination. The stages of the adoption process are shown in Figure 1.

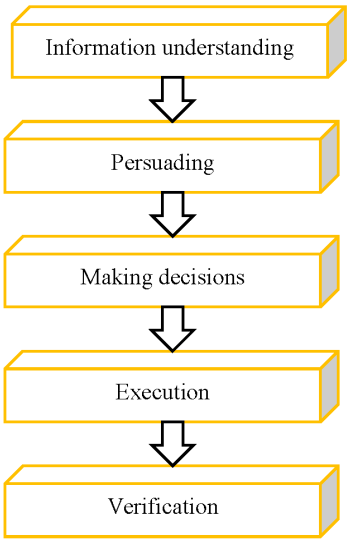

Figure 1. An overview of ANN's design.

A significant contribution to the procedure for creative dispersion is made by the promotional elements of the invention. A business's capacity to monitor and oversee these factors over an indefinite length of time is the only factor that determines whether an invention will flourish in the short term or over the long run. This part provides an overview of three important market factors—sale price, advertising, and guarantee term—used in the current research to obtain crucial management insights. Pricing may be an especially complicated and effective marketing tactic, and it is the primary factor in consumers' purchasing choices [11]. Market share and profitability are both significantly affected by sale price. Commercial enterprises therefore endeavour to maintain a tight grip on the prices at which they sell products while catering to diverse customer requirements.

An appealing price can make it easier for manufacturers to increase demand over the long run. Companies should therefore maintain reasonable control over their selling prices. Sales research has shown that demand tends to decline whenever the sale price increases [12]. Because of the correlation between sale price and desire for the good, selling price is a primary consideration in determining continued popularity. Marketing has been described as "any monetary means of informal dissemination of products, services, or ideas by a recognised sponsor," according to the American Marketing Association. A commercial enterprise uses advertising to inform, convince, or remind its intended customer base about the products or services it offers. Promotions and advertising

help businesses improve the personal worth of their innovations and are among the most significant forms of marketing communication. Marketing techniques for new items therefore serve as an effective medium for increasing demand without creating a danger of failure [13]. Manufacturers likewise need to give their promotional and advertising tactics close attention because these methods significantly affect the profitability of newly launched items.

2. RELATED WORK

Innovation diffusion modelling has developed actively as a distinct field during recent decades, with key work centred on the Bass diffusion model. The Bass model provided a deterministic framework describing innovation adoption through two key parameters: the innovation and imitation coefficients. However, the assumptions inherent in the Bass model were static and could not capture such dynamics, hence the need to explore stochastic influences. Subsequent research elaborated diffusion models incorporating dynamic market sizes and attempted to capture temporal changes in adopter potential [15]. Although these models included variability to some degree, they remained mostly deterministic in their estimation of adoption rates. Future work was therefore directed toward more sophisticated stochastic models, such as geometric Brownian motion and other probabilistic tools, to model market fluctuations more efficiently and accurately.

Later work incorporated stochastic characteristics into diffusion models in order to capture uneven fluctuations in adoption rates. This research focused on randomness in consumer behaviour by prescribing models that combine deterministic adoption functions with stochastic noise [16]. The integration marked a shift toward more realistic modelling frameworks that reconcile theoretical models with observed market conditions. It led to further improvements related to the suggested approach, namely the stochastic diffusion models SDM1 and SDM2. Stochastic differential equations and Itô calculus provide a sound and versatile mathematical foundation for incorporating randomness into market-growth and adoption functions.

Other researchers extended this line of work using stochastic processes, including Wiener processes and geometric Brownian motion, to model dynamic uncertainty. These methodologies are central to SDM1 and SDM2, which deploy SDEs to model both adoption rates and market size under uncertainty. Recent work has also highlighted the need to incorporate dynamic market size into available diffusion models. Market expansion is often formulated using exponential growth functions that demonstrate rapid expansion when a product is newly introduced [17]. These functions are incorporated directly into SDM1, which assumes an exponential distribution for market size. Research examining S-shaped logistic growth in market expansion provides the theoretical premise for the Erlang distribution employed in SDM2.

