Multiple Opinions in a Fuzzy Soft Expert Set and Their Application to Decision-Making Issues

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Abstract

Decision-making procedures frequently encounter disputes or contradictions between expert perspectives, and thus we need ways to resolve them. When working with many viewpoints in a decision-making environment, this method enables dynamic, changing input from different experts, which may be particularly helpful in real-world situations like expert systems and group decision-making. In this work, we provide multiple opinions in the fuzzy soft expert set and their application to decision-making issues (MO-FSES), which is an extension of the fuzzy soft set, and multiple opinions in time fuzzy soft expert set. The characteristics of its primary operations complement, union intersection, AND, and OR will also be defined and examined by me. Lastly, we will use this strategy for decision-making challenges.

Keywords: Soft set; Fuzzy soft set; Soft expert set; Fuzzy soft expert set; Multiple opinions

1 Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov[1]] introduced the notion of soft set theory as a tool in math for coping with such uncertainty. Following Molodtsov's work[2] and[3] researched several soft set operations and applications. Also[4] they presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji[5] also applied this idea to handle decision-making challenges. There are numerous important benefits to using several viewpoints from various experts or sources, particularly when there is uncertainty, complexity, or subjective judgment involved. We may discover the benefits of employing numerous viewpoints via professor reviews. For instance, by merging several expert perspectives, multiple opinions can assist eliminate bias, uncover mistakes, and boost decision accuracy[6] .By combining several viewpoints, fuzzy logic enables the handling of ambiguity and uncertainty, resulting in more complex and knowledgeable conclusions.[7] In order to ensure that the final conclusion represents a wider consensus, the decision-making process becomes more flexible and resilient against outliers.[8] It is simpler to handle ambiguous or subjective circumstances when expert viewpoints are combined to provide a more thorough grasp of difficult issues.[9] By weighing varying degrees of opinion certainty, fuzzy aggregation approaches aid in resolving expert disputes and fostering consensus.[10] Several points of view guarantee more equitable, moral, and balanced decision-making, particularly in domains such as the social sciences, healthcare, and law.[11 14] proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures. Wang[22]showed that in many real situations, immediate sensory data is insufficient for decision making. Additionally, researchers introduced using a neutrosophic fuzzy soft set to solve decision-making problems, like in,[13],[21] also[17] introduced the definition of Mapping on complex multi-fuzzy soft expert classes other researchers introduced topics in complex fuzzy as.[23 12] intrduced the definition of time fuzzy soft set investigate some of its features, and apply this notion to a decision-making problem and he show It is critical to understand the history of the parameters under consideration in order to ensure the

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credibility of the information provided by specialists. The experts' previous experiences are gathered in the number of periods (years, months, etc.) in which they are involved in a certain decision-making circumstance, and by looking at the time component, individuals are more confident in the conclusion that they make. We must examine the influence of time on fuzzy soft set applications, not only for the present period, but also for the past and future periods (forecasting information). Approaches for creating fuzzy equivalency relations in matrix form have been developed by recent research on fuzzy relations, such as the work of Shakhatreh and Qawasmeh.[16] These approaches are essential for applications in a variety of domains, including control systems and decision-making. The associativity of the max-min composition of three fuzzy relations[19] was also investigated, and the results shed important light on the characteristics of these compositions and their use in fuzzy logic. In this paper, we will present the notion of fuzzy soft expert set with multi opinions and time fuzzy soft expert set with multi opinions,Which is more performant and valuable, as we will see and the decisions made will be more precise, the information's component time value will be taken into account when we make decisions. We will also describe and analyze the essential operations that form the basis of this strategy: complement, union, and intersection. Finally, these techniques will be used to tackle decision-making difficulties. In this work, we provide a set of time intervals, generalize them into what we refer to as a Time-Fuzzy Soft Expert Set and Multiple Opinions in a Fuzzy Soft Expert Set (MO-FSES), explore some of their characteristics, and use this concept in a decision-making task.

2 Preliminaries

This section discusses a number of core ideas in soft set theory. Molodtsov[1] defined soft sets as follows over U: Let U represent the universe set and P the A collection of parameters. $P(U)$ signifies the power set of U and $J \subseteq P$.

Definition 2.1. [1] Think about this mapping

$$J : M \to P(U).$$

Any A pair (J, M) is considered a *soft set* over U. A parameterized collection of subsets of the universe set U is called a soft set over U. For $\delta \in J$, $J(\delta)$ may be thought of as the set of δ-approximate members of the soft set (J, M).

