



Łukasiewicz Intuitionistic Fuzzy Filters in Hoops and its Application in Medical Diagnosis

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Abstract

The new theory of Łukasiewicz intuitionistic fuzzy set and Łukasiewicz intuitionistic fuzzy filter is introduced. Some properties of Łukasiewicz intuitionistic fuzzy filter is presented. It is explored that under what circumstances, the Łukasiewicz intuitionistic fuzzy set can be a Łukasiewicz intuitionistic fuzzy filter. An algorithm for diagnosing disease is developed and provided with demonstration.

Keywords: Łukasiewicz intuitionistic fuzzy set; Łukasiewicz intuitionistic fuzzy filter; Conjunction and disjunction of Łukasiewicz intuitionistic fuzzy sets; Intuitionistic fuzzy filters, hoops

1. Introduction

In 1969, Bosbach introduced an algebraic structure in hoop algebra, relaxes the requirement of distributivism, which is the fundamental property of Boolean algebra. Hoop is a naturally ordered procrim, which is partially ordered commutative residuated integral monoid. L. A. Zadeh introduced the fuzzy sets in 1965. The application of fuzzy filters to the hoops was studied by Borzooei and Aaly Kologani [3]. Kologani M.A, Borzooei R.A, Kim H.S, Jun Y.B and Ahn S.S [8] studied intuitionistic fuzzy filters on hoops. On hoops, several mathematician defined the congruence relation [9]. Jan Łukasiewicz introduced Łukasiewicz logic in the early 20th century. Łukasiewicz logic is a classical many-valued logic, a logic of the Łukasiewicz t-norm. Jun and Ahn applied Łukasiewicz logic to BE-algebras [7]. Atanassov provided the concept of intuitionistic fuzzy set in 1986. In 2024, Mohseni Takallo M, Aaly Kologani M, Jun Y. B and Borzooei R. A examined the properties of filter in hoops by applying the Łukasiewicz fuzzy set idea [11].

In real life, there may be a situation where uncertainty, imprecision and complexity is crucial for decision making, modelling and optimizing process. To contribute to the advancement of theoretical developments and in practical applications, the concept of Łukasiewicz intuitionistic fuzzy filter is presented in this study. The properties of a Łukasiewicz intuitionistic fuzzy filter is investigated. Using the concept and operations of Łukasiewicz intuitionistic filter, we developed an algorithm for medical diagnosis. We provide a demonstration for the developed algorithm.

2. Preliminaries

Definition 2.1.[11]

An algebra $(\mathcal{H}, \circ, \rightsquigarrow, 1)$ is said to be **hoop** if it satisfies:

$$(\mathcal{H}1) \forall s \in \mathcal{H}, s \rightsquigarrow s = 1$$

$$(\mathcal{H}2) \forall s, t \in \mathcal{H}, s \circ (s \rightsquigarrow t) = t \circ (t \rightsquigarrow s)$$

$$(\mathcal{H}3) \forall s, t, u \in \mathcal{H}, s \rightsquigarrow (t \rightsquigarrow u) = (s \rightsquigarrow t) \rightsquigarrow u$$

$$(\mathcal{H}4) (\mathcal{H}, \circ, 1) \text{ is a commutative monoid}$$

Define $s \leq t$ on a hoop $\mathcal{H}$ if and only if $s \rightsquigarrow t = 1$.

Proposition 2.2.[11]

The following assertions are true for every hoop $\mathcal{H}$.

$$\forall s, t \in \mathcal{H}, s \circ t \leq u \Leftrightarrow s \leq t \rightsquigarrow u \quad (1)$$

$$\forall s, t \in \mathcal{H}, s \circ t \leq s, t \quad (2)$$

$$\forall s, t \in \mathcal{H}, s \leq t \rightsquigarrow s \quad (3)$$

$$\forall s \in \mathcal{H}, s \rightsquigarrow 1 = 1, 1 \rightsquigarrow s = s \quad (4)$$

$$\forall s, t, u \in \mathcal{H}, (s \rightsquigarrow t) \circ (t \rightsquigarrow u) \leq s \rightsquigarrow u \quad (5)$$

$$\forall s, t, u \in \mathcal{H}, s \leq t \Rightarrow s \circ u \leq t \circ u \quad (6)$$

$$\forall s, t \in \mathcal{H}, s \circ (s \rightsquigarrow t) \leq t \quad (7)$$

Definition 2.3.[11]

A subset $\mathcal{F}$ of $\mathcal{H}$ is called filter of hoop if for every $s, t \in \mathcal{H}$ satisfies the following two condition.

- $s, t \in \mathcal{F} \Rightarrow s \circ t \in \mathcal{F}$
- $s \leq t, s \in \mathcal{F} \Rightarrow t \in \mathcal{F}$

Proposition 2.4.[11]

A subset $\mathcal{F}$ of $\mathcal{H}$ is called filter of $\mathcal{H}$ if and only if the following are satisfied.

- $1 \in \mathcal{F}$
- $s, s \rightsquigarrow t \in \mathcal{H} \Rightarrow t \in \mathcal{H}, \quad \forall s, t \in \mathcal{H}$

Definition 2.5. (Atanassov, 1999):

Let $\mathcal{H}$ be a nonempty set. An intuitionistic fuzzy set A in $\mathcal{H}$ is an object having the form $A = \{h, \mu(h), \nu(h) | h \in \mathcal{H}\}$, provided $0 \leq \mu(h) + \nu(h) \leq 1$.

The functions $\mu': \mathcal{H} \rightarrow [0,1]$, $\nu': \mathcal{H} \rightarrow [0,1]$ represent the degree of membership and degree of non-membership of the element $h \in \mathcal{H}$ to the set A .