Stochastic innovation diffusion models have been especially widely used in industries with high levels of technology implementation [18]. Researchers have used stochastic models to predict consumer-electronics and automobile markets, emphasising the need to introduce stochastic components in order to account for external causes such as economic activity, technologies, and campaigns. SDM1 and SDM2 include stochastic factors in both adoption and growth curves to prevent overly optimistic predictions. Empirical evidence also indicates that stochastic diffusion models are superior to deterministic alternatives. Tests using actual sales data have repeatedly shown that stochastic models with dynamic market size and random disturbance provide better parameter estimates and goodness of fit [19]. These empirical validations support the broader development and use of complex stochastic models such as SDM1 and SDM2.

Stochastic diffusion models have also been considered from the perspective of marketing and inventory control. Forecasts that determine inventory needs and production plans benefit from stochastic models. Such models can help businesses predict market occurrences and align production plans accordingly [20]. Stochastic modelling is similarly critical to achieving marketing objectives for technology products that are adopted at different rates and times. These observations demonstrate the relevance of SDM1 and SDM2 to real-life decision making.

This line of work has also emphasized the need to achieve an optimal level of model complexity [21]. Excessive complexity has become a potential problem in model development. Research therefore supports models that are theoretically attractive as well as realistically feasible, an objective reflected in SDM1 and SDM2. Predictions from these models do not necessarily require overly complex program modelling, even though geometric Brownian motion and logistic growth functions remain demonstrative and efficient.

The development of other computational methods provides further prospects for stochastic diffusion modelling [22]. Genetic algorithms have been applied to estimate parameters of complex diffusion models and improve predictions. These tools are important for the empirical evaluation of SDM1 and SDM2, which use a genetic algorithm to estimate model parameters and optimise the model. Cross-validation approaches, including rolling cross-validation, have also been used to assess predictive accuracy and competitiveness under different market conditions.

In summary, a large number of publications on stochastic diffusion modelling provide ample conceptual support for the advancement of SDM1 and SDM2. Diffusion modelling has progressed through stochastic structures designed to respond effectively to real markets [23]. The use of stochastic factors in market-size and adoption functions is a major step in this evolution, providing greater precision and realism in predicting the uptake of technological change. On this basis, SDM1 and SDM2 represent state-of-the-art models that account for practical requirements while remaining grounded in the academic tradition.

Cross-validation approaches have been used to assess the predictive accuracy of stochastic models and their competitiveness under different market conditions. The available literature provides conceptual support for the advancement of SDM1 and SDM2 [24]. The use of stochastic factors in market-size and adoption functions is a major advancement that offers greater precision and realism in predicting technological uptake. One difficulty in stochastic modelling is parameter estimation under uncertainty. Investigators have addressed this problem using sophisticated computational methods such as neural networks. When data are inadequate

or noisy, these techniques make it possible to estimate parameters for complex models. SDM1 and SDM2 build on these accomplishments by using genetic algorithms to optimise their settings and increase predictive accuracy.

This computational resilience helps ensure that predictions remain trustworthy even in market settings fraught with uncertainty. Another key contribution to stochastic diffusion modelling is the investigation of noise and its influence on adoption behaviour. Noise, modelled as Gaussian white noise or geometric Brownian motion, can represent unpredictable surroundings that affect adoption rates. Such random elements are integral to SDM1 and SDM2 and allow the models to attain a degree of authenticity that deterministic equivalents cannot achieve.

Empirical validation has repeatedly shown stochastic diffusion models to be better than older deterministic techniques. Studies have found that stochastic models represent real-world data more accurately, particularly in technology-driven sectors. These findings highlight the need for models such as SDM1 and SDM2, which are designed to manage uncertainties inherent in markets for mobile phones, vehicles, and consumer electronics. In addition to their theoretical and empirical merits, stochastic diffusion equations have practical consequences for corporate strategy and public policy. They may provide precise demand estimates that inform inventory management and manufacturing planning, as well as promotional strategies adapted to changing market conditions.