Definition 2.2. [4] Let P be the A collection of parameters and U be the initial universal set. Let the power set of all fuzzy subsets of U be I^U. Let F be the mapping

$$F : A \to I^U.$$

and $J \subseteq P$. A *fuzzy soft set* over U is a pair (J, P).

Definition 2.3. [4] Regarding two FSE (J, M) and (K, N) over U, (J, M) is known as a fuzzy soft subset of. (K, N) if

1. $M \subset N$ and
2. $\forall \delta \in J, J(\delta)$ is fuzzy subset of $K(\delta)$.

The association is represented by $(F,\ A)\, \tilde{\subset}\, (K, N)$. In this situation, (K, N) is known as a fuzzy soft superset of. (J, M).

Definition 2.4. [4] $(J, M)^c$ represents the complement of a FSE (J, M), which has been described by $(J, M)^c$ = $(J^c, \rceil A)$ where $J^c : \rceil A \to P(U)$ is a mapping provided by

$$J^c(\Gamma) = c(J(\rceil\Gamma)), \forall \Gamma \in \rceil M.$$

c describes any fuzzy complement.

Definition 2.5. [4] If (J, M) and (K, N) are two FSE then (J, M) AND (K, N) indicted by $(J,\ M) \wedge (K, N)$ is defined by

$$(J, M) \wedge (K, N) = (C, M \times N)$$

such that $C(\Gamma,\lambda) = t(J(\Gamma), K(\lambda)), \forall(\Gamma,\lambda) \in J \times N$,with t being any t-norm.

Definition 2.6. [4] If (J, M) and (K, N) are two FSE then (J, M) OR (K, N) indicted by $(J, M) \vee (K, N)$ is defined by

$$(J, M) \vee (K, N) = (O, M \times N)$$

such that $O(\Gamma,\lambda) = s(J(\Gamma), G(\lambda)), \forall(\Gamma,\lambda) \in M \times N$, Here, s stands for any s-norm.

Definition 2.7. [4] The union of two FSE (J, M) and (K, N) over a common universe U is the FSE (H, C) where $C = M \cup N$, and $\forall\delta \in C$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cup N. \end{cases}$$

Here, s stands for any s-norm.

Definition 2.8. [4] a FSE (J, M) and another FSE (K, N) intersecting over a common universe U is the fuzzy soft set (H, C) where $C = M \cup N$, and $\forall\delta \in R$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cap N. \end{cases}$$

Definition 2.9. .[14] Let U be a set of universes, P a A collection of parameters, X A team of specialists. Let $O = \{o_1, o_2, ..., o_n\}$ be A collection of viewpoints, $Z = E \times X \times O$ and $M \subseteq Z$. A pair $(F,\ M)$referred to be a soft expert set above U,in which F is a mapping provided by

$$F\ :\ A\ \rightarrow\ P(U)$$

where $P(U)$ denoted the power set of U.

Definition 2.10. .[15] Let U be a set of universes, P a A collection of parameters, X a set of experts. Let O be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$. A pair (J, M) referred to be a fuzzy soft expert set above U, where F is a mapping given by

$$J : M \rightarrow I^U$$

Which I^U provided fuzzy subsets of U.

Definition 2.11. [12] Let P be a collection of parameters and U be an initial universal set. Let the power set of all fuzzy subsets of U be represented by I^U. let $J \subseteq P$ and T represent a time period where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F, E)_t\, \forall\, t \in T$ is called a *time-fuzzy soft set* $T - FSS$ over U where J_t is a mapping given by

