Exponential Laws and Aggregation Operators on Neutrosophic Cubic Sets

Majid Khan [1], **Ismat Beg** [2,*] **and Muhammad Gulistan** [1]

[1] Hazara University, Mansehra, Pakistan majid_swati@yahoo.com , gulistanmath@hu.edu.pk
[2] Lahore School of Economics, Lahore, Pakistan ibeg@lahoreschool.edu.pk

* Correspondence: ibeg@lahoreschool.edu.pk

Abstract

This paper presents operational laws along with their cosine measure for the numbers whose base is an interval value and study their properties. Consequent upon these definitions and properties neutrosophic cubic weighted exponential averaging and dual neutrosophic cubic weighted exponential averaging aggregation operators are defined. A multi attribute decision making method is then developed for proposed aggregation operators. An example is constructed as an application. The validity of multi attribute decision making method is also tested and comparative analysis is provided to compare these aggregation operators with existing results.

Keywords: : Neutrosophic cubic number; dual neutrosophic cubic number; neutrosophic cubic exponential weighted averaging; dual neutrosophic cubic exponential weighted averaging ; multi attribute decision making.

1.Introduction

The decision making is an imperative part of cognitive based human activity. Multi attribute decision making method (MADM) provides a better environment to rank the set of alternatives under different criteria. The problem arises when vague and insufficient data is available. This uncertainty in data can be handle by fuzzy set theory (FS) presented by Zadeh [1] and its extensions like interval valued fuzzy set (IVFS) [2,3], intuitionistic fuzzy set [4], interval valued intuitionistic fuzzy set (IVIFS) [5], cubic set (CS) [6]. The intuitionistic fuzzy set attracted the researcher due to its structure of both membership, non-membership and hesitant component. Over the last few decades, several researchers used it for decision making problems [7,8,9,10,11,12]. In IFS the hesitancy component is depended upon the choice of membership degree and non-membership degree which restricted the choice of choosing. Smarandache defined a novel tool, neutrosophic set NS [13] to deal with vagueness in more desirable way. NS are generalization of IFS [14]. In NS all the components are independent. Soon after its presentation, it is further extended into INS [15], NCS [16] etc. The collection and manipulation of data is a hard job. In daily life problems we are often in a situation that the extraction of accurate and precise information is not possible due to the vague nature of problem, communication gaps, hesitancy etc. Thus NS and its extension are better tools to deal with such situation in comparison with IFS and cubic set. This characteristic of NS and INS attracted the researchers to apply it in different field of decision making process [17,18,19,20,21,22,23,24,25]. Neutrosophic Cubic Number (NCN) is a combination of INS and NS which makes it a better choice so that expert can choose the value in the form of an interval value and single value. NCS provides a better plate form to deal with vague and insufficient data. In recent pass researchers have

DOI: 10.5281/zenodo.3758466
Received: January 12, 2020 Revised: March 28 2020 Accepted: April 16, 2020

applied NCS to aggregation operators and decision making problem. Zhan *et al.* [26] worked on multi criteria decision making on neutrosophic cubic sets. Banerjee *et al.* [27] used GRA for multi criteria decision making on neutrosophic cubic set. Lu and Ye [28] defined cosine measure to neutrosophic cubic set. Pramanik *et al*. [29] used similarity measure to neutrosophic cubic set in 2017. Majid *et al.* [30] presented neutrosophic cubic Einstein geometric aggreagtion operators. Khalid *et al.* [31] introduced MBJ – Neutrosophic Translation on G-Algebra and Schweizer [32] studied two probabilities for the three states of neutrosophy. In their works all these researchers defined arithematic and Einstein operation and aggregations operators under neutrosophic cubic environment. It is observed in decision making process that the weight is based on the assumption to be in [0,1] and their sum is one. The aim of this work is to deal the situation where the weight is crisp or interval value. To support this task some new operational laws along with some aggregation operators are defined on NCS.

Contribution In this work, we propose methodologies to measures the aggregate value of neutrosophic cubic values using exponential form.

- The neutrosophic cubic exponential operational laws are defined along with some important properties.
- Based on these operations and properties neutrosophic cubic exponential aggregations operators are defined. Validity and comparative analysis are discussed.
- The neutrosophic cubic set is the combination of both neutrosophic and interval neutrosophic, this characteristic enable us to deal both interval valued neutrosophic and neutrosophic set at the same time.

Organization This study consist of nine sections. Section 1 covers the introductory work of researchers over the some past decades. In section 2 the preliminaries work is reviewed which enable us to start this work. In section 3 some exponential operational laws with base crisp and interval value are introduced, dual neutrosophic cubic number (DNCN) are defined and some useful properties are obtained. Cosine measures is defined to compare two NCN and DNCN. Section 4, presents neutrosophic cubic weighted exponential averaging (NCWEA) operators and dual neutrosophic cubic weighted exponential averaging (DNCWEA) aggregations operators are defined in which the weight is in the form of crisp value or interval valued and the exponents are NCS. Section 5 develops a decision making process for the proposed aggregations operators. An illustrative example is provided as an application in section 6. In Section 7, the validity test is performed for DM problem to check the validity of MADM. In section 8, the MADM based upon proposed aggregation operators is compared with some neutrosophic cubic weighted averaging (NCWA) and neutrosophic cubic Einstein weighted averaging (NCEWA) aggregation operators. The paper ends with conclusion in section 9.

2. Preliminaries

Definition 2.1 [13] A structure $N=\left\{\left(T_N(y),I_N(y),F_N(y)\right)\mid y\in Y\right\}$ NS, where T_N, I_N and F_N are fuzzy sets and respectively called truth, indeterminacy and falsity functions.

Definition 2.2 [15] An INS in Y is a structure $N=\left\{\left(\tilde{T}_N(y),\tilde{I}_N(y),\tilde{F}_N(y)\right)\mid y\in Y\right\}$ where $\tilde{T}_N(y)$, $\tilde{I}_N(y)$ and $\tilde{F}_N(y)$ are interval valued fuzzy truth, indeterminacy an falsity function in Y respectively.

Definition 2.3 [16] A structure $A=\left\{\left(y,\tilde{T}_N(y),\tilde{I}_N(y),\tilde{F}_N(y),T_N(y),I_N(y),F_N(y)\right)/\,y\in Y\right\}$ is NCS in Y in which $\left(\tilde{T}_N(y)=\left[T_N^L,T_N^U\right],\tilde{I}_N(y)=\left[I_N^L,I_N^U\right],\tilde{F}_N(y)=\left[F_N^L,F_N^U\right]\right)$ is an INS and $\left(T_N,I_N,F_N\right)$ is NS in Y. Simply denoted by $N=\left(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N,T_N,I_N,F_N\right)$ where $,[0,0]\leq\tilde{T}_N+\tilde{I}_N+\tilde{F}_N\leq[3,3],0\leq T_N+I_N+F_N\leq 3.$ N^Y denotes the collection of NCS in Y .

For the sake of convenience the NCS are written as $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$

DOI: 10.5281/zenodo.3758466

Definition 2.4 [30] The sum of two NCN, $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ and $B=([T_B^L,T_B^U],[I_B^L,I_B^U],[F_B^L,F_B^U],T_B,I_B,F_B)$ is defined as

$$A\oplus B=\left(\begin{array}{c}[T_A^L+T_B^L-T_A^LT_B^L,T_A^U+T_B^U-T_A^UT_B^U],[I_A^L+I_B^L-I_A^LI_B^L,I_A^U+I_B^U-I_A^UI_B^U],\\ [F_A^LF_B^L,F_A^UF_B^U],T_AT_B,I_AI_B,F_A+F_B-F_AF_B\end{array}\right)$$

Definition 2.5 [30] The product of two NCN, $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ and $B=\left(\left[T_B^L,T_B^U\right],\left[I_B^L,I_B^U\right],\left[F_B^L,F_B^U\right],T_B,I_B,F_B\right)$ is defined as

$$A\otimes B=\left(\begin{array}{c}[T_A^LT_B^L,T_A^UT_B^U],[I_A^LI_B^L,I_A^UI_B^U],[F_A^L+F_B^L-F_A^LF_B^L,F_A^U+F_B^U-F_A^UF_B^U],\\ T_A+T_B-T_AT_B,I_A+I_B-I_AI_B,F_AF_B\end{array}\right)$$

Definition 2.6 [30] The scalar multiplication on a NCN $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ and a Scalar ϖ is defined

$$\varpi A=\left(\begin{array}{c}[1-(1-T_A^L)^{\varpi},1-(1-T_A^U)^{\varpi}],[1-(1-I_A^L)^{\varpi},1-(1-I_A^U)^{\varpi}],\\ [\left(F_A^L\right)^{\varpi},\left(F_A^U\right)^{\varpi}],\left(T_A\right)^{\varpi},\left(I_A\right)^{\varpi},1-\left(1-F_A\right)^{\varpi}\end{array}\right)$$

Theorem 2.7 [30] Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ be a NCN, then the exponential operation is defined by

$$A^{\varpi}=\left(\begin{array}{c}[(T_A^L)^{\varpi},(T_A^U)^{\varpi}],[(I_A^L)^{\varpi},(I_A^U)^{\varpi}],[1-\left(1-F_A^L\right)^{\varpi},1-\left(1-F_A^U\right)^{\varpi}],\\ 1-\left(1-T_A\right)^{\varpi},1-\left(1-I_A\right)^{\varpi},\left(F_A\right)^{\varpi}\end{array}\right)$$

3. Exponential operational laws with crisp and interval parameters on neutrosophic cubic sets

Operational laws has a key role in any work. In this section we develop some exponential laws in neutrosophic cubic environment in which the exponential parameters are crisp and interval value.

Definition 3.1 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ be a NCN the exponential law for crisp value ∇ is defined as

$$\nabla^A=\begin{cases}\begin{pmatrix}\left[\nabla^{1-(T_A)^L},\nabla^{1-(T_A)^U}\right],\left[\nabla^{1-(I_A)^L},\nabla^{1-(I_A)^U}\right],\\ \left[1-\nabla^{(F_A)^L},1-\nabla^{(F_A)^U}\right],1-\nabla^{T_A},1-\nabla^{I_A},\nabla^{1-F_A}\end{pmatrix},\nabla\in[0,1]\\ \begin{pmatrix}\left[(1/\nabla)^{1-(T_A)^L},(1/\nabla)^{1-(T_A)^U}\right],\left[(1/\nabla)^{1-(I_A)^L},(1/\nabla)^{1-(I_A)^U}\right],\\ \left[1-(1/\nabla)^{(F_A)^L},1-(1/\nabla)^{(F_A)^U}\right],1-(1/\nabla)^{T_A},1-(1/\nabla)^{I_A},(1/\nabla)^{1-F_A}\end{pmatrix},\nabla>1\end{cases}$$

In both cases ∇^A is a NCN.

