



MADM-Strategy using Grey Relational Analysis under Rough Single-Valued Pentapartitioned Neutrosophic Set Environment

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Abstract

This paper aims to introduce various operations in the context of the Rough Single-Valued Pentapartitioned Neutrosophic Set (RSVPNS) environment. Then, based on Grey Relational Analysis (GRA), we propose a Multi-Attribute Decision-Making (MADM) technique. Additionally, we present a practical numerical example to validate the proposed MADM technique in the context of selecting a tourist place for government initiatives aimed at enhancing its attractiveness to tourists.

Keywords: SVNS; SVPNS; RSVPNS; GRA; Hamming Distance; MADM

1. Introduction

In 1965, Prof. L.A. Zadeh proposed the concept of a fuzzy set (FS), in which each element has a membership value ranging from 0 to 1. Later, Prof. K. Atanassov expanded on the concept of FS by introducing Intuitionistic Fuzzy Set (IFS). In 1998, Smarandache [8] introduced the concept of Neutrosophic Set (NS) theory. After that, Wang et al. [9] looked into the Single-Valued NS (SVNS) concept. Afterward, many researchers have incorporated the concept of SVNS and its extensions into their MADM techniques. Deng [4] introduced the concept of the grey system theory in 1989.

Rough set theory, which Pawlak [6] first proposed in 1982, is a very helpful approach for addressing situations involving incomplete, ambiguous, or imprecise information. Later, utilizing rough set theory, Pawlak and Sowinski [7] presented a MADM technique. Wu [10] first suggested the rough sets approximation in the grey information system in 2010. Rough set theory has recently been successfully applied to FS, IFS, SVNS, and other systems. In 2014, Broumi et al. [1] introduced the idea of rough NS (RNS). In 2020, Mallick and Pramanik [5] proposed the Single-Valued Pentapartitioned NS (SVPNS) concept, which was likewise a very potent mathematical tool to cope with data having ignorance, unknown incomplete and uncertain information. Every element in an SVPNS has membership values based on the degree of truth, contradiction, ignorance, unknown, and falsity. Later, Das et al. [2] presented the notion of rough SVPNS (RSVPNS) and applied the concept of topology to it.

The remaining section of this article is structured as follows:

In Section 2, we present some definitions and findings that will be very useful in developing the main findings of this study. In Section 3, we develop a MADM technique based on GRA for the context of RSVPNS. Section 4 includes a real-world numerical example to validate the proposed MADM technique. Finally, in Section 5, we outline some future research scope and conclude the paper.

2. Preliminaries & Definitions

For the development of the major results of this paper, we present some existing definitions and findings in this section.

Single Valued Pentapartitioned Neutrosophic Set (SVPNS)

Let W be a non-empty fixed set. Then, an SVPNS [5] C over W is defined as follows:

$$C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\},$$

where $\Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})$ are the degree of truth membership, degree of contradiction membership, degree of ignorance membership, degree of unknown membership and degree of falsity membership of each $\ddot{e} \in W$. So, $0 \leq \Pi_C(\ddot{e}) + \Sigma_C(\ddot{e}) + \Omega_C(\ddot{e}) + \Psi_C(\ddot{e}) + \Theta_C(\ddot{e}) \leq 5$, for all $\ddot{e} \in W$.

The absolute SVPNS (1_W) [5] and the null SVPNS (0_W) over W are defined as follows:

$$(i) 1_W = \{(\ddot{e}, 1, 1, 0, 0, 0) : \ddot{e} \in W\};$$

$$(ii) 0_W = \{(\ddot{e}, 0, 0, 1, 1, 1) : \ddot{e} \in W\}.$$

Let us consider two SVPNSs $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ and $Q = \{(\ddot{e}, \Pi_Q(\ddot{e}), \Sigma_Q(\ddot{e}), \Omega_Q(\ddot{e}), \Psi_Q(\ddot{e}), \Theta_Q(\ddot{e})) : \ddot{e} \in W\}$ over W . Then, $C \subseteq Q$ [5] if and only if $\Pi_C(\ddot{e}) \leq \Pi_Q(\ddot{e}), \Sigma_C(\ddot{e}) \leq \Sigma_Q(\ddot{e}), \Omega_C(\ddot{e}) \geq \Omega_Q(\ddot{e}), \Psi_C(\ddot{e}) \geq \Psi_Q(\ddot{e}), \Theta_C(\ddot{e}) \geq \Theta_Q(\ddot{e}), \forall \ddot{e} \in W$.

Let us consider two SVPNSs $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ and $E = \{(\ddot{e}, \Pi_E(\ddot{e}), \Sigma_E(\ddot{e}), \Omega_E(\ddot{e}), \Psi_E(\ddot{e}), \Theta_E(\ddot{e})) : \ddot{e} \in W\}$ over W . Then, the intersection [5] of C and E is $C \cap E = \{(\ddot{e}, \min\{\Pi_C(\ddot{e}), \Pi_E(\ddot{e})\}, \min\{\Sigma_C(\ddot{e}), \Sigma_E(\ddot{e})\}, \max\{\Omega_C(\ddot{e}), \Omega_E(\ddot{e})\}, \max\{\Psi_C(\ddot{e}), \Psi_E(\ddot{e})\}, \max\{\Theta_C(\ddot{e}), \Theta_E(\ddot{e})\}) : \ddot{e} \in W\}$.

Assume that $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ and $E = \{(\ddot{e}, \Pi_E(\ddot{e}), \Sigma_E(\ddot{e}), \Omega_E(\ddot{e}), \Psi_E(\ddot{e}), \Theta_E(\ddot{e})) : \ddot{e} \in W\}$ be two SVPNSs over W . Then, the union [5] of C and E is $C \cup E = \{(\ddot{e}, \max\{\Pi_C(\ddot{e}), \Pi_E(\ddot{e})\}, \max\{\Sigma_C(\ddot{e}), \Sigma_E(\ddot{e})\}, \min\{\Omega_C(\ddot{e}), \Omega_E(\ddot{e})\}, \min\{\Psi_C(\ddot{e}), \Psi_E(\ddot{e})\}, \min\{\Theta_C(\ddot{e}), \Theta_E(\ddot{e})\}) : \ddot{e} \in W\}$.