$$F_t : A \rightarrow I^U.$$

3 Multiple Opinions in a Fuzzy Soft Expert Set and Their Application to Decision-Making Issues (MO-FSES)

3.1 General Introduction

We discovered the advantages of using several perspectives in the decision-making problem based on the literature studies covered in the introduction. We also discovered that the fuzzy soft expert set is an appropriate mathematical tool for handling uncertainty and the fuzzy soft set model. The fact that the models are developed by several experts makes this notion significant. Enhancing this idea to make it more practical and efficient is our aim in this study. A group of experts and a generalized fuzzy soft expert with two viewpoints were examined by.[15] The set O is defined as follows: $O = \{1 = \text{agree}, 0 = \text{disagree}\}$. However, in certain situations, two viewpoints might not be enough to make a conclusion. In order to make accurate judgments and learn how to make better ones, we might need to make sure that the state has a variety of viewpoints. Taking a collection of opinions as a starting point, we generalize the idea of a fuzzy soft expert set with multiple opinions in the set O, where $O = \{a = \text{strongly agree}, b = \text{agree}, c = \text{disagree}, d = \text{strongly disagree}\}$, and describe how this idea is applied to the decision-making problem. By combining the fuzzy soft expert set with multiple opinions in this research, we introduce the concept of multi-opinions in a fuzzy soft expert set (MO-FSES) as a generalization. of fuzzy soft expert set. We shall also define its basic operations, complement, and union intersection. Also, we shall give hypothetical applications of this concept in decision-making problems.

3.2 Main Definition

Definition 3.1. . Let U represent a group of universes, E represent a collection of parameters, and X represent a group of experts. Let O be a collection of many viewpoints., $Z = E \times X \times O$ and $A \subseteq Z$. A pair $(F, A)^{MO}$ is called multiple opinions in a fuzzy soft expert set (MO-FSES) over U, where F is a mapping given by

$$F : A \to I^U$$

Where I^U indicates all fuzzy subsets of U.

Definition 3.2. . With regard to two MO-FSES $(F, A)^{MO}$ and $(G,\ B)^{MO}$ over U, $(F, A)^{MO}$ known as a MO-FSES. subset of $(G, B)^{MO}$ if

1. $A \subseteq\ B$,
2. $\forall \varepsilon \in B, G(\varepsilon)$ is fuzzy subset of $F(\varepsilon)$.

This relationship is shown by $(F, A)^{MO} \tilde{\subseteq} (G, B)^{MO}$. In this case $(G, B)^{MO}$ is called a fuzzy soft expert superset of $(F,\ A)$.

Definition 3.3. . Two MO-FSES $(F, A)^{MO}$ and $(G,\ B)^{MO}$ over U are considered equal if $(F, A)^{MO}$ is a soft expert subset of $(G, B)^{MO}$ and $(G, B)^{MO}$ is a soft expert subset of (F, A) .

Definition 3.4. . Accepted MO-FSES, indicted by, $(F, A)_{\mu^+}$ over U is a MO-FSES subset from $(F, A)^{MO}$ defined as follow:

$$(F, A)_{\mu^+} = \left\{F_{\mu^+}(\alpha) : \alpha \in E \times X \times \left\{\mu^{++}\right\}\right\},$$

and

$$(F, A)_{\mu^+} = \left\{F_{\mu^+}(\alpha) : \alpha \in E \times X \times \left\{\mu^{+-}\right\}\right\}.$$

Definition 3.5. . Rejected MO-FSES, indicted by, $(F, A)_{\mu^-}$ over U is a MO-FSES subset from $(F, A)^{MO}$ defined as follow:

$$(F, A)_{\mu^-} = \left\{F_{\mu^-}(\alpha) : \alpha \in E \times X \times \left\{\mu^{-+}\right\}\right\},$$

and

$$(F, A)_{\mu^-} = \left\{F_{\mu^-}(\alpha) : \alpha \in E \times X \times \left\{\mu^{--}\right\}\right\}.$$

Definition 3.6. . The complement of MO-FSES $(F, A)^{MO}$, indicated by $((F, A)^{MO})^c$, is defined by $((F, A)^{MO})^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping given by

$$F^c(\alpha) = c(F(\rceil\alpha)), \forall \alpha \subset \rceil A,$$

. The fuzzy complement is indicted by c.

Proposition 3.7. . *If $(F, A)^{MO}$ is MO-FSES over U, then*

1. $\left((F, A)^{MO})^c\right)^c = (F, A)^{MO}$,

Definition 3.8. . $(F,A)^{MO}$ and $(G,B)^{MO}$ over U, the union of two MO-FSES, is represented by $(F,A)^{MO}\,\tilde{\cup}\,(G,B)^{MO}$ is the MO-FSES $(H,C)^{MO}$ where $C = A \cup B$, and $\forall\, \varepsilon \subset C$,

$$H(\varepsilon)^{MO} = \begin{cases} F(\varepsilon), & if \varepsilon \subset A - B \\ G(\varepsilon), & if \varepsilon \subset B - A \\ s(F(\varepsilon), G(\varepsilon)), & if \varepsilon \subset A \cap\ B. \end{cases}$$