Recent research has highlighted the importance of continuously evaluating and validating models to ensure resilience. Rolling cross-validation evaluates predictive performance on previously unseen data and lowers the danger of overfitting. This approach is particularly pertinent to SDM1 and SDM2 because they are intended to function across volatile market conditions. Such validation preserves predictive effectiveness and flexibility. The models' adaptability also derives from their capacity to depict a wide range of market phenomena. Recurrent purchasing, for example, is addressed by SDM2, whose Erlang distribution allows progressive market expansion and interactions with returning customers. SDM1's focus on exponential expansion makes it suitable for products that demonstrate rapid early adoption, such as products driven by viral advertising. The interaction between theoretical development and practical use remains central to the field. SDM1 and SDM2 exemplify this approach by combining stochastic elements, variable market size, and modern computational techniques [25].

3. OBJECTIVE OF THE RESEARCH WORK

The ultimate goal of the SDM1 and SDM2 research is to build sophisticated stochastic diffusion models that reliably predict the uptake of technical breakthroughs in unpredictable and ever-changing market conditions. By including unpredictable components such as random changes in market expansion and adoption rates, these models address the constraints of classic deterministic paradigms. SDM1 uses an exponential distribution to capture rapid early-stage market growth, while SDM2 uses an Erlang distribution to depict slow but steady S-shaped market development. The project synthesises logistic development processes, geometric Brownian motion, and stochastic differential equations into a flexible and plausible model of innovation dissemination. Its final objective is to support superior advertising, inventory, and manufacturing decisions for new technology products by improving forecasting precision and providing insight into customer behaviour.

4. MOTIVATION FOR THE RESEARCH WORK

The inadequacies of conventional deterministic diffusion models in representing the intricacies of actual innovation-acceptance procedures motivated the development of SDM1 and SDM2. Deterministic frameworks often presume constant market sizes and consistent adoption rates, in contrast to the ever-changing and unpredictable character of modern markets driven by socio-economic variables, technical developments, and exogenous variations. The proliferation of technological disruption and unpredictable customer behaviour creates an urgent need for reliable models that include uncertainty and changing market dynamics. SDM1 and SDM2 fill this need by mimicking erratic market growth and adoption rates using stochastic components such as geometric Brownian motion and stochastic differential equations. These frameworks provide an enhanced structure for understanding technology dissemination, conducting accurate forecasts, and making informed strategic decisions.

5. THE PROJECTED METHOD

Unpredictability in a customer's purchase choice may be caused by fluctuations in the components that influence the diffusion process, regardless of how small or extensive the variations may be. Uncertainty in consumers' purchasing decisions can significantly affect ordering and reserve-holding practices among producers, distributors, and sellers. Consequently, a reliable forecast requires stochastic analysis of the diffusion process that accounts for rapidly changing internal and external characteristics.

This section builds an adaptive innovation-diffusion framework that includes variable market size through stochastic differential equations. The total popularity of innovations is assumed to change over time because of rapid developments in high-technology products. Three diffusion frameworks that account for market expansion are developed to explain the fundamental market behaviour of target purchasers. The suggested model evolves in a stochastic setting according to a dynamic process in order to demonstrate that acceptance rates are subject to random variation. Wiener processes and Itô calculus are used to solve the stochastic differential equations. The proposed models are then used to investigate the development trends of consumer durable products.

The block diagram in Figure 2 illustrates the intricate procedures underlying the proposed stochastic diffusion approaches. These procedures model and forecast innovation uptake by adding stochastic elements and dynamic marketplace behaviour. Input variables for SDM1 and SDM2 are established as the basis for both models. These data points are essential for understanding the changing nature of the marketplace and the uncertainty underlying consumer acceptance.