Example 3.2 Let $A=([0.2,0.8],[0.4,0.7],[0.1,0.5],0.7,0.2,0.6)$ be a NCN, $\nabla=0.5$ and $\nabla=3$, then

$$\nabla^A=\begin{cases}([0.574,0.870],[0.659,0.812],[0.066,0.292],0.384,0.129,0.757),\nabla=0.5\\ ([0.801,0.411],[0.641,0.460],[0.104,0.425],0.539,0.198,0.514),\nabla=3\end{cases}$$

Definition 3.3 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$, be a NCN and

$A^*=([1,1],[1,1],[0,0],0,0,1)$ be maximum NCN, then the cosine measure(C_m) is defined as

$$C_m(A)=\left\{\cos\frac{\pi}{18}\left(1-T_A^L+1-T_A^U+1-I_A^L+1-I_A^U+F_A^L+F_A^U+T_A+I_A+1-F_A\right)\right\},C_m(A)\in[0,1]$$

Remark 3.4 If C_m *(A)* and C_m *(B)* be the cosine measures of two NCN then $C_m(A)>C_m(B)\Rightarrow A>B$ *and* $C_m(A)=C_m(B)\Rightarrow A=B$

Theorem 3.5 Let $A=\left(\left[\nabla^{1-T_A^L},\nabla^{1-T_A^U}\right],\left[\nabla^{1-I_A^L},\nabla^{1-I_A^U}\right],\left[1-\nabla^{F_A^L},1-\nabla^{F_A^U}\right],1-\nabla^{T_A},1-\nabla^{I_A},\nabla^{1-F_A}\right)$ be a neutrosophic cubic value and $\nabla_1\geq\nabla_2$, then $(\nabla_1)^A\geq(\nabla_2)^A$ for $\nabla_1,\nabla_2\in[0,1]$ and $(\nabla_1)^A\leq(\nabla_2)^A$ for $\nabla_1,\nabla_2>1.$

Proof: Let $\nabla_1\geq\nabla_2$ and $\nabla_1,\nabla_2\in[0,1]$ then

$$(\nabla_1)^A=\begin{pmatrix}\left[(\nabla_1)^{1-T_A^L},(\nabla_1)^{1-T_A^U}\right],\left[(\nabla_1)^{1-I_A^L},(\nabla_1)^{1-I_A^U}\right],\\ \left[1-(\nabla_1)^{F_A^L},1-(\nabla_1)^{F_A^U}\right],1-(\nabla_1)^{T_A},1-(\nabla_1)^{I_A},(\nabla_1)^{1-F_A}\end{pmatrix}$$

and

$$(\nabla_2)^A=\begin{pmatrix}\left[(\nabla_2)^{1-T_A^L},(\nabla_2)^{1-T_A^U}\right],\left[(\nabla_2)^{1-I_A^L},(\nabla_2)^{1-I_A^U}\right],\\ \left[1-(\nabla_2)^{F_A^L},1-(\nabla_2)^{F_A^U}\right],1-(\nabla_2)^{T_A},1-(\nabla_2)^{I_A},(\nabla_2)^{1-F_A}\end{pmatrix}$$

Since $(\nabla_1)^{1-T_A^L}\geq(\nabla_2)^{1-T_A^L},(\nabla_1)^{1-T_A^U}\geq(\nabla_2)^{1-T_A^U}$, $(\nabla_1)^{1-I_A^L}\geq(\nabla_2)^{1-I_A^L},(\nabla_1)^{1-I_A^U}\geq(\nabla_2)^{1-I_A^U}$, $1-(\nabla_1)^{F_A^L}\leq1-(\nabla_2)^{F_A^L},1-(\nabla_1)^{F_A^U}\leq1-(\nabla_2)^{F_A^U}$, $1-(\nabla_1)^{T_A}\leq1-(\nabla_2)^{T_A}$, $1-(\nabla_1)^{I_A}\leq1-(\nabla_2)^{I_A}$, $(\nabla_1)^{1-F_A}\geq(\nabla_2)^{1-F_A}$, so

$$\cos\left((\nabla_1)^A\right)=\left\{\frac{\pi}{18}\begin{pmatrix}1-(\nabla_1)^{1-T_A^L}+1-(\nabla_1)^{1-I_A^L}+(\nabla_1)^{F_A^L}+\\ 1-(\nabla_1)^{1-T_A^U}+1-(\nabla_1)^{1-I_A^U}+(\nabla_1)^{F_A^U}+\\ (\nabla_1)^{T_A}+(\nabla_1)^{I_A}+1-(\nabla_1)^{1-F_A}\end{pmatrix}\right\}\geq$$

$$\cos\left((\nabla_2)^A\right)=\left\{\frac{\pi}{18}\begin{pmatrix}1-(\nabla_2)^{1-T_A^L}+1-(\nabla_2)^{1-I_A^L}+(\nabla_2)^{F_A^L}\\ +1-(\nabla_2)^{1-T_A^U}+1-(\nabla_2)^{1-I_A^U}+(\nabla_2)^{F_A^U}+\\ (\nabla_2)^{T_A}+(\nabla_2)^{I_A}+1-(\nabla_2)^{1-F_A}\end{pmatrix}\right\}$$

obviously $(\nabla_1)^A\geq(\nabla_2)^A$, if $\nabla_1\geq\nabla_2$ and $\nabla_1,\nabla_2>1,$ then $0</\nabla_1,1/\nabla_2\leq 1.$

Remark 3.6 Considering some values of ∇ , we can affirm some special cases of $(\nabla)^A$.

1. If $\nabla=1,$ then $(\nabla)^A=\left([1,1],[1,1],[0,0],0,0,1\right).$

2. If $A=\left([1,1],[1,1],[0,0],0,0,1\right)$, then $(\nabla)^A=\left([1,1],[1,1],[0,0],0,0,1\right).$ For each value of ∇.

3. If $A=\left([0,0],[0,0],[1,1],1,1,0\right)$,then $(\nabla)^A=\left([\nabla,\nabla],[\nabla,\nabla],\left[1-(\nabla)^{F_A^L},1-(\nabla)^{F_A^U}\right],1-(\nabla)^{T_A},1-(\nabla)^{I_A},\nabla\right).$

Theorem 3.7 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$, $B=\left(\left[T_B^L,T_B^U\right],\left[I_B^L,I_B^U\right],\left[F_B^L,F_B^U\right],T_B,I_B,F_B\right)$ and $C=\left(\left[T_C^L,T_C^U\right],\left[I_C^L,I_C^U\right],\left[F_C^L,F_C^U\right],T_C,I_C,F_C\right)$ be three NCNs and $\nabla\in[0,1]$ $(if\ \nabla>1, then 1/\nabla)$ then the following holds.

- ➢ $\nabla^A\oplus\nabla^B=\nabla^B\oplus\nabla^A$
- ➢ $\nabla^A\otimes\nabla^B=\nabla^B\otimes\nabla^A$
- ➢ $\left(\nabla^A\oplus\nabla^B\right)\oplus\nabla^C=\nabla^A\oplus\left(\nabla^B\oplus\nabla^C\right)$
- ➢ $\nabla^A\otimes\nabla^B\otimes\nabla^C=\nabla^A\otimes\left(\nabla^B\otimes\nabla^C\right)$

Proof Straight forward, so omitted.

Theorem 3.8 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$, $B=\left(\left[T_B^L,T_B^U\right],\left[I_B^L,I_B^U\right],\left[F_B^L,F_B^U\right],T_B,I_B,F_B\right)$ be two NCNs and ϖ be a scalar then the following holds.

- ➢ $\varpi(\triangledown^A\oplus\triangledown^B)=\varpi\triangledown^A\oplus\varpi\triangledown^B$
- ➢ $(\triangledown^A\oplus\triangledown^B)^{\varpi}=(\triangledown^A)^{\varpi}\oplus(\triangledown^B)^{\varpi}$
- ➢ $(\varpi_1\oplus\varpi_2)\triangledown^A=\varpi_1\triangledown^A\oplus\varpi_2\triangledown^A$
- ➢ $((\triangledown^A)^{\varpi_1})^{\varpi_2}=(\triangledown^A)^{\varpi_1\varpi_2}$

Proof: Consider

DOI: 10.5281/zenodo.3758466

$$\varpi\left(\nabla^{A}\oplus\nabla^{B}\right)=\varpi\left\{\left(\begin{array}{c}\left[\nabla^{1-T_A^L},\nabla^{1-T_A^U}\right],\left[\nabla^{1-I_A^L},\nabla^{1-I_A^U}\right],\\ \left[1-\nabla^{F_A^L},1-\nabla^{F_A^U}\right],\\ 1-\nabla^{T_A},1-\nabla^{I_A},\nabla^{1-F_A}\end{array}\right)\oplus\left(\begin{array}{c}\left[\nabla^{1-T_B^L},\nabla^{1-T_B^U}\right],\left[\nabla^{1-I_B^L},\nabla^{1-I_B^U}\right],\\ \left[1-\nabla^{F_B^L},1-\nabla^{F_B^U}\right],\\ 1-\nabla^{T_B},1-\nabla^{I_B},\nabla^{1-F_B}\end{array}\right)\right\}$$

$$=\varpi\left\{\left(\begin{array}{c}\left[\nabla^{1-T_A^L}+\nabla^{1-T_B^L}-\nabla^{1-T_A^L}\ \nabla^{1-T_B^L},\nabla^{1-T_A^U}+\nabla^{1-T_B^U}-\nabla^{1-T_A^U}\ \nabla^{1-T_B^U}\right],\\ \left[\nabla^{1-I_A^L}+\nabla^{1-I_B^L}-\nabla^{1-I_A^L}\ \nabla^{1-I_B^L},\nabla^{1-I_A^U}+\nabla^{1-I_B^U}-\nabla^{1-I_A^U}\ \nabla^{1-I_B^U}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)\left(1-\nabla^{F_B^L}\right),\left(1-\nabla^{F_B^U}\right)\left(1-\nabla^{F_B^U}\right)\right],\\ \left(1-\nabla^{T_A}\right)\left(1-\nabla^{T_B}\right),\left(1-\nabla^{I_A}\right)\left(1-\nabla^{I_B}\right),\nabla^{1-F_A}+\nabla^{1-F_B}-\nabla^{1-F_A}\ \nabla^{1-F_B}\end{array}\right)\right\}$$

$$=\left\{\begin{array}{c}\left[\begin{array}{c}1-\left(1-\nabla^{1-T_A^L}-\nabla^{1-T_B^L}\ \nabla^{1-T_A^L}\ \nabla^{1-T_B^L}\right)^{\varpi},\\ 1-\left(1-\nabla^{1-T_A^U}-\nabla^{1-T_B^U}\ \nabla^{1-T_A^U}\ \nabla^{1-T_B^U}\right)^{\varpi}\end{array}\right],\left[\begin{array}{c}1-\left(1-\nabla^{1-I_A^L}-\nabla^{1-I_B^L}\ \nabla^{1-I_A^L}\ \nabla^{1-I_B^L}\right)^{\varpi},\\ 1-\left(1-\nabla^{1-I_A^U}-\nabla^{1-I_B^U}\ \nabla^{1-I_A^U}\ \nabla^{1-I_B^U}\right)^{\varpi}\end{array}\right],\\ \left[\left(\left(1-\nabla^{F_A^L}\right)\left(1-\nabla^{F_B^L}\right)\right)^{\varpi},\left(\left(1-\nabla^{F_B^U}\right)\left(1-\nabla^{F_B^U}\right)\right)^{\varpi}\right],\\ \left(1-\nabla^{T_A}\right)\left(1-\nabla^{T_B}\right)^{\varpi},\left(\left(1-\nabla^{I_A}\right)\left(1-\nabla^{I_B}\right)^{\varpi}\right),\\ \left(1-\left(1-\nabla^{1-F_A}\ \nabla^{1-F_B}+\nabla^{1-F_A}\ \nabla^{1-F_B}\right)^{\varpi}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left[\begin{array}{c}1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}\left(1-\nabla^{1-T_B^L}\right)^{\varpi},\\ 1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}\left(1-\nabla^{1-T_B^L}\right)^{\varpi}\end{array}\right],\left[\begin{array}{c}1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}\left(1-\nabla^{1-I_B^L}\right)^{\varpi},\\ 1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}\left(1-\nabla^{1-I_B^L}\right)^{\varpi}\end{array}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)^{\varpi}\left(1-\nabla^{F_B^L}\right)^{\varpi},\left(1-\nabla^{F_B^U}\right)^{\varpi}\left(1-\nabla^{F_B^U}\right)^{\varpi}\right],\\ \left(1-\nabla^{T_A}\right)^{\varpi}\left(1-\nabla^{T_B}\right)^{\varpi},\left(1-\nabla^{I_A}\right)^{\varpi}\left(1-\nabla^{I_B}\right)^{\varpi},\\ 1-\left(1-\nabla^{1-F_A}\right)^{\varpi}\left(1-\nabla^{1-F_B}\right)^{\varpi}\end{array}\right\}$$