Proposition 3.9. . *If $(F,A)^{MO}$, $(G,B)^{MO}$ and $(H,C)^{MO}$ are three MO-FSES over U, then*

1. $(F,A)^{MO}\,\tilde{\cup}\,((G,B)^{MO}\,\tilde{\cup}\,(H,C))^{MO} = ((F,A)^{MO}\,\tilde{\cup}\,(G,B))^{MO}\,\tilde{\cup}\,(H,C)^{MO}$,
2. $(F,A)^{MO}\,\tilde{\cup}\,(F,A)^{MO} = (F,A)^{MO}$,

Definition 3.10. . The intersection of two MO-FSES$(F,A)^{MO}$ and $(G,B)^{MO}$ over U, indicted by $(F,A)^{MO}\,\tilde{\cup}\,(G,B)^{MO}$, is the fuzzy soft expert set $(H,C)^{MO}$ where $C = A \cup B$, and $\forall\, \varepsilon \subset C$,

$$H(\varepsilon)^{MO} = \begin{cases} F(\varepsilon), & if \varepsilon \subset A - B \\ G(\varepsilon), & if \varepsilon \subset B - A \\ t(F(\varepsilon), G(\varepsilon)), & if \varepsilon \subset A \cap\ B. \end{cases}$$

Proposition 3.11. . *If $(F,A)^{MO}$, $(G,B)^{MO}$ and $(H,C)^{MO}$ are three MO-FSES over U, then*

1. $(F,A)^{MO}\,\tilde{\cap}\,((G,B)^{MO}\,\tilde{\cap}\,(H,C))^{MO} = ((F,A)^{MO}\,\tilde{\cap}\,(G,B))\,\tilde{\cap}\,(H,C)^{MO}$,
2. $(F,A)^{MO}\,\tilde{\cap}\,(F,A)^{MO} = (F,A)^{MO}$,

Proposition 3.12. . *If $(F,A)^{MO}$, $(G,B)^{MO}$ and $(H,C)^{MO}$ are three MO-FSES over U, then*

1. $(F,A)^{MO}\,\tilde{\cup}\,((G,B)\,\tilde{\cap}\,(H,C)) = \left((F,A)^{MO}\,\tilde{\cup}\,(G,B)\right)\tilde{\cap}\left((F,A)^{MO}\,\tilde{\cup}\,(H,C)\right)$,
2. $(F,A)^{MO}\,\tilde{\cap}\,((G,B)\,\tilde{\cup}\,(H,C)) = \left((F,A)^{MO}\,\tilde{\cap}\,(G,B)\right)\tilde{\cup}\left((F,A)^{MO}\,\tilde{\cap}\,(H,C)\right)$.

Definition 3.13. . If $(F,A)^{MO}$ and $(G,B)^{MO}$ are two MO-FSES over U then "$(F,A)^{MO}$ AND $(G,B)^{MO}$", indicted by $(F,A) \wedge (G,B)^{MO}$, is defined by

$$(F,A) \wedge (G,B) = (H, A \times B)$$

where $H(\alpha,\beta) = t(F(\alpha), G(\beta)), \forall (\alpha,\beta) \in A \times B$.

Definition 3.14. . If $(F,A)^{MO}$ and $(G,B)^{MO}$ are two MO-FSES over U then "$(F,A)^{MO}$ OR $(G,B)^{MO}$", indicted by $(F,A) \vee (G,B)^{MO}$, is defined by

$$(F,A) \vee (G,B) = (H, A \times B)$$

where $H(\alpha,\beta) = s(F(\alpha), G(\beta)), \forall (\alpha,\beta) \in A \times B$.

3.3 An Algorithm

Shifting that MO-FSES to FSS is our goal, and we use the Roy and Maji algorithm (R&M) to determine the best course of action.
Note: Here, we state that the expression rejected (μ^{-}) implies disagree (μ^{-+}) and strongly disagree (μ^{--}), while accepted (μ^{+}) means strongly agree (μ^{++}) and agree and (μ^{+-}).
The steps of the Algorithm are as follow:

1. Input the FSESMO (F,Z).

2. Find the strongly agree opinions of FSESMO and agree opinions of FSESMO, Tables (1, 2).
3. Find the disagree opinions of FSESMO and strongly disagree opinions of FSESMO, Tables (4, 5).
4. Find the simplified accepted opinions of FSESMO, Table (3).
5. Find the simplified 1- rejected opinions of FSESMO, (Table 6).
6. Find (G, E) .In this case, $G(e)$ is defined as:

$$G(e) = \left\{ \frac{u}{\frac{1}{\|R\|} \sum_{p \in R} ((\mu^{++}(p) + \mu^{+-}(p)) + (1 - \mu^{-+}(p)) + (1 - \mu^{--}(p)))} : u \in U, e \in E \right\} \quad (1)$$

where $R = O \times X$, (Table 7).