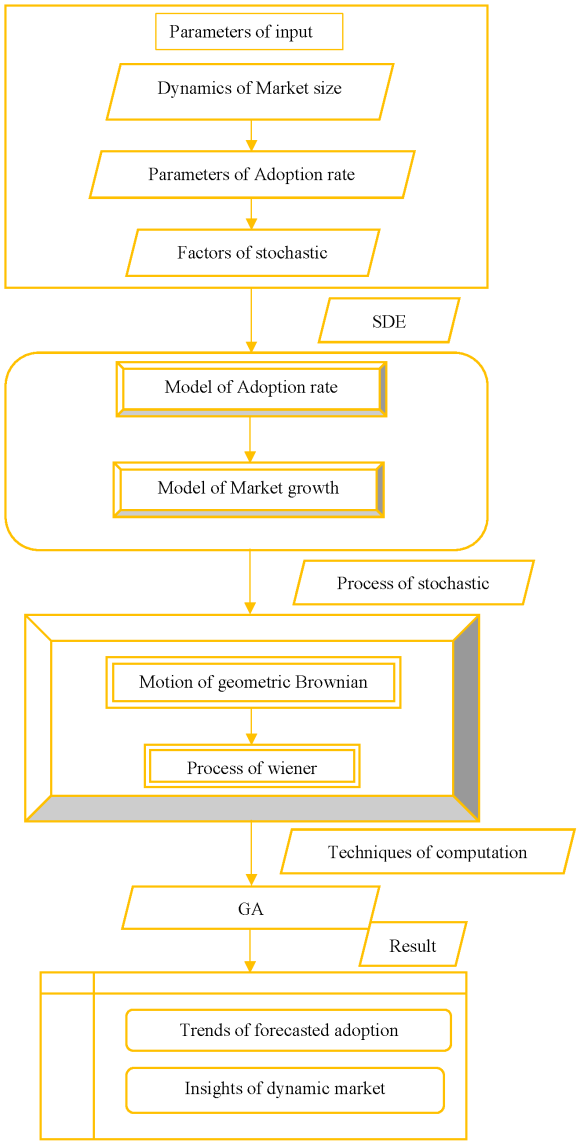

Figure 2. Structure of the suggested approach.

5.1 Dynamics of Market Size

Market-size dynamics describe the segment of new consumers who have the opportunity to acquire an invention during its lifespan. Inputs consist of the initial market size and the greatest feasible market size (m_M), determined by parameters such as product type, methods for market expansion, and customer availability. Adoption rates can be greatly influenced by the stochastic modelling of dynamic market development, which represents practical uncertainty.

5.2 Parameters of Adoption Rate

Logistic growth operations, characterised by parameters such as the implementation-rate factor (c) and learning coefficient (β), regulate the rate of adoption. Within an adoption function, these characteristics represent the effects of social contacts, word-of-mouth, and general media. For example, an elevated value of c signifies rapid acceptance, while a low value of β implies slower market spread that may be affected by customer hesitancy or lack of knowledge.

5.3 Stochastic Factors

Market behaviour is affected by economic swings, technological improvements, and outside events. The real world is essentially unforeseeable. Random variables obtained from stochastic procedures, such as geometric Brownian motion and Gaussian white noise, are used to describe these ambiguities. These stochastic inputs increase the realism of the systems and ensure that they can be used under a wide variety of market conditions.

Stochastic differential equations are at the heart of both SDM1 and SDM2. They provide a conceptual structure for depicting adoption and market expansion under uncertainty.

