Stochastic Diffusion Process Models for Driving Innovation in Market-Driven Product Development

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Abstract

In order to analyses the diffusion of new technological products in rapidly changing market environments, this paper presents two new stochastic diffusion models: SDM1 and SDM2. The two models also utilize stochastic market size function in capturing rather random growth of potential users, inherent in most real-world markets. SDM1 apply the exponential distribution to model the market growth rate to consider the cases characterized by the high increase, while SDM2 adapt the Erlang distribution to reflect the S-shape to consider the long-term adoptions. The presented models rely on stochastic differential equations with recourse to calculus, and they adopt stochastic geometric Brownian motion and logistic growth function for adoption rates. This makes it possible to capture effects of learning as well as the non-regularity of adoption over time. The empirical results of benchmark models by using Apple iPhones and Samsung Galaxy smartphones sales data show the better performance of SDM1 and SDM2. The performance of the methodologies is measured using parameters, the goodness-of-fit tests and the forecast accuracy that all show that the proposed methods are very efficient. These models have a rich theoretical background, which comprises the foundation for explaining adoption patterns, which in turn will facilitate the behaviour of managers and policymakers towards understanding consumers, controlling inventory, and designing significant marketing strategies for technology products in a stochastic world. Both SDM1 and SDM2, the suggested algorithms, outperform the state-of-the-art techniques in terms of accuracy. SDM1 outperforms the other models with an accuracy of 95.32 percent. SDM2's greater accuracy in forecasting is shown by its outperformance of all techniques, which stands at 97.3%.

Keywords: SDM2; SDM1; MAPE; SDE; RMSE; MSE; MAE

1. Introduction

Adoption of technical breakthroughs is a significant factor that plays a significant influence in changing the structure of the market including the behaviour of consumers. On the other hand, projecting the development of new goods in real-world settings is a difficult undertaking because of the inherent unpredictability that are present in customer tastes, changes in markets, and external variables [1-2]. However, despite their widespread use, conventional probabilistic diffusion models are not capable of effectively capturing the unpredictable and ever-changing character of market expansion. This work presents two sophisticated stochastic diffusion models, SDM1 and SDM2, which combine dynamically market shares and spurious changes to offer an accurate illustration of the method of adoption. The purpose of this investigation is to solve the shortcomings that have been identified. The purpose of these frameworks is to improve our knowledge of diffusion processes, especially in markets that are characterised by fast technical improvements and unpredictable customer behaviour. It is crucial for organisations to have a solid understanding of the diffusion process in order to optimise their advertising approaches, management of inventory, and manufacturing scheduling for optimal results. The vast majority of traditional models use the assumption that the market size is fixed and that rate of adoption are predetermined. This restricts their application in real-world settings, which are characterised by revenue potential and adaptation

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behaviour that are impacted by externalities [3-4]. Through the incorporation of stochastic components into the modelling framework, SDM1 and SDM2 are able to tackle these deficiencies. The incorporation of mathematical Brownian motions and changing development parameters into these frameworks results in a strong technique to projecting the widespread use of breakthroughs over the course of time.

Perhaps the most important aspects of SDM1 are that it use an exponential range to depict the expansion of the market. This hypothesis depicts the fast initial growth that is often seen in the early phases of technological acceptance [5-6]. This is the period in which prospective adopters rise rapidly because of vigorous advertisements and growing consumer awareness. On the other hand, SDM2 makes use of an Erlang distribution, which represents market expansion with an S-shaped curve. This curve reflects a more gradual diffusion procedure that takes into consideration the earliest adopter and late-stage customers. These theories, when combined, are able to accommodate a wide variety of market conditions, which enables them to be extremely flexible to a wide variety of goods and businesses. The use of Itô calculus and stochastic differential equations (SDEs) is what allows for the stochastic aspect of SDM1 and SDM2 to be realised. A realistic picture of the diffusion process may be obtained via the use of these computational methods, which make it possible to include unplanned changes into the rate of penetration and increasing market size [7]. The use of geometrical Brown's equations guarantees that the estimates take into account quirks and ambiguities in the functioning of the market. These include variables such as shifts in customer preferences, variations in financial situations, and developments in technology capabilities. With this method, the capacity for forecasting and preciseness of the algorithms are greatly improved, hence becoming them acceptable for the analysis of market situations that are both unpredictable and complicated.

The scientific verification of SDM1 and SDM2 reveals that both of them are preferable than the conventional diffusion models. By using practical sales information from goods such as Apple iPhones and Samsung's Galaxy mobile devices, these projections supply a more accurate representation of observed usage trends and exceed benchmarking simulations in terms of the accuracy of their forecasts [8]. There are a number of key measures that illustrate the usefulness of the presented frameworks for collecting current market conditions and projecting anticipated adoption patterns. These metrics include root mean squared error (RMSE), mean absolute percentage error (MAPE), and correlation of reliability (R2). Notwithstanding the mathematical soundness of SDM1 and SDM2, the relevance of these two models goes further. Businesses are able to construct focused marketing efforts, optimise manufacturing schedules, and establish efficient stock control methods with the help of those theories, which provide substantial insight into the behaviour of prospective adopters. SDM1 and SDM2 are crucial tools for executives and lawmakers because they include stochastic aspects, which give an achievable and complete structure for comprehending diffusion of innovation in a chaotic market context. The structure is provided by the incorporation of unpredictable factors. A comprehensive examination of the approach behind SDM1 and SDM2 is presented in the two subsequent sections [9]. This examination covers its mathematics structure, essential presumptions, and its implementation situations. Furthermore, the statistical evaluation and outcome assessment of these frameworks are explored, proving their actual importance and usefulness in forecasting the acceptance of technical breakthroughs. This is done in order to demonstrate that these approaches are successful. This investigation intends to enhance the area of diffusion modelling and offer a solid basis for future studies and their utilisation in managing innovations. These advances will be made possible by the advancements that are made in the present research.