DOI: 10.5281/zenodo.3758466

$$
=\left\{\begin{array}{c}
\left[\begin{array}{c}
1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}-\left(1-\nabla^{1-T_B^L}\right)^{\varpi}+\left(1-\nabla^{1-T_A^L}\right)^{\varpi}+\\
\left(1-\nabla^{1-T_B^L}\right)^{\varpi}-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}\left(1-\nabla^{1-T_B^L}\right)^{\varpi},\\
1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi}-\left(1-\nabla^{1-T_B^U}\right)^{\varpi}+\left(1-\nabla^{1-T_A^U}\right)^{\varpi}+\\
\left(1-\nabla^{1-T_B^U}\right)^{\varpi}-\left(1-\nabla^{1-T_A^U}\right)^{\varpi}\left(1-\nabla^{1-T_B^U}\right)^{\varpi}
\end{array}\right],\\
\left[\begin{array}{c}
1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}-\left(1-\nabla^{1-I_B^L}\right)^{\varpi}+\left(1-\nabla^{1-I_A^L}\right)^{\varpi}+\\
\left(1-\nabla^{1-I_B^L}\right)^{\varpi}-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}\left(1-\nabla^{1-I_B^L}\right)^{\varpi},\\
1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi}-\left(1-\nabla^{1-I_B^U}\right)^{\varpi}+\left(1-\nabla^{1-I_A^U}\right)^{\varpi}+\\
\left(1-\nabla^{1-I_B^U}\right)^{\varpi}-\left(1-\nabla^{1-I_A^U}\right)^{\varpi}\left(1-\nabla^{1-I_B^U}\right)^{\varpi}
\end{array}\right],\\
\left[\varpi\,\nabla^{F_A^L}\oplus\varpi\,\nabla^{F_B^L},\varpi\,\nabla^{F_A^U}\oplus\varpi\,\nabla^{F_B^U}\right],\varpi\,\nabla^{T_A}\oplus\varpi\,\nabla^{T_B},\varpi\,\nabla^{I_A}\oplus\varpi\,\nabla^{I_B},\\
1-\left(1-\nabla^{1-T_A}\right)^{\varpi}-\left(1-\nabla^{1-T_B}\right)^{\varpi}+\left(1-\nabla^{1-T_A}\right)^{\varpi}+\\
\left(1-\nabla^{1-T_B}\right)^{\varpi}-\left(1-\nabla^{1-T_A}\right)^{\varpi}\left(1-\nabla^{1-T_B}\right)^{\varpi}
\end{array}\right\}
$$

$$
=\left\{\begin{array}{c}
\left[\begin{array}{c}
1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}+1-\left(1-\nabla^{1-T_B^L}\right)^{\varpi}-\left(1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}\left(1-\nabla^{1-T_B^L}\right)^{\varpi}\right),\\
1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi}+1-\left(1-\nabla^{1-T_B^U}\right)^{\varpi}-\left(1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi}\left(1-\nabla^{1-T_B^U}\right)^{\varpi}\right)
\end{array}\right],\\
\left[\begin{array}{c}
1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}+1-\left(1-\nabla^{1-I_B^L}\right)^{\varpi}-\left(1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}\left(1-\nabla^{1-I_B^L}\right)^{\varpi}\right),\\
1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi}+1-\left(1-\nabla^{1-I_B^U}\right)^{\varpi}-\left(1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi}\left(1-\nabla^{1-I_B^U}\right)^{\varpi}\right)
\end{array}\right],\\
\left[\varpi\,\nabla^{F_A^L}\oplus\varpi\,\nabla^{F_B^L},\varpi\,\nabla^{F_A^U}\oplus\varpi\,\nabla^{F_B^U}\right],\varpi\,\nabla^{T_A}\oplus\varpi\,\nabla^{T_B},\varpi\,\nabla^{I_A}\oplus\varpi\,\nabla^{I_B}
\end{array}\right\}
$$

$$=\left\{\begin{array}{c}\left[\begin{array}{c}\left(1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi}\right)\oplus\left(1-\left(1-\nabla^{1-T_B^L}\right)^{\varpi}\right),\\ \left(1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi}\right)\oplus\left(1-\left(1-\nabla^{1-T_B^U}\right)^{\varpi}\right)\end{array}\right],\\ \left[\begin{array}{c}\left(1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi}\right)\oplus\left(1-\left(1-\nabla^{1-I_B^L}\right)^{\varpi}\right),\\ \left(1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi}\right)\oplus\left(1-\left(1-\nabla^{1-I_B^U}\right)^{\varpi}\right)\end{array}\right],\\ \left[\varpi\,\nabla^{F_A^L}\oplus\varpi\,\nabla^{F_B^L},\varpi\,\nabla^{F_A^U}\oplus\varpi\,\nabla^{F_B^U}\right],\varpi\,\nabla^{T_A}\oplus\varpi\,\nabla^{T_B},\varpi\,\nabla^{I_A}\oplus\varpi\,\nabla^{I_B},\\ 1-\left(1-\nabla^{1-T_A}\right)^{\varpi}+1-\left(1-\nabla^{1-T_B}\right)^{\varpi}-\left(1-\left(1-\nabla^{1-T_A}\right)^{\varpi}\left(1-\nabla^{1-T_B}\right)^{\varpi}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left[\varpi\,\nabla^{T_A^L}\oplus\varpi\,\nabla^{T_B^L},\varpi\,\nabla^{T_A^U}\oplus\varpi\,\nabla^{T_B^U}\right],\left[\varpi\,\nabla^{I_A^L}\oplus\varpi\,\nabla^{I_B^L},\varpi\,\nabla^{I_A^U}\oplus\varpi\,\nabla^{I_B^U}\right],\\ \left[\varpi\,\nabla^{F_A^L}\oplus\varpi\,\nabla^{F_B^L},\varpi\,\nabla^{F_A^U}\oplus\varpi\,\nabla^{F_B^U}\right],\varpi\,\nabla^{T_A}\oplus\varpi\,\nabla^{T_B},\varpi\,\nabla^{I_A}\oplus\varpi\,\nabla^{I_B},\varpi\,\nabla^{F_A}\oplus\varpi\,\nabla^{F_B}\end{array}\right\}$$

$$=\varpi\,\nabla^{A}\oplus\varpi\,\nabla^{B}$$

- $$\left(\varpi_1+\varpi_2\right)\nabla^{A}=\left(\varpi_1+\varpi_2\right)\left\{\begin{array}{c}\left[\nabla^{1-T_A^L},\nabla^{1-T_A^U}\right],\left[\nabla^{1-I_A^L},\nabla^{1-I_A^U}\right],\\ \left[1-\nabla^{F_A^L},1-\nabla^{F_A^U}\right],1-\nabla^{T_A},1-\nabla^{I_A},\nabla^{1-F_A}\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left[1-\left(1-\nabla^{1-T_A^L}\right)^{(\varpi_1+\varpi_2)},1-\left(1-\nabla^{1-T_A^U}\right)^{(\varpi_1+\varpi_2)}\right],\\ \left[1-\left(1-\nabla^{1-I_A^L}\right)^{(\varpi_1+\varpi_2)},1-\left(1-\nabla^{1-I_A^U}\right)^{(\varpi_1+\varpi_2)}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)^{(\varpi_1+\varpi_2)},\left(1-\nabla^{F_A^U}\right)^{(\varpi_1+\varpi_2)}\right],\\ \left(1-\nabla^{T_A}\right)^{(\varpi_1+\varpi_2)},\left(1-\nabla^{I_A}\right)^{(\varpi_1+\varpi_2)},1-\left(1-\nabla^{1-F_A}\right)^{(\varpi_1+\varpi_2)}\end{array}\right\}$$

DOI: 10.5281/zenodo.3758466

$$=\left\{\begin{array}{c}\left[\begin{array}{c}1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi_1}+\left(1-\nabla^{1-T_A^L}\right)^{\varpi_1}-\left(1-\nabla^{1-T_A^L}\right)^{\varpi_2}+\\ \left(1-\nabla^{1-T_A^L}\right)^{\varpi_2}-\left(1-\nabla^{1-T_A^L}\right)^{\varpi_1}\left(1-\nabla^{1-T_A^L}\right)^{\varpi_2},\\ 1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi_1}+\left(1-\nabla^{1-T_A^U}\right)^{\varpi_1}-\left(1-\nabla^{1-T_A^U}\right)^{\varpi_2}+\\ \left(1-\nabla^{1-T_A^U}\right)^{\varpi_2}-\left(1-\nabla^{1-T_A^U}\right)^{\varpi_1}\left(1-\nabla^{1-T_A^U}\right)^{\varpi_2}\end{array}\right],\\ \left[\begin{array}{c}1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi_1}+\left(1-\nabla^{1-I_A^L}\right)^{\varpi_1}-\left(1-\nabla^{1-I_A^L}\right)^{\varpi_2}+\\ \left(1-\nabla^{1-I_A^L}\right)^{\varpi_2}-\left(1-\nabla^{1-I_A^L}\right)^{\varpi_1}\left(1-\nabla^{1-I_A^L}\right)^{\varpi_2},\\ 1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi_1}+\left(1-\nabla^{1-I_A^U}\right)^{\varpi_1}-\left(1-\nabla^{1-I_A^U}\right)^{\varpi_2}+\\ \left(1-\nabla^{1-I_A^U}\right)^{\varpi_2}-\left(1-\nabla^{1-I_A^U}\right)^{\varpi_1}\left(1-\nabla^{1-I_A^U}\right)^{\varpi_2}\end{array}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)^{\varpi_1}\left(1-\nabla^{F_A^L}\right)^{\varpi_2},\left(1-\nabla^{F_A^U}\right)^{\varpi_1}\left(1-\nabla^{F_A^U}\right)^{\varpi_2}\right],\\ \left(1-\nabla^{T_A}\right)^{\varpi_1}\left(1-\nabla^{T_A}\right)^{\varpi_2},\left(1-\nabla^{I_A}\right)^{\varpi_1}\left(1-\nabla^{I_A}\right)^{\varpi_2},\\ 1-\left(1-\nabla^{1-F_A}\right)^{\varpi_1}+\left(1-\nabla^{1-F_A}\right)^{\varpi_1}-\\ \left(1-\nabla^{1-F_A}\right)^{\varpi_2}\left(1-\nabla^{1-F_A}\right)^{\varpi_2}-\left(1-\nabla^{1-F_A}\right)^{\varpi_1}\left(1-\nabla^{1-F_A}\right)^{\varpi_2}\end{array}\right\}$$