5.4 Model of Adoption Rate

The adoption-rate model describes how the speed of adoption changes over time and depends on the logistic growth curve. A stochastic component is included in the SDE to compensate for unpredictable variations, while a deterministic component describes the underlying expansion pattern:

$$\frac{dM(t)}{dt} = c\left[F[n(t)] - M(t)\right] + \sigma\gamma(t). \tag{1}$$

$$\frac{dM(t)}{dt} = t\left(n(t) - M(t)\right). \tag{2}$$

$$t = u(t) + n'. \tag{3}$$

$$t = g(t) + \sigma\gamma(t). \tag{4}$$

$$\frac{dM(t)}{dt} = g(t)\left(n(t) - M(t)\right) + \sigma\gamma(t)\left(n(t) - M(t)\right). \tag{5}$$

In the above equations, $M(t)$ reflects the total number of consumers, $F[n(t)]$ indicates the anticipated size of the marketplace, and $\sigma\gamma(t)$ represents random variation in adoption rates. The term $n(t) - M(t)$ shows the remaining potential purchasers who gradually embrace innovations. The function $u(t)$ is taken as a constant adoption rate, while n' causes the rate to fluctuate. Ordinary Gaussian white noise $\gamma(t)$ is a randomised signal, and σ is a positive value representing the size of uneven fluctuations. In other words, the random value is computed using a one-dimensional Wiener process, or Brownian motion.

5.5 Model of Market Growth

The total size of the market is dynamic and changes over time, with unpredictable elements playing a role. In SDM1, marketplace growth follows an exponential distribution:

$$n(v) = n_0 + N\left(1 - e^{-\alpha v}\right). \tag{6}$$

$$n(v) = n_0 e^{\alpha v}, \qquad \alpha > 0. \tag{7}$$

$$\frac{dM(t)}{dt} = g(t)\left(n - M(t)\right). \tag{8}$$

$$g(t) = \frac{e(t)}{1-e(t)}. \tag{9}$$

$$g(t) = \frac{e(t)}{1-e(t)} + n'. \tag{10}$$

Here, n represents the initial size of the marketplace and α is a positive indicator reflecting the pace at which the market is expanding. The symbol t represents the adoption or hazard rate at a given moment, $e(t)$ represents the rate of adoption, and $n - M(t)$ denotes the unrealised market potential not yet purchased by customers. The term n' refers to fluctuations in the adoption rate, while α denotes the increase in that rate. In SDM2, an Erlang distribution represents development in an S-shaped pattern:

$$n(v) = n_0 + N\left[1 - e^{-\alpha v}\left(1+\alpha v\right)\right]. \tag{11}$$

$$g(t) = \frac{e(t)}{1-e(t)} + \sigma\omega(t). \tag{12}$$

Here $\omega(t)$ is a random value known as conventional Gaussian white noise, while σ is a positive parameter specifying the degree of unequal oscillation. These growth functions are incorporated into the SDEs, ensuring that market-size dynamics align with the nature of the product and target audience. By including unpredictability, stochastic procedures capture the intrinsic uncertainty present in real-world market behaviour. An example of a continuous random procedure is the Wiener process, denoted by $Z(t)$:

$$\frac{dZ(t)}{dt} = \omega(t). \tag{13}$$

$$\alpha(t) = \frac{h(t)}{1-H(t)} + \sigma\gamma_t. \tag{14}$$

$$H(t) = 1 - e^{-\alpha t}. \tag{15}$$

Equation (14) depicts the time-dependent rate of market growth, denoted by $h(t)/(1-H(t))$. The symbol σ is a positive value indicating the magnitude of irregular fluctuations, and γ_t is typical Gaussian white noise corresponding to a stochastic randomised process.

5.6 Geometric Brownian Motion

Geometric Brownian motion (GBM) models erratic variations in market share and adoption rates. It is characterised by a drift term, which reflects the typical expansion trend, and a dispersion term, which represents random disturbance. GBM ensures that the models can replicate situations in which market circumstances change unpredictably over time, such as economic recession or technological advancement.