It is possible for consumers to embrace an invention or service via a series of steps that are collectively referred to as the implementation procedure. The fundamental behavioural theory suggests that there is a discernible amount of time that elapses from the moment a potential purchaser becomes aware of the invention and the moment he decides to accept it at that point in time. The technique of adoption had been determined to be a time-varying sequence operation, since this notion is now established. The prospective purchaser undergoes a number of phases in the method of adoption until finally deciding to embrace the innovation. Through his research, he has categorised the procedure of adopting into five distinct phases that are arranged in a hierarchy structure: understanding, argumentation, assessment, execution, and verification [10]. It is possible that the outcomes of each step will have an effect on the pattern of invention dissemination. Because of this, the procedure of dissemination has to be shown by using several stages included within the method of adoption. The numerous steps of the adopting process are broken down into their respective stages and shown in Figure 1.

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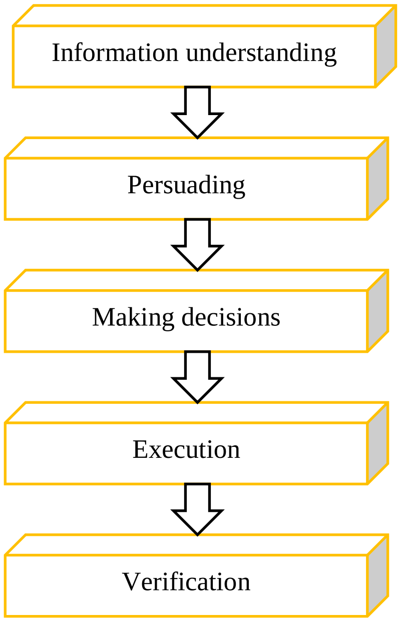

Figure 1. An Overview of ANN's Design

A significant contribution to the procedure for creative dispersion is made by the promotional elements of the invention. A business's capacity to monitor and oversee these factors over an indefinite length of time is the only factor that determines whether an invention will flourish in the short term or over the long run. The purpose of this part is to provide an overview of three most important market factors, namely the price of sale, ads, and guarantees term, which were used in the current research in order to get crucial management insights. It is possible that pricing is an especially complicated and effective marketing tactic, and it is the primary thing that factors into the choice that consumers make about their purchases [11]. The marketplace shares and profitability measurement for the firm are both significantly impacted because of the cost of sale, which has a significant impact. Therefore, in order to achieve the highest possible profit while simultaneously catering to the diverse requirements of their clientele, commercial enterprises endeavour to maintain a tight grip on the prices at which they sell their products.

In actuality, a pricing that is appealing might make it easier for manufacturers to increase the demand for their products over the long run. Companies should have reasonable control over the prices at which they sell their products. From the research that has been done in the field of sales, it has been shown the demand function has a downward tendency whenever the cost of sale is increased [12]. Because of the correlation that is present among the cost of sale and the desire for the good itself, the selling pricing is the primary consideration in determining the continued popularity of the good. Marketing has been described as "any monetary means of informal dissemination of products, services, or ideas by a recognised sponsor," according to the American Marketing Association (AMA). A commercial enterprise use advertisement as a means of informing, convincing, or reminding its intended customer base of the items or services it offers. Businesses are able to improve the personal worth of their innovations with the assistance of promotions and advertising, which are the two most significant forms of communication for marketing. Therefore, marketing and marketing techniques for new items serve as an effective medium to increase demands without creating any danger of failure to be stimulated [13]. In a similar vein, manufactures need to give their promotional and advertising tactics a closer look since these methods have a significant impact on the profitability of their newly launched item.

2. Related Work

The innovation diffusion modelling as a distinct field was actively developing during the last decades, and the investigator, who presented the Bass diffusion model, has performed the key works. The Bass model provided a deterministic framework to describe the adoption of innovations through two key parameters: Since its analysis of the innovation and imitation coefficients [15]. However, as the author were keen to point out, the assumptions inherent in the Bass model were static and it was unable to capture such dynamics hence the need to explore stochastic influences.

Taking from this need for stochastic frameworks, the researcher has elaborated the diffusion models, which incorporated dynamic market sizes and thus tried to capture the temporal change in the behaviour of adopter

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potentialities [16]. Although they both included variability in some degree within their model, both the investigator used mostly deterministic to estimate adoption rates. They suggested the future researchers might advance their work, using more highly sophisticated stochastic models like the geometric Brownian motion and other probabilistic tools in order to model the fluctuations of the market with more efficiency and accuracy.

The author advanced the course by incorporating stochastic characteristics into diffusion models in order to capture uneven nature of fluctuations in adoption rates. Their work focused on randomness prediction of consumer's behaviour prescribing models that include deterministic adoption functions with stochastic noise [17]. This integration was important into a historical shift towards more realistic modelling frameworks, thus reconciling theoretical models with the observed real market conditions. Singh et al.'s work led to further improvements of the methods related to the suggested approach, namely the stochastic diffusion models SDM1 and SDM2. The investigator brought into focalization of innovation diffusion stochastic differential equations through Itô Calculus as a sound and versatile mathematical modelling. Their contribution offers the theoretical basis for incorporating randomness into functions of market growth and adoption.

Other researchers continued on this by using stochastic processes, Including Wiener processes as well as geometric Brownian motion to model stochastic dynamic uncertainties. These methodologies are core to the derivation of SDM1 and SDM2, which deploy SDEs to model both adoption rates and market size where there is uncertainty. As many authors pointed out recently, there is a need for more research that incorporates dynamic market size into diffusion models available. Market expansion is normally formulated early on by the investigator in the form of exponential growth functions, which demonstrate the essence of rapid market expansion when the product is newly introduced in the market [18]. These are incorporated directly into the SDM1 model that assumes an exponential distribution for the market size. In the following, we discuss and present empirical evidence for studies by the author that examined S-shaped logistic growth patterns in the market expansion. Like the time-to-market and the number of countries scaled models, the researcher developed and tested further the theoretical premise of the Erlang distributions that we have employed in the SDM2 model.

Stochastic innovation diffusion models have been particularly widely used in industries where high implementation of technology is observed [19]. The author also used stochastic models and confirmed the potential of this approach for the prediction of consumer electronics and automobiles. They emphasised the need to introduce stochastic components of adoption in the sources separated from golden waves in order to consider the external causes like economic activity, technologies, and campaigns. Its precursor SDM1, and successional model SDM2, includes stochastic factors in both adoption and growth curves to pre-empt overly optimistic predictions. These literatures have also supported other empirical evidence indicating that stochastic diffusion models are superior to deterministic models. Notably, the researcher tested the two classes of models on actual sales data; the author [20] conducted the same. These confirmed in every instance that the stochastic models especially those with dynamism in market size and random disturbance presented better parameter estimations and better fitness than the deterministic models. These empirical validations make a powerful statement for the general advancement and use of more complex stochastic models such as SDM1 and SDM2.

