Data-Driven Pricing Decisions for Ensuring the Success of Strategic Product Development

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Abstract

By effectively including diffusion into the framework, this study further illustrates whether Optimal Control Theory may be used to identify and address the control of prices issue of technology items. There is a three-stage paradigm that it uses to describe the procedure of adoption process: consciousness, inspiration, and adopting itself. The process is described by the diffusion logistic functions; furthermore, the model takes into account price fluctuations' sensitivity. The selling price is a choice variable constrained such that the total profit over the relevant planning horizon is optimised. In this paper, the Hamiltonian function is used in obtaining necessary optimality conditions with spectral learning effects and Costate equations complemented by the adoption rates by use of Pontryagin's Maximum Principle. As the problem is formulated as the continuous optimization problem, it is discretized for its practical applications, and the model is solved with the help of LINGO 15.0 software. The data used to validate the model implemented was derived from historical sales records of the electronics and semiconductor industries to obtain a measure of realism. Analysed sensitivity studies show how variations in adoption parameters including the price elasticity and customer attrition affect adoption rates and profitability. As such, the study offers managerial implications for the management of private sector schemes to focus on the application of dynamic pricing strategies as the optimal balance between consumers' perceived value and firm revenues. It provides managers with strong tools for the implementation of adoption into a new generation of technology-enabled markets, maximization of revenues, and sustaining of competitive advantage. Outperforming all analysed models, the suggested technique employing Optimal Control Theory obtains an accuracy of 96%. This proves that the suggested strategy is the best at forecasting when a product will be adopted.

Keywords: LINGO; SVM; LR; KNN; RF; Optimal Control Theory

1. Introduction

One of the most effective mathematics frameworks, Optimal Control Theory is used to ascertain the most effective method for directing an unstable system over a period [1]. From the Calculus of Modifications that concentrates on optimising functions by establishing the circumstances during where they acquire extremum standards it arose as a component of the Calculus of Variability. By handling changing systems that are controlled by mathematical formulas, Optimal Control Theory can optimise trajectory or choices that develop over a specific time range. This contrasts with static optimisation, which only manages static systems. Several fields, including economics, finance, biology, and business research are all affected by its significance. Optimal Control Theory might be marked as originated approximately in the middle of the 20th century and recognized as an important constituent of modern theories of control. It was a race for space, in which scientists sought to provide the optimal course charts for spaceships that moved the theory forward. The theory mostly leaned on the elementary instruments provided by the publication of Richard Bellman on dynamism and Lev Pontryagin on Maximal Principles particularly [2]. The

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approach employed by Bellman was to divide the problem into smaller related issues that can be solved easily the idea by Pontryagin provided set standards that ensured that the control techniques developed were as optimal as possible.

Another feature of Optimal Control Theory that can be underlined is the ability to model systems that develop using state factors and control parameters. Control parameters are indications of force, strength or price of input or choice while state variables are indications of the growing conditions of the structure. As concerns state parameters, examples include place, speed, and heat [3]. Optimal Control Theory provides means for constructing objectives of function that characterizes the performance indicators of an expense, time, or profit. Such a situation is achieved by coupling differential equations expressing dynamics of the whole system. It is always believed that an optimum control issue is made up of three parts in its theoretical definition. These are a conditional the formula, a price function and two simple constraints. Analogous to conventional differential equations, the state equation sets out how the whole thing behaves. The cost function is used to assess an efficiency of a definite method with applying integration with the time horizon. The governments and the agencies can be placed in circumstances based on equity and disparity, which are the manifestation of the material, the financial or the technological constraints.

Optimal Control Theory has a great influence on a view of technology such as Robotics, Aircraft, and Automobile [4]. It is applied in robotics for instance to develop proper paths for the robots that are supposed to perform activities like welder or paint. An important function within the Aerospace Industry is to effectively control the flight paths of aeroplanes and spaceships and the amount of fuel they consume. Similarly, it rains in new trends in automated automobiles in automotive technology that enhances the scheduling of appropriate courses and uses of power. Of all theoretical approaches, the Optimal Control Theory is particularly significant in deciding about allocation of resources, manufacturing schedules and pricing strategies. It is used for getting most of the money, either for revenues or for expenses, in which businesses use to determine how to best allocate the resources such as labor, capital etc over a period. Besides, in the macroeconomic guidelines modeling, elements of the framework are applied, including fiscal and quantitative ones aimed at preserving industrial growth and inflation rate. This work also defines the functions operation of Hamiltonian used to link the cost functional and the fluctuations of a given system [4]. It may be possible to build control techniques, which either make use of the goal variable, or keep it to the lowest level, through an assessment of the dynamics of the Hamiltonian theory.