Definition 2.6.[6]

Let c be a point of $\mathcal{H}$ and $\alpha \in (0,1]$, $\beta \in [0,1)$ be two real numbers such that $\alpha + \beta \leq 1$.

An intuitionistic fuzzy subset of $\mathcal{H}$ is said to be intuitionistic fuzzy point if it is of the form

$$C_{(\alpha, \beta)}(s) = \begin{cases} (\alpha, \beta) & \text{if } s = c \\ (0, 1) & \text{otherwise} \end{cases}$$

Definition 2.7.[6]

For an intuitionistic fuzzy set $\rho(s, \mu(s), \nu(s))$ in $\mathcal{H}$, $C_{(\alpha, \beta)}$ an intuitionistic fuzzy point is contained in ρ , denoted by $C_{(\alpha, \beta)} \in (\mu, \nu)$ if it satisfies $\mu(x) \geq \alpha$ and $\nu(x) \leq \beta$.

An intuitionistic fuzzy point is quasi-coincident with ρ denoted by $C_{(\alpha, \beta)} q \rho$ if and only if

$$\alpha > \nu(c) \text{ or } \beta < \mu(c)$$

Definition 2.8.[8]

An intuitionistic fuzzy set $\rho(s, \mu(s), \nu(s))$ is said to be an intuitionistic fuzzy filter of $\mathcal{H}$ if ρ satisfies

$$\mu(s) \leq \mu(1) \text{ and } \nu(s) \geq \nu(1), \quad \forall s \in \mathcal{H}$$

$$\mu(s) \geq \min\{\mu(t), \mu(t \rightsquigarrow s)\},$$

$$\nu(s) \leq \max\{\nu(t), \nu(t \rightsquigarrow s)\} \quad \forall s, t \in \mathcal{H}$$

Definition 2.9.[14]

The conjunction and disjunction of Łukasiewicz intuitionistic fuzzy sets A and B of a hoop $\mathcal{H}$ are defined by

$$A \oplus B = \{s, \min(1, \mu_A'(s) + \mu_B'(s)), \max(0, \nu_A'(s) + \nu_B'(s) - 1) \mid s \in \mathcal{H}\}$$

$$A \odot B = \{s, \max(0, \mu_A'(s) + \mu_B'(s) - 1), \min(1, \nu_A'(s) + \nu_B'(s)) \mid s \in \mathcal{H}\}$$

3. Łukasiewicz Intuitionistic Fuzzy Filters**Definition 3.1.**

Let $\rho(s, \mu(s), \nu(s))$ be an intuitionistic fuzzy set in $\mathcal{H}$ and $\theta \in (0, 1)$. $L_\rho^\theta(s, \mu', \nu')$ is called Łukasiewicz intuitionistic fuzzy set of ρ in $\mathcal{H}$ if μ' and ν' are the functions given by

$$\mu': \mathcal{H} \rightarrow [0, 1], s \mapsto \max\{0, \mu(s) + \theta - 1\}$$

$$\nu': \mathcal{H} \rightarrow [0, 1], s \mapsto \max\{0, \nu(s) + \theta - 1\}$$

Definition 3.2.

Łukasiewicz intuitionistic fuzzy set L_ρ^θ of an intuitionistic fuzzy set ρ in $\mathcal{H}$ is called a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$ if

$$\forall s, t \in \mathcal{H}, \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1),$$

$$s_{(\alpha, \beta)} \in L_\rho^\theta, t_{(\gamma, \delta)} \in L_\rho^\theta \Rightarrow (s \circ t)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (8)$$

$$\forall s, t \in \mathcal{H}, \alpha \in (0, 1], \beta \in [0, 1),$$

$$s \leq t, s_{(\alpha, \beta)} \in L_\rho^\theta \Rightarrow t_{(\alpha, \beta)} \in L_\rho^\theta \quad (9)$$

Example 3.3.

Assume that the set $\mathcal{H} = \{s, t, u\}$ has binary operations " $\circ$ " and " $\rightsquigarrow$ " as in Tables 3.3.1. and 3.3.2.

Table 3.3.1

$\circ$	s	t	u
s	s	s	s
t	s	s	t
u	s	t	u

Table 3.3.2

$\rightsquigarrow$	s	t	u
s	u	u	u
t	t	u	u
u	s	t	u

Clearly $\mathcal{H}$ is hoop.

Define an intuitionistic fuzzy set $\rho: \mathcal{H} \rightarrow [0, 1]$ by

$$\rho(x) = \{(s, 0.4, 0.5), (t, 0.4, 0.5), (u, 0.7, 0.1)\}$$

For $\theta = 0.8$, the Łukasiewicz intuitionistic fuzzy set L_ρ^θ of ρ in $\mathcal{H}$ as follows

$$L_\rho^\theta(x) = \{(s, 0.2, 0.3), (t, 0.2, 0.3), (u, 0.5, 0)\}$$

The reader can now confirm that L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.4.

Łukasiewicz intuitionistic fuzzy set $L_\rho^\theta(s, \mu', \nu')$ of an intuitionistic fuzzy set $\rho(s, \mu(s), \nu(s))$ in $\mathcal{H}$ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$ if and only if the following are satisfied.

- $\mu'(1)$ is an upper bound of $\{ \mu'(s) | s \in \mathcal{H} \}$
- $\nu'(1)$ is a lower bound of $\{ \nu'(s) | s \in \mathcal{H} \}$

(10)

- $\forall s, t \in \mathcal{H}, \forall \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1]$

$$s_{(\alpha, \beta)} \in L_\rho^\theta, (s \rightsquigarrow t)_{(\gamma, \delta)} \in L_\rho^\theta \implies t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (11)$$

Proof Let us assume that L_ρ^θ be a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

If $\mu'(1)$ is not an upper bound of $\{ \mu'(s) | s \in \mathcal{H} \}$ and $\nu'(1)$ is not a lower bound of $\{ \nu'(s) | s \in \mathcal{H} \}$, then $\mu'(1) < \mu'(z)$ & $\nu'(1) > \nu'(z)$ for some $z \in \mathcal{H}$.