Suppose that $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ be an SVPNS over a universal set W . Then, the complement [5] of C i.e., $C^c = \{(\ddot{e}, \Theta_C(\ddot{e}), \Psi_C(\ddot{e}), 1 - \Omega_C(\ddot{e}), \Sigma_C(\ddot{e}), \Pi_C(\ddot{e})) : \ddot{e} \in W\}$.

Hamming Distance between SVPNSs

Let us consider two SVPNSs $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ and $A = \{(\ddot{e}, \Pi_A(\ddot{e}), \Sigma_A(\ddot{e}), \Omega_A(\ddot{e}), \Psi_A(\ddot{e}), \Theta_A(\ddot{e})) : \ddot{e} \in W\}$ over W . Let $|W| = n$, where $|W|$ denotes the cardinality of W . Then, the Hamming distance (H_d) [3] between C and A is defined by:

$$H_d(C, A) = \sum_{\ddot{e} \in \Psi} (|\Pi_C(\ddot{e}) - \Pi_A(\ddot{e})| + |\Sigma_C(\ddot{e}) - \Sigma_A(\ddot{e})| + |\Omega_C(\ddot{e}) - \Omega_A(\ddot{e})| + |\Psi_C(\ddot{e}) - \Psi_A(\ddot{e})| + |\Theta_C(\ddot{e}) - \Theta_A(\ddot{e})|), \quad (1)$$

where $0 \leq H_d(C, A) \leq 5n$.

Normalized Hamming Distance between SVPNSs

Let us consider two SVPNSs $C = \{(\ddot{e}, \Pi_C(\ddot{e}), \Sigma_C(\ddot{e}), \Omega_C(\ddot{e}), \Psi_C(\ddot{e}), \Theta_C(\ddot{e})) : \ddot{e} \in W\}$ and $A = \{(\ddot{e}, \Pi_A(\ddot{e}), \Sigma_A(\ddot{e}), \Omega_A(\ddot{e}), \Psi_A(\ddot{e}), \Theta_A(\ddot{e})) : \ddot{e} \in W\}$ over W . Assume that $|W| = n$, where $|W|$ denotes the cardinality of the fixed set W . Then, the normalized Hamming distance ($N-H_d$) [3] between C and A is defined by:

$$N-H_d(C, A) = \frac{1}{5n} \sum_{\ddot{e} \in \Psi} (|\Pi_C(\ddot{e}) - \Pi_A(\ddot{e})| + |\Sigma_C(\ddot{e}) - \Sigma_A(\ddot{e})| + |\Omega_C(\ddot{e}) - \Omega_A(\ddot{e})| + |\Psi_C(\ddot{e}) - \Psi_A(\ddot{e})| + |\Theta_C(\ddot{e}) - \Theta_A(\ddot{e})|), \quad (2)$$

where $0 \leq N-H_d(C, A) \leq 1$.

Rough Single-Valued Pentapartitioned Neutrosophic Set (RSVPNS)

Let W be a fixed set, and ρ is an equivalence relation defined over it. Let $C = \{(\check{e}, \Pi_C(\check{e}), \Sigma_C(\check{e}), \Omega_C(\check{e}), \Psi_C(\check{e}), \Theta_C(\check{e})) : \check{e} \in W\}$ be an SVPNS over W . In the approximation space (W, ρ) , the lower approximation set $[\underline{T}(C)]$ and the upper approximation set $[\overline{T}(C)]$ of C are then defined as follows:

$$\underline{T}(C) = \{(\check{e}, \Pi_{\underline{T}(C)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\}$$

$$\text{and } \overline{T}(C) = \{(\check{e}, \Pi_{\overline{T}(C)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\},$$

where $\Pi_{\underline{T}(C)} = \wedge_{C \in [\check{e}]_\rho} \Pi_C(\check{e}), \Sigma_{\underline{T}(C)} = \wedge_{C \in [\check{e}]_\rho} \Sigma_C(\check{e}), \Omega_{\underline{T}(C)} = \wedge_{C \in [\check{e}]_\rho} \Omega_C(\check{e}), \Psi_{\underline{T}(C)} = \wedge_{C \in [\check{e}]_\rho} \Psi_C(\check{e}), \Theta_{\underline{T}(C)} = \wedge_{C \in [\check{e}]_\rho} \Theta_C(\check{e}),$
 $\Pi_{\overline{T}(C)} = \vee_{C \in [\check{e}]_\rho} \Pi_C(\check{e}), \Sigma_{\overline{T}(C)} = \vee_{C \in [\check{e}]_\rho} \Sigma_C(\check{e}), \Omega_{\overline{T}(C)} = \vee_{C \in [\check{e}]_\rho} \Omega_C(\check{e}), \Psi_{\overline{T}(C)} = \vee_{C \in [\check{e}]_\rho} \Psi_C(\check{e}), \Theta_{\overline{T}(C)} = \vee_{C \in [\check{e}]_\rho} \Theta_C(\check{e}).$

So, $0 \leq \Pi_{\underline{T}(C)}(\check{e}) + \Sigma_{\underline{T}(C)}(\check{e}) + \Omega_{\underline{T}(C)}(\check{e}) + \Psi_{\underline{T}(C)}(\check{e}) + \Theta_{\underline{T}(C)}(\check{e}) \leq 5$, for all $\check{e} \in W$;

and $0 \leq \Pi_{\overline{T}(C)}(\check{e}) + \Sigma_{\overline{T}(C)}(\check{e}) + \Omega_{\overline{T}(C)}(\check{e}) + \Psi_{\overline{T}(C)}(\check{e}) + \Theta_{\overline{T}(C)}(\check{e}) \leq 5$, for all $\check{e} \in W$.