Moreover, the theory enables making additions to costate variables that can be described as Lagrange multiplier coefficients in relation to the shifts in requirements and which contain information about the prices of state factors that are not directly attributable. Since then, with the appearance of the Dynamic Programming by Richard E. Bellman Optimal Control Theory was supplemented with another valuable tool [5]. The optimisation concept according to which the smaller issues of an optimal approach are really optimum in and of one another is used by this technique. Because of the backward solution from the last attempt, determining the function values at each step, African Dynamics comes up with improved control tactics to respond to the challenges for businesses. It is quite useful in cases with discrete-time methods network and circumstances that has complicating restrictions.

It has also increased its usage in Health care and biological structures by the help of Optimal Control Theory. This is particularly so as far as passing on viral diseases, and formulating immunisation plans that possibly place little pressure on the public's health care system is concerned. In addition, it aids in enhancing the effectiveness of therapy for chronic conditions like cancer or bouts of insulin resistance; an essential element in guaranteeing positive effects while, at the same time, reducing detrimental impacts. More recent advances in computing techniques have led to a major expansion of the use of Optimal Control Theory. Some examples of quantitative solvers are; Gradient Descent Method and Sequential Quadratic Programming. These solvers enable easy identification to solution for problems like those stated in Mathematics for instance MATLAB, GAMS and LINGO are examples of software programmes that contain strong groundwork ideal for successful carrying out of and analysing optimum control issues [6]. In fields such as solar energy where optimum control is applied in either electricity storage operation or in maintaining stability of the electrical grid, these technologies are very useful.

Challenges arise from Optimal Control Theory bearing in mind that theory has many advantages. Whenever the motions of the structure are other than linear or contain elements of unpredictability, the problem of dealing with multi-dimensional issues mathematically is one of the major concerns that must be singled out as an issue of major difficulty [7]. For cases such as these, the solutions can be approximated using approximation approaches like Model Predictive Control (MPC) or strengthened learning. The immediacy of data ensures continuous adjustments that ensure workability in unpredictable circumstances creating validity for the strategy.

In the final analysis, Optimal Control Theory is an essential part of analysis that serves to optimise the peculiarity of structures in the given field of study. The versatility and importance of the technology in question is indicated by its applications area, which can span from engineering and finance to medicine and environment [8]. Even

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today, the idea still acts as a key factor in the creation of technology because it allows the decision-makers to achieve their desired end in the right manner. The continued development of computer instruments and interdisciplinary studies that ensure that Optimal Control Theory will continue to be a valuable tool for solving difficult problems in the environment that is becoming increasingly dynamic.

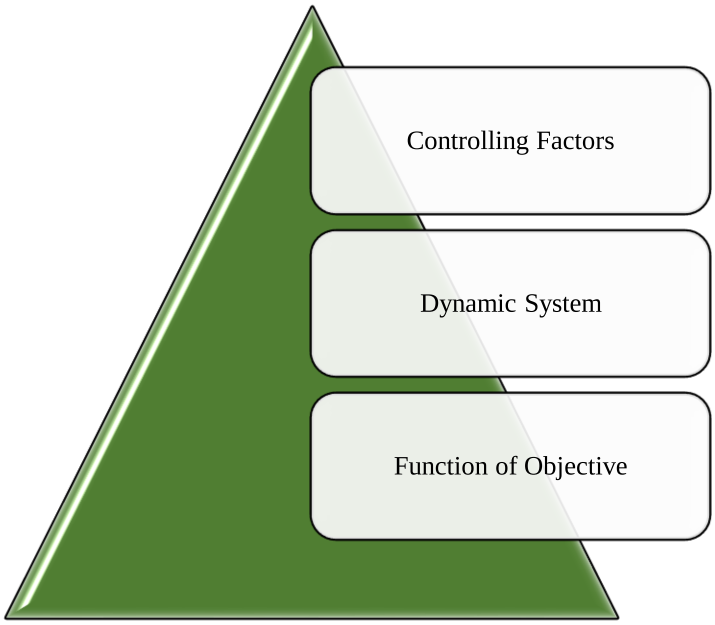

Figure 1. Dynamical Systems Optimisation Flow Diagram

The optimal control theory is shown in a visual format by the block schematic, which shows the fundamental elements and flow of the concept. The control parameters block, which is located on the left side of the screen, will serve as the starting point for the procedure [9]. Examples of deciding parameters are force within a system that moves as well as price within a simulation of economics. These data inputs are used to make decisions. In order to obtain the results that are wanted, they are modified and have direct impact on the present condition of the framework. This block is connected to the centre block by arrows, which illustrate when these signals cause alterations to the functioning of the system.