$$=\left\{\left(\begin{array}{c}\left[1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi_1},1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi_1}\right],\\ \left[1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi_1},1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi_1}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)^{\varpi_1},\left(1-\nabla^{F_A^U}\right)^{\varpi_1}\right],\\ \left(1-\nabla^{T_A}\right)^{\varpi_1},\left(1-\nabla^{I_A}\right)^{\varpi_1},\\ 1-\left(1-\nabla^{1-F_A}\right)^{\varpi_1}\end{array}\right)\oplus\left(\begin{array}{c}\left[1-\left(1-\nabla^{1-T_A^L}\right)^{\varpi_2},1-\left(1-\nabla^{1-T_A^U}\right)^{\varpi_2}\right],\\ \left[1-\left(1-\nabla^{1-I_A^L}\right)^{\varpi_2},1-\left(1-\nabla^{1-I_A^U}\right)^{\varpi_2}\right],\\ \left[\left(1-\nabla^{F_A^L}\right)^{\varpi_2},\left(1-\nabla^{F_A^U}\right)^{\varpi_2}\right],\\ \left(1-\nabla^{T_A}\right)^{\varpi_2},\left(1-\nabla^{I_A}\right)^{\varpi_2},\\ 1-\left(1-\nabla^{1-F_A}\right)^{\varpi_2}\end{array}\right)\right\}$$

$$=\varpi_1\,\nabla^A\oplus\varpi_2\,\nabla^A$$

DOI: 10.5281/zenodo.3758466

➢ $$\left(\left(\nabla^{A}\right)^{\varpi_1}\right)^{\varpi_2}=\begin{pmatrix}\left[\left(\nabla^{1-T_A^L}\right)^{\varpi_1},\left(\nabla^{1-T_A^U}\right)^{\varpi_1}\right],\left[\left(\nabla^{1-I_A^L}\right)^{\varpi_1},\left(\nabla^{1-I_A^U}\right)^{\varpi_1}\right],\\ \left[1-\left(1-\nabla^{F_A^L}\right)^{\varpi_1},1-\left(1-\nabla^{F_A^U}\right)^{\varpi_1}\right],\\ 1-\left(1-\nabla^{T_A}\right)^{\varpi_1},1-\left(1-\nabla^{I_A}\right)^{\varpi_1},\left(\nabla^{1-F_A}\right)^{\varpi_1}\end{pmatrix}^{\varpi_2}$$

$$=\begin{pmatrix}\left[\left(\nabla^{1-T_A^L}\right)^{\varpi_1\varpi_2},\left(\nabla^{1-T_A^U}\right)^{\varpi_1\varpi_2}\right],\left[\left(\nabla^{1-I_A^L}\right)^{\varpi_1\varpi_2},\left(\nabla^{1-I_A^U}\right)^{\varpi_1\varpi_2}\right],\\ \left[1-\left(1-\left(1-\left(1-\nabla^{F_A^L}\right)\right)^{\varpi_1}\right)^{\varpi_2},1-\left(1-\left(1-\left(1-\nabla^{F_A^U}\right)\right)^{\varpi_1}\right)^{\varpi_2}\right],\\ 1-\left(1-\left(1-\left(1-\nabla^{T_A}\right)\right)^{\varpi_1}\right)^{\varpi_2},\\ 1-\left(1-\left(1-\left(1-\nabla^{I_A}\right)\right)^{\varpi_1}\right)^{\varpi_2},\left(\left(\nabla^{1-F_A}\right)^{\varpi_1}\right)^{\varpi_2}\end{pmatrix}$$

$$=\begin{pmatrix}\left[\left(\nabla^{1-T_A^L}\right)^{\varpi_1\varpi_2},\left(\nabla^{1-T_A^U}\right)^{\varpi_1\varpi_2}\right],\left[\left(\nabla^{1-I_A^L}\right)^{\varpi_1\varpi_2},\left(\nabla^{1-I_A^U}\right)^{\varpi_1\varpi_2}\right],\\ \left[1-\left(1-\nabla^{F_A^L}\right)^{\varpi_1\varpi_2},1-\left(1-\nabla^{F_A^U}\right)^{\varpi_1\varpi_2}\right],\\ 1-\left(1-\nabla^{T_A}\right)^{\varpi_1\varpi_2},1-\left(1-\nabla^{I_A}\right)^{\varpi_1\varpi_2},\left(\nabla^{1-F_A}\right)^{\varpi_1\varpi_2}\end{pmatrix}$$

$$=\left(\nabla^{A}\right)^{\varpi_1\varpi_2}$$

Definition 3.9 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$, $B=\left(\left[T_B^L,T_B^U\right],\left[I_B^L,I_B^U\right],\left[F_B^L,F_B^U\right],T_B,I_B,F_B\right)$ be two NCN, then $\tilde{d}=\left[A,B\right]$ is referred as DNCN.

Definition 3.10 Let $A=\left(\left[T_A^L,T_A^U\right],\left[I_A^L,I_A^U\right],\left[F_A^L,F_A^U\right],T_A,I_A,F_A\right)$ be a NCN the exponential law for interval value parameter for $\tilde{\nabla}=\left[\nabla^L,\nabla^U\right]$ is defined as

$$(\tilde{\nabla})^{A}=\begin{cases}\left\{\begin{array}{c}\left(\begin{array}{c}\left[(\nabla^{L})^{1-(T_A)^L},(\nabla^{L})^{1-(T_A)^U}\right],\\ \left[(\nabla^{L})^{1-(I_A)^L},(\nabla^{L})^{1-(I_A)^U}\right],\\ \left[1-(\nabla^{L})^{(F_A)^L},1-(\nabla^{L})^{(F_A)^U}\right],\\ 1-(\nabla^{L})^{T_A},1-(\nabla^{L})^{I_A},(\nabla^{L})^{1-F_A}\end{array}\right),\\ \left(\begin{array}{c}\left[(\nabla^{U})^{1-(T_A)^L},(\nabla^{U})^{1-(T_A)^U}\right],\\ \left[(\nabla^{U})^{1-(I_A)^L},(\nabla^{U})^{1-(I_A)^U}\right],\\ \left[1-(\nabla^{U})^{(F_A)^L},1-(\nabla^{U})^{(F_A)^U}\right],\\ 1-(\nabla^{U})^{T_A},1-(\nabla^{U})^{I_A},(\nabla^{U})^{1-F_A}\end{array}\right)\end{array}\right\},\nabla^{L},\nabla^{U}\in[0,1]\\ \left\{\begin{array}{c}\left(\begin{array}{c}\left[(1/(\nabla^{L}))^{1-(T_A)^L},(1/(\nabla^{L}))^{1-(T_A)^U}\right],\\ \left[(1/(\nabla^{L}))^{1-(I_A)^L},(1/(\nabla^{L}))^{1-(I_A)^U}\right],\\ \left[1-(1/(\nabla^{L}))^{(F_A)^L},1-(1/(\nabla^{L}))^{(F_A)^U}\right],\\ 1-(1/(\nabla^{L}))^{T_A},1-(1/(\nabla^{L}))^{I_A},(1/(\nabla^{L}))^{1-F_A}\end{array}\right),\\ \left(\begin{array}{c}\left[(1/(\nabla^{U}))^{1-(T_A)^L},(1/(\nabla^{U}))^{1-(T_A)^U}\right],\\ \left[(1/(\nabla^{U}))^{1-(I_A)^L},(1/(\nabla^{U}))^{1-(I_A)^U}\right],\\ \left[1-(1/(\nabla^{U}))^{(F_A)^L},1-(1/(\nabla^{U}))^{(F_A)^U}\right],\\ 1-(1/(\nabla^{U}))^{T_A},1-(1/(\nabla^{U}))^{I_A},(1/(\nabla^{U}))^{1-F_A}\end{array}\right)\end{array}\right\},\nabla^{L},\nabla^{U}>1\end{cases}$$

In both cases it is neutrosophic cubic dual.

Example 3.11 Let $A=\left([0.3,0.7],[0.2,0.7],[0.3,0.8],0.5,0.5,0.6\right)$ be a NCN and $\tilde{\nabla}=[0.3,0.7]$, $\tilde{\nabla}=[4,8]$, then

$$\left(\tilde{\nabla}\right)^{A}=\begin{cases}\left\{\begin{array}{c}\left(\begin{array}{c}[0.430,0.696],[0.381,0.696],\\ [0.303,0.618],0.514,0.452,0.617\end{array}\right)\\ \left(\begin{array}{c}[0.779,0.898],[0.751,0.989],\\ [0.101,0.248],0.807,0.836,0.867\end{array}\right)\end{array}\right\},\tilde{\nabla}=[0.3,0.7]\\ \left\{\begin{array}{c}\left(\begin{array}{c}[0.378,0.659],[0.329,0.659],\\ [0.340,0.670],0.564,0.500,0.574\end{array}\right)\\ \left(\begin{array}{c}[0.233,0.535],[0.189,0.535],\\ [0.464,0.810],0.712,0.646,0.435\end{array}\right)\end{array}\right\},\tilde{\nabla}=[4,8]\end{cases}$$

Definition 3.12 Let $\tilde{d}_i=\left[A_i,B_i\right],\left(i=1,2\right)$ be two DNCN and ϖ be a real number then the algebraic operations are defined as

- $\tilde{d}_i\oplus\tilde{d}_j=\left[A_i\oplus A_j,B_i\oplus B_j\right]$
- $\tilde{d}_i\otimes\tilde{d}_j=\left[A_i\otimes A_j,B_i\otimes B_j\right]$
- $\varpi\tilde{d}_1=\left[\varpi A_1,\varpi B_1\right]$
- $\left(\tilde{d}_1\right)^{\varpi}=\left[\left(A_1\right)^{\varpi},\left(B_1\right)^{\varpi}\right]$

In these operations, first we use an interval exponential operational laws and then operation between NCNs or

DOI: 10.5281/zenodo.3758466

real numbers are used. Hence it provide both the rationality of interval exponential operational laws and NCNs or real number operational laws as well.

Definition 3.13 Let $\tilde{d}=\left\{\left(\left[T_{A^L}^{L},T_{A^L}^{U}\right],\left[I_{A^L}^{L},I_{A^L}^{U}\right],\left[F_{A^L}^{L},F_{A^L}^{U}\right],T_{A^L},I_{A^L},F_{A^L}\right),\left(\left[T_{A^U}^{L},T_{A^U}^{U}\right],\left[I_{A^U}^{L},I_{A^U}^{U}\right],\left[F_{A^U}^{L},F_{A^U}^{U}\right],T_{A^U},I_{A^U},F_{A^U}\right)\right\}$ be a DNCN and $\tilde{d}^{*}=\left\{\left([1,1],[1,1],[0,0,],0,0,1\right),\left([1,1],[1,1],[0,0,],0,0,1\right)\right\}$ be the maximum DNCN, then the cosine measure(C_m) is defined as

$$C_m\left(\tilde{d}\right)=\left\{\cos\tfrac{\pi}{36}\left(1-T_{A^L}^{L}+1-T_{A^L}^{U}+1-I_{A^L}^{L}+1-I_{A^L}^{U}+F_{A^L}^{L}+F_{A^L}^{U}+T_{A^L}+I_{A^L}+1-F_{A^L}+1-T_{A^U}^{L}+1-T_{A^U}^{U}+1-I_{A^U}^{L}+1-I_{A^U}^{U}+F_{A^U}^{L}+F_{A^U}^{U}+T_{A^U}+I_{A^U}+1-F_{A^U}\right)\right\},$$

$$C_m\left(\tilde{d}\right)\in[0,1]$$

Definition 3.14 Let $C_m\left(\tilde{d}_1\right)$ *and* $C_m\left(\tilde{d}_2\right)$ be the cosine measures of two NCN then $C_m\left(\tilde{d}_1\right)>C_m\left(\tilde{d}_2\right)\Rightarrow\tilde{d}_1>\tilde{d}_2$ and $C_m\left(\tilde{d}_1\right)=C_m\left(\tilde{d}_2\right)\Rightarrow\tilde{d}_1=\tilde{d}_2$.