Because of $z \leq 1$ and $z_{(\mu'(z), \nu'(z))} \in L_\rho^\theta$, by (9) we have $1_{(\mu'(z), \nu'(z))} \in L_\rho^\theta$.

That is $\mu'(1) \geq \mu'(z)$ & $\nu'(1) \leq \nu'(z)$ which contradicts the assumption. Thus $\mu'(1)$ is an upper bound of $\{ \mu'(s) | s \in \mathcal{H} \}$ and $\nu'(1)$ is a lower bound of $\{ \nu'(s) | s \in \mathcal{H} \}$.

Now assume that for $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1]$,

$$s_{(\alpha, \beta)} \in L_\rho^\theta \text{ and } (s \rightsquigarrow t)_{(\gamma, \delta)} \in L_\rho^\theta.$$

Then by (8), we have $((s \rightsquigarrow t) \circ s)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Since $(s \rightsquigarrow t) \circ s \leq t$, it follows from (9) that $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$.

Conversely assume that L_ρ^θ satisfies (10) and (11). Let $s, t \in \mathcal{H}$ and $\alpha \in (0, 1], \beta \in [0, 1]$ be such that $s \leq t$ and $s_{(\alpha, \beta)} \in L_\rho^\theta$.

Then by definition of hoop and (10), we have $\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \alpha$ and $\nu'(s \rightsquigarrow t) = \nu'(1) \leq \nu'(s) \leq \beta$. That is $(s \rightsquigarrow t)_{(\alpha, \beta)} \in L_\rho^\theta$. Using (11), we have $t_{(\alpha, \beta)} \in L_\rho^\theta$ which proves (9).

Let $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1]$ be such that $s_{(\alpha, \beta)} \in L_\rho^\theta$ and $t_{(\gamma, \delta)} \in L_\rho^\theta$. Since, we have $s \rightsquigarrow (t \rightsquigarrow s \circ t) = s \circ t \rightsquigarrow s \circ t = 1$ by the definition of hoop, we have

$$\mu'(s \rightsquigarrow (t \rightsquigarrow s \circ t)) = \mu'(1) \geq \mu'(t) \geq \gamma \text{ and}$$

$$\nu'(s \rightsquigarrow (t \rightsquigarrow s \circ t)) = \nu'(1) \leq \nu'(t) \leq \delta.$$

Therefore, we have $(s \rightsquigarrow (t \rightsquigarrow s \circ t))_{(\gamma, \delta)} \in L_\rho^\theta$. By using (11), we have $(t \rightsquigarrow s \circ t)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Again using (11), we have $(s \circ t)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ which proves (8).

Therefore L_ρ^θ in $\mathcal{H}$ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.5.

Łukasiewicz intuitionistic fuzzy set L_ρ^θ of an intuitionistic fuzzy set ρ in $\mathcal{H}$ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$ if and only if

- $\forall s \in \mathcal{H}, \forall \alpha \in (0, 1], \beta \in [0, 1], s_{(\alpha, \beta)} \in L_\rho^\theta \implies 1_{(\alpha, \beta)} \in L_\rho^\theta$

(12)

- $\forall s, t \in \mathcal{H}, \mu'(t) \geq \min\{\mu'(s), \mu'(s \rightsquigarrow t)\}$ and

$$\nu'(t) \leq \max\{\nu'(s), \nu'(s \rightsquigarrow t)\} \quad (13)$$

Proof Suppose L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Let $s \in \mathcal{H}$ and $\alpha \in (0, 1], \beta \in [0, 1]$ be such that $s_{(\alpha, \beta)} \in L_\rho^\theta$. From (10), it follows that $\mu'(1) \geq \mu'(s) \geq \alpha$ and $\nu'(1) \leq \nu'(s) \leq \beta$ and so $1_{(\alpha, \beta)} \in L_\rho^\theta$ which proves (12). Note that $s_{(\mu'(s), \nu'(s))} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\mu'(s \rightsquigarrow t), \nu'(s \rightsquigarrow t))} \in L_\rho^\theta, \forall s, t \in \mathcal{H}$. By (11), we have

$$t_{(\mu'(s) \wedge \mu'(s \rightsquigarrow t), \nu'(s) \vee \nu'(s \rightsquigarrow t))} \in L_\rho^\theta$$

$$\Rightarrow \mu'(t) \geq \min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \text{ and } v'(t) \leq \max\{v'(s), v'(s \rightsquigarrow t)\}, \forall s, t \in \mathcal{H}.$$

Conversely, suppose that L_ρ^θ satisfies (12) and (13). Since $s_{(\mu'(s), v'(s))} \in L_\rho^\theta, \forall s \in \mathcal{H}$ and by (12), we have $1_{(\mu'(s), v'(s))} \in L_\rho^\theta$ and so $\mu'(1) \geq \mu'(s), v'(1) \leq v'(s), \forall s \in \mathcal{H}$.

$\therefore \mu'(1)$ is an upper bound of $\{\mu'(s) | s \in \mathcal{H}\}$ and $v'(1)$ is a lower bound of $\{v'(s) | s \in \mathcal{H}\}$. Let $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1)$ be such that $s_{(\alpha, \beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma, \delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha, v'(s) \leq \beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma, v'(s \rightsquigarrow t) \leq \delta$. By using (13), we have,

$$\mu'(t) \geq \min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \geq \min\{\alpha, \gamma\} \text{ and}$$

$$v'(t) \leq \max\{v'(s), v'(s \rightsquigarrow t)\} \leq \max\{\beta, \delta\}$$

Thus $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. By Theorem 3.4, L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proposition 3.6.