Here, the operators “ \vee ” and “ \wedge ” denote “max” and “min” operators respectively. Clearly, $\underline{T}(C)$ and $\overline{T}(C)$ are two SVPNSs over W . The pair $(\underline{T}(C), \overline{T}(C))$ is called the RSVPNS [2] in the approximation space (W, ρ) .

Let $T(C) = (\underline{T}(C), \overline{T}(C)) = \{(\check{e}, \Pi_{\underline{T}(C)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\}, \{(\check{e}, \Pi_{\overline{T}(C)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\}$ be a RSVPNS in the approximation space (W, ρ) . Then, $[(\Pi_{\underline{T}(C)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e})), (\Pi_{\overline{T}(C)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e}))]$ is called a Rough Single-Valued Pentapartitioned Neutrosophic Number (RSVPNN) [2] for all $\check{e} \in W$.

Assume that $T(A) = (\underline{T}(A), \overline{T}(A))$ be a RSVPNS in the approximation space (W, ρ) . Then, the complement [2] of $T(A) = (\underline{T}(A), \overline{T}(A))$ is defined by:

$$T(A)^c = (\underline{T}(A)^c, \overline{T}(A)^c),$$

where $\underline{T}(A)^c = \{(\check{e}, \Theta_{\underline{T}(A)}(\check{e}), \Psi_{\underline{T}(A)}(\check{e}), 1 - \Omega_{\underline{T}(A)}(\check{e}), \Sigma_{\underline{T}(A)}(\check{e}), \Pi_{\underline{T}(A)}(\check{e})) : A \in [\check{e}]_\rho, \check{e} \in W\}$

and $\overline{T}(A)^c = \{(\check{e}, \Theta_{\overline{T}(A)}(\check{e}), \Psi_{\overline{T}(A)}(\check{e}), 1 - \Omega_{\overline{T}(A)}(\check{e}), \Sigma_{\overline{T}(A)}(\check{e}), \Pi_{\overline{T}(A)}(\check{e})) : A \in [\check{e}]_\rho, \check{e} \in W\}.$

Suppose that $T(C) = (\underline{T}(C), \overline{T}(C))$ and $T(A) = (\underline{T}(A), \overline{T}(A))$ be two RSVPNSs [2] in (W, ρ) . Then, $T(C) \subseteq T(A)$ if and only if $\underline{T}(C) \subseteq \underline{T}(A)$ and $\overline{T}(C) \subseteq \overline{T}(A)$ i.e., $\Pi_{\underline{T}(C)}(\check{e}) \leq \Pi_{\underline{T}(A)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}) \leq \Sigma_{\underline{T}(A)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}) \geq \Omega_{\underline{T}(A)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}) \geq \Psi_{\underline{T}(A)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e}) \geq \Theta_{\underline{T}(A)}(\check{e}), \Pi_{\overline{T}(C)}(\check{e}) \leq \Pi_{\overline{T}(A)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}) \leq \Sigma_{\overline{T}(A)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}) \geq \Omega_{\overline{T}(A)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}) \geq \Psi_{\overline{T}(A)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e}) \geq \Theta_{\overline{T}(A)}(\check{e})$, for all $\check{e} \in W$.

Assume that $T(C) = (\underline{T}(C), \overline{T}(C))$ and $T(E) = (\underline{T}(E), \overline{T}(E))$ be two RSVPNSs in the approximation space (W, ρ) . Then, $T(C) = T(E)$ [2] if and only if $\underline{T}(C) = \underline{T}(E)$ and $\overline{T}(C) = \overline{T}(E)$ i.e., $\Pi_{\underline{T}(C)}(\check{e}) = \Pi_{\underline{T}(E)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}) = \Sigma_{\underline{T}(E)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}) = \Omega_{\underline{T}(E)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}) = \Psi_{\underline{T}(E)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e}) = \Theta_{\underline{T}(E)}(\check{e}), \Pi_{\overline{T}(C)}(\check{e}) = \Pi_{\overline{T}(E)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}) = \Sigma_{\overline{T}(E)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}) = \Omega_{\overline{T}(E)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}) = \Psi_{\overline{T}(E)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e}) = \Theta_{\overline{T}(E)}(\check{e})$, for all $\check{e} \in W$.

Let $T(C) = (\underline{T}(C), \overline{T}(C))$ and $T(E) = (\underline{T}(E), \overline{T}(E))$ be two RSVPNSs in (W, ρ) . Then, the union and intersection [2] of $T(C)$ and $T(E)$ are defined as follows:

$$T(C \cap E) = (\underline{T}(C \cap E), \overline{T}(C \cap E)) \text{ and } T(C \cup E) = (\underline{T}(C \cup E), \overline{T}(C \cup E)),$$

where $\underline{T}(C \cap E) = \{(\check{e}, \Pi_{\underline{T}(C)}(\check{e}) \wedge \Pi_{\underline{T}(E)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}) \wedge \Sigma_{\underline{T}(E)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}) \vee \Omega_{\underline{T}(E)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}) \vee \Psi_{\underline{T}(E)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e}) \vee \Theta_{\underline{T}(E)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\};$

$\overline{T}(C \cap E) = \{(\check{e}, \Pi_{\overline{T}(C)}(\check{e}) \wedge \Pi_{\overline{T}(E)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}) \wedge \Sigma_{\overline{T}(E)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}) \vee \Omega_{\overline{T}(E)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}) \vee \Psi_{\overline{T}(E)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e}) \vee \Theta_{\overline{T}(E)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\};$