4. Neutrosophic Cubic Exponential Weighted Aggregation operator

Using definitions 3.1 and 3.10, in this section we propose the NCWEA and DNCWEA operators, where the base is crisp value or an interval numbers and the exponent is a NCNs.

Definition 4.1 We define the Neutrosophic cubic weighted exponential averaging operator (NCWEA) as

$$NCWEA(N_1,N_2,\dots N_m)=\bigotimes_{i=1}^{m}(\nabla_i)^{N_i}$$

where $N_i\ (i=1,2,\dots,m)$ are weight and $\nabla_i(i=1,2,\dots,m)$ real numbers respectively.

Theorem 4.2 Let $N_i=\left(\left[T_{N_i}^{L},T_{N_i}^{U}\right],\left[I_{N_i}^{L},I_{N_i}^{U}\right],\left[F_{N_i}^{L},F_{N_i}^{U}\right],T_{N_i},I_{N_i},F_{N_i}\right)$ for $(i=1,2,\dots,m)$ be the collection of NCs and $\nabla_i(i=1,2,\dots,m)$ are real numbers respectively, then the NCWEA is a NCs, where

$$NCWEA\left(N_1,N_2,...,N_m\right)=\begin{cases}\left(\begin{array}{c}\left[\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}},\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}}\right],\\ \left[\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}},\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}}\right],\\ \left[1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{L}},1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{U}}\right],\\ 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{T_{N_i}},1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{I_{N_i}},\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-F_{N_i}}\end{array}\right),if\quad\nabla\in\left[0,1\right]\\ \left(\begin{array}{c}\left[\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{1-T_{N_i}^{L}},\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{1-T_{N_i}^{U}}\right],\\ \left[\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{1-I_{N_i}^{L}},\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{1-I_{N_i}^{U}}\right],\\ \left[1-\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{F_{N_i}^{L}},1-\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{F_{N_i}^{U}}\right],\\ 1-\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{T_{N_i}},1-\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{I_{N_i}},\bigotimes_{i=1}^{m}\left(1/\nabla_i\right)^{1-F_{N_i}}\end{array}\right),if\quad\nabla>1\end{cases}$$

where N_i $(i=1,2,...,m)$ is the weight of $\nabla_i (i=1,2,...,m)$.

Proof To prove the theorem we use mathematical induction, let $\nabla_i \in [0,1]$ where $i=1,2,...,m$

For $m=2,$ we have

$$NCWEA\left(N_1,N_2\right)=\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{N_i}$$

$$=\left(\nabla_1\right)^{N_1}\otimes\left(\nabla_2\right)^{N_2}$$

$$=\left\{\left(\begin{array}{c}\left[\left(\nabla_1\right)^{1-T_{N_1}^{L}},\left(\nabla_1\right)^{1-T_{N_1}^{L}}\right],\\ \left[\left(\nabla_1\right)^{1-I_{N_1}^{L}},\left(\nabla_1\right)^{1-I_{N_1}^{L}}\right],\\ \left[1-\left(\nabla_1\right)^{F_{N_1}^{L}},1-\left(\nabla_1\right)^{F_{N_1}^{U}}\right],\\ 1-\left(\nabla_1\right)^{T_{N_1}},1-\left(\nabla_1\right)^{I_{N_1}},\left(\nabla_1\right)^{1-F_{N_1}}\end{array}\right)\otimes\left(\begin{array}{c}\left[\left(\nabla_2\right)^{1-T_{N_2}^{L}},\left(\nabla_2\right)^{1-T_{N_2}^{L}}\right],\\ \left[\left(\nabla_2\right)^{1-I_{N_2}^{L}},\left(\nabla_2\right)^{1-I_{N_2}^{L}}\right],\\ \left[1-\left(\nabla_2\right)^{F_{N_2}^{L}},1-\left(\nabla_2\right)^{F_{N_2}^{U}}\right],\\ 1-\left(\nabla_2\right)^{T_{N_2}},1-\left(\nabla_2\right)^{I_{N_2}},\left(\nabla_2\right)^{1-F_{N_2}}\end{array}\right)\right\}$$

DOI: 10.5281/zenodo.3758466

$$
=\left(\begin{array}{c}
\left[\left(\nabla_1\right)^{1-T_{N_1}^L}\left(\nabla_2\right)^{1-T_{N_2}^L},\left(\nabla_1\right)^{1-T_{N_1}^U}\left(\nabla_2\right)^{1-T_{N_2}^U}\right],\left[\left(\nabla_1\right)^{1-I_{N_1}^L}\left(\nabla_2\right)^{1-I_{N_2}^L},\left(\nabla_1\right)^{1-I_{N_1}^U}\left(\nabla_2\right)^{1-I_{N_2}^U}\right], \\
\left[\begin{array}{c}
1-\left(\nabla_1\right)^{F_{N_1}^L}+1-\left(\nabla_2\right)^{F_{N_2}^L}-\left(1-\left(\nabla_1\right)^{F_{N_1}^L}\right)\left(1-\left(\nabla_2\right)^{F_{N_2}^L}\right), \\
1-\left(\nabla_1\right)^{F_{N_1}^U}+1-\left(\nabla_2\right)^{F_{N_2}^U}-\left(1-\left(\nabla_1\right)^{F_{N_1}^U}\right)\left(1-\left(\nabla_2\right)^{F_{N_2}^U}\right)
\end{array}\right], \\
1-\left(\nabla_1\right)^{T_{N_1}}+1-\left(\nabla_2\right)^{T_{N_2}}-\left(1-\left(\nabla_1\right)^{T_{N_1}}\right)\left(1-\left(\nabla_2\right)^{T_{N_2}}\right), \\
1-\left(\nabla_1\right)^{I_{N_1}}+1-\left(\nabla_2\right)^{I_{N_2}}-\left(1-\left(\nabla_1\right)^{I_{N_1}}\right)\left(1-\left(\nabla_2\right)^{I_{N_2}}\right),\left(\nabla_1\right)^{1-F_{N_1}}\left(\nabla_2\right)^{1-F_{N_2}}
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L}\right],\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L}\right], \\
\left[\begin{array}{c}
1-\left(\nabla_1\right)^{F_{N_1}^L}+\left(1-\left(\nabla_2\right)^{F_{N_2}^L}\right)\left(\nabla_1\right)^{F_{N_1}^L}, \\
1-\left(\nabla_1\right)^{F_{N_1}^U}+\left(1-\left(\nabla_2\right)^{F_{N_2}^U}\right)\left(\nabla_1\right)^{F_{N_1}^U}
\end{array}\right], \\
1-\left(\nabla_1\right)^{T_{N_1}}+\left(1-\left(\nabla_2\right)^{T_{N_2}}\right)\left(\nabla_1\right)^{T_{N_1}}, \\
1-\left(\nabla_1\right)^{I_{N_1}}+\left(1-\left(\nabla_2\right)^{I_{N_2}}\right)\left(\nabla_1\right)^{I_{N_1}},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-F_{N_i}}
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L}\right],\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L}\right], \\
\left[1-\left(\nabla_1\right)^{F_{N_1}^L}\left(\nabla_2\right)^{F_{N_2}^L},1-\left(\nabla_1\right)^{F_{N_1}^U}\left(\nabla_2\right)^{F_{N_2}^U}\right], \\
1-\left(\nabla_1\right)^{T_{N_1}}\left(\nabla_2\right)^{T_{N_2}},1-\left(\nabla_1\right)^{I_{N_1}}\left(\nabla_2\right)^{I_{N_2}},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-F_{N_i}}
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-T_{N_i}^L}\right],\left[\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-I_{N_i}^L}\right], \\
\left[1-\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{F_{N_i}^L},1-\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{F_{N_i}^U}\right],1-\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{T_{N_i}},1-\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{I_{N_i}},\bigotimes_{i=1}^{2}\left(\nabla_i\right)^{1-F_{N_i}}
\end{array}\right)
$$

Assuming for $n = m,$ is a NCs that is

DOI: 10.5281/zenodo.3758466

$$NCWEA\left(N_1, N_2, \ldots, N_m\right) = \begin{pmatrix} \left[\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}}, \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}}\right], \left[\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}}, \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}}\right], \\ \left[1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{L}}, 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{U}}\right], \\ 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{T_{N_i}}, 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{I_{N_i}}, \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-F_{N_i}} \end{pmatrix}$$

We prove the result for $n = m+1,$ is a NCs.

$$NCWEA\left(N_1, N_2, \ldots, N_{m+1}\right) = \left\{ \begin{pmatrix} \begin{bmatrix} \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}}, \\ \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-T_{N_i}^{L}} \end{bmatrix}, \\ \begin{bmatrix} \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}}, \\ \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-I_{N_i}^{L}} \end{bmatrix}, \\ \begin{bmatrix} 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{L}}, \\ 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{F_{N_i}^{U}} \end{bmatrix}, \\ 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{T_{N_i}}, \\ 1-\bigotimes_{i=1}^{m}\left(\nabla_i\right)^{I_{N_i}}, \\ \bigotimes_{i=1}^{m}\left(\nabla_i\right)^{1-F_{N_i}} \end{pmatrix} \otimes \begin{pmatrix} \begin{bmatrix} \left(\nabla_{\varpi+1}\right)^{1-T_{N_{\varpi+1}}^{L}}, \\ \left(\nabla_{\varpi+1}\right)^{1-T_{N_{\varpi+1}}^{L}} \end{bmatrix}, \\ \begin{bmatrix} \left(\nabla_{\varpi+1}\right)^{1-I_{N_{\varpi+1}}^{L}}, \\ \left(\nabla_{\varpi+1}\right)^{1-I_{N_{\varpi+1}}^{L}} \end{bmatrix}, \\ \begin{bmatrix} 1-\left(\nabla_{\varpi+1}\right)^{F_{N_{\varpi+1}}^{L}}, \\ 1-\left(\nabla_{\varpi+1}\right)^{F_{N_{\varpi+1}}^{U}} \end{bmatrix}, \\ 1-\left(\nabla_{\varpi+1}\right)^{T_{N_{\varpi+1}}}, \\ 1-\left(\nabla_{\varpi+1}\right)^{I_{N_{\varpi+1}}}, \\ \left(\nabla_{\varpi+1}\right)^{1-F_{N_{\varpi+1}}} \end{pmatrix} \right\}$$

$$= \begin{pmatrix} \left[\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{1-T_{N_i}^{L}}, \bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{1-T_{N_i}^{L}}\right], \\ \left[\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{1-I_{N_i}^{L}}, \bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{1-I_{N_i}^{L}}\right], \\ \left[1-\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{F_{N_i}^{L}}, 1-\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{F_{N_i}^{U}}\right], \\ 1-\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{T_{N_i}}, 1-\bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{I_{N_i}}, \bigotimes_{i=1}^{\varpi+1}\left(\nabla_i\right)^{1-F_{N_i}} \end{pmatrix}$$

If $\nabla_i > 1,$ then $0 < 1/\nabla_i < 1,$ and then using above procedure a similar proof can be obtained for the following aggregation operators

DOI: 10.5281/zenodo.3758466

$$NCWEA\left(N_1, N_2, ..., N_m\right) = \begin{pmatrix} \left[\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{1-T^L_{N_i}}, \overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{1-T^U_{N_i}} \right], \\ \left[\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{1-I^L_{N_i}}, \overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{1-I^U_{N_i}} \right], \\ \left[1-\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{F^L_{N_i}}, 1-\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{F^U_{N_i}} \right], \\ 1-\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{T_{N_i}}, 1-\overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{I_{N_i}}, \overset{m}{\underset{i=1}{\otimes}} \left(1/\nabla_i\right)^{1-F_{N_i}} \end{pmatrix}$$

This complete the proof.