The block that reflects the behavioural patterns the system exhibits over time is called the dynamics of the system block, and it is located in the middle of the diagram. This block is identified by state parameters such as location, the speed, or state of energy [10]. Calculated differential equations that are guided by controlled inputs bring about the evolution of those parameters. The centre of the changing structure is reflected in this block, which is where the changes are defined by the rules of the materialistic, economical, or technological world. To indicate how the shifting circumstances have an effect on the general outcome or objectives of the whole thing, arrows pointing from the whole system dynamical section to the opposite side are shown.

The last step in the optimisation process is the goal function block, which is located on the right side of the diagram. The performance of the system is evaluated based on certain criteria, such as minimising costs or maximising efficiency, both of which are mentioned [11]. Other annotations, such as restrictions and optimisation strategies, bring attention to important aspects and approaches that are used to control inputs and arrive at the best possible solution. Methods of optimisation, such as Pontryagin's maximal principle, direct decision-making to ensure that it is both efficient and practicable. Constraints, on the other hand, place restrictions on the solutions that are plausible. These components, when combined, create an integrated structure into which optimum control theory can be understood and implemented.

2. Existing Work Done

The branch of mathematics called Optimal Control Theory is focused on defining a control procedure for a constantly changing environment to achieve the objective that fulfils the specified criteria. These is the mathematical models seeking the theory that define the options which varies over time. It is applicable in large flexibility in numerous domains such as financial, economic, biology, and others because processes evolve with time under the influence of control factors [12]. This is actually a new methodology, despite the fact that it has its

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basis in classical optimisation approaches, and it has evolved dramatically since it was initially conceived. Optimal Control Theory could even be extended back around Calculus of Variations category, which is a portion of computational mathematics that is concerned mainly with optimization of functions. The scholar and others did extend it to so-called fluid systems from the range of possible applications of the Calculus of Variability. Among the utilization developments is the Maximum Principle presented by the researcher first decode PMP. This concept, which provides the required circumstances for an optimum control issue, continues to be very relevant. A fundamental component of his theory is a collection of differential equations and constraints that may be applied in the process of determining the best possible solution regarding control issues.

Perhaps the first uses of optimal oversight were in the aviation sector, wherein researchers were tasked with designing the most efficient flight routes for spacecraft and aircraft. The origin of the broad implementation of the concepts in the field of engineering may be traced back to the utilisation of the Calculus of Variations and optimum management of real-world systems such as those. Optimal control theory continues to rely heavily on the Maximum Principle as one of its foundational principles [13]. The researcher's Optimum Principles offers both a conceptual structure and useful tools for resolving control issues. It does this by providing the requirements that must be met for a control approach to be considered optimum. In order to do this, a Hamiltonian function must be defined. This function must integrate the fluctuations of the system with the goal function that must be maximised or minimised.

The Hamiltonian is a significant idea given that it incorporates both the controller inputs and the status parameters. This ensures that the variables that govern at every single step are selected in a manner that maximises the overall aim. There are additional costate factors that are included into the Optimal Principle. These costate variables are added as Lagrange multiplies in order to impose the limitations of the system. The investigator was pioneering in the area of dynamic programming (DP), which developed as another effective instrument for addressing optimum control issues. This discipline arose concurrently with the investigator's work [14]. According to the investigator's Principles of Maximum efficiency, an optimum choice at any step of an operation ought to put into consideration all possible options for following stages. This allows the issue to be broken down into more manageable subdivisions that may be handled in a recursive manner. When dealing with issues in which the period of time range becomes discontinuous and the status and control elements are susceptible to uncertainty, dynamism is an especially beneficial technique.

Modern Dynamic Programming Algorithmic methods such as value repetition for policy repetition were developed as a result of the work done by the investigator. These approaches handle optimum control issues by addressing the Bellman equations. Up to the point when converge is reached, these techniques perform repeated updates on valued variables [15]. In situations when the issue is too complicated for analytical approaches to handle, such as in probabilistic control structures and when dealing with non-linear factors, DP serves as an optimal solution. Immediately after the presentation of the researcher's greatest concept, a number of researchers tried to generalise the concept in order to apply it to a wider range of situations. Probabilistic mechanisms have been included into Optimal Control Theory, which is a significant expansion of the theory. Although majorities of the structures that exist in everyday life are not completely predictable, the incorporation of randomisation enables the optimisation of processes that are subject to unpredictability.