5.7 Wiener Process

The Wiener process, frequently referred to as Brownian motion, is an essential component in modelling stochastic noise in adoption rates. It depicts unpredictable disturbances caused by customer choices, promotional efforts, or other unanticipated external influences. Including the Wiener process allows the algorithms to account for anomalies in adoption behaviour, increasing their realism and predictive reliability. The deterministic term of the adoption rate, denoted by $g(v)$, follows a logistic distribution:

$$g(v) = \frac{c}{1+\beta e^{-cv}}. \tag{16}$$

$$H(t) = 1 - \sum_{m=0}^{l-1} \frac{(\alpha t)^m}{m!} e^{-\alpha t}. \tag{17}$$

Whenever $l = 1$, the Erlang distribution reduces to the exponential variation function. In the current investigation, the shape parameter is assumed to be $l = 2$. The distribution function for market expansion can then be written as:

$$H(t) = 1 - \left(1+\alpha t\right)e^{-\alpha t}. \tag{18}$$

Sophisticated computational approaches are used to estimate variables and optimise the models. These methods ensure that SDM1 and SDM2 agree with real-world data and are able to make accurate predictions.

5.8 Genetic Algorithms

Parameter estimation is accomplished using genetic algorithms (GA), enabling the models to match historical patterns precisely. This optimisation approach simulates random selection and iteratively refines variables such as b, β, α, and σ in order to minimise forecast errors. Genetic algorithms are especially helpful for complicated models such as SDM1 and SDM2, where standard optimisation approaches may not converge.

Rolling cross-validation is used to validate model effectiveness. The data are divided into testing and training sets, and the models are rigorously evaluated on previously unseen data. This procedure ensures that the models generalise effectively to new circumstances, preventing overfitting while preserving high predictive accuracy. By capturing adoption patterns and market trends over time, the final outputs of SDM1 and SDM2 provide decision-makers with actionable information.

5.9 Trends of Forecasted Adoption

At each moment, the models provide a comprehensive perspective on how the market reacts to innovations by predicting the average amount of adoption, $M(t)$. These patterns allow firms to forecast spikes in demand, determine points of market saturation, and organise manufacturing and distribution plans accordingly.

5.10 Insights into the Dynamic Market

In addition to adoption projections, the mathematical models provide information about market behaviour under ambiguity. The rapid initial expansion scenarios highlighted by SDM1 make it suitable for technologies motivated by viral advertising or early-adopter excitement. In contrast, SDM2 captures progressive adoption trends, making it appropriate for goods with longer life-cycle phases or substantial barriers to entry.

6. RESULT AND ANALYSIS

Business managers receive useful inputs for controlling critical management choices when they properly understand diffusion dynamics over time. The current work contributes to forecasting the diffusion process for both researchers and practitioners. Computational results support the contention that the suggested models have superior predictive potential compared with benchmark systems. Consequently, the presented approach can provide a complete depiction of future demand patterns. Although the framework is built on assumptions strongly related to the actual market environment, it contains significant recommendations for practitioners.

Research on innovation-diffusion modelling has given little consideration to incorporating randomness when determining the number of prospective buyers. Market size is commonly treated as either constant or time-varying, while current research does not adequately include randomness or ambiguity in market-potential growth functions. Previous probabilistic definitions of market size are therefore restricted. In the real world, however, a development curve may experience oscillations as new advances enter the market.

6.1 Mean Absolute Error

Mean absolute error (MAE) measures the average magnitude of errors between actual and anticipated values without considering their direction.

6.2 Mean Absolute Percentage Error

Mean absolute percentage error (MAPE) provides a scale-independent assessment by expressing errors as a proportion of real values. It is useful for comparative analysis because it shows the magnitude of errors relative to actual values.

6.3 Root-Mean-Squared Error

Root-mean-squared error (RMSE) measures the square root of the average squared deviation between actual and anticipated values. It is sensitive to outliers because it penalises larger variances more heavily than smaller ones.

6.4 Mean Squared Error

Mean squared error (MSE) quantifies the average squared deviation between anticipated and actual values, serving as an indicator of error magnitude.

6.5 Accuracy

Accuracy is the ratio of correct predictions to the total number of forecasts, expressed as a percentage. It is a straightforward and intuitive statistic commonly used to assess model effectiveness.