7. Use the Roy & Maji's Algorithm on (G, E) since:
   - Create the fuzzy soft set's comparison table.(G, E) and calculate r_i and t_j for u_i,$\forall_i$, (Table 8). where r_i represent the row sum of an object u_i and is calculated by using the formula, $r_i = \sum_{j=1}^{n} c_{ij}$, (Table 9).
     Likewise, t_j represent the column sum of an object u_j and computed by using the formula, $t_j = \sum_{i=1}^{n} c_{ij}$, (Table 9).
   - Compute the score of u_i,$\forall_i$, where the score of an object u_i is s_i may be given as s_i=r_i-t_i, (Table 9).
   - The decision is s_k if, $s_k = max_i s_i$.
   - Any one of u_k can be selected if k has several values.

3.4 An application of multi-opinions in fuzzy soft expert set in decision-making

This section shows that the MO-FSES theory may be used fictitiously to a decision-making issue, proving that it is a viable approach. and it is applicable to issues in a wide range of uncertain fields. This enables us to make accurate judgments and develop decision-making skills and introduce an algorithm to convert the MO-FSES to FSS then find the decision. The application consists of a set of opinions $\{\text{strongly agree}, \text{agree}, \text{disagree}, \text{strongly disagree}\}$, we can see that giving more consideration to the expert opinions (multi opinions) will affect the result in decision making problems based on FSESMO.

Example 3.15. Now, let $U = \{u_1, u_2, u_3, u_4\}$ be a set of universe, let $E = \{e_1, e_2, e_3\}$ a A collection of parameters and let X be a set of experts (agents), and $O = \{a = \text{strongly agree}, b = \text{agree}, c = \text{disagree}, d = \text{strongly disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. μ be a fuzzy set of Z, i.e. $\mu : Z \to I = [0, 1]$

Define the function

$$F : A \to I^U \times I$$

as follows:

$F(e_1, m, a) = \left\{ \frac{u_1}{0.8}, \frac{u_2}{0.6}, \frac{u_3}{0.4}, \frac{u_4}{0.3} \right\}, F(e_1, n, a) = \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.7}, \frac{u_3}{0.2}, \frac{u_4}{0.4} \right\},$

$F(e_1, r, a) = \left\{ \frac{u_1}{0.7}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.2} \right\}, F(e_2, m, a) = \left\{ \frac{u_1}{0.2}, \frac{u_2}{0.4}, \frac{u_3}{0.7}, \frac{u_4}{0.5} \right\},$

$F(e_2, n, a) = \left\{ \frac{u_1}{0.4}, \frac{u_2}{0.3}, \frac{u_3}{0.6}, \frac{u_4}{0.6} \right\}, F(e_2, r, a) = \left\{ \frac{u_1}{0.1}, \frac{u_2}{0.2}, \frac{u_3}{0.9}, \frac{u_4}{0.7} \right\},$

$F(e_3, m, a) = \left\{ \frac{u_1}{0.2}, \frac{u_2}{0.7}, \frac{u_3}{0.4}, \frac{u_4}{0.8} \right\}, F(e_3, n, a) = \left\{ \frac{u_1}{0.3}, \frac{u_2}{0.5}, \frac{u_3}{0.2}, \frac{u_4}{0.6} \right\},$

$F(e_3, r, a) = \left\{ \frac{u_1}{0.1}, \frac{u_2}{0.9}, \frac{u_3}{0.5}, \frac{u_4}{0.9} \right\}, F(e_1, m, b) = \left\{ \frac{u_1}{0.9}, \frac{u_2}{0.7}, \frac{u_3}{0.6}, \frac{u_4}{0.4} \right\},$