Any Łukasiewicz intuitionistic fuzzy filter L_ρ^θ of $\mathcal{H}$ satisfies:

$$\triangleright \forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1),$$

$$u \leq s \rightsquigarrow t, s_{(\alpha, \beta)} \in L_\rho^\theta, u_{(\gamma, \delta)} \in L_\rho^\theta \Rightarrow t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (14)$$

$$\triangleright \forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1),$$

$$s \leq t \rightsquigarrow u, s_{(\alpha, \beta)} \in L_\rho^\theta, t_{(\gamma, \delta)} \in L_\rho^\theta \Rightarrow u_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (15)$$

$$\triangleright \forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1),$$

$$(s \rightsquigarrow t)_{(\alpha, \beta)} \in L_\rho^\theta, (t \rightsquigarrow u)_{(\gamma, \delta)} \in L_\rho^\theta \Rightarrow (s \rightsquigarrow u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (16)$$

$$\triangleright \forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1), (s \rightsquigarrow t)_{(\alpha, \beta)} \in L_\rho^\theta,$$

$$(s \circ u)_{(\gamma, \delta)} \in L_\rho^\theta \Rightarrow (t \circ u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta \quad (17)$$

Proof Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1)$ be such that $u \leq s \rightsquigarrow t$,

$s_{(\alpha, \beta)} \in L_\rho^\theta$ and $u_{(\gamma, \delta)} \in L_\rho^\theta$. Since $u \leq s \rightsquigarrow t$, then $u \rightsquigarrow (s \rightsquigarrow t) = 1$,

$\mu'(s) \geq \alpha, v'(s) \leq \beta$ and $\mu'(u) \geq \gamma, v'(u) \leq \delta$. Hence by (13) we have,

$$\begin{aligned} \mu'(t) &\geq \min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \\ &\geq \min\{\mu'(s), \min\{\mu'(u), \mu'(u \rightsquigarrow (s \rightsquigarrow t))\}\} \\ &= \min\{\mu'(s), \min\{\mu'(u), \mu'(1)\}\} \\ &= \min\{\mu'(s), \mu'(u)\} \\ &\geq \min\{\alpha, \gamma\} \text{ and} \\ v'(t) &\leq \max\{v'(s), v'(s \rightsquigarrow t)\} \\ &\leq \max\{v'(s), \max\{v'(u), v'(u \rightsquigarrow (s \rightsquigarrow t))\}\} \\ &= \max\{v'(s), \max\{\mu'(u), v'(1)\}\} \\ &= \max\{v'(s), v'(u)\} \\ &\leq \max\{\beta, \delta\} \end{aligned}$$

and so $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Therefore (14) is true.

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1)$ be such that $s \leq t \rightsquigarrow u, s_{(\alpha, \beta)} \in L_\rho^\theta$ and $t_{(\gamma, \delta)} \in L_\rho^\theta$. Since $s \leq t \rightsquigarrow u$, using (1) we have, $s \circ t \leq u$ and by (8) we have $(s \circ t)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ and so by (9) we have $u_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ which proves (15).

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1)$ be such that $(s \rightsquigarrow t)_{(\alpha, \beta)} \in L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma, \delta)} \in L_\rho^\theta$. Using (8) we have,

$$((s \rightsquigarrow t) \circ (t \rightsquigarrow u))_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$$

Since $(s \rightsquigarrow t) \circ (t \rightsquigarrow u) \leq s \rightsquigarrow u$, we have from (9) that, $(s \rightsquigarrow u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. This proves (16).

Let $s, t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1]$ be such that $(s \rightsquigarrow t)_{(\alpha, \beta)} \in L_\rho^\theta$, and $(s \circ u)_{(\gamma, \delta)} \in L_\rho^\theta$. By using (8) we have, $(s \circ u \circ (s \rightsquigarrow t))_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ and by commutativity of $\circ$, we have $(u \circ s \circ (s \rightsquigarrow t))_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Since $s \circ (s \rightsquigarrow t) \leq t$, we have $u \circ s \circ (s \rightsquigarrow t) \leq u \circ t = t \circ u$ by (6).

By (9), we have $(t \circ u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ which proves (17).

Theorem 3.7.

Let ρ be an intuitionistic fuzzy filter of $\mathcal{H}$. Then the Łukasiewicz intuitionistic fuzzy set L_ρ^θ of ρ in $\mathcal{H}$ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be Łukasiewicz intuitionistic fuzzy set of an intuitionistic fuzzy filter ρ in $\mathcal{H}$. Then

$$\begin{aligned} \mu'(1) &= \max\{0, \mu(1) + \theta - 1\} \\ &\geq \max\{0, \mu(s) + \theta - 1\} \\ &= \mu'(s) \quad \text{and} \end{aligned}$$

$$\begin{aligned} v'(1) &= \max\{0, v(1) + \theta - 1\} \\ &\leq \max\{0, v(s) + \theta - 1\} \\ &= v'(s) \quad \forall s \in \mathcal{H}. \end{aligned}$$