6.6 F1-Score

The F1-score is the harmonic mean of precision and recall, providing a single measure that balances these two performance aspects. It is particularly advantageous when precision and recall differ substantially.

Table 1. Assessing the Performance of Existing and Proposed Techniques

Models	MAE	MAPE	RMSE
Logistic Growth Algorithm	11.45	9.64	13.45
Dynamic Diffusion Algorithm	10.67	8.34	12.45
Bass Diffusion Algorithm	9.34	8.13	11.56
SDM1	7.34	6.98	9.75
SDM2	6.24	5.74	8.53

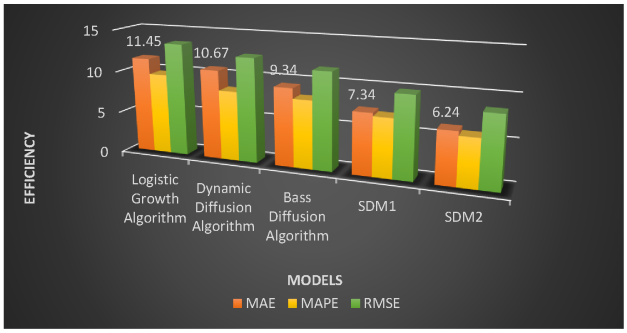

Figure 3. Evaluation of ML models in comparison with more traditional approaches.

The comparison based on MAE, MAPE, and RMSE demonstrates the clear superiority of the proposed methods, SDM1 and SDM2, over existing approaches. Among the existing models, the Bass Diffusion Algorithm performs better than the Logistic Growth Algorithm and the Dynamic Diffusion Algorithm, with an MAE of 9.34, MAPE of 8.13, and RMSE of 11.56. SDM1 significantly reduces these errors, achieving an MAE of 7.34, MAPE of 6.98, and RMSE of 9.75. SDM2 further refines performance with the lowest error values: MAE of 6.24, MAPE of 5.74, and RMSE of 8.53. These results highlight the robustness of the proposed methods in minimising prediction errors and providing a more reliable representation of the data.

Table 2. Results of MSE Statistical Measures

Models	MSE
Logistic Growth Algorithm	163.76
Dynamic Diffusion Algorithm	142.76
Bass Diffusion Algorithm	123.76
SDM1	70.77
SDM2	59.75

The analysis using MSE and MAPE further highlights the superiority of SDM1 and SDM2 over current methods. Although the Bass Diffusion Algorithm outperforms the Logistic Growth Algorithm and Dynamic Diffusion Algorithm, it cannot compete with the suggested designs, with a MAPE of 8.13 and MSE of 123.76. SDM1 achieves a MAPE of 6.98 and an MSE of 70.77, whereas SDM2 further reduces these errors to 5.74 and 59.75, respectively. These data show that SDM1 and SDM2 provide more accurate and dependable modelling than the alternatives, with lower error rates and greater predictive precision.

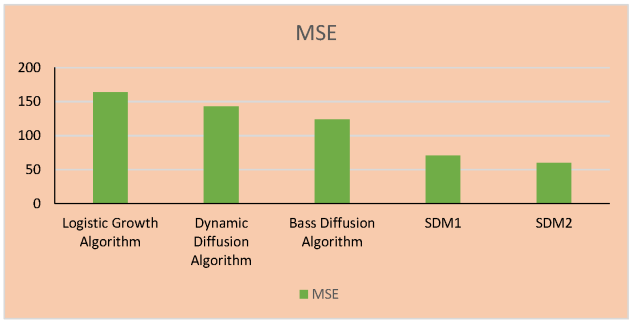

Figure 4. Comparing ML models with more conventional methods for evaluation.