Stochastic diffusion models have also been considered from the angle of utility in the field of marketing and inventory control. The researcher emphasized that forecasts determining the need for inventory and production plans involve stochastic models. They showed with their work how uses of stochastic diffusion models in businesses could predict the occurrences in the market and subsequently align the plans for production [21]. For a similar reason, the investigator argued that stochastic modelling is critical to achieving marketing objectives, particularly when marketing technology products, which are characteristically adopted at different rates at different time points. Relevance of SDM1 and SDM2 to real-life decision making is evidenced by these down to earth observations.

This line of work has also emphasized the need to achieve an optimal level of model complexity in stochastic innovation diffusion models [22]. The investigator also highlighted the potential problem with excessive model complexity that seems to be the current trend in model development. Their work speaks for models that are theoretically attractive as well as realistically feasible, or what is underlined in the development of SDM1/SDM2. The strength of these models is that making predictions based on them does not necessarily require complex calculations or program modelling, despite the fact that such tools as geometric Brownian motion and logistic growth functions are rather demonstrative and efficient.

The development of other computational methods also recommends further prospects in stochastic diffusion to be implemented [23]. The author has tried applying genetic algorithms to estimate parameters of complex forms of

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diffusion models for making better predictions. These computational tools are important for the empirical evaluation of SDM1 and SDM2, which utilizes a genetic algorithm, to estimate model parameters and to optimize the model. Cross-validation approaches, including those described by the researcher as rolling cross-validation, have also been used to assess the predictive accuracy of stochastic models and check their competitiveness in terms of different market conditions.

In summary, this paper finds that there are a large number of publications on stochastic diffusion modelling, thus providing ample conceptual support for the advancement of SDM1 and SDM2. Diffusion modelling has advanced with a multitude of scholars contributing to the technology of stochastic structures to effectively respond to real markets [24]. The use of stochastic factors in market size and adoption functions is therefore a major advancement in this evolution, and provides the extraordinary precision and realism in the prediction of the take up of technological change. On this basis SDM1 and SDM2, offer a definition of state-of-the-art models, which takes into account the applicative requirements according to the academic tradition.

Cross-validation approaches, including those described by the author as rolling cross-validation, have also been used to assess the predictive accuracy of stochastic models and check their competitiveness in terms of different market conditions. In summary, this paper finds that there are a large number of publications on stochastic diffusion modelling, thus providing ample conceptual support for the advancement of SDM1 and SDM2 [25]. Diffusion modelling has advanced with a multitude of scholars contributing to the technology of stochastic structures to effectively respond to real markets. The use of stochastic factors in market size and adoption functions is therefore a major advancement in this evolution, and provides the extraordinary precision and realism in the prediction of the take up of technological change. On this basis SDM1 and SDM2, offer a definition of state-of-the-art models, which takes into account the applicative requirements according to the academic tradition. In the field of stochastic modelling, one of the difficulties that has been encountered is the estimate of values under uncertain. A number of investigators, including, have taken on this problem by using sophisticated computer methods, such as neural networks. In situations when the data is either inadequate or noisy, these techniques make it possible to estimate the variable values of complicated models. By using algorithmic genetics to optimise their settings and increase their predicted accuracy, SDM1 and SDM2 build upon these accomplishments and continue to expand upon them. This computationally resilience guarantees that the predictions will continue to be trustworthy even in market settings that are fraught with a great deal of uncertainty. The investigation of noisy terms and the influence that they have on adaptation behaviour is yet another key addition that has been made to stochastic diffusion modelling. The author provided evidence that noise, which may be modelled as Gaussian white noise or cubic Brownian motion, can serve as a representation of unpredictable surroundings that have an impact on adoption rates. The stochastic dynamics models (SDM1 and SDM2), in which stochastic variables are employed to reflect irregular oscillations in the market, are not complete without these revelations. Those random aspects are included into the models, which allows them to attain a degree of authenticity that their determinism equivalents are unable to accomplish.

The empirical validation of stochastic diffusion models has repeatedly shown that these models are better than the old deterministic techniques. Studies conducted by the investigator, for instance, brought to light the fact that stochastic models provide a more accurate representation of the data that is collected in the actual world, especially in technology-driven sectors. These results highlight the need of models such as SDM1 and SDM2, which are especially intended to manage the uncertainties that are inherent in markets for items such as cell phones, vehicles, and consumer electronics based on the unique characteristics of those markets. Furthermore, to their merits in theory and empirical research, stochastic diffusion equations also have practical consequences for the formulation of corporate strategies and public policy. The author investigated how these kinds of models may be used to provide precise requirement estimates, which can then be used to influence inventory management and manufacturing planning. Within a similar vein, the researcher proved their usefulness in the process of building promotional strategies that are adapted to the ever-changing circumstances of the market. These applications demonstrate the importance of SDM1 and SDM2 for policymakers and managers who are attempting to maximise the effectiveness of decision-making in contexts that are fraught with uncertainty.

Recently conducted research has also highlighted the need of continuously evaluating and validating models in order to guarantee their resilience. Rolled the cross-validation is an algorithm that was presented by the author as a method to evaluate the effectiveness of a predictive model on data that has not been seen before, hence lowering the danger of overfitting? This approach is especially pertinent for SDM1 and SDM2, which are intended to function in a variety of market conditions that are characterised by high levels of volatility. The prediction effectiveness and flexibility of these models are preserved by the use of validation methods of this kind. One aspect of stochastic diffusion models that contributes to their adaptability is their capacity to accurately depict a wide range of market occurrences. For example, the investigator investigated the phenomenon of recurrent purchase,

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which is an important aspect in a variety of different businesses. Specifically, this facet is handled in SDM2, in which Erlang distribution makes it possible to include progressive market expansion as well as contacts with returning customers. In a similar vein, the fact that SDM1 is centred on an exponential expansion makes it an excellent choice for products that demonstrate fast adoption quickly, for example those that are driven by viral advertising efforts. The interaction among developments in theory with practical uses continues to be a primary subject of interest because stochastic diffusion models are always undergoing modifications. A number of researchers, including, have brought attention to the significance of alignment theoretical bases using data from the actual world in order to improve the usefulness of models. SDM1 and SDM2 are examples of this method since they include stochastic aspects, variable size of markets, and modern computing approaches. This ensures that they continue to be useful and effective instruments for comprehending and forecasting the spread of innovations.