Therefore, $\mu'(1)$ is an upper bound of $\{\mu'(s) | s \in \mathcal{H}\}$

$$v'(1) \text{ is a lower bound of } \{v'(s) | s \in \mathcal{H}\}.$$

Let $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0, 1], \beta, \delta \in [0, 1]$ be such that $s_{(\alpha, \beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma, \delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha$, $v'(s) \leq \beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma$, $v'(s \rightsquigarrow t) \leq \delta$ which imply that

$$\begin{aligned} \mu'(t) &= \max\{0, \mu(t) + \theta - 1\} \\ &\geq \max\{0, \min(\mu(s), \mu(s \rightsquigarrow t)) + \theta - 1\} \\ &= \max\{0, \min\{\mu(s) + \theta - 1, \mu(s \rightsquigarrow t) + \theta - 1\}\} \\ &= \min\{\max\{0, \mu(s) + \theta - 1\}, \max\{0, \mu(s \rightsquigarrow t) + \theta - 1\}\} \\ &= \min\{\mu'(s), \mu'(s \rightsquigarrow t)\} \\ &\geq \min\{\alpha, \gamma\} \text{ and} \end{aligned}$$

$$\begin{aligned} v'(t) &= \max\{0, v(t) + \theta - 1\} \\ &\leq \max\{0, \max(v(s), v(s \rightsquigarrow t)) + \theta - 1\} \\ &= \max\{0, \max\{v(s) + \theta - 1, v(s \rightsquigarrow t) + \theta - 1\}\} \\ &= \max\{\max\{0, v(s) + \theta - 1\}, \max\{0, v(s \rightsquigarrow t) + \theta - 1\}\} \\ &= \max\{v'(s), v'(s \rightsquigarrow t)\} \\ &\leq \max\{\beta, \delta\} \end{aligned}$$

Therefore $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ and so L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

The converse need not be true as in the example given below.

Example 3.8.

In the Example 3.3., L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Since $\rho(u) = 0.4 \not\geq 0.7 = \min\{\rho(u), \rho(t \rightarrow u)\}$, ρ fails to be an intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.9.

A Łukasiewicz intuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (15) is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof For $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

$$s_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } (s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta.$$

Since $s \rightsquigarrow t \leq s \rightsquigarrow t$, using (15) we have $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$.

Hence by Theorem (3.4), L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.10.

A Łukasiewicz intuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (16) is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof For $t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

$$t_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } (t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta.$$

Since $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$. Using (16) we have, $(1 \rightsquigarrow u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$ and it follows from (4) that $u_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$.

Hence by Theorem(3.4), L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.11.

A Łukasiewicz intuitionistic fuzzy set L_ρ^θ in $\mathcal{H}$ satisfying (10) and (17) is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof For $t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0,1], \beta, \delta \in [0,1)$,

$$t_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } u_{(\gamma,\delta)} \in L_\rho^\theta.$$

Then $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$ and $u_{(\gamma,\delta)} = (1 \circ u)_{(\gamma,\delta)} \in L_\rho^\theta$. Using (17) we have $(t \circ u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$.

Let $s, t \in \mathcal{H}$ and $\sigma \in (0,1], \tau \in [0,1)$ be such that $s \leq t$ and $s_{(\sigma,\tau)} \in L_\rho^\theta$. The condition (10) leads to

$$\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \sigma \text{ and } v'(s \rightsquigarrow t) = v'(1) \leq v'(s) \leq \tau$$

Hence $(s \rightsquigarrow t)_{(\sigma,\tau)} \in L_\rho^\theta$. Since $s_{(\sigma,\tau)} = (s \circ 1)_{(\sigma,\tau)} \in L_\rho^\theta$, (3.10) implies that $(t \circ 1)_{(\sigma,\tau)} = t_{(\sigma,\tau)} \in L_\rho^\theta$. Therefore L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.12.

L_ρ^θ be Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ which satisfies the condition(10). If L_ρ^θ satisfies

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0,0.5], \forall \beta, \delta \in [0,0.5), s \leq t \rightsquigarrow u, s_{(\alpha,\beta)} \in L_\rho^\theta, t_{(\gamma,\delta)} \in L_\rho^\theta \\ \Rightarrow u_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta. \quad (18)$$

(or)

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0.5,1], \forall \beta, \delta \in [0.5,1), s \leq t \rightsquigarrow u, s_{(\alpha,\beta)} \in L_\rho^\theta, t_{(\gamma,\delta)} \in L_\rho^\theta \\ \Rightarrow u_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta. \quad (19)$$

then L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ and satisfies the condition(10). Assume that L_ρ^θ satisfies (18). For $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0,0.5] \subseteq (0,1], \beta, \delta \in [0,0.5) \subseteq [0,1)$ $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$.

Since $s \rightsquigarrow t \leq s \rightsquigarrow t$, the condition (18) leads to $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Since $\alpha \wedge \gamma \leq 0.5, \beta \vee \delta < 0.5$, it follows that $\mu'(t) > \beta \vee \delta \geq \alpha \wedge \gamma$ and $v'(t) \leq \beta \vee \delta$. i.e., $t_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Therefore L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Now, let L_ρ^θ satisfies (19) and $s, t \in \mathcal{H}$ and $\alpha, \gamma \in (0.5,1] \subseteq (0,1), \beta, \delta \in [0.5,1) \subseteq [0,1)$ be such that $s_{(\alpha,\beta)} \in L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} \in L_\rho^\theta$. Then $\mu'(s) \geq \alpha > 1 - \alpha, v'(s) \leq \beta < 1 - \beta$ and $\mu'(s \rightsquigarrow t) \geq \gamma > 1 - \gamma$,

$v'(s \rightsquigarrow t) \leq \delta < 1 - \delta$. Therefore, $s_{(\alpha,\beta)} q L_\rho^\theta$ and $(s \rightsquigarrow t)_{(\gamma,\delta)} q L_\rho^\theta$. Using the condition (19), we have $t_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$ since $s \rightsquigarrow t \leq s \rightsquigarrow t$.

Hence L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.13.