$F(e_1,n,b)=\left\{\frac{u_1}{0.8},\frac{u_2}{0.9},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\}, F(e_1,r,b)=\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},$
$F(e_2,m,b)=\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\}, F(e_2,n,b)=\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},$
$F(e_2,r,b)=\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{1.0},\frac{u_4}{0.8}\right\}, F(e_3,m,b)=\left\{\frac{u_1}{0.3},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\},$
$F(e_3,n,b)=\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\}, F(e_3,r,b)=\left\{\frac{u_1}{0.3},\frac{u_2}{1.0},\frac{u_3}{0.5},\frac{u_4}{1.0}\right\},$
$F(e_1,m,c)=\left\{\frac{u_1}{0.5},\frac{u_2}{0.5},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\}, F(e_1,n,c)=\left\{\frac{u_1}{0.6},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},$
$F(e_1,r,c)=\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\}, F(e_2,m,c)=\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},$
$F(e_2,n,c)=\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\}, F(e_2,r,c)=\left\{\frac{u_1}{0.8},\frac{u_2}{0.7},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\},$
$F(e_3,m,c)=\left\{\frac{u_1}{0.7},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.1}\right\}, F(e_3,n,c)=\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.8},\frac{u_4}{0.4}\right\},$
$F(e_3,r,c)=\left\{\frac{u_1}{0.9},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.1}\right\}, F(e_1,m,d)=\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},$
$F(e_1,n,d)=\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\}, F(e_1,r,d)=\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.6}\right\},$
$F(e_2,m,d)=\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\}, F(e_2,n,d)=\left\{\frac{u_1}{0.2},\frac{u_2}{0.5},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\},$
$F(e_2,r,d)=\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.0},\frac{u_4}{0.1}\right\}, F(e_3,m,d)=\left\{\frac{u_1}{0.5},\frac{u_2}{0.0},\frac{u_3}{0.2},\frac{u_4}{0.0}\right\},$
$F(e_3,n,d)=\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.1}\right\}, F(e_3,r,d)=\left\{\frac{u_1}{0.7},\frac{u_2}{0.0},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\}.$

Then we can view the a MO-FSES (F,Z) as including the following set of estimates:

$$
\begin{aligned}
(F,Z)=\Big\{ & \left((e_1,m,a),\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right),\left((e_1,n,a),\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\}\right),\\
& \left((e_1,r,a),\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.2}\right\}\right),\left((e_2,m,a),\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.5}\right\}\right),\\
& \left((e_2,n,a),\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.6},\frac{u_4}{0.6}\right\}\right),\left((e_2,r,a),\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.9},\frac{u_4}{0.7}\right\}\right),\\
& \left((e_3,m,a),\left\{\frac{u_1}{0.2},\frac{u_2}{0.7},\frac{u_3}{0.4},\frac{u_4}{0.8}\right\}\right),\left((e_3,n,a),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.2},\frac{u_4}{0.6}\right\}\right),\\
& \left((e_3,r,a),\left\{\frac{u_1}{0.1},\frac{u_2}{0.9},\frac{u_3}{0.5},\frac{u_4}{0.9}\right\}\right),\left((e_1,m,b),\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\}\right),\\
& \left((e_1,n,b),\left\{\frac{u_1}{0.8},\frac{u_2}{0.9},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\}\right),\left((e_1,r,b),\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\}\right),\\
& \left((e_2,m,b),\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\}\right),\left((e_2,n,b),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right),\\
& \left((e_2,r,b),\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{1.0},\frac{u_4}{0.8}\right\}\right),\left((e_3,m,b),\left\{\frac{u_1}{0.3},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\}\right),\\
& \left((e_3,n,b),\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\}\right),\left((e_3,r,b),\left\{\frac{u_1}{0.3},\frac{u_2}{1.0},\frac{u_3}{0.5},\frac{u_4}{1.0}\right\}\right),\\
& \left((e_1,m,c),\left\{\frac{u_1}{0.5},\frac{u_2}{0.5},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\}\right),\left((e_1,n,c),\left\{\frac{u_1}{0.6},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),\\
& \left((e_1,r,c),\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\}\right),\left((e_2,m,c),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),\\
& \left((e_2,n,c),\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\}\right),\left((e_2,r,c),\left\{\frac{u_1}{0.8},\frac{u_2}{0.7},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\}\right),\\
& \left((e_3,m,c),\left\{\frac{u_1}{0.7},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.1}\right\}\right),\left((e_3,n,c),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.8},\frac{u_4}{0.4}\right\}\right),\\
& \left((e_3,r,c),\left\{\frac{u_1}{0.9},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.1}\right\}\right),\left((e_1,m,d),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),\\
& \left((e_1,n,d),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\}\right),\left((e_1,r,d),\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.6}\right\}\right),\\
& \left((e_2,m,d),\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\}\right),\left((e_2,n,d),\left\{\frac{u_1}{0.2},\frac{u_2}{0.5},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\}\right),\\
& \left((e_2,r,d),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.0},\frac{u_4}{0.1}\right\}\right),\left((e_3,m,d),\left\{\frac{u_1}{0.5},\frac{u_2}{0.0},\frac{u_3}{0.2},\frac{u_4}{0.0}\right\}\right),\\
& \left((e_3,n,d),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.1}\right\}\right),\left((e_3,r,d),\left\{\frac{u_1}{0.7},\frac{u_2}{0.0},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\}\right)\Big\}.
\end{aligned}
$$