Definition 4.3 Let $N_i = \left(\left[T^L_{N_i}, T^U_{N_i}\right], \left[I^L_{N_i}, I^U_{N_i}\right], \left[F^L_{N_i}, F^U_{N_i}\right], T_{N_i}, I_{N_i}, F_{N_i}\right)$ for $(i = 1, 2, ..., m)$ be the collection of NCs and $\tilde{\nabla} = \left[\nabla^L_i, \nabla^U_i\right](i = 1, 2, ..., m)$ be the collection of interval numbers, then the DNCWEA operator is defined as

$$DNCWEA\left(N_1, N_2, ..., N_m\right) = \overset{m}{\underset{i=1}{\otimes}} \left(\tilde{\nabla}_i\right)^{N_i}$$

where $N_i (i = 1, 2, ..., m)$ is the weight corresponding to $\tilde{\nabla}_i = \left[\nabla^L_i, \nabla^U_i\right](i = 1, 2, ..., m)$.

Theorem 4.4 *Let* $N_i = \left(\left[T^L_{N_i}, T^U_{N_i}\right], \left[I^L_{N_i}, I^U_{N_i}\right], \left[F^L_{N_i}, F^U_{N_i}\right], T_{N_i}, I_{N_i}, F_{N_i}\right)$ for $(i = 1, 2, ..., m)$ be the collection of NCs and $\tilde{\nabla} = \left[\nabla^L_i, \nabla^U_i\right](i = 1, 2, ..., m)$ be collection of interval numbers, then the DNCWEA operator is given by

$$NCWEA\left(N_1, N_2, ..., N_m\right) =$$

$$\begin{cases} \left\{ \begin{array}{l} \left(\begin{array}{c} \left[\bigotimes_{i=1}^{m} (\triangledown_i^L)^{1-T_{N_i}^L}, \bigotimes_{i=1}^{m} (\triangledown_i^L)^{1-T_{N_i}^L} \right], \\ \left[\bigotimes_{i=1}^{m} (\triangledown_i^L)^{1-I_{N_i}^L}, \bigotimes_{i=1}^{m} (\triangledown_i^L)^{1-I_{N_i}^L} \right], \\ \left[1-\bigotimes_{i=1}^{m} (\triangledown_i^L)^{F_{N_i}^L}, 1-\bigotimes_{i=1}^{m} (\triangledown_i^L)^{F_{N_i}^U} \right], \\ 1-\bigotimes_{i=1}^{m} (\triangledown_i^L)^{T_{N_i}}, 1-\bigotimes_{i=1}^{m} (\triangledown_i^L)^{I_{N_i}}, \\ \bigotimes_{i=1}^{m} (\triangledown_i^L)^{1-F_{N_i}} \end{array} \right) \\ \left(\begin{array}{c} \left[\bigotimes_{i=1}^{m} (\triangledown_i^U)^{1-T_{N_i}^L}, \bigotimes_{i=1}^{m} (\triangledown_i^U)^{1-T_{N_i}^L} \right], \\ \left[\bigotimes_{i=1}^{m} (\triangledown_i^U)^{1-I_{N_i}^L}, \bigotimes_{i=1}^{m} (\triangledown_i^U)^{1-I_{N_i}^L} \right], \\ \left[1-\bigotimes_{i=1}^{m} (\triangledown_i^U)^{F_{N_i}^L}, 1-\bigotimes_{i=1}^{m} (\triangledown_i^U)^{F_{N_i}^U} \right], \\ 1-\bigotimes_{i=1}^{m} (\triangledown_i^U)^{T_{N_i}}, 1-\bigotimes_{i=1}^{m} (\triangledown_i^U)^{I_{N_i}}, \\ \bigotimes_{i=1}^{m} (\triangledown_i^U)^{1-F_{N_i}} \end{array} \right) \end{array} \right\}, if\ 0 \le \triangledown_i^L \le \triangledown_i^U \le 1 \\ \left\{ \begin{array}{l} \left(\begin{array}{c} \left[\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{1-T_{N_i}^L}, \bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{1-T_{N_i}^U} \right], \\ \left[\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{1-I_{N_i}^L}, \bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{1-I_{N_i}^U} \right], \\ \left[1-\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{F_{N_i}^L}, 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{F_{N_i}^U} \right], \\ 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{T_{N_i}}, 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{I_{N_i}}, \\ \bigotimes_{i=1}^{m} (1/\triangledown_i^L)^{1-F_{N_i}} \end{array} \right) \\ \left(\begin{array}{c} \left[\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{1-T_{N_i}^L}, \bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{1-T_{N_i}^U} \right], \\ \left[\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{1-I_{N_i}^L}, \bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{1-I_{N_i}^U} \right], \\ \left[1-\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{F_{N_i}^L}, 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{F_{N_i}^U} \right], \\ 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{T_{N_i}}, \\ 1-\bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{I_{N_i}}, \bigotimes_{i=1}^{m} (1/\triangledown_i^U)^{1-F_{N_i}} \end{array} \right) \end{array} \right\}, if\ \triangledown_i^U \ge \triangledown_i^L > 1 \end{cases}$$

The proof is analogous to Theorem 4.2.

5. Decision making method based on the NCWEA and DNCWEA operators

Based on NCWEA and DNCWEA operators, a decision making problem can be dealt. In such MADM problem the weight is NCs and alternative value are crisp or interval numbers.

For this consider the MADM problem with m alternative $A = \{a_1, a_2, ..., a_m\}$ and $C = \{y_1, y_2, ..., y_n\}$ be n attributes. An expert has evaluated these attributes in the form of NCs and the suitable alternative in the form of crisp value $\nabla_{ij} \in [0,1] \left(1/\nabla_{ij}, \text{if } \nabla_{ij} > 1\right)$ or interval numbers. $\nabla_{ij} = \left[\nabla_{ij}^{L}, \nabla_{ij}^{U}\right] \subseteq [0,1]\left(1/\nabla_{ij}^{L}, \text{if } \nabla_{ij}^{L} > 1\,;\, 1/\nabla_{ij}^{U}, \text{if } \nabla_{ij}^{U} > 1\right)$ for $\left(i = 1,2,...,m; j = 1,2,...,n\right)$.

Step 1: A preference value decision matrix $D = \left[\nabla_{ij}\right]$ or $D = \left[\tilde{\nabla}_{ij}\right]$ is constructed for m alternatives and n attributes, where weight is expressed as $N_j = \left(\left[T_{N_j}^{L}, T_{N_j}^{U}\right], \left[I_{N_j}^{L}, I_{N_j}^{U}\right], \left[F_{N_j}^{L}, F_{N_j}^{U}\right], T_{N_j}, I_{N_j}, F_{N_j}\right)$ $\left(j = 1,2,...,n\right)$ in NCs form for corresponding attributes.

Step 2: Using the suitable aggregation operator like NCEWA or DNCEWA the overall aggregated value is obtained.

Step 3: Using the measurement function of definition 3.3 or definition 3.14, values are ranked.

Step 4: The best alternative is chosen amongst the ranked.

6. Example

Next we provide an illustrative example as an application to our aggregation operators.

Example 6.1

Our example from daily life is an application to pick the best alternative using the decision making matrix with base either crisp values or interval numbers and weight as neutrosophic cubic number.

A steering committee is interested to prioritize the set of information improvement project using a multi-attribute decision making method. The committee must prioritized the implementation and development of set of six information technologies improvement projects $a_j (j = 1,2,...,6)$. The weight of these six attributes are expressed in term of NCs

$$\left\{a_1, a_2, a_3, a_4, a_5, a_6\right\} = \left\{\begin{matrix} ([0.5,0.6],[0.2,0.5],[0.4,0.8],0.7,0.8,0.4),([0.2,0.5],[0.7,0.9],[0.3,0.7],0.8,0.5,0.3), \\ ([0.4,0.7],[0.2,0.5],[0.5,0.7],0.3,0.6,0.2),([0.3,0.6],[0.4,0.7],[0.2,0.5],0.6,0.4,0.7), \\ ([0.2,0.5],[0.3,0.7],[0.2,0.6],0.5,0.3,0.8),([0.1,0.6],[0.3,0.6],[0.4,0.8],0.6,0.9,0.4) \end{matrix}\right\}$$

by decision maker. The three factors (alternatives) $\left\{y_1, y_2, y_3\right\}$, y_1 - productivity to maximize the efficiency and effectiveness, y_2 differentiation from products and services of competitors, and - y_3 management to assist the managers in enhancing the planning, are considered to assess the contribution of these project. The goal of committee is to choose best alternative among them.

Step 1: The decision maker(s) is (are) required to make the suitable judgement of alternatives $y_i \left(i = 1,2,3\right)$ with respect to these attributes $a_j (j = 1,2,...,6)$ and give the evaluated information of the crisp values $\nabla_{ij} \in \left[0,1\right]$, which is structured as follow.

$$D = \left[\nabla_{ij}\right] = \begin{bmatrix} 0.7 & 0.6 & 0.5 & 0.8 & 0.5 & 0.4 \\ 0.5 & 0.7 & 0.9 & 0.4 & 0.6 & 0.7 \\ 0.2 & 0.3 & 0.6 & 0.5 & 0.7 & 0.6 \end{bmatrix}$$

Step 2: Utilizing the NCEWA operator to evaluate these preferences of alternatives

$$d_1 = NCWEA(a_1,a_2,...,a_6) = \begin{pmatrix} \left[\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{1-T_{N_j}^L}, \underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{1-T_{N_j}^U}\right], \\ \left[\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{1-I_{N_j}^L}, \underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{1-I_{N_j}^U}\right], \\ \left[1-\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{F_{N_j}^L}, 1-\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{F_{N_j}^U}\right], \\ 1-\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{T_{N_j}}, 1-\underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{I_{N_j}}, \underset{j=1}{\overset{6}{\otimes}}\left(\nabla_{1j}\right)^{1-F_{N_j}} \end{pmatrix}$$

$$=\left([0.0790,0.2445],[0.0874,0.2959],[0.6964,0.9082],0.8499,0.8748,0.1523\right)$$

Similarly $d_2=\left([0.1266,0.2859],[0.1491,0.3657],[0.5789,0.8545],0.8381,0.8054,0.2616\right)$

$$d_3=\left([0.0367,0.1275],[0.0459,0.1827],[0.8126,0.9684],0.9568,0.9521,0.0606\right)$$

Step 3: We have $C_m(d_1)=0.9998067$, $C_m(d_2)=0.9998217$ and $C_m(d_3)=0.9997450$, here $C_m(d_2)>C_m(d_1)>C_m(d_3)$ the ranking order of alternatives are $y_2>y_1>y_3$.