Let L_ρ^θ be a Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ and satisfy the condition(10). If L_ρ^θ satisfies

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 0.5], \forall \beta, \delta \in [0, 0.5], (s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta, \\ (t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow (s \rightsquigarrow u)_{(\alpha\wedge\gamma, \beta\vee\delta)} q L_\rho^\theta. \quad (20)$$

(or)

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0.5, 1], \forall \beta, \delta \in [0.5, 1], (s \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta, \\ (t \rightsquigarrow u)_{(\gamma,\delta)} q L_\rho^\theta \Rightarrow (s \rightsquigarrow u)_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta. \quad (21)$$

then L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof L_ρ^θ be Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ satisfying (10). Assume that L_ρ^θ satisfies (20). Assume that for $t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 0.5] \subseteq (0, 1]$, $\beta, \delta \in [0, 0.5] \subseteq [0, 1]$,

$$t_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } (t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta.$$

Then the condition (4) implies that $t_{(\alpha,\beta)} = (1 \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$ and (20) implies $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma, \beta\vee\delta)} = u_{(\alpha\wedge\gamma, \beta\vee\delta)} q L_\rho^\theta$. Since $\alpha\wedge\gamma \leq 0.5$, $\beta\vee\delta < 0.5$, it follows that $\mu'(1 \rightsquigarrow u) > \beta\vee\delta \geq \alpha\wedge\gamma$ or $v'(1 \rightsquigarrow u) \leq \beta\vee\delta$. i.e., $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma, \beta\vee\delta)} = u_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$. Thus L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Suppose that L_ρ^θ satisfy (21) and for $t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0.5, 1] \subseteq (0, 1]$, $\beta, \delta \in [0.5, 1] \subseteq [0, 1]$,

$$t_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } (t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta.$$

Since $\alpha, \gamma > 0.5$ and $\beta, \delta > 0.5$, it follows that $\mu'(t) = \mu'(1 \rightsquigarrow t) > \beta$, $v'(t) = v'(1 \rightsquigarrow t) < \alpha$ and

$\mu'(t \rightsquigarrow u) > \delta$, $v'(t \rightsquigarrow u) < \gamma$. Therefore, $(1 \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta$ and $(t \rightsquigarrow u)_{(\gamma,\delta)} \in L_\rho^\theta$. Hence $(1 \rightsquigarrow u)_{(\alpha\wedge\gamma, \beta\vee\delta)} = u_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$ since (21). Thus L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Theorem 3.14.

Let L_ρ^θ be a Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ which satisfies (10). If L_ρ^θ satisfies

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0, 0.5], \forall \beta, \delta \in [0, 0.5], (s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta, \\ (s \circ u)_{(\gamma,\delta)} \in L_\rho^\theta \Rightarrow (t \circ u)_{(\alpha\wedge\gamma, \beta\vee\delta)} q L_\rho^\theta. \quad (22)$$

(or)

$$\forall s, t, u \in \mathcal{H}, \forall \alpha, \gamma \in (0.5, 1], \forall \beta, \delta \in [0.5, 1], (s \rightsquigarrow t)_{(\alpha,\beta)} q L_\rho^\theta, \\ (s \circ u)_{(\gamma,\delta)} q L_\rho^\theta \Rightarrow (t \circ u)_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta. \quad (23)$$

then L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Proof Suppose that L_ρ^θ is a Łukasiewicz intuitionistic fuzzy set in $\mathcal{H}$ satisfying(10). Assume that L_ρ^θ satisfies the condition(22). Assume that for $t, u \in \mathcal{H}$ and $\alpha, \gamma \in (0, 0.5] \subseteq (0, 1]$, $\beta, \delta \in [0, 0.5] \subseteq [0, 1]$,

$$t_{(\alpha,\beta)} \in L_\rho^\theta \text{ and } u_{(\gamma,\delta)} \in L_\rho^\theta.$$

Then $1 \rightsquigarrow t_{(\alpha,\beta)} = t_{(\alpha,\beta)} \in L_\rho^\theta$ and $1 \circ u_{(\gamma,\delta)} = u_{(\gamma,\delta)} \in L_\rho^\theta$. The condition (22) leads to $(t \circ u)_{(\alpha\wedge\gamma, \beta\vee\delta)} q L_\rho^\theta$. Since $\alpha\wedge\gamma \leq 0.5$, $\beta\vee\delta < 0.5$, it follows that $v'(t \circ u) \leq \beta\vee\delta$ and $\mu'(t \circ u) \geq \alpha\wedge\gamma$. i.e., $(t \circ u)_{(\alpha\wedge\gamma, \beta\vee\delta)} \in L_\rho^\theta$. Let $s, t \in \mathcal{H}$ and $\alpha \in (0, 0.5] \subseteq (0, 1]$, $\beta \in [0, 0.5] \subseteq [0, 1]$ be such that $s \leq t$ and $s_{(\alpha,\beta)} \in L_\rho^\theta$. It follows from the condition (10) that $\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \alpha$ and $v'(s \rightsquigarrow t) = v'(1) \leq v'(s) \leq \beta$. i.e., $(s \rightsquigarrow t)_{(\alpha,\beta)} \in L_\rho^\theta$. Since $s_{(\alpha,\beta)} = (s \circ 1)_{(\alpha,\beta)} \in L_\rho^\theta$, using the condition (22) we have $t_{(\alpha,\beta)} = (t \circ 1)_{(\alpha,\beta)} q L_\rho^\theta$. Since $\alpha \leq 0.5$ and $\beta < 0.5$, $\mu'(t) \geq \alpha$ and $v'(t) \leq \beta$. Therefore, $t_{(\alpha,\beta)} \in L_\rho^\theta$. Hence L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

Now assume L_ρ^θ satisfies (23). For $t, u \in \mathcal{H}$, $\alpha, \gamma \in (0.5, 1] \subseteq (0, 1]$, $\beta, \delta \in [0.5, 1] \subseteq [0, 1]$,

$$t_{(\alpha, \beta)} \in L_\rho^\theta \text{ and } u_{(\gamma, \delta)} \in L_\rho^\theta.$$