Then we have the following results of strongly agree opinions of MO-FSES (Table 1).

Table 1: Strongly agree opinions of FSESMO

U	$\mu_{u_1}^{++}$	$\mu_{u_2}^{++}$	$\mu_{u_3}^{++}$	$\mu_{u_4}^{++}$
(e_1, m, a)	0.8	0.6	0.4	0.3
(e_1, n, a)	0.5	0.7	0.2	0.4
(e_1, r, a)	0.7	0.6	0.5	0.2
(e_2, m, a)	0.2	0.4	0.7	0.5
(e_2, n, a)	0.4	0.3	0.6	0.6
(e_2, r, a)	0.1	0.2	0.9	0.7
(e_3, m, a)	0.2	0.7	0.4	0.8
(e_3, n, a)	0.3	0.5	0.2	0.6
(e_3, r, a)	0.1	0.9	0.5	0.9

Then we have the following results of agree opinions of MO-FSES (Table 2).

Table 2: Agree opinions of FSESMO

U	$\mu_{u_1}^{+-}$	$\mu_{u_2}^{+-}$	$\mu_{u_3}^{+-}$	$\mu_{u_4}^{+-}$
(e_1, m, b)	0.9	0.7	0.6	0.4
(e_1, n, b)	0.8	0.9	0.3	0.5
(e_1, r, b)	0.7	0.6	0.6	0.4
(e_2, m, b)	0.4	0.5	0.8	0.6
(e_2, n, b)	0.6	0.4	0.7	0.6
(e_2, r, b)	0.2	0.3	1.0	0.8
(e_3, m, b)	0.3	0.8	0.6	0.9
(e_3, n, b)	0.5	0.7	0.4	0.7
(e_3, r, b)	0.3	1.0	0.5	1.0

Next, we simplified the accepted opinions of FSESMO by computing c_j for u_i,$\forall_i$, where c_j represent the column sum of an object u_i and is calculated by using the formula, $c_j = \sum_{i=1}^{6} \mu_{u_i}^{+}$ (Table 3).

Table 3: Simplified accepted opinions of FSESMO

U	$\sum \mu_{u_1}^{+}$	$\sum \mu_{u_2}^{+}$	$\sum \mu_{u_3}^{+}$	$\sum \mu_{u_4}^{+}$
$(e_1, X, +)$	4.4	4.1	2.6	2.2
$(e_2, X, +)$	1.9	2.1	4.7	3.8
$(e_3, X, +)$	1.7	4.6	2.6	4.9

And we have the following results of 1-Disagree opinions of MO-FSES, (Table 4).

Table 4: 1-Disagree opinions of FSESMO

U	$1 - \mu_{u_1}^{-+}$	$1 - \mu_{u_2}^{-+}$	$1 - \mu_{u_3}^{-+}$	$1 - \mu_{u_4}^{-+}$
(e_1, m, c)	0.5	0.5	0.2	0.1
(e_1, n, c)	0.4	0.5	0.3	0.2
(e_1, r, c)	0.7	0.6	0.5	0.3
(e_2, m, c)	0.3	0.2	0.7	0.6
(e_2, n, c)	0.6	0.4	0.7	0.5
(e_2, r, c)	0.2	0.3	0.9	0.8
(e_3, m, c)	0.3	0.8	0.6	0.9
(e_3, n, c)	0.4	0.6	0.2	0.6
(e_3, r, c)	0.1	0.8	0.6	0.9

And we have the following results of 1-Strongly disagree opinions of MO-FSES, (Table 5).