$\underline{T}(C \cup E) = \{(\check{e}, \Pi_{\underline{T}(C)}(\check{e}) \vee \Pi_{\underline{T}(E)}(\check{e}), \Sigma_{\underline{T}(C)}(\check{e}) \vee \Sigma_{\underline{T}(E)}(\check{e}), \Omega_{\underline{T}(C)}(\check{e}) \wedge \Omega_{\underline{T}(E)}(\check{e}), \Psi_{\underline{T}(C)}(\check{e}) \wedge \Psi_{\underline{T}(E)}(\check{e}), \Theta_{\underline{T}(C)}(\check{e}) \wedge \Theta_{\underline{T}(E)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\};$

and $\overline{T}(C \cup E) = \{(\check{e}, \Pi_{\overline{T}(C)}(\check{e}) \vee \Pi_{\overline{T}(E)}(\check{e}), \Sigma_{\overline{T}(C)}(\check{e}) \vee \Sigma_{\overline{T}(E)}(\check{e}), \Omega_{\overline{T}(C)}(\check{e}) \wedge \Omega_{\overline{T}(E)}(\check{e}), \Psi_{\overline{T}(C)}(\check{e}) \wedge \Psi_{\overline{T}(E)}(\check{e}), \Theta_{\overline{T}(C)}(\check{e}) \wedge \Theta_{\overline{T}(E)}(\check{e})) : C \in [\check{e}]_\rho, \check{e} \in W\}.$

3. GRA-Based MADM Strategy under RSV PNS Environment:

Any decision maker (DM) faces a difficult problem when selecting an option from a family of potential alternatives against a set of certain attributes. In order to quickly make the right judgement, the DM must plan a MADM technique. Assume that $L = \{Q_1, Q_2, \dots, Q_p\}$ is the group of p alternatives and that $S = \{P_1, P_2, \dots, P_q\}$ is the family of q attributes. According to RSV PNNs, the DM offers evaluation data for each alternative Q_k ($k = 1, 2, \dots, p$) based on the attribute P_n ($n = 1, 2, \dots, q$). A decision matrix can thus express all of the evaluation information for all alternatives.

The steps of the proposed MADM strategy under the RSV PNS environment are presented as follows:

Step-1: Construction of RSV PNN Based Decision Matrix

We generate the new decision matrix using the decision maker's evaluation data as follows:

Table 1: The new decision matrix using the decision maker's evaluation data

DM	P_1	P_2	...	P_q
Q_1	$(\underline{S}_{11}, \bar{S}_{11})$	$(\underline{S}_{12}, \bar{S}_{12})$...	$(\underline{S}_{1q}, \bar{S}_{1q})$
Q_2	$(\underline{S}_{21}, \bar{S}_{21})$	$(\underline{S}_{22}, \bar{S}_{22})$...	$(\underline{S}_{2q}, \bar{S}_{2q})$
...
Q_p	$(\underline{S}_{p1}, \bar{S}_{p1})$	$(\underline{S}_{p2}, \bar{S}_{p2})$...	$(\underline{S}_{pq}, \bar{S}_{pq})$

Here, the RSV PNN used by the decision-maker to evaluate the alternative Q_k ($k = 1, 2, \dots, p$) in relation to the attribute P_n ($n = 1, 2, \dots, q$) is $\langle \underline{S}_{kn}, \bar{S}_{kn} \rangle$ ($k = 1, 2, \dots, p; n = 1, 2, \dots, q$).

Step-2: Determination of the AGO

In this step, we apply the AGO to RSV PNNs (shown in Table-1) to convert the evaluation information into the SVPNN. The following is the definition of the AGO:

$$S_{kn} = S_{kn}(\Pi_{kn}, \Sigma_{kn}, \Omega_{kn}, \Psi_{kn}, \Theta_{kn}) = S_{kn}((\underline{\Pi}_{kn}, \bar{\Pi}_{kn})^{0.5}, (\underline{\Sigma}_{kn}, \bar{\Sigma}_{kn})^{0.5}, (\underline{\Omega}_{kn}, \bar{\Omega}_{kn})^{0.5}, (\underline{\Psi}_{kn}, \bar{\Psi}_{kn})^{0.5}, (\underline{\Theta}_{kn}, \bar{\Theta}_{kn})^{0.5}) \tag{3}$$

Now, the single-valued pentapartitioned neutrosophic decision matrix is transformed in the form of SVPNNs as follows:

Table 2: The single-valued pentapartitioned neutrosophic decision matrix is transformed in the form of SVPNNs

DM	P_1	P_2	...	P_q
Q_1	S_{11}	S_{12}	...	S_{1q}
Q_2	S_{21}	S_{22}	...	S_{2q}
...
Q_p	S_{p1}	S_{p1}	...	S_{pq}

Step-3: Determination of the Weights of the Attribute

DMs frequently run against partially known or unknowable attribute weights while making decisions. Therefore, determining attribute weight is essential for sound decision-making. In that instance, the DM can assess the attribute weights using the CF.

The CF is defined as follows:

$$\xi_n = \sum_{k=1}^p (3 + \Pi_{kn}(Q_k, P_n) + \Sigma_{kn}(Q_k, P_n) - \Omega_{kn}(Q_k, P_n) - \Psi_{kn}(Q_k, P_n) - \Theta_{kn}(Q_k, P_n))/5 \tag{4}$$

Then, the weight of the n^{th} attribute is defined by $w_n = \frac{\xi_n}{\sum_{n=1}^q \xi_n}$ (5)

Here, $\sum_{n=1}^q w_n = 1$.

Step-4: Determination of the IRSVPNERS and IRSVPNEURS for the Decision Matrix

The IRSVPNERS for the decision matrix is presented as follows:

$$R^+ = [(\Pi_1^+, \Sigma_1^+, \Omega_1^+, \Psi_1^+, \Theta_1^+), (\Pi_2^+, \Sigma_2^+, \Omega_2^+, \Psi_2^+, \Theta_2^+), \dots, (\Pi_q^+, \Sigma_q^+, \Omega_q^+, \Psi_q^+, \Theta_q^+)], \tag{6}$$

where $\Pi_n^+ = \max \{\Pi_{kn}(Q_k, P_n): k = 1, 2, 3, \dots, p\}$, $\Sigma_n^+ = \max \{\Sigma_{kn}(Q_k, P_n): k = 1, 2, 3, \dots, p\}$, $\Omega_n^+ = \min \{\Omega_{kn}(Q_k, P_n): k = 1, 2, 3, \dots, p\}$, $\Psi_n^+ = \min \{\Psi_{kn}(Q_k, P_n): k = 1, 2, 3, \dots, p\}$ and $\Theta_n^+ = \min \{\Theta_{kn}(Q_k, P_n): k = 1, 2, 3, \dots, p\}$.