Then $1 \rightsquigarrow t_{(\alpha, \beta)} = t_{(\alpha, \beta)} \in L_\rho^\theta$ and $1 \circ u_{(\gamma, \delta)} = u_{(\gamma, \delta)} \in L_\rho^\theta$. Since $\alpha, \gamma \in (0.5, 1]$, $\beta, \delta \in [0.5, 1]$, $\mu'(1 \rightsquigarrow t) > \beta$, $\nu'(1 \rightsquigarrow t) < \alpha$ and $\mu'(1 \circ u) > \delta$, $\nu'(1 \circ u) < \gamma$. That is $(1 \rightsquigarrow t)_{(\alpha, \beta)} q L_\rho^\theta$ and $(1 \circ u)_{(\gamma, \delta)} q L_\rho^\theta$. The condition (23) induces that $(t \circ u)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in L_\rho^\theta$. Let $s, t \in \mathcal{H}$ and $\alpha \in (0.5, 1] \subseteq (0, 1]$, $\beta \in [0.5, 1] \subseteq [0, 1]$ be such that $s \leq t$ and $s_{(\alpha, \beta)} \in L_\rho^\theta$. It follows from the condition (10) that

$$\mu'(s \rightsquigarrow t) = \mu'(1) \geq \mu'(s) \geq \alpha \text{ and } \nu'(s \rightsquigarrow t) = \nu'(1) \leq \nu'(s) \leq \beta$$

which imply that $\nu'(s \rightsquigarrow t) < \alpha$ or $\mu'(s \rightsquigarrow t) > \beta$. i.e., $(s \rightsquigarrow t)_{(\alpha, \beta)} q L_\rho^\theta$. Since $\mu'(s \circ 1) = \mu'(s) \geq \alpha$, $\nu'(s \circ 1) = \nu'(s) \leq \beta$, we have $\mu'(s \circ 1) > \beta$, $\nu'(s \circ 1) < \alpha$. Therefore $(s \circ 1)_{(\alpha, \beta)} q L_\rho^\theta$. Using (23), we get $(t \circ 1)_{(\alpha, \beta)} = t_{(\alpha, \beta)} \in L_\rho^\theta$. Hence L_ρ^θ is a Łukasiewicz intuitionistic fuzzy filter of $\mathcal{H}$.

4. Application in Medical Diagnosis

The application of Łukasiewicz intuitionistic fuzzy set spans various fields such as complex decision-making, mathematical modelling, financial forecasting etc...

Now, we will focus on the application of Łukasiewicz intuitionistic fuzzy sets in medical diagnosis. We will diagnose the diseases using the following algorithm.

4.1. Algorithm

- Consider the set of patients $P = \{\dot{P}_i\}$.
- Consider the factors such as set of symptoms and test results as $S = \{\dot{S}_i\}$
- From the data collected from the patients, define the intuitionistic fuzzy set for each criteria $\dot{S}_i = \{p_i, \mu(p_i), \nu(p_i)\}$ by

$$\mu(p_i), \nu(p_i) : P \rightarrow [0, 1]$$

- For an arbitrary value of $\theta \in (0, 1)$, find the Łukasiewicz intuitionistic fuzzy set.
- Fix the rules or conditions of a disease to be diagnosed using the operations of Łukasiewicz intuitionistic fuzzy set.
- Computing according to the condition, we get the resulting Łukasiewicz intuitionistic fuzzy set.
- Calculate score by subtracting the non-membership value from the membership value of the resulting Łukasiewicz intuitionistic fuzzy set.
- By the value of score, we get the possibility of the patients having that disease.

4.2. Demonstration

Suppose if we are to diagnose the possibility of a person having heart disease.

Let us consider the Kaggle data set (<https://www.kaggle.com/datasets/rishidamarla/heart-disease-prediction>). From the above dataset, we consider set of 14 patients $P = \{\dot{P}_1, \dot{P}_2, \dot{P}_3, \dot{P}_4, \dot{P}_5, \dot{P}_6, \dot{P}_7, \dot{P}_8, \dot{P}_9, \dot{P}_{10}, \dot{P}_{11}, \dot{P}_{12}, \dot{P}_{13}, \dot{P}_{14}\}$ going to be diagnosed.

Let us consider the two criteria. $S = \{\dot{S}_1, \dot{S}_2\}$ be the set of test results of the patients for diagnosing heart disease, where $\dot{S}_1 = ST \text{ depression}$, $\dot{S}_2 = \text{Number of vessels fluoro}$.

To predict the heart disease, let us use the fuzzy rule:

If ST depression is moderate and Number of vessels fluoro is moderate then heart disease is present.

Consider the data set,

Patients	ST depression	Number of vessels fluoro
$\dot{P}_1$	2.4	3
$\dot{P}_2$	1.6	0
$\dot{P}_3$	0.3	0

$\dot{P}_4$	0.2	1
$\dot{P}_5$	0.2	1
$\dot{P}_6$	0.4	0
$\dot{P}_7$	0.6	1
$\dot{P}_8$	1.2	1
$\dot{P}_9$	1.2	2
$\dot{P}_{10}$	4	3
$\dot{P}_{11}$	0.5	0
$\dot{P}_{12}$	0	0
$\dot{P}_{13}$	0	0
$\dot{P}_{14}$	2.6	2

From the above data, the intuitionistic fuzzy sets $S_i : P \rightarrow [0,1]$ are defined by the membership function.