Table 5: 1-Strongly disagree of FSESMO

U	$1-\mu_{u_1}^{--}$	$1-\mu_{u_2}^{--}$	$1-\mu_{u_3}^{--}$	$1-\mu_{u_4}^{--}$
(e_1, m, d)	0.7	0.5	0.3	0.2
(e_1, n, d)	0.6	0.8	0.4	0.6
(e_1, r, d)	0.8	0.7	0.6	0.4
(e_2, m, d)	0.4	0.3	0.9	0.8
(e_2, n, d)	0.8	0.5	0.9	0.8
(e_2, r, d)	0.3	0.2	1.0	0.9
(e_3, m, d)	0.5	1.0	0.8	1.0
(e_3, n, d)	0.6	0.8	0.4	0.9
(e_3, r, d)	0.3	1.0	0.7	0.9

Likewise, we simplified 1-rejected opinions of FSESMO by computing k_j for u_i,$\forall_i$, where k_j represent the column sum of an object u_i and is calculated by using the formula, $k_j = \sum_{i=1}^{6}(1-\mu_{u_i}^{-})$, (Table 6).

Table 6: Simplified 1-rejected opinions of FSES

U	$\sum(1-\mu_{u_1}^{-})$	$\sum(1-\mu_{u_2}^{-})$	$\sum(1-\mu_{u_3}^{-})$	$\sum(1-\mu_{u_4}^{-})$
$(e_1, X, -)$	3.7	3.6	2.3	1.8
$(e_2, X, -)$	2.6	1.9	5.1	4.4
$(e_3, X, -)$	2.2	5.0	3.3	5.2

Next, we compute the (G, E) to convert the MO-FSES to a FSS using relation (1). We compute $G(e_1)$ for u_1 as shown below to demonstrate this step.

$$G(e_1) = \left\{ \frac{u_1}{\frac{1}{12}\sum_{p\in R}((\mu^{+}(p)) + (1-\mu^{-}(p)))} : u \in U, e \in E \right\}$$

$$= \left\{ \frac{u_1}{\frac{1}{12}(4.4+3.7)} \right\}$$

$$= \left\{ \frac{u_1}{\frac{1}{12}(8.1)} \right\}$$

$$= \frac{u_1}{0.675}$$

A similar method may be used to convert u_i with all parameters. Table 7 displays the conversion results.

Table 7: Converting FSSMO to FSS

U	u_1	u_2	u_3	u_4
$G(e_1)$	0.675	0.641	0.408	0.333
$G(e_2)$	0.375	0.333	0.816	0.683
$G(e_3)$	0.325	0.800	0.491	0.841

Once the fuzzy soft expert with multiple points of view has been transformed to a fuzzy soft set, we use the R&M method to create a comparison table and calculate the score value in order to choose the best choice. According to the data contained in table 7, u_1 and u_2 equal 2. because two membership value of u_1 exceeds the membership value of $u_2(0.675 > 0.641$ and $0.375 > 0.333)$ and the comparison for u_1 with u_3 equal 1 because one membership value of u_1 exceeds the membership value of u_3 , however, in contrasting the membership values of u_1 with u_4 we get $u_1 = 1$ because there are one value for u_1 exceeds the membership

value of $u_4(0.675 > 0.333)$.
Table 8 is the comparison table of the MO-FSES that was produced above.

Table 8: Comparison table of FSES

U	u_1	u_2	u_3	u_4
u_1	3	2	1	1
u_2	1	3	2	1
u_3	2	1	3	2
u_4	2	2	1	3

We now compute the row-sum (r_i), column-sum (t_j), and score (S_i) for each u_i as shown below:

Table 9: Score table of FSS

U	r_i	t_j	S_i
u_1	7	8	-1
u_2	7	8	-1
u_3	8	7	1
u_4	8	7	1

The choice is in favor of choosing u_3 or u_4, as can be seen from the score table above, which shows that the maximum score is 1.

4 Conclusion

In this research we have introduced the concept of fuzzy soft expert set with multi opinions, we give hypothetical application of this concepts in decision making problem has been shown and we researched some of its characteristics. The operations of complement, union, and intersection have been established.. A hypothetical applications of this theory in solving a decision making problems was given.

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