The IRSVPNEURS for the decision matrix is presented as follows:

$$R^- = [(\Pi_1^-, \Sigma_1^-, \Omega_1^-, \Psi_1^-, \Theta_1^-), (\Pi_2^-, \Sigma_2^-, \Omega_2^-, \Psi_2^-, \Theta_2^-), \dots, (\Pi_q^-, \Sigma_q^-, \Omega_q^-, \Psi_q^-, \Theta_q^-)], \tag{7}$$

where $\Pi_n^- = \min \{\Pi_{kn}(L_k, S_n): k=1, 2, 3, \dots, p\}$, $\Sigma_n^- = \min \{\Sigma_{kn}(L_k, S_n): k=1, 2, 3, \dots, p\}$, $\Omega_n^- = \max \{\Omega_{kn}(L_k, S_n): k=1, 2, 3, \dots, p\}$, $\Psi_n^- = \max \{\Psi_{kn}(L_k, S_n): k=1, 2, 3, \dots, p\}$ and $\Theta_n^- = \max \{\Theta_{kn}(L_k, S_n): k=1, 2, 3, \dots, p\}$.

Step-5: Determination of the RSVPNGRC of Each Alternative from IRSVPNERS & IRSVPNEURS

The RSVPNGRC of each alternative from IRSVPNERS is presented as follows:

$$\Delta_{kn}^+ = \frac{\min_k \min_n \lambda_{kn}^+ + y \cdot \max_k \max_n \lambda_{kn}^+}{\lambda_{kn}^+ + y \cdot \max_k \max_n \lambda_{kn}^+}, \tag{8}$$

where $\lambda_{kn}^+ = Hd((\Pi_n^+, \Sigma_n^+, \Omega_n^+, \Psi_n^+, \Theta_n^+), (\Pi_{kn}, \Sigma_{kn}, \Omega_{kn}, \Psi_{kn}, \Theta_{kn}))$, $k = 1, 2, \dots, p$; $n = 1, 2, \dots, q$, and $y \in [0, 1]$.

The RSVPNGRC of each alternative from IRSVPNEURS is given below:

$$\Delta_{kn}^- = \frac{\min_k \min_n \lambda_{kn}^- + y \cdot \max_k \max_n \lambda_{kn}^-}{\lambda_{kn}^- + y \cdot \max_k \max_n \lambda_{kn}^-}, \tag{9}$$

where $\lambda_{kn}^- = Hd((\Pi_{kn}, \Sigma_{kn}, \Omega_{kn}, \Psi_{kn}, \Theta_{kn}), (\Pi_n^-, \Sigma_n^-, \Omega_n^-, \Psi_n^-, \Theta_n^-))$, $k=1, 2, \dots, p$; $n=1, 2, \dots, q$, and $y \in [0, 1]$.

Here, the identification coefficients Δ_{kn}^+ and Δ_{kn}^- are employed to modify the comparison environment's range and regulate the intensity of relation coefficient differences. When $y=1$ and $y=0$, the comparison environment doesn't change and vanishes, respectively. The range of the grey relationship coefficient will get much larger if the identification coefficient is less. Typically, $y=0.5$ is taken into account in decision-making situations.

Step-6: Determine the RSVPNGRC

The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS are defined as follows:

$$G_k^+ = \sum_{n=1}^q w_n \Delta_{kn}^+, \tag{10}$$

where $k = 1, 2, \dots, p$,

and

$$G_k^- = \sum_{n=1}^q w_n \Delta_{kn}^-, \tag{11}$$

where $k = 1, 2, \dots, p$.

Step-7: Determination of the RSVPNRRD

The RSVPNRRD of each alternative can be defined as follows:

$$\mathfrak{R}_k = \frac{G_k^+}{G_k^+ + G_k^-} \tag{12}$$

where $k = 1, 2, \dots, p$.

Step-8: Ranking of the alternatives

The RSVPNRRDs of the alternatives can be arranged in ascending order to establish the order in which they should be ranked. The best alternative is the one with the highest RSVPNRRD value.

Step-9: End

The flowchart of the proposed MADM strategy is given below:

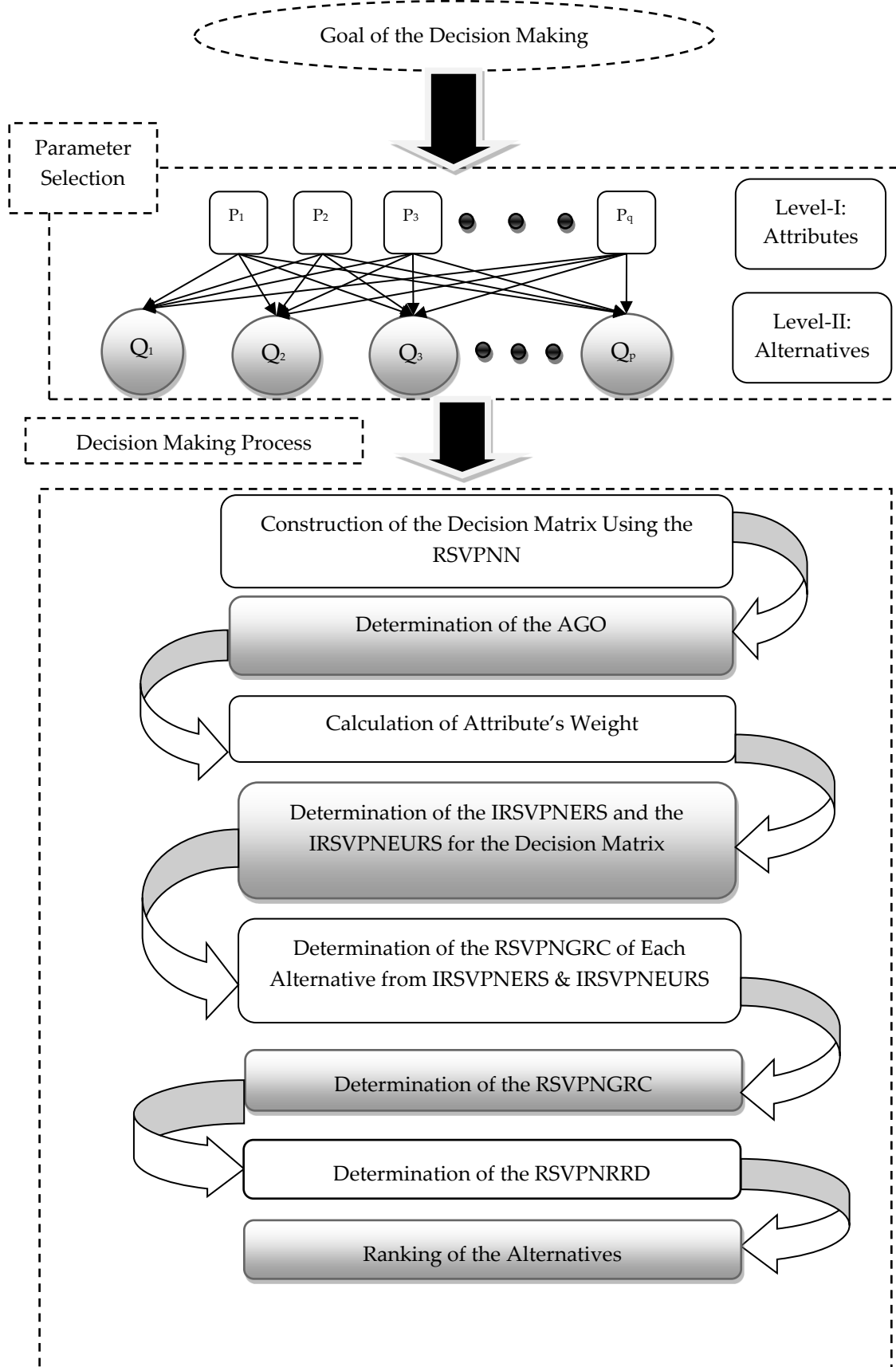


Figure 1. Proposed MADM Technique

4. Validation of the Proposed MADM Technique:

This section includes a numerical example from the actual world to demonstrate the value and efficacy of the suggested MADM technique.

Example 4.1. "Selection of a Tourist Place by Government Initiative to Make It Attractive for Tourists".

To encourage tourism in the country, it is critical to make it appealing in a variety of ways. International tourism, as we all know, is critical to every country's economic, cultural, and social development. The country's architecture and wealth of heritage treasures are among the most appealing aspects of Indian ecotourism. In addition, other historic sites are in poor condition. It would be a fantastic idea to outsource the maintenance and external illumination of cultural structures to corporate behemoths in exchange for some marketing exposure at the monuments. For example, tour guides, children's entertainment, gourmet excursions, and tourist connections with local culture could all be included. It would be completely incorrect to simply say, "Make important connections." This is a broad assertion. Such a tourist destination, however, requires appropriate roadways and approach spaces. Although they may not appear ideal, packaged rail travel, convenient bus connections, and secure car rental services with qualified employees, combined with excellent roads and motorways, are critical components of an excellent tourist experience. Many people seem to have difficulty-eroticizing India for the West, but it will happen at some point. Although we are no longer portraying India as a land of snakes and charmers, the tourism ministry should certainly pursue aggressive web and other marketing strategies to promote India as a must-see destination. Whether it is airing "Incredible India" advertisements in other countries, organizing tourism seminars, or providing Indian locations with facilities to support international film productions, India's government is doing everything it can to promote the country. Vigorous marketing is required to effectively be seen and heard. India can now profit from providing personalized experiences, rare animal sanctuaries, religious pilgrimage trips, and extreme Himalayan adventures. With so many points of differentiation, Indian tourism should focus on what it has to offer that makes it appealing in every way. Despite the fact that the government has started to make significant progress in this area, there is still a long way to go before tourism can generate significant revenue for the country. We must promote India as a vacation destination where the regal Bengal tiger, a common worker, a few billionaires, religious dichotomies, and unique gastronomic delights can be found. When deciding where to go, how to go, when to go, and what to bring with us, there are a few other factors to consider. For example, when we want to know where to go, how to get there, when to go, what to bring, and when to go, we need complete information or a suitable package that fits within our budget. More importantly, we want to know about the location; what kinds of views are available, and whether or not essential amenities are available. The tourism industry should prioritize factors that attract visitors. The most prevalent concerns to which the government must pay attention are:

(i) Communication facility of such tourist place, (ii) Accommodation, (iii) Basic health facility, (iv) Online booking facilities for different services, and (v) Safety.

So, the selection of tourist places by the Government can be considered as a MADM problem. After initializing, the decision maker selects four major alternatives, namely Q_1 , Q_2 , Q_3 , and Q_4 . For the selection of a suitable alternative, the decision maker selects seven attributes such as

P₁: Communication facility of such tourist place,

P₂: Accommodation,

P₃: Basic health facility,

P₄: Online booking facilities for different services,

P₅: Safety.

The following is the presentation of the current MADM problem:

We construct a rough single-valued pentapartitioned neutrosophic decision matrix as follows, using the evaluation data for each alternative against the attributes provided by the DM:

Table 3: Rough single-valued pentapartitioned neutrosophic decision matrix

	P ₁	P ₂	P ₃	P ₄	P ₅
Q ₁	[(0.9,0.3,0.1,0.1,0.0), (1.0,0.4,0.1,0.0,0.0)]	[(0.7,0.2,0.1,0.1,0.1), (0.9,0.4,0.0,0.0,0.1)]	[(0.8,0.2,0.3,0.2,0.2), (1.0,0.4,0.1,0.2,0.0)]	[(1.0,0.2,0.2,0.1,0.1), (1.0,0.4,0.1,0.1,0.0)]	[(0.6,0.4,0.1,0.0,0.2), (0.8,0.5,0.0,0.0,0.1)]
Q ₂	[(0.8,0.1,0.2,0.2,0.1), (0.9,0.3,0.1,0.1,0.0)]	[(0.8,0.2,0.2,0.2,0.1), (0.8,0.3,0.1,0.1,0.0)]	[(0.8,0.1,0.4,0.2,0.1), (0.9,0.4,0.1,0.1,0.1)]	[(0.9,0.5,0.3,0.2,0.1), (1.0,0.6,0.1,0.1,0.1)]	[(0.8,0.1,0.0,0.2,0.0), (1.0,0.4,0.0,0.1,0.0)]
Q ₃	[(0.8,0.2,0.2,0.3,0.3), (1.0,0.4,0.0,0.1,0.2)]	[(0.8,0.0,0.1,0.1,0.2), (1.0,0.2,0.1,0.0,0.1)]	[(0.7,0.2,0.2,0.1,0.2), (0.9,0.3,0.1,0.1,0.1)]	[(0.8,0.1,0.2,0.1,0.1), (0.9,0.4,0.1,0.0,0.1)]	[(0.7,0.2,0.2,0.1,0.2), (0.9,0.4,0.1,0.0,0.1)]
Q ₄	[(0.9,0.3,0.1,0.1,0.2), ,)	[(0.7,0.2,0.3,0.2,0.1), ,)	[(0.9,0.1,0.1,0.1,0.1), ,)	[(0.6,0.2,0.3,0.2,0.1), ,)	[(0.8,0.1,0.1,0.2,0.1), ,)

By using the eq. (3), we form the single-valued pentapartitioned neutrosophic decision matrix as follows:

Table 4: The single-valued pentapartitioned neutrosophic decision matrix

	P ₁	P ₂	P ₃	P ₄	P ₅
Q ₁	(0.949,0.346,0.1,0.0,0.0)	(0.794,0.283,0.0,0.0,0.1)	(0.894,0.283,0.173,0.2,0.0)	(1.0,0.283,0.141,0.1,0.0)	(0.693,0.447,0.0,0.0,0.141)
Q ₂	(0.848,0.173,0.141,0.141,0.0)	(0.8,0.245,0.141,0.141,0.0)	(0.848,0.2,0.2,0.141,0.1)	(0.949,0.548,0.173,0.141,0.1)	(0.894,0.2,0.0,0.141,0.0)
Q ₃	(0.894,0.283,0.0,0.173,0.245)	(0.894,0.0,0.1,0.0,0.141)	(0.794,0.245,0.141,0.1,0.141)	(0.848,0.2,0.141,0.0,0.1)	(0.794,0.283,0.141,0.0,0.141)
Q ₄	(0.9,0.387,0.0,0.1,0.0)	(0.837,0.283,0.245,0.141,0.0)	(0.949,0.2,0.0,0.0,0.1)	(0.693,0.283,0.173,0.141,0.1)	(0.894,0.224,0.0,0.2,0.1)

Now, by using eq. (4) & eq. (5), we find the weight vector as follows:

$$(w_1, w_2, w_3, w_4, w_5) = (0.2057448, 0.1959888, 0.1958592, 0.2007437, 0.2016636)$$

The IRSVPNERS (R⁺) and IRSVPNEURS (R⁻) for the decision matrix are presented as follows:

Table 5: The IRSVPNERS (R⁺) and IRSVPNEURS (R⁻) for the decision matrix

	P ₁	P ₂	P ₃	P ₄	P ₅
Q ₁	(0.949,0.346,0.1,0.0,0.0)	(0.794,0.283,0.0,0.0,0.1)	(0.894,0.283,0.173,0.2,0.0)	(1.0,0.283,0.141,0.1,0.0)	(0.693,0.447,0.0,0.0,0.141)
Q ₂	(0.848,0.173,0.141,0.141,0.0)	(0.8,0.245,0.141,0.141,0.0)	(0.848,0.2,0.2,0.141,0.1)	(0.949,0.548,0.173,0.141,0.1)	(0.894,0.2,0.0,0.141,0.0)
Q ₃	(0.894,0.283,0.0,0.173,0.245)	(0.894,0.0,0.1,0.0,0.141)	(0.794,0.245,0.141,0.1,0.141)	(0.848,0.2,0.141,0.0,0.1)	(0.794,0.283,0.141,0.0,0.141)

Q ₄	(0.9,0.387,0.0,0.1,0.0)	(0.837,0.283,0.245,0.141,0.0)	(0.949,0.2,0.0,0.0,0.1)	(0.693,0.283,0.173,0.141,0.1)	(0.894,0.224,0.0,0.2,0.1)
R ₊	(0.949,0.387,0.0,0.0,0.0)	(0.894,0.283,0.0,0.0,0.0)	(0.949,0.283,0.0,0.0,0.0)	(1.0,0.548,0.141,0.0,0.0)	(0.894,0.447,0.0,0.0,0.0)
R ₋	(0.848,0.173,0.141,0.173,0.245)	(0.794,0.0,0.245,0.141,0.141)	(0.794,0.2,0.2,0.2,0.141)	(0.693,0.2,0.173,0.141,0.1)	(0.693,0.2,0.141,0.2,0.141)

The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS is presented in the Table-6, Table-7, Table-8 and Table-9, respectively.