The Optimal Principles of the researcher is extended to systems in which the dynamics are impacted by unplanned events via the use of stochastic optimal control (SOC) models. Notable examples of such models include those developed by the investigator and others. In industries such as financing, wherein stock values fluctuate in response to randomised events, and robots, where modelling is necessary to take into account variability in data from sensors and actuation noise, models as if these are crucial. When it comes to implementing Optimal Control Theory, one of the major obstacles is the mathematical difficulty of solving multidimensional networks [16]. This is particularly true in situations where the dynamics of the system are nonlinear or contain a large number of state controls and factors that communicate with one another. Several computational methods have been created developed in order to solve this issue. Automatic Collocations approaches, Sequential Quadratic Programming (SQP), and Gradient-Based Optimisation have all made major improvements to this field. After discretizing the optimum control problem across the time horizon, straight geolocation techniques proceed to solve the resultant arrangement of algebraic formulas by using irregular solvers. This strategy is very useful for tackling non-linear challenges, which are situations in which other approaches could prove to be as successful.

The Optimal Control Theory has been shown to have important applicability in a variety of professional domains, especially in networks wherein operations shift over time and management choices require to be implemented in an evolving way. In the field of aerospace technology, the method was first used for optimising flight paths for

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spacecraft's trajectory [17]. By taking into account the forces of gravity and the amount of fuel that was used, the researcher demonstrated that the Highest Principles could be used to predict the most efficient course for the spaceship. Throughout the course of time, the concept was expanded to include a variety of technological fields, such as mechanics, electronics, and robots, where it was first used. Automating tasks requiring computers to discover the most efficient paths while considering obstacles, consumption of electricity, and other factors—such as movement planning—has proven to be an essential part of optimal control. The Optimal Control Theory is utilised for electricity systems in the electrical engineering sector with the goals of distributing loads and effectiveness optimising. Simulations were developed for the purpose of the investigation for optimising controlling choices related to the operation of switches, electrical lines, and conversions [18]. In this way, needs may be met at the lowest feasible cost. The researcher's methods were utilised as well in the development of optimal controls for vehicle technology. In the realm of vehicle autonomous navigation, where optimising safety, fuel efficiency, and travel time necessitates making control decisions in real-time, this is particularly true.

Some of the most notable and significant applications of Optimal Control Theory is in the domains of finances and economics [19]. The researcher and others looked into the potential applications of Optimal Management to model the global economy. Actions, such as managing economies and regulating inflationary. The goal of this debate is to find the optimal strategy that, over a period, minimises the negative effects of unemployment, inflation, and the national debt overall the marketplace's financial stability. As an illustration, the researcher showed ways governments may stability the financial sector over an additional period by adjusting spending, taxation, and loan rates. When it comes to managing investment portfolios, the Optimal Control Theory has been put to good use in the financial sector. This is especially true when it comes to limiting risk and maximising earnings under uncertain circumstances. The researcher constructed investment-optimising algorithms using these approaches [20]. One component of these theoretical frameworks is the gradual reallocation of commodity like stocks, bonds, or real estate with the goal of maximising revenue while mitigating risks associated with market fluctuations. Optimal Control Theory provides a framework for managing long-term investing plans, hedge plans, and risk management, making its application crucial in modern finance.

The area of biological structures and medicinal application has also experienced a great growth for research dedicated to Optimal Control Theory. Researchers in the area of epidemic research utilised the best management to model the emergence of infectious illnesses such as HIV, malaria, or tuberculosis and to determine the most effective vaccination schedules. Using the most effective control and coefficients of variability, the researcher was able to simulate the progression of the disease as an ecosystem and determine the best vaccination method [21]. Taking into account the easily accessible features and the limits, the researcher was able to reduce the total amount of injuries and fatalities. The area of personalised medicine is now making use of the Optimal Control Theory to plan the administration of drugs, such as radiation or insulin, in order to manage diabetes and associated complications. The investigator-devised models in which control inputs (such as the dose of a medicine or the injection of insulin) are modified over time in order to maximise the effectiveness of therapy while simultaneously minimising adverse effects. These simulations are especially useful in situations in which the patient's condition changes during the course of care and therapy has to be proactively adapted in order to achieve the best possible long-lasting medical results.