Table 3. Statistical Measures for MAD Outcomes

Models	Accuracy (%)
Logistic Growth Algorithm	86.3
Dynamic Diffusion Algorithm	89.3
Bass Diffusion Algorithm	91.3
SDM1	95.32
SDM2	97.3

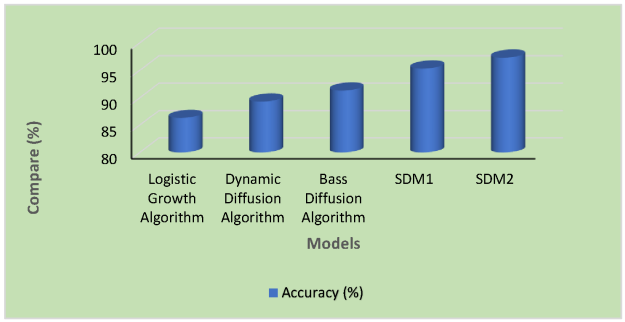

Figure 5. Efficacy of different systems.

The suggested techniques, SDM1 and SDM2, outperform current methods when compared by accuracy. The Bass Diffusion Algorithm reaches 91.3%, compared with 86.3% for the Logistic Growth Algorithm and 89.3% for the Dynamic Diffusion Algorithm. SDM1 reaches 95.32%, while SDM2 reaches 97.3%. These outcomes demonstrate the efficacy of the suggested strategies in practical settings by showing that they are more precise and more successful in identifying foundational trends. SDM1 and SDM2 are therefore more suitable for modelling innovation dissemination and making precise predictions of market behaviour.

Table 4. Exploratory DL Models in Relation to Proposed Methods

Models	F1-score
Logistic Growth Algorithm	0.83
Dynamic Diffusion Algorithm	0.86
Bass Diffusion Algorithm	0.89
SDM1	0.93
SDM2	0.96

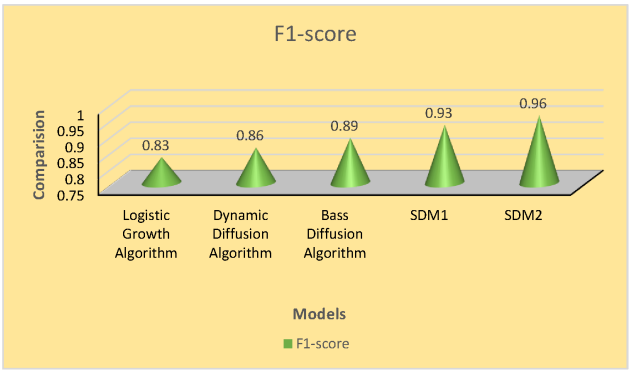

Figure 6. Effectiveness of different models.

Compared with current models, the suggested techniques perform much better in terms of F1-score. The Logistic Growth Algorithm reaches 0.83, the Dynamic Diffusion Algorithm 0.86, and the Bass Diffusion Algorithm 0.89. With improved recall and precision, SDM1 achieves an F1-score of 0.93. SDM2 outperforms all other models with an F1-score of 0.96, demonstrating the best balance of recall and precision for generating accurate forecasts.

7. CONCLUSION

The SDM1 and SDM2 models are a significant step forward in innovation-diffusion modelling because they include stochastic components and dynamic industry growth. SDM1 is well suited to products experiencing rapid adoption because it uses an exponential growth function to represent fast initial market development. SDM2 uses the Erlang distribution to represent S-shaped development patterns, allowing smooth and continuous market expansion. By using stochastic differential equations and geometric Brownian motion to include inconsistency, these models improve on existing deterministic frameworks and accurately portray uncertain markets and inconsistent adoption behaviour.

Verification with real-world sales data shows the reliability and accuracy of these algorithms in anticipating acceptance patterns, which is valuable to companies and policymakers. If SDM1 and SDM2 are developed further, they may be applied to situations involving several products that either compete with or complement one another in shared marketplaces. The models' predictive power may be enhanced by including pricing tactics, advertising efforts, and interpersonal dynamics. Modern computational algorithms such as machine learning and neural networks may support more precise optimisation and parameterisation. Applying the models to new technologies and developing markets can further increase their influence across sectors.

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