Membership function for ST depression is moderate:

$$\mu_{\text{moderate}} = \begin{cases} 0 & \text{if } x \leq 1 \\ \frac{x-1}{1.5} & \text{if } 1 < x \leq 2.5 \\ \frac{4-x}{1.5} & \text{if } 2.5 < x \leq 4 \\ 0 & \text{if } x > 4 \end{cases}$$

Membership function for Number of vessels fluoro:

$$\mu_{\text{moderate}} = \begin{cases} 0 & \text{for } x < 2 \\ 3-x & \text{for } 2 \leq x \leq 3 \\ 0 & \text{for } x > 3 \end{cases}$$

$$\dot{S}_1 = \left\{ (\dot{P}_1, 0.93, 0.07), (\dot{P}_2, 0.4, 0.6), (\dot{P}_3, 0, 1), (\dot{P}_4, 0, 1), (\dot{P}_5, 0, 1), \right. \\ \left. (\dot{P}_6, 0, 1), (\dot{P}_7, 0, 1), (\dot{P}_8, 0.13, 0.87), (\dot{P}_9, 0.13, 0.87), (\dot{P}_{10}, 0, 1), \right. \\ \left. (\dot{P}_{11}, 0, 1), (\dot{P}_{12}, 0, 1), (\dot{P}_{13}, 0, 1), (\dot{P}_{14}, 0.93, 0.07) \right\}$$

$$\dot{S}_2 = \left\{ (\dot{P}_1, 0, 1), (\dot{P}_2, 0, 1), (\dot{P}_3, 0, 1), (\dot{P}_4, 0, 1), (\dot{P}_5, 0, 1), \right. \\ \left. (\dot{P}_6, 0, 1), (\dot{P}_7, 0, 1), (\dot{P}_8, 0, 1), (\dot{P}_9, 1, 0), (\dot{P}_{10}, 0, 1), \right. \\ \left. (\dot{P}_{11}, 0, 1), (\dot{P}_{12}, 0, 1), (\dot{P}_{13}, 0, 1), (\dot{P}_{14}, 1, 0) \right\}$$

For $\theta = 0.9$, the Łukasiewicz intuitionistic fuzzy sets $L_{\dot{S}_i}^\theta$ of $\dot{S}_i$ as follows

$$L_{\dot{S}_1}^\theta = \left\{ (\dot{P}_1, 0.83, 0), (\dot{P}_2, 0.3, 0.5), (\dot{P}_3, 0, 0.9), (\dot{P}_4, 0, 0.9), (\dot{P}_5, 0, 0.9), \right. \\ \left. (\dot{P}_6, 0, 0.9), (\dot{P}_7, 0, 0.9), (\dot{P}_8, 0.03, 0.77), (\dot{P}_9, 0.03, 0.77), (\dot{P}_{10}, 0, 0.9), \right. \\ \left. (\dot{P}_{11}, 0, 0.9), (\dot{P}_{12}, 0, 0.9), (\dot{P}_{13}, 0, 0.9), (\dot{P}_{14}, 0.83, 0) \right\}$$

$$L_{\dot{S}_2}^\theta = \left\{ (\dot{P}_1, 0, 0.9), (\dot{P}_2, 0, 0.9), (\dot{P}_3, 0, 0.9), (\dot{P}_4, 0, 0.9), (\dot{P}_5, 0, 0.9), (\dot{P}_6, 0, 0.9), (\dot{P}_7, 0, 0.9), \right. \\ \left. (\dot{P}_8, 0, 0.9), (\dot{P}_9, 0.9, 0), (\dot{P}_{10}, 0, 0.9), (\dot{P}_{11}, 0, 0.9), (\dot{P}_{12}, 0, 0.9), (\dot{P}_{13}, 0, 0.9), (\dot{P}_{14}, 0.9, 0) \right\}$$

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$$L_{S_1}^\theta \oplus L_{S_2}^\theta = \left\{ (\dot{P}_1, 0.83, 0), (\dot{P}_2, 0.3, 0.4), (\dot{P}_3, 0, 0.8), (\dot{P}_4, 0, 0.8), (\dot{P}_5, 0, 0.8), \right. \\ \left. (\dot{P}_6, 0, 0.8), (\dot{P}_7, 0.9, 0), (\dot{P}_8, 0.03, 0.67), (\dot{P}_9, 0.93, 0), (\dot{P}_{10}, 0, 0.8), \right. \\ \left. (\dot{P}_{11}, 0, 0.8), (\dot{P}_{12}, 0, 0.8), (\dot{P}_{13}, 0, 0.8), (\dot{P}_{14}, 1, 0) \right\}$$

Patients	Score
$\dot{P}_1$	0.83
$\dot{P}_2$	-0.1
$\dot{P}_3$	-0.8
$\dot{P}_4$	-0.8
$\dot{P}_5$	-0.8
$\dot{P}_6$	-0.8
$\dot{P}_7$	0.9
$\dot{P}_8$	-0.64
$\dot{P}_9$	0.93
$\dot{P}_{10}$	-0.8
$\dot{P}_{11}$	-0.8
$\dot{P}_{12}$	-0.8
$\dot{P}_{13}$	-0.8
$\dot{P}_{14}$	1

Patients $\dot{P}_1, \dot{P}_7, \dot{P}_9, \dot{P}_{14}$ are more likely to have heart disease and $\dot{P}_2, \dot{P}_3, \dot{P}_4, \dot{P}_5, \dot{P}_6, \dot{P}_8, \dot{P}_{10}, \dot{P}_{11}, \dot{P}_{12}, \dot{P}_{13}$ are unlikely to have heart disease.

The accuracy depends on the rules we determine. Here we provide a demonstration with 80% accuracy. To enhance the accuracy, work on a more generalized fuzzy rule for diagnosing the disease.

6. Conclusion

In this paper, we introduced the notion of Łukasiewicz intuitionistic fuzzy filter based on intuitionistic fuzzy point. The properties of Łukasiewicz intuitionistic fuzzy filter is investigated. An algorithm is developed for medical diagnosis using the concept of Łukasiewicz intuitionistic fuzzy set and its operation. A demonstration is provided for our developed algorithm. In future, we want to make our algorithm applicable for large data sets using programming languages with better accuracy.

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