Although it has a broad range of programmes, Optimal Control Theory is confronted with a number of difficulties. The computational complexity that is required in addressing large-scale systems that change is one of the most significant challenges. This is particularly true in situations where the functions of the entire system become highly unpredictable or include a huge number of variables that interact with one another [22]. To provide an example, the optimisation issue may be analytically laborious and expensive with robot & aerospace projects since the systems involved might comprise hundreds of state and operational parameters. The development of approximation approaches and heuristic algorithms has been done for the purpose to overcome this issue. Current systems are being used to estimate the best solution via the use of methods such as Model Predictive Control (MPC), which includes solving a succession of optimising issues over a limited horizon. Reinforcement learning (RL) is an additional method that facilitates the acquisition of regulations via the process of trial and error. In recent years, reinforcement learning has grown in popularity because to its ability to optimise networks despite necessitating an exact model of the system. This enables it to be used in scenarios in which modelling are either difficult to get or not accessible [23]. There is a good chance that developments in machine learning, AI, and data processing in real-time will have a significant impact on the path that Optimal Control Theory will take in the future. These developments will make it possible to implement optimisation strategies that are both more advanced and efficacious. Furthermore, as the sector develops, there shall be a larger emphasis placed on stochastic optimisation approaches to deal with ambiguities in applications running in real time [24-25]. This is especially true in fields such as auto autonomy, banking, and medicine.

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3. Objective of the research work

- Improving the long-term efficiency of complex structures requires the creation of computational models and methods.
- With certain limitations in mind, develop control techniques that aim to maximise or minimise a predetermined objective function, such as expenditure, effectiveness, or profitability.
- In order to tackle contemporary management system problems, it is necessary to study and expand upon traditional ideas like flexible programming and Pontryagin's Minimum Theorem.

4. Motivation for the research work

- When dealing with complicated dynamical optimisation issues, real-world systems often turn to Optimal Control Theory for a theoretical foundation.
- The significance of this study is driven by the necessity for effective decision-making in ever-changing domains including the aerospace sector, robots, and financial.
- To accomplish goals like maximising performance or minimising costs, the methodology allows for the modelling and optimisation of systems that evolve gradually.

5. The Projected method

A framework for the spread of innovations and their impact on sales growth is established in this section. Incorporating a dynamic study of the product's pricing in influencing customer purchase choices, its adoption procedure is conceptualised as a three-stage integrative procedure. Present modelling approach additionally incorporates logistic training curves for product acceptance rate. It is also assumed that the corporation has already decided on its intended demographic and is planning to use the best pricing method to maximise income. We have included the influence of the cost training curve, which varies as innovative sales do. Utilising the progressive implementation of Pontryagin's maximal concept in marketing materials, the optimal pricing approach is derived. Through the use of the phase framework, the procedure for diffusion has been modelled. Similar to the integral diffusion model that has been developed, a potential customer first becomes aware of the product, followed by the accurate person becomes attracted about it and creates a desire to purchase it following being happy with the retail cost of that item. When it comes to the acceptance of the inventiveness, the most significant factor is the recognition of the invention. Therefore, the first step of the method of diffusion is the dissemination of awareness, which is the stage in which prospective buyers are educated regarding the presence of the invention and the features that it has. It is possible to express the computational meaning behind the transmission of data as follows:

$$\frac{d}{dt}M_k(v) = c(n - M_k(v)) \qquad (1)$$

where c is the percentage of people that are aware of the product. Equation (1) explains the immediate pace through which the prospective market is becoming educated, and it is proportionate to the quantity of prospective consumers who are still uninformed of the information already available. When the equation (1) is solved via the starting constraint M (0) 0, the analytical outcome that follows is generated:

$$M_k(v) = n(1 - f^{-cv}) \qquad (2)$$

Computed at time t, the aggregate number of aware customers is given by Equation (2).There is a significant relationship between the retail price of the product and the desires of the client to purchase it. Undoubtedly, among the many important aspects of advertisement is the pricing of the product or service. In light of this, after a person has a thorough understanding of the product, they will be more likely to accept it, but only after, they have considered the cost of the product. Due to the fact, consumers would rather pay an affordable rate than a greater price; the reliability that consumers associate with the price is clearly dropping. Therefore, the value of the product is a significant factor in determining whether an individual is motivated to acquire innovation. The total number of prospective adopters who are motivated to make a purchase is thus stated as:

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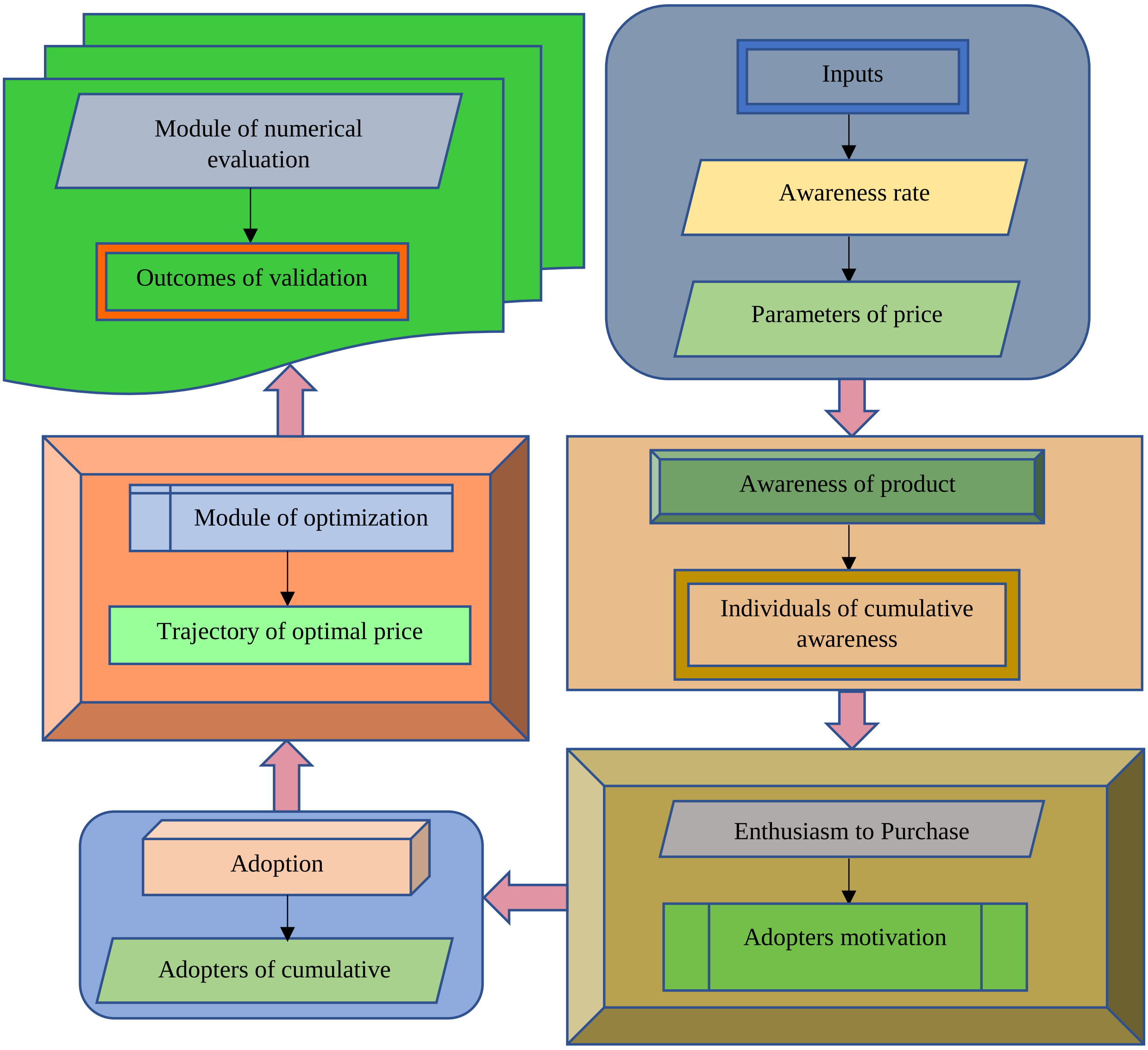

Figure 2. Architecture of proposed method

$$M_2(v) = M_2(v)\varphi(v) \quad (3)$$

The price function, denoted by $\varphi(v)$, is characterised by its subsequent negatively exponential structure, which is as follows:

$$\varphi(v) = f^{-\theta Q(v)} \quad (4)$$

where the variable parameter that represents the price component is denoted by the symbol θ. The total number of enthusiastic adopters may be calculated using formula (5), which is produced by putting the expressions of (2) and (4) into a formula (3).

$$M_2(v) = n(1 - f^{-cv})f^{-\theta Q(v)} \quad (5)$$

The inspired individual is encouraged to proceed with the purchase and ultimately becomes the adopter of the technology when they have obtained certainty regarding the pricing of the item being bought. Additionally, it is being noticed that as the process of diffusion continues, the scope of the market expands in tandem and the

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increasing quantity of purchasers. Consequently, the level of adoption is believed to be a representation of the logistical character, which is to express.