Table 6: The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS (a)

	P ₁	P ₂	P ₃	P ₄	P ₅	min _m λ ⁺ _{mn}	max _m λ ⁺ _{mn}
λ ⁺ _{1n}	0.141	0.586	0.611	0.48	0.342	0.141	0.611
λ ⁺ _{2n}	0.597	0.414	0.625	0.327	0.388	0.327	0.625
λ ⁺ _{3n}	0.577	0.524	0.575	0.6	0.546	0.524	0.6
λ ⁺ _{4n}	0.149	0.443	0.183	0.845	0.523	0.149	0.845
min _m min _n λ ⁺ _{mn}	0.141						
max _m max _n λ ⁺ _{mn}	0.845						

Table 7: The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS (b)

	P ₁	P ₂	P ₃	P ₄	P ₅	min _m λ ⁻ _{mn}	max _m λ ⁻ _{mn}
λ ⁻ _{1n}	0.733	0.755	0.351	0.563	0.588	0.351	0.755
λ ⁻ _{2n}	0.277	0.496	0.154	0.604	0.542	0.154	0.604
λ ⁻ _{3n}	0.297	0.386	0.204	0.325	0.384	0.204	0.386
λ ⁻ _{4n}	0.725	0.467	0.596	0.083	0.407	0.083	0.725
min _m min _n λ ⁻ _{mn}	0.083						
max _m max _n λ ⁻ _{mn}	0.755						

Table 8: The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS (c)

Δ ⁺ _{mn}	P ₁	P ₂	P ₃	P ₄	P ₅
Q ₁	1	0.55875	0.545235	0.624377	0.737083
Q ₂	0.552722	0.67364	0.537947	0.751835	0.69525
Q ₃	0.563782	0.595351	0.564912	0.5511	0.581828
Q ₄	0.986002	0.651069	0.930636	0.444576	0.595981

Table 9: The RSVPNGRC of each alternative from IRSVPNERS and IRSVPNEURS (d)

Δ_{mn}^-	P_1	P_2	P_3	P_4	P_5
Q ₁	0.414678	0.406623	0.632121	0.489633	0.476955
Q ₂	0.703591	0.527189	0.866416	0.46918	0.500816
Q ₃	0.682728	0.603143	0.791917	0.655516	0.604728
Q ₄	0.417687	0.545293	0.473035	1	0.586998

The RSVPNGRCs G_m^+ and G_m^- of each alternative (Q_m, m=1, 2, 3, 4) from IRSVPNERS and IRSVPNEURS respectively are presented in Table-10.

Table 10: The RSVPNGRCs G_m^+ and G_m^- of each alternative (Q_m, m=1, 2, 3, 4) from IRSVPNERS and IRSVPNEURS

	G_m^+	G_m^-
Q ₁	0.696025	0.483293
Q ₂	0.64224	0.61296
Q ₃	0.571284	0.667324
Q ₄	0.722174	0.604576

The RSVPNRRD (\mathfrak{R}_m) of each alternative Q_m (m = 1, 2, 3, 4) is presented in Table-11.

Table 11: The RSVPNRRD (\mathfrak{R}_m) of each alternative Q_m (m = 1, 2, 3, 4)

	$\mathfrak{R}_m = \frac{G_m^+}{G_m^+ + G_m^-}$
Q ₁	0.590193
Q ₂	0.511663
Q ₃	0.461231
Q ₄	0.544318

From Table-11, it is clear that $\mathfrak{R}_3 < \mathfrak{R}_2 < \mathfrak{R}_4 < \mathfrak{R}_1$. Therefore, Q₁ is the most appropriate alternative i.e., Tourist Place, to choose for making it attractive for tourists.

5. Conclusion

In this paper, we have developed a MADM technique based on grey relational analysis for the RSVPN environment. In addition, we have solved a real-world numerical example to validate our suggested MADM method. The proposed MADM technique is also intended to be able to address other real-world issues, such as flat selection, weaver selection, Ph.D. guide selection, teacher selection and medical diagnostic.

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Authors Contribution: All the authors have equal contributions to the preparation of this article.

For the sake of presenting clarity, the following abbreviations are used throughout this article.

Short Terms	
Neutrosophic Set	NS
Single-Valued NS	SVNS
Rough NS	RNS
Single-Valued Pentapartitioned NS	SVPNS
Rough SVPNS	RSVPNS
Grey Relational Analysis	GRA
Multi-Attribute Decision-Making	MADM
Accumulated Geometric Operator	AGO
Compromise Function	CF
Ideal Rough Single-Valued Pentapartitioned Neutrosophic Estimates Reliability Solution	IRSVPNERS
Ideal Rough Single-Valued Pentapartitioned Neutrosophic Estimates Unreliability Solution	IRSVPNEURS
Rough Single-Valued Pentapartitioned Neutrosophic Grey Relational Coefficient	RSVPNGRC
Rough Single-Valued Pentapartitioned Neutrosophic Relative Relational Degree	RSVPNRRD

References

- [1] S. Broumi, F. Smarandache, and M. Dhar, "Rough Neutrosophic Sets," *Neutrosophic Sets and Systems*, vol. 3, pp. 60-65, 2014.
- [2] S. Das, R. Das, and B. C. Tripathy, "Topology on Rough Pentapartitioned Neutrosophic Set," *Iraqi Journal of Science*, vol. 63, no. 6, pp. 2630-2640, 2022.
- [3] S. Das, B. Shil, and S. Pramanik, "A Novel Decision-Making Framework Using Neutrosophic Set Theory," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 1-12, 2021.
- [4] J. L. Deng, "Introduction to Grey System Theory," *The Journal of Grey System*, vol. 1, no. 1, pp. 1-24, 1989.
- [5] R. Mallick and S. Pramanik, "Pentapartitioned Neutrosophic Set and Its Properties," *Neutrosophic Sets and Systems*, vol. 36, pp. 184-192, 2020.
- [6] Z. Pawlak, "Rough Sets," *International Journal of Information and Computer Sciences*, vol. 11, no. 5, pp. 341-356, 1982.
- [7] Z. Pawlak and R. Sowinski, "Rough Set Approach to Multi Attribute Decision Analysis," *European Journal of Operational Research*, vol. 72, no. 3, pp. 443-459, 1994.
- [8] F. Smarandache and J. Dezert, "Generalized Neutrosophic Logic: A New Approach to Uncertainty," *Journal of Uncertainty Analysis and Applications*, vol. 8, no. 2, pp. 1-12, 2020.
- [9] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, "Single Valued Neutrosophic Sets," *Multispace and Multistructure*, vol. 4, pp. 410-413, 2010.
- [10] Q. Wu, "Rough Set Approximations in Grey Information System," *Journal of Computational Information Systems*, vol. 6, no. 9, pp. 3057-3065, 2010.