3. Objective of the research work

Building sophisticated stochastic diffusion models that reliably predict the uptake of technical breakthroughs in unpredictable and ever-changing market conditions is the ultimate goal of the SDM1 and SDM2 research projects. By include unpredictable components like unpredictable changes in expansion in the market and acceptance rates, these models attempt to solve the constraints of classic determinism paradigms. While SDM2 utilises an Erlang distribution to depict slow but steady S-shaped market development, SDM1 utilises an exponential statistic to capture fast early stages growth in markets. This project aims to synthesise logistic development processes, geometrical Brownian motions, and stochastic inequality equations into a flexible and plausible model of innovative dissemination. The end objective is to make superior advertising, supplies, and manufacturing decisions for new tech items by improving precision in forecasting and providing knowledge of customer behaviour.

4. Motivation for the research work

The inadequacies of conventional determinism diffusion models in adequately representing the intricacies of actual innovation acceptance procedures were the impetus for creating the SDM1 and SDM2 models. In contrast to the ever-changing and unpredictable character of contemporary markets driven by socio-economic variables, technical developments, and exogenous variations, deterministic frameworks often presume constant market sizes and consistent adoption rates. There is an urgent need for reliable models that include uncertainty and changing market dynamics due to the proliferation of technological disruption and unpredictable customer behaviour. To fill this need, SDM1 and SDM2 mimic erratic market growth and rate of adoption using stochastic components such geometrical Brownian dynamics and probabilistic linear equations. To more closely correlate with the intricacies of fuelled by technology marketplaces, make more informed strategic decisions, and conduct accurate forecasts, these frameworks supply an enhanced structure to comprehending the dissemination of technologies.

5. The Projected method

The unpredictability in the customer's purchase choice may be caused by the fluctuation in the components that influence the diffusion process, regardless of how minute or extensive the variation may be. One of the most significant factors that may significantly affect the ordering and reserve holding practices of producers, distributors, and sellers is the uncertainty that exists in the consumer's purchasing evaluation. Consequently, in order to provide a reliable forecast of the future, it is important to conduct a stochastic analysis of the diffusion process, taking into account the quickly changing internal as well as external characteristics. This part builds an adaptive diffusion of innovation framework that includes the variable market size by making use of the stochastic differential equation. This is done in light of the fact that it is necessary to portray a genuine adopting phenomenon. In the technique that has been provided, it is believed that the overall popularity of inventions would alter over the years because of the fast developments that are being made in the functioning of high-tech items. In the current part, three diffusion frameworks that take into account market expansion are being developed in order to comprehend the fundamental market behaviour of target purchasers. The suggested model develops in a stochastic setting according to a dynamic process in order to demonstrate that the acceptance rate is subject to variations at random. In order to resolve the stochastic differential equations, applications of the Wiener process and the Itô calculus are used. In addition to this, the models that were suggested were actually used in order to investigate the development trend of various consumers' disposable items.

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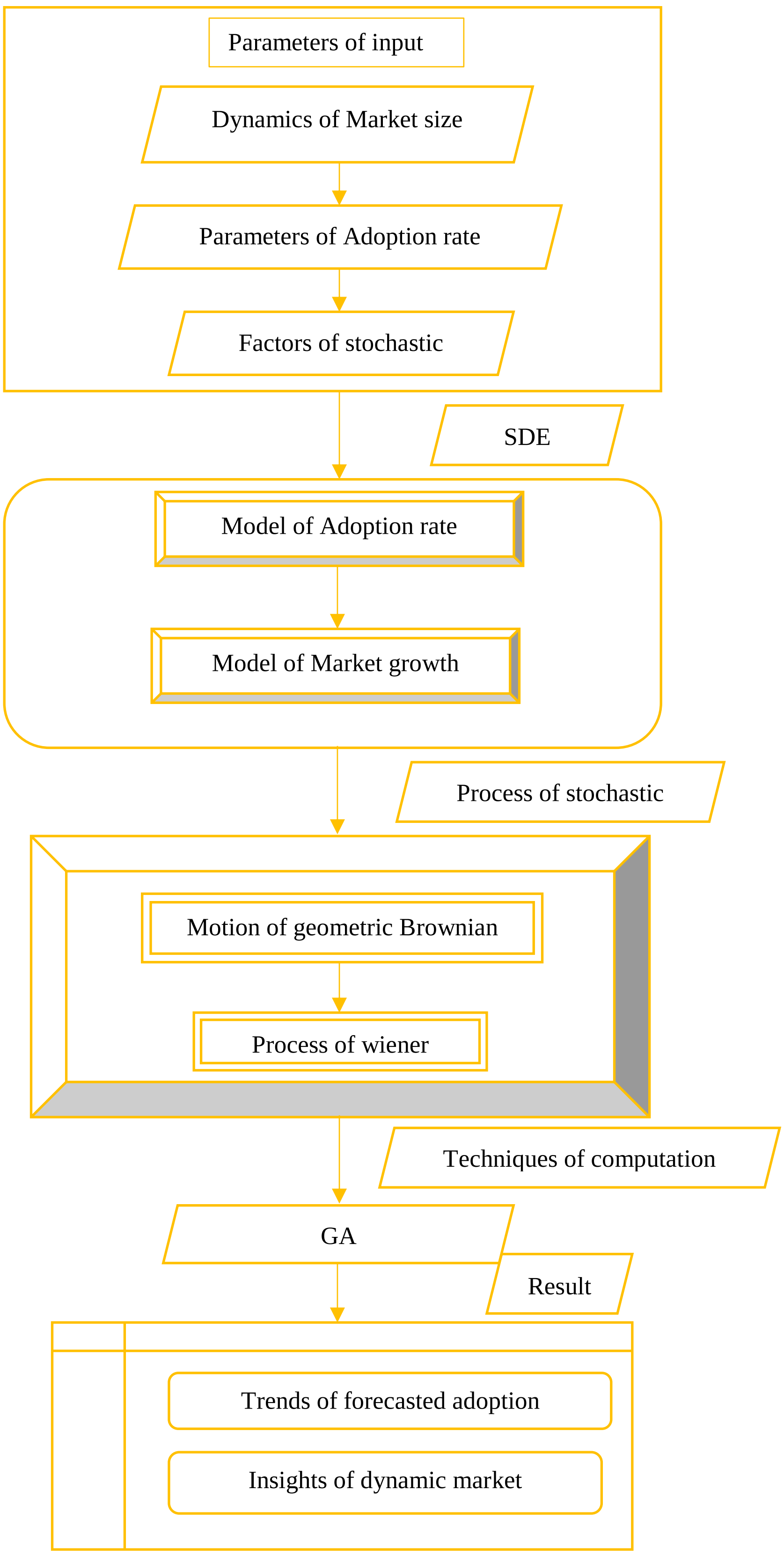

Figure 2. Structure of the suggested approach

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The block diagrams for the SDM1 and SDM2 models provide an illustration of the intricate procedures that are the basis for the stochastic diffusion approaches that have been presented. This set of procedures is intended to model and foresee the uptake of innovations by adding stochastic aspects and dynamical marketplace behaviours. The input variables of the SDM1 and SDM2 methods have been meticulously established which serves as the basis for both models. When it comes to understanding the changing nature of the marketplace and the uncertainty underlying consumer acceptance, these data points are very necessary.

5.1 Dynamics of Market size: The patterns of market size describe every segment of new consumers who has the opportunity to acquire the invention during the course of its lifespan. The inputs consist of the starting size of the market and the greatest feasible market size (m M), which is determined by parameters such as the kind of product, the methods for market expansion, and the availability of the customer. It is possible for adoption rates to be greatly influenced by stochastic modelling of dynamical market development, which is done in order to represent practical uncertainty.

5.2 Parameters of Adoption rate: Logical growth operations, which are characterised by characteristics such as the rate of implementation factor (c) and the learning coefficients (β), are responsible for regulation of the rate of adoption. Within the context of an adoption manipulate, these characteristics represent the effect of social contacts, word-of-mouth, and the general media. For example, an elevated amount of c signifies a quick acceptance, while a low figure of β implies a slower spread in the market, which may be impacted by customer hesitancy or a lack of knowledge.

5.3 Factors of stochastic: Market behaviour is impacted by variables such as economic swings, technology improvements, and outside events. The actual world is essentially unforeseeable, and this is a truth that cannot be changed. The use of random variables that are obtained from stochastic procedures, such as geometric Brownian movement and Gaussian white noise, is utilised in order to describe these ambiguities. These stochastic inputs increase the systems' level of realism, which in turn ensures that they may be used to a wide variety of market conditions.

Stochastic differential equations (SDEs) are at the heart of both SDM1 and SDM2. These formulas provide a conceptual structure that can be used to depict the method of adoption and market expansion in the face of uncertainties.

5.4 Model of Adoption rate: A framework, which describes how the speed of adopting changes over time is called an adoption rate approach. This approach depends on the logistical growth curve. A stochastic component is included into the SDE in order to compensate for unpredictable variations, and the deterministic component is used to describe the underlying expansion pattern with the SDE. It is written as the equation:

$$\frac{dM(t)}{dt} = c.[F[n(t)] - M(t)] + \sigma.\gamma(t) \quad (1)$$

$$\frac{dM(t)}{dt} = t(n(t) - M(t)) \quad (2)$$

$$t = u(t) + 'n' \quad (3)$$

$$t = g(t) + \sigma\gamma(t) \quad (4)$$

$$\frac{dM(t)}{dt} = g(t)\big(n(t) - M(t)\big) + \sigma\gamma(t)(n(t) - M(t)) \quad (5)$$

In the above equation, the variable $M(t)$ reflects the total amount of consumers, $F[n(t)]$ indicates the anticipated size of the marketplace, and σ γ(t) is a representation of the random variations in adopting rates. The ratio of growth during any given period 't', and the potential that remains for purchasers who gradually embrace innovations are shown by the $n(t) - M(t)$. where $u(t)$ is taken for a constant rate of adoption. The 'n' in the second part is what causes the adoption rate to fluctuate. When ordinary Gaussian white noise $\gamma(t)$ is a randomised signal and σ is a value that is positive that represents the size of the uneven fluctuations. In other words, the inverse of a one-dimensional Wiener processes (or a Brownian motions) is used to compute the random value.

5.5 Model of Market growth: The total size of the market is dynamic and changing over time, with unpredictable elements playing a role in this process. When it comes to SDM1, the growth of the marketplace followed an exponential distribution:

$$n(v) = n_0 + N.(1 - f^{-\propto v}) \quad (6)$$

$$n(v) = n_0 f^{\propto v}; \propto > 0 \quad (7)$$

$$\frac{dM(t)}{dt} = g(t)\big(n - M(t)\big) \quad (8)$$

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$$g(t) = \frac{e(t)}{1-e(t)} \quad (9)$$

$$g(t) = \frac{e(t)}{1-e(t)} + 'n' \quad (10)$$

where n represents the initial size of the marketplace and α is a positive indicator that reflects the pace at which the market is currently expanding. where the symbol 't' represents the rate of adopting or hazards rate at any given moment, and the symbol ' $e(t)$' represents the rate of adopting and $n - M(t)$ symbolises the unrealised prospective of the market that has not yet been purchased by the customer.

In this context, "n" refers to the fluctuations in the rate of adoption. The increase in the rate is denoted by the symbol α. When it comes to SDM2, an Erlang distributed represents development in an S-shaped pattern:

$$n(v) = n_0 + N.(1 - f^{-\propto v}.(1+\propto v)) \quad (11)$$

$$g(t) = \frac{e(t)}{1-e(t)} + \sigma\omega(t) \quad (12)$$

where $\omega(t)$ is a random value that is known to be a conventional Gaussian white noise; σ is a positive parameter that specifies the degree to which the unequal oscillation occurs. These growth functions are incorporated into the SDEs, ensuring that market size dynamics align with the nature of the product and target audience. By including unpredictability into the models, stochastic procedures are able to capture the intrinsic uncertainty that is present in the behaviour of markets in the actual world. An example of a continuously random procedure is the Wiener process, which is denoted by the function $Z(t)$.

$$\frac{dZ(t)}{dt} = \omega(t) \quad (13)$$

$$\propto (t) = \frac{h(t)}{1-H(t)} + \sigma\gamma t \quad (14)$$

$$H(t) = (1 - f^{-\propto t}) \quad (15)$$

This equation depicts the influenced by time rate of market growth, denoted by $\frac{h(t)}{1-H(t)}$. The symbol σ is a positive value that indicates the amount of irregularity fluctuations, and γt is a typical Gaussian white noise, corresponding to a stochastic randomised process.

5.6 Motion of geometric Brownian: Modelling the erratic variations in market share and rates of adoption is accomplished via the application of geometrical Brownian motion, sometimes known as GBM. The drift term, which reflects the typical expansion trend, and the dispersion term, whose represents random mistakes, are the two terms that characterise this process so that it can be understood. GBM guarantees that the models are able to replicate situations in which market circumstances change in an unpredictable manner as time goes on, such as when the economy is experiencing a recession or when technical advancements are being made.

5.7 Process of wiener: The Wiener process, which is frequently referred to as Brown's motion, is an essential component in the modelling of stochastic noise seen in adoption rates. In the procedure of being adopted, it depicts unpredictable disturbances that may occur because of customer choices, promotional efforts, or other unanticipated external influences. The Wiener process is included into the algorithms, which allows them to take into consideration anomalies in adoption behaviour. This results in an increase in the algorithms' level of reality & reliability in making predictions. Additionally, it is taken into consideration that the logical distribution equation is followed by the determinism term of adopting rate, which is denoted by g(v) respectively.

$$g(v) = \frac{c}{1+\beta f^{-cv}} \quad (16)$$

$$H(t) = 1 - \sum_{m=0}^{l-1} \frac{(\propto t)^m}{m!} f^{-\propto t} \quad (17)$$

Whenever the value of l=1, the Erlang distributing may be reduced to the exponential variation function. With regard to the current investigation, the actual value of the parameter shape is assumed to be l=2. One such equation that may be used to define the distributional function for the expansion of the market is equation (18):

$$H(t) = (1 - (1+\propto t)f^{-\propto t} \quad (18)$$

For the purpose of estimating variables and optimising the models, sophisticated computational approaches are used. These methods guarantee that SDM1 and SDM2 are in agreement with data from the actual world and in a position to make accurate predictions.

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5.8 Genetic Algorithms (GA): The estimate of parameters is accomplished via the use genetic methods, which enables the models to precisely match historical patterns. With this optimisation approach, the procedure of random selection is simulated, variables like b, β, α, and σ are refined in an iterative manner in order to minimise forecast mistakes. On the other hand, GAs are especially helpful for complicated models such as SDM1 and SDM2, which are examples of situations in which standard optimisation approaches may not convergence.

For validating the effectiveness of the models, rolling cross-validations is taken into consideration. Using this method, the data is divided into testing and training sets, and the models are rigorously tested on data that they have not before seen. Through this process, it guarantees that the models are able to generalise effectively to new circumstances, so preventing the overfitting while preserving a high level of predicted accuracy. By capturing adoption patterns and market trends as time passes, the final outputs of SDM1 and SDM2 offer decision-makers with information that can be put into action.

5.9 Trends of forecasted adoption: At each given moment, the models provide a comprehensive perspective on the way the market reacts to the innovations by predicting the average amount of adoption (M(t)) at the moment in time. With the use of these patterns, firms are able to forecast spikes in demand, determine points at which the market has reached saturation, and organise their manufacturing and distribution plans correspondingly.

5.10 Insights of dynamic market: The mathematical models, in along with providing adoption projections, also provide information about the behaviour of the market when there is ambiguity. The quick initial expansion scenarios that are highlighted by SDM1 make it a suitable modelling technique for technology that are motivated by viral advertising or early adopter excitement. On the contrary, side, SDM2 is able to capture progressive adoption trends, making it appropriate for goods that have longer phases of life or have substantial barriers to entry.

6. Result and Analysis

Business managers are provided with useful inputs for the purpose of controlling critical management choices when they have proper understanding of the dynamics of dissemination with respect to the time. The current work makes a substantial contribution to the knowledge of the forecasting the diffusion process, which is important for researchers as well as practitioners. In light of the outcomes of the computational analysis, data analysis was presented to support the contention that the suggested models had superior predictive potential in comparison to the benchmarking systems. As a result, the approach that has been provided is able to provide a full depiction of the future patterns in demand. Although this framework is built on expectations that have a strong relationship to the actual market environment, the approach that has been given contains significant suggestions that are directed towards the practitioner. As far as the research on diffusion of innovations modelling is concerned, there have been no consideration given to the incorporation of randomness in determining the rate of prospective buyers. There is a consensus that the size of the market for innovations is either constant or time varying. When it comes to the development functional associated with market possibilities, the current amount of research does not adequately include the element of random or ambiguity. Because of this, the descriptions of the probabilistic definition of market size that prior research have provided are restricted. On the other hand, in the actual world, the curve of development may experience oscillations because of the introduction of new advances into the market.

6.1. Mean Absolute Error (MAE): MAE takes into account the average size of errors among actual and anticipated values, but not their trajectory.

6.2. Mean Absolute Percentage Error (MAPE): MAPE provides an assessment measure that is independent of scale by expressing mistakes as a proportion of real values. It is helpful for comparison analysis since it shows how big the mistakes are in relation to the real values.

6.3. Root-mean-squared error (RMSE): RMSE is a measure of the average of the squared deviation between actual and anticipated values. It is sensitive to outliers since it punishes bigger variances compared to smaller ones.

6.4. Mean Squared mistake (MSE): It quantifies the average of the squared deviations between anticipated and actual values, serving as an indicator of mistake size.

6.5. Accuracy: It is the ratio of accurate guesses to the entire number of forecasts expressed as a percentage. This is a straightforward and obvious statistic often used to assess the effectiveness of models.

6.6. F1-score: The F1-score is the balanced average of accuracy and recall, offering a singular measure that equilibrates these two performing facets. It is particularly advantageous when the model's accuracy and recall exhibit substantial disparity.

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Table 1: Assessing the Performance of Existing and Future Techniques and Methods

Models	MAE	MAPE	RMSE
Logistic Growth Algorithm	11.45	9.64	13.45
Dynamic Diffusion Algorithm	10.67	8.34	12.45
Bass Diffusion Algorithm	9.34	8.13	11.56
SDM1	7.34	6.98	9.75
SDM2	6.24	5.74	8.53

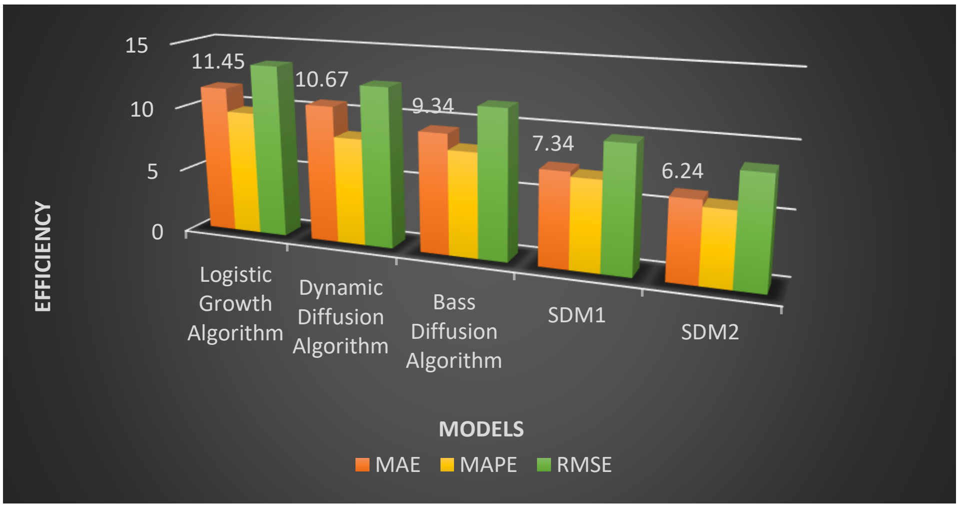

Figure 4. Evaluation of ML models in comparison to more approaches that are traditional

The comparison of the models based on MAE, MAPE, and RMSE demonstrates the clear superiority of the proposed methods, SDM1 and SDM2, over the existing approaches. Among the existing models, the Bass Diffusion Algorithm performs better than the Logistic Growth Algorithm and the Dynamic Diffusion Algorithm, with a MAE of 9.34, MAPE of 8.13, and RMSE of 11.56. However, SDM1 significantly reduces these errors, achieving a MAE of 7.34, MAPE of 6.98, and RMSE of 9.75, indicating its improved accuracy. SDM2 further refines performance with the lowest error values: MAE of 6.24, MAPE of 5.74, and RMSE of 8.53, making it the most effective model. These results highlight the robustness of the proposed methods in minimizing prediction errors and providing a more reliable representation of the data compared to traditional diffusion algorithms.

Table 2: Results of MSE Statistical Measures

Models	MSE
Logistic Growth Algorithm	163.76
Dynamic Diffusion Algorithm	142.76
Bass Diffusion Algorithm	123.76
SDM1	70.77
SDM2	59.75

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The model's analysis using Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) further highlights the superiority of the suggested techniques, SDM1 and SDM2, over the current methods. Although it outperforms the Logistic Growth Algorithm and the Dynamic Diffusion Method, the Bass Diffusion Algorithms still can't compete with the suggested designs (MAPE: 8.13 and MSE: 123.76, respectively). A MAPE of 6.98 and an MSE of 70.77 were achieved by SDM1, whereas SDM2 further reduced these errors to 5.74 for MAPE and 59.75 for MSE, demonstrating substantial gains for both methods. This data shows that SDM1 and SDM2 provide more accurate and dependable modelling than the alternatives, with lower error rates and greater precision in predictions. With lower MAPE and MSE values, SDM1 and SDM2 provide a more resilient and precise means of addressing the issue than standard diffusion methods by more efficiently absorbing the deeper trends in the data.

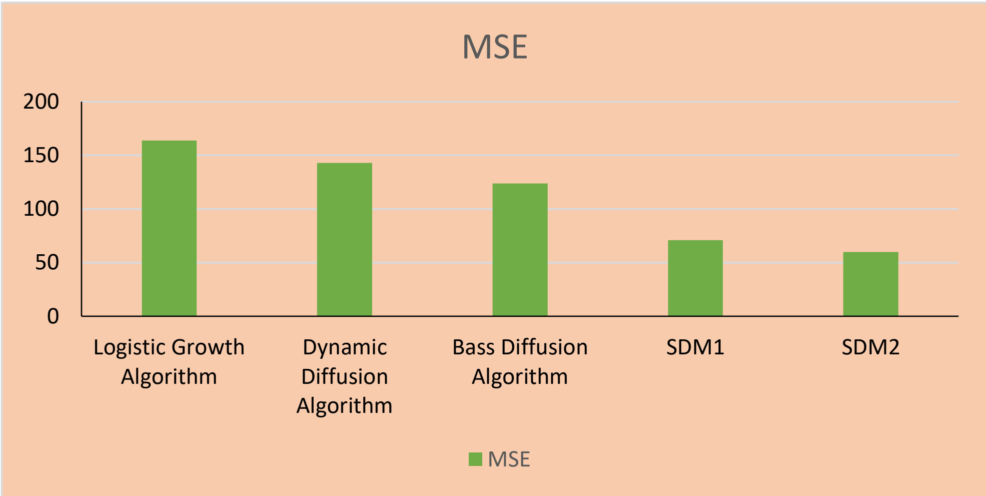

Figure 5. Comparing ML models to more conventional methods for evaluation.

Table 3: Statistical Measures for MAD Outcomes

Models	Accuracy (%)
Logistic Growth Algorithm	86.3
Dynamic Diffusion Algorithm	89.3
Bass Diffusion Algorithm	91.3
SDM1	95.32
SDM2	97.3

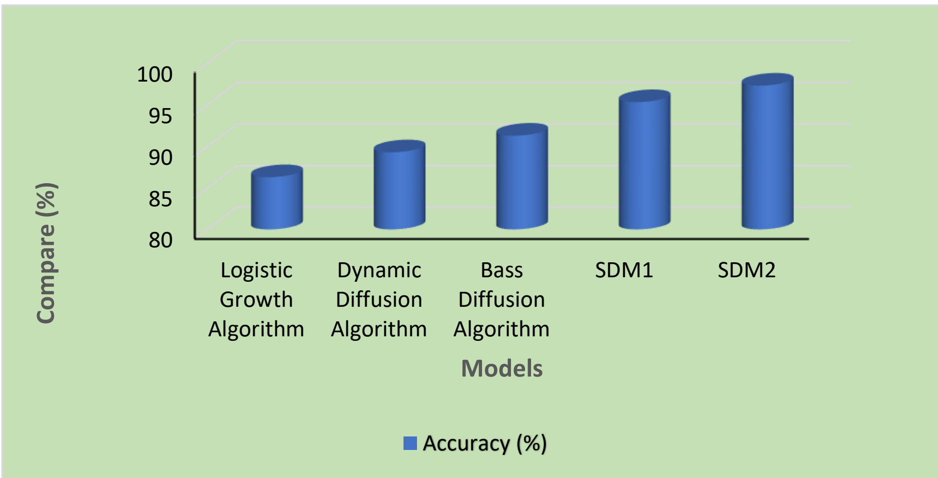

Figure 6. Efficacy of different systems

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The suggested techniques, SDM1 and SDM2, outperform the current methods when compared with the models that were determined by Accuracy (%). With a greater accuracy of 91.3% compared to the Logistic Growth Algorithm (86.3%) & the Dynamic Diffusion Algorithm (89.3%), the Bass Diffusion Algorithm outperforms the other two algorithms when it comes to forecasting adoption patterns. Having said that, SDM1's accuracy of 95.32% and SDM2's 97.3% both much surpass the performance of the current models. These outcomes prove the efficacy of the suggested strategies in practical settings by showing that they are more precise as well as successful in identifying the evidence's foundational trends. It seems that SDM1 and SDM2 are more suitable to modelling innovation dissemination and making more precise predictions of market behaviours due to their increased accuracy.

Table 4: Exploratory DL models in relation to proposed methods

Models	F1-score
Logistic Growth Algorithm	0.83
Dynamic Diffusion Algorithm	0.86
Bass Diffusion Algorithm	0.89
SDM1	0.93
SDM2	0.96

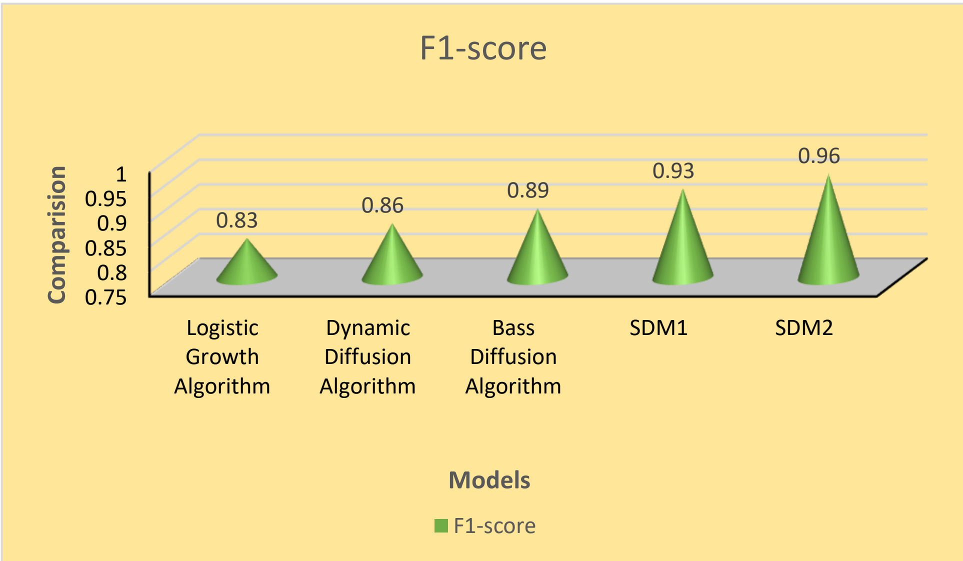

Figure 7. Effectiveness of different models

When compared to the current models, the suggested techniques, SDM1 and SDM2, performed much better in terms of F1-scores. The Logistic Growth Algorithm (0.83) and the Dynamic Diffusion Algorithm (0.86) nevertheless surpass the suggested approaches, while the Bass Diffusion Algorithm fares worse with an F1-score of 0.89. With its improved recall and accuracy, SDM1 earns an outstanding F1-score of 0.93. With an F1-score of 0.96, SDM2 outperforms all other models, proving that it has the best combination of recall and accuracy for generating accurate forecasts.

7. Conclusion

The SDM1 and SDM2 models are a huge step forward in the diffusion of innovation modelling since they include stochastic components and dynamical industry growth. SDM1 is well suited for items going through a period of vigorous adoption quickly as it uses a function with exponential growth to represent the fast initial development of markets. The Erlang distributed is used by SDM2 to represent S-shaped development patterns, which allows for smooth and continuous expansion of the market. By using the stochastic differential equations and geometrically Brownian dynamics to include inconsistency, these models improve upon existing predictable frameworks and accurately portray uncertain markets and inconsistent adopting behaviours. Verification with real-world sales data shows how reliable and accurate these algorithms are in anticipating acceptance patterns, which is great news for companies and politicians. If SDM1 and SDM2 are further developed, they might be used to situations involving

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the spread of several items that either compete with or complement one another in shared marketplaces. Enhancing the models' prediction powers might be achieved by including other aspects like pricing tactics, advertising efforts, and interpersonal dynamics. They might be even more precisely optimised and parameterized with the help of modern computer algorithms like machine instruction and neural networks. It is possible to increase the models' influence in many other sectors by applying them to new technology and developing markets.

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