$$c(v) = \frac{c}{1+\beta f^{-cv}} \quad (6)$$

Consequently, the request function's differential formulation is expressed as:

$$\frac{d}{dt} M_3(v) = \frac{c}{1+\beta f^{-cv}} [M_2(v) - M_3(v)] \quad (7)$$

The subsequent equation for the selling rate is obtained by putting formulas (5) and (6) into formula (7):

$$M_3(v) = \frac{c}{1+\beta f^{-cv}} [n(1 - f^{-cv}) f^{-\theta Q(v)} - M_3(v)] \quad (8)$$

The innovation's end users are shown in Equation (8). The impact of pricing on the demand for a product and its effect on the purchasing behaviour of adoption has been the subject of study conducted by a number of scholars over the years. Within the scope of the present investigation, the impact of price fluctuation on the popularity of an item is taken into consideration. Based on the assumption that customer views on their acceptance of the item may be changed to some degree by managing the cost of the item, it is considered that this is possible. Taking into consideration this philosophy, an optimisation model is constructed from the point of view of the producer in order to establish the best pricing of the item, which ultimately results in the greatest possible number of sales of the good or service. Neither the expense nor the price are considered to be dynamic factors in this particular research. Additionally, the expenditure of training phenomena is integrated by taking into account the marginal price that is based on sales and lowers as the quantity of adoption increases, i.e.

$$D(v) - D\big(M_3(v)\big), \frac{eD\big(M_3(v)\big)}{eM_3(v)} = D(M_3(v)) \leq 0 \quad (9)$$

Additionally, it presumes that the maker wishes to maximise the overall value of its profits by regulating the selling price of the product throughout the course of the specified planned period 'V' at a given discounted rate s. Structured as an optimum control approach, the current issue of profit maximisation is presented here. When it comes to the issue that has been suggested, the selling price Q(v) is regarded to be an indicator variable that has to be optimised in order to manage the total amount of sales of the product, $M_3(v)$ which is further referred to as the systems (or state) variables. The purpose of the optimisation model is to maximise the overall earnings of the company while taking into account the constraints of the rate of sale. It is possible to organise the optimum control issue for maximising the benefit function as follows:

$$y(v) = M_3(v) = \frac{c}{1+\beta f^{-cv}} \{n(1 - f^{-cv}) f^{-\theta Q(v)} - M_3(v)\} \quad (10)$$

The link that exists among the control parameter, $Q(v)$, and the current state factor, $M_3(v)$ (, is characterised by the limitation of adopting rate, which can be found in equation (10). The primary purpose y is doubly differentiable with regard to price, and it fulfils the relationships that are listed below:

$$y_q > 0 y_{qq} < 0 \quad (11)$$

Where $y_q = \frac{\partial y(v)}{\partial Q(v)}$ and $y_{qq} = \frac{\partial^2 y(v)}{\partial Q(v)^2}$ (12)

With Pontryagin's Optimal principle, the vital requirement of optimum theory of control is utilised in order to find a solution to the management issue that has been presented. In the first step of the process, the restricted issue is transformed into an unrestricted the formula, and the calculation that is produced as a result is referred to as the Hamiltonian value for short (G). The formula that follows is characteristic of the current-value Hamilton for the issue that has been presented:

$$G = \big\{\big(Q(v) - D(v)\big) + \mu(v)\big\} M_3(v) \quad (13)$$

Where $\mu(v)$ represents the current-value costate variables that exhibits minimal fluctuation in the desired value for an adjustment in the state variables that is relatively small, and $M_3(v)$ is the number of times the state variables are changed.

$$\frac{d}{dt}\mu(v) = \mu(v) = s\mu - \frac{\partial G}{\partial M_3(v)} = 0 \quad (14)$$

$$\mu(v) = s\mu = \{(Q(v) - D(v) + \mu)\} \frac{\partial y(v)}{\partial M_3(v)} + \frac{\partial D(v)}{\partial M_3(v)} y(v) \quad (15)$$

$$\mu(V) = 0 \quad (16)$$

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The transversally circumstances, often known as the border condition or the costate variables is represented by the equation (16). The Hamiltonian operation, denoted by the letter G, is interpreted in the literal sense as a representation of the entire profit, taking consideration the immediate advantages, which are $Q(v) - D(v)M_3(v)$, as well as the upcoming advantages which are $\mu(v)M_3(v)$. Because of this, the required (first-order) requirement is as follows in order to find the best possible approach:

$$\frac{\partial G}{\partial Q(v)} = 0 \qquad (17)$$

$$M_3(v) + \{(Q(v) - D(v) + \mu(v))\}y_q = 0 \qquad (18)$$

A numerical instance is used in this part to show the practicality of the technique that has been provided. The actual sales and pricing data of the consumer-durable item are taken into consideration for performing a statistical evaluation of the oversight issue that was described earlier. When it comes to finding a solution to the numerical issue, the optimisation programme LINGO 15.0 is used. In accordance to the suggested research, the goal of the optimisation model issue is to achieve the optimum selling cost approach for the novel product that controls sales activities and maximises the overall profit of the company. This is the purpose of this issue. It is believed that there are twelve distinct points in time that are equal over the whole planned period. When it comes to the objective of providing a numerical demonstration, the marginal price is supposed to be constant, which means that it is regarded to be detached from the sales of the product. The technology diffusion process have been regarded as an important topic of study by researchers and practitioners for a considerable amount of time. An innovative technology is required to possess effective reasoning skills as well as technical understanding in order to be successful in this field. The findings presented in this part, which are based on instances taken from real life, have several management implications that are significant. The approach that was provided proved the significance of awareness of products and price approach in relation to the final acceptance of the technology. The findings of the empirical investigation that were created on the real sale and pricing data of four distinct goods might be of assistance to management in gaining a more accurate grasp of the mechanisms of adoption. One of the most important aspects of innovation that helps to accelerate the procedure of its dissemination is the pricing of the product. As a result, the pricing approach is an essential component in the process of advertising the product. Additionally, the cost of selling has a direct influence on the amount of profit that the company makes. It is for that reason that leaders are required to conduct a thorough examination of their pricing strategy in order to maximise the profitability of the company, as stated in this section. In light of this, the conclusions of the current optimisation issue will make it easier for managers to stimulate the sales paradigms by concentrating on the changing pricing strategy. In the majority of the techniques to modelling diffusion, it is expected that the customer base size of the invention would remain consistent over time and that those individuals will ultimately develop into the end-user. With that being said, not every potential purchaser will be persuaded to go through with the deal. Potential customers are presented with the product in order to get information that may either be used to support or oppose the implementation of that item.

Their choice to purchase the product is not made until they have reached a point where they are content with the knowledge that they have received about it. As a result, the purpose of this section is to analyse a three-stage diffusion of innovative ideas model that investigates the role of discontent on the part of the likely customer in the procedure of creativity dissemination. The first step of the method of diffusion is responsible for contributing to the dissemination of information about the invention among the target market. In addition to this, the model that has been suggested takes into account the consequences of unfavourable data as well as the good word-of-mouth impact that occurs when creativity is acknowledged. In addition to this, a maximisation model is developed, which results in a strategy for optimising the price choices for the invention. An ideal management challenge is created by transforming the non-linear optimisation issue into the optimal control issue with the purpose of maximising the total projected profit functions. The rate of discount is taken into consideration in order to accommodate the current preferences of the consumer for payment. The network or state elements, which are optimised by managing the price varying, are represented by the presumably engaged demographic and overall sales in the regulation equation. These parameters are optimised by manipulating the pricing variable. An ideal adaptive pricing strategy for a new invention is modelled in this segment with the goal of maximising the overall profit for the company. Furthermore, incremental the adoption is a potent metric that almost all businesses are required to be concerned about. As a result, the three-step diffusion process that was developed is implemented in order to characterise the total sales and to decide the price approaches.

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6. Results and Analysis

To optimise pricing techniques for fresh product acceptance, the suggested technique employs a three-stage diffusion model via optimal control theory. Market acceptance, product desire to buy, and awareness of the product are the three phases that make up the diffusion process. In the introduction phase, various forms of dissemination and personal recommendations educate prospective clients. Using price techniques to gauge consumer interest is part of the motivational stage, and a logistics expansion function is used to predict choices for purchases in the adherence stage. To maximise profit over a defined planned perspective, the framework uses dynamic optimisation using price as the variable of control. In order to guarantee an ideal price trajectory that adjusts to market circumstances and consumer behaviour, the approach is determined using Pontryagin's Maximum Principles. Verification using real-world data shows that the method works to increase rates of adoption and profits.

6.1. Accuracy: The share of observation that were accurately anticipated, including both positive and negative results. This number represents how well the model does at classifying users as either adopters or non-adopters.

6.2. F1-Score: A harmonic average of accuracy and memory. Whenever the data is unbalanced, it is especially helpful since it gives an even assessment of a model's true beneficial and false negative prediction abilities.

6.3. Recall (Sensitivity): This metric measures how well the model recognises real adopters. This metric evaluates the model's capacity to include all prospective users in the specified market.

6.4. Precision: True positive rate, or the percentage of expected adopters who really do so. Without which includes many false positives, it shows that the model is reliable in forecasting adoption.

Table 1: Outcome of Statistical Measures of Accuracy

Models	Accuracy (%