Step 4: The best alternative on the basis of these calculations is y_2 management to assist the managers in improving their planning.

If the suitable judgment of each attribute $y_i\ (i=1,2,3)$ is made for interval numbers of interval valued decision making matrix:

In such a case the proposed MADM is based on DNCWEA operator may be applied to choose the suitable alternative, described in the following steps:

Step 1: First of all the interval valued decision making matrix is formed by decision maker:

$$\tilde{D}=\left(\tilde{\nabla}_{ij}\right)=\begin{bmatrix} [0.5,0.8] & [0.4,0.6] & [0.2,0.5] & [0.7,0.9] & [0.4,0.6] & [0.3,0.4] \\ [0.4,0.5] & [0.6,0.8] & [0.8,0.9] & [0.4,0.6] & [0.5,0.6] & [0.7,0.8] \\ [0.2,0.3] & [0.3,0.5] & [0.5,0.7] & [0.4,0.5] & [0.6,0.8] & [0.5,0.8] \end{bmatrix}$$

Step 2: The proposed MADM based on DNCWEA operator is applied to decision making matrix considering NC values of attributes $a_j\ (j=1,2,...,6)$ as weight for alternatives $y_i\ (i=1,2,3)$.

$$\tilde{d}_1 = NCWEA\left(a_1, a_2, \ldots, a_6\right) = \left\{ \begin{array}{c} \left(\begin{array}{c} \left[\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{1-T_{N_j}^{L}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{1-T_{N_j}^{U}} \right], \\ \left[\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{1-I_{N_j}^{L}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{1-I_{N_j}^{U}} \right], \\ \left[1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{F_{N_j}^{L}}, 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{F_{N_j}^{U}} \right], \\ 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{T_{N_j}}, 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{I_{N_j}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{L}\right)^{1-F_{N_j}} \end{array} \right) \\ \left(\begin{array}{c} \left[\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{1-T_{N_j}^{L}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{1-T_{N_j}^{U}} \right], \\ \left[\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{1-I_{N_j}^{L}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{1-I_{N_j}^{U}} \right], \\ \left[1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{F_{N_j}^{L}}, 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{F_{N_j}^{U}} \right], \\ 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{T_{N_j}}, 1-\bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{I_{N_j}}, \bigotimes_{j=1}^{6} \left(\nabla_{1j}^{U}\right)^{1-F_{N_j}} \end{array} \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left(\left[0.0163, 0.1001\right], \left[0.0220, 0.1216\right], \left[0.8766, 0.9819\right], 0.9547, 0.9691, 0.0348\right), \\ \left(\left[0.1061, 0.2962\right], \left[0.1424, 0.3700\right], \left[0.1524, 0.4290\right], 0.2675, 0.8458, 0.1773\right) \end{array} \right\}$$

$$\tilde{d}_2 = \left\{ \begin{array}{l} \left(\left[0.0835, 0.2133\right], \left[0.09542, 0.2875\right], \left[0.6657, 0.9098\right], 0.8921, 0.8609, 0.1802\right) \\ \left(\left[0.2221, 0.3792\right], \left[0.2173, 0.4416\right], \left[0.4986, 0.7823\right], 0.7512, 0.7240, 0.2018\right) \end{array} \right\}$$

$$\tilde{d}_3 = \left\{ \begin{array}{l} \left(\left[0.0211, 0.0951\right], \left[0.0274, 0.1384\right], \left[0.8525, 0.9804\right], 0.9703, 0.9682, 0.0426\right) \\ \left(\left[0.1069, 0.2433\right], \left[0.1125, 0.2970\right], \left[0.7165, 0.9052\right], 0.8853, 0.8736, 0.1526\right) \end{array} \right\}$$

Step 3: To rank the value the cosine measure is determined for the values computed in Step 2, $C_m\left(\tilde{d}_1\right) = 0.3346, C_m\left(\tilde{d}_2\right) = 0.3613, C_m\left(\tilde{d}_3\right) = 0.2460.$ The ranked alternatives are as $y_2 > y_1 > y_3$.

Step 4: The best alternative on the basis of these calculations is y_2 management to assist the managers in improving their planning.

7. Validity Test

Wang and Triantaphyllou [33] proposed criteria to figure out the validity of a MADM method.

Test Criterion 1: "The replacement of a non-optimal alternative with an arbitrary worse value does not change the index of best alternative".

Test Criterion 2: "The transitive property is satisfied by an effective MADM method ".

Test Criterion 3: "If MADM problem is decomposed into the sub DM problem and the same MADM procedure is applied to sub problem for ranking of alternatives, the order ranking of the alternatives must be similar to ranking of un decomposed DM problem".

DOI: 10.5281/zenodo.3758466

Validity Test by Criterion 1

We change the rating value of non-optimal alternative y_3 by $y_3^{'} = [0.3 \quad 0.8 \quad 0.4 \quad 0.2 \quad 0.9 \quad 0.5]$, we have $d_3 = ([0.0307, 0.1155], [0.0272, 0.1118], [0.8034, 0.9585], 0.9348, 0.9462, 0.0579)$ and $C_m(d_3) = 0.1695$, made no changes in said method.

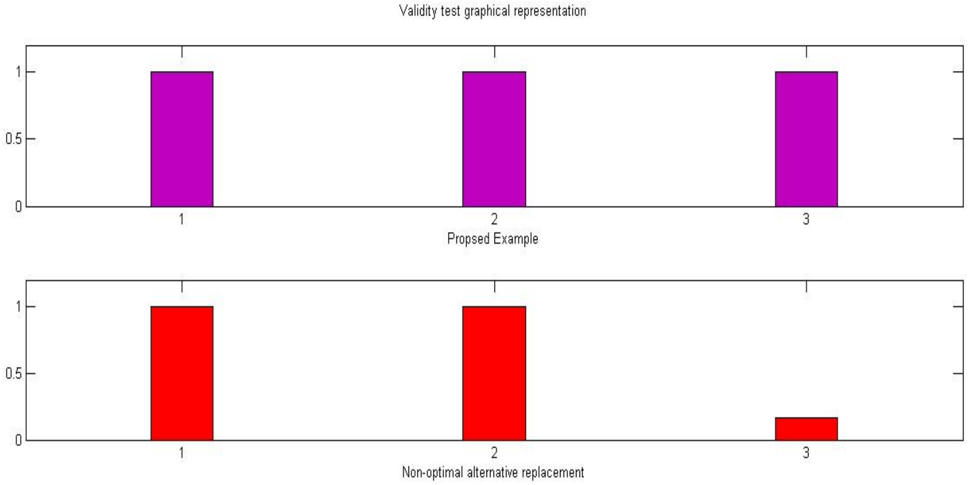

Fig.1: Comparison of graph by changing an alternative with non-optimal alternative.

The graph indicates that if the non-optimal value does not cause any change in optimal alternative, which is 1 (y_1) in this graph and 3(y_3) is non-optimal alternative.

Validity Test by Criterion 2

Under this criterion we decompose the decision matrix into $\{y_1, y_2\}, \{y_1, y_3\}$ and $\{y_2, y_3\}$, we observe that $y_2 > y_1$, $y_1 > y_3$ and $y_2 > y_3$. That is transitive property is satisfied.

Validity Test by Criterion 3

By validity test in criterion 2, we observe that the sub DM satisfy the original ranking order, that is $y_2 > y_1 > y_3$.

Hence validity test 1,2 and 3 are satisfied by MADM.

8. Comparison Analysis

In this section we compare the exponential aggregation operator with NCWA operator.

$$d_1^{'} = NCWA(a_1, a_2, ..., a_6) = \begin{pmatrix} \left[1-\bigotimes_{j=1}^{6}(1-T_{N_j}^{L})^{w_j}, 1-\bigotimes_{j=1}^{6}(1-T_{N_j}^{U})^{w_j}\right], \\ \left[1-\bigotimes_{j=1}^{6}(1-I_{N_j}^{L})^{w_j}, 1-\bigotimes_{j=1}^{6}(1-I_{N_j}^{U})^{w_j}\right], \\ \left[\bigotimes_{j=1}^{6}\left(F_{N_j}^{L}\right)^{w_j}, \bigotimes_{j=1}^{6}\left(F_{N_j}^{U}\right)^{w_j}\right], \\ \bigotimes_{j=1}^{6}\left(T_{N_j}\right)^{w_j}, \bigotimes_{j=1}^{6}\left(I_{N_j}\right)_I^{w_j}, 1-\bigotimes_{j=1}^{6}(1-\left(F_{N_j}\right)_F)^{w_j} \end{pmatrix}$$

$$d_1^{'} = \left([0.7311, 0.9551], [0.8277, 0.9352], [0.0154, 0.2351], 0.1429, 0.1102, 0.9072\right)$$

$$d_2^{'} = \left([0.7309, 0.9682], [0.8384, 0.9880], [0.0153, 0.2411], 0.0910, 0.1086, 0.9062\right)$$

$$d_3^{'} = \left([0.5974, 0.9262], [0.7160, 0.9608], [0.0334, 0.3001], 0.1484, 0.1461, 0.9072\right)$$

In this case $C_m\left(d_1^{'}\right) = 0.9831, C_m\left(d_2^{'}\right) = 0.9851,$ and $C_m\left(d_3^{'}\right) = 0.9663.$ This yields that $y_2 > y_1 > y_3.$

Here the role of real number (base) is change by weight in neutrosophic cubic weighted aggregation operator (NCWA) operator, we observe the same result as by NCEWA operator.

Now we compare the exponential aggregation operator with NCWA operator.

$$d_1^{'} = NCWA(a_1, a_2, ..., a_6) = \begin{pmatrix} \left[\dfrac{\bigotimes_{j=1}^{6}\left(1+T_{N_j}^{L}\right)^{w_j} - \bigotimes_{j=1}^{6}\left(1-T_{N_j}^{L}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(1+T_{N_j}^{L}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(1-T_{N_j}^{L}\right)^{w_j}}, \dfrac{\bigotimes_{j=1}^{6}\left(1+T_{N_j}^{U}\right)^{w_j} - \bigotimes_{j=1}^{6}\left(1-T_{N_j}^{U}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(1+T_{N_j}^{U}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(1-T_{N_j}^{U}\right)^{w_j}}\right], \left[\dfrac{\bigotimes_{j=1}^{6}\left(1+I_{N_j}^{L}\right)^{w_j} - \bigotimes_{j=1}^{6}\left(1-I_{N_j}^{L}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(1+I_{N_j}^{L}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(1-I_{N_j}^{L}\right)^{w_j}}, \dfrac{\bigotimes_{j=1}^{6}\left(1+I_{N_j}^{U}\right)^{w_j} - \bigotimes_{j=1}^{6}\left(1-I_{N_j}^{U}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(1+I_{N_j}^{U}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(1-I_{N_j}^{U}\right)^{w_j}}\right], \\ \left[\dfrac{2\bigotimes_{j=1}^{6}\left(F_{N_j}^{L}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(2-F_{N_j}^{L}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(F_{N_j}^{L}\right)^{w_j}}, \dfrac{2\bigotimes_{j=1}^{6}\left(F_{N_j}^{U}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(2-F_{N_j}^{U}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(F_{N_j}^{U}\right)^{w_j}}\right], \\ \dfrac{2\bigotimes_{j=1}^{6}\left(T_{N_j}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(2-T_{N_j}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(T_{N_j}\right)^{w_j}}, \dfrac{2\bigotimes_{j=1}^{6}\left(I_{N_j}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(2-I_{N_j}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(I_{N_j}\right)^{w_j}}, \dfrac{\bigotimes_{j=1}^{6}\left(1+F_{N_j}\right)^{w_j} - \bigotimes_{j=1}^{6}\left(1-F_{N_j}\right)^{w_j}}{\bigotimes_{j=1}^{6}\left(1+F_{N_j}\right)^{w_j} + \bigotimes_{j=1}^{6}\left(1-F_{N_j}\right)^{w_j}} \end{pmatrix}$$

$$d_1^{'} = \left([0.7231, 0.9424], [0.8123, 0.9042], [0.0167, 0.2413], 0.1325, 0.1245, 0.8952\right)$$

$$d_2^{'} = \left([0.7241, 0.9523], [0.8258, 0.9752], [0.0149, 0.2253], 0.0985, 0.1436, 0.8967\right)$$

$$d_3^{'} = \left([0.5862, 0.9155], [0.7012, 0.9624], [0.0337, 0.3001], 0.1325, 0.1453, 0.9157\right)$$

In this case $C_m\left(d_1^{'}\right) = 0.9725, C_m\left(d_2^{'}\right) = 0.9753,$ and $C_m\left(d_3^{'}\right) = 0.9574$ This yields that $y_2 > y_1 > y_3.$ Here the role of real number (base) is change by weight in neutrosophic cubic Einstein weighted aggregation operator (NCEWA) operator, we observe the we get the same result as by NCEWA operator.

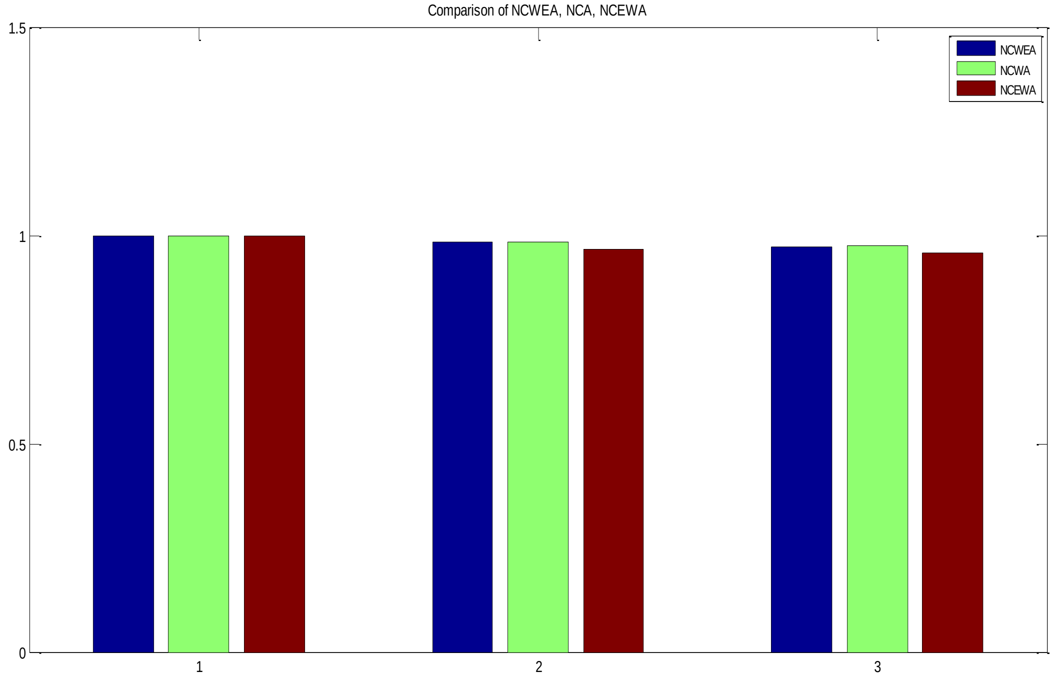

Fig.2: Graphical comparison of NCWEA with NCWA and NCEWA.

From the graph it is clear that NCWEA produce same result as NCWA and NCEWA, so we have a good tool to deal the cases in which neutrosophic cubic values appears in exponential form.

9. Conclusion

This manuscript presents a novel exponential operational laws (EOL) on NCSs with base crisp value and interval value which is an effective addition to the existing laws. We evaluated some properties and relations. Based upon these EOLs, we established NCWEA and DNCWEA operators. These aggregation operators are applied to establish a MADM for solving the daily life problem with the neutrosophic cubic information. The proposed method is used upon a daily life problem as an application. A comparative analysis with neutrosophic cubic weighted aggregation operator (NCWA) and neutrosophic cubic Einstein weighted aggregation operator (NCEWA) is provided to show the effectiveness of the approach. The graphical representation is accomplished between these operators. It is concluded that these newly defined operational laws and the proposed aggregation operators can parallelly be used to solve the MADM problems in more accomplished manner.

Acknowledgement; The authors are grateful to learned referees for their constructive suggestions, which greatly helped us to improve the paper significantly.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

References

[1] L. A. Zadeh. Fuzzy sets, *Inform. Control,* 8; 338-353, 1965.

[2] L. A. Zadeh. Outline of a new approach to the analysis of complex system and decision processes interval-valued fuzzy sets, *IEEE Trans*, 1968.

[3] I. B. Turksen. Interval-valued strict preference with Zadeh triples, *Fuzzy Sets and Systems*, 78;183-195, 1996.

[4] K. Atanassov. Intuitionistic fuzzy sets, *Fuzzy Sets Syst*., 20:87-96, 1986.

[5] K. Atanassov; G. Gargov. Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets Syst*., 31:343-349, 1989.

[6] Y. B. Jun; C. S. Kim; K. O.Yang. Cubic sets, *Ann. Fuzzy Math. Inform,* 1, 83-98, 2012.

[7] K. Kumar; H. Garg. TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment, *Computational and Applied Mathematics*, 37(2); 1319-1329, 2018.

[8] G. Kaur; H. Garg. Cubic intuitionistic fuzzy aggregation operators, *International Journal for Uncertainty Quantification*, 8(5); 405-427, 2018.

[9] H. Garg; S. Singh. A novel triangular interval type-2 intuitionistic fuzzy sets and their aggregation operators, *Iranian Journal of Fuzzy Systems*, 15(5); 69-93, 2018.

[10] Z. S. Xu, "Intuitionistic fuzzy aggregation operators", *IEEE Transactions on Fuzzy Systems*, 15; 1179-1187, 2007.

[11] Z. S. Xu; R. R. Yager. Some geometric aggregation operators based on intuitionistic fuzzy sets, *International Journal of General Systems*, 35; 417-433, 2006.

[12] D. Rani and H. Garg. Distance measures between the complex intuitionistic fuzzy sets and its applications to the decision - making process, *International Journal for Uncertainty Quantification*, 7(5); 423- 439, 2017.

[13] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press, Rehoboth, NM, 1999.

[14] F. Smarandache. NeutroAlgebra is a Generalization of Partial Algebra, International Journal of Neutrosophic Science, Volume 2 , Issue 1, pp.08-17 , 2020.

[15] H. Wang; F. Smarandache; Y.Q. Zhang. Interval neutrosophic sets and logic, Theory and Application in Computing, Hexis, Phoenix, AZ, 2005.

[16] Y. B. Jun; F. Smarandache; C.S. Kim. Neutrosophic cubic sets, New Math. Nat. Comput. 8:41 SPC/INSTRUCTION FILE JSK-151001R0-1108, 2015.

[17] Nancy; H. Garg. Novel single-valued neutrosophic decision making operators under frank norm operations and its application, *International Journal for Uncertainty Quantification,* 6(4), 361-375, 2016.

[18] P. Liu; Y. Wang. Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean, *Neural Computing & Applications*, 25 (7-8); 2001-2010, 2014.

[19] X. D. Peng; J. G. Dai. A bibliometric analysis of neutrosophic set: Two decades review from 1998--2017. Artificial Intelligence Review, 2018. https://doi.org/10.1007/s10462-018-9652-0.

[20] H. Garg; Nancy. Multi-criteria decision-making method based on prioritized muirhead mean aggregation operator under neutrosophic set environment, *Symmetry*, 10(7), 280, 2018.

[21] S. Jha; R. Kumar; J.M Chatterjee; M. Khari; N. Yadav; F. Smarandache, "Neutrosophic soft set decision making for stock trending analysis", *Evolving Systems*, 1-7.

[22] H. Garg; Nancy. Some hybrid weighted aggregation operators under neutrosophic set environment and their applications to multicriteria decision-making, *Applied Intelligence*, 1-18, 2018

[23] P. Biswas; S. Pramanik; B.C Giri. Topsis method for multiattribute group decision-making under single-valued neutrosophic environment", *Neural Computing and Applications*, 27(3); 727-737.

[24] M. Abdel-Basset; Y. Zhou; M. Mohamed; V. Chang. A group decision making framework based on neutrosophic vikor approach for e-government website evaluation. *Journal of Intelligent & Fuzzy Systems*, 34(6); 40213-4224, 2018.

[25] J. Ye. Exponential operations and aggregation operators of interval neutrosophic sets and their decision making methods", SpringerPlus, 5; 1488, 2016.

[26] J. Zhan; M. Khan; M. Gulistan;A. Ali. Applications of neutrosophic cubic sets in multi-criteria decision making, Int. J. Uncertain. Quanti, 7; 377-394, 2017.

[27] D. Banerjee; B.C. Giri; S. Pramanik; F. Smarandache. GRA for multi attribute decision making in neutrosophic cubic set environment. Neutrosophic Sets Syst., 15; 64-73, 2017.

[28] Z. Lu; J. Ye; Cosine measures of neutrosophic cubic sets for multiple attribute decision-making, Symmetry, 9; 2017.

[29] S. Pramanik; S. Dalapati; S. Alam; T.K. Roy; F. Smarandache. Neutrosophic cubic MCGDM method based on similarity measure, Neutrosophic Sets Syst., 16; 44-56, 2017.

[30] M. Khan; M. Gulistan; M. Khan; F.Smaradache. Neutrosophic cubic Einstein geometric aggregation operators with application to multi-creiteria decision making theory method, Symmetry, 11, 2019.

[31] M. Khalid; Y. B. Jun; M. M. Takallo; N.A.Khalid. Magnification of MBJ-Neutrosophic Translation on G-Algerbra, Volume 2 , Issue 1, pp. 27-37 , 2020

[32] P. Schweizer, Uncertainty: two probabilities for the three states of neutrosophy, International Journal of Neutrosophic Science, Volume 2 , Issue 1, pp.18-26 , 2020.

[33] X. Wang; E. Triantaphyllou. Ranking irregularities when evaluating alternatives by using some electre methods, *Omega - International Journal of Management Science*, 36; 45-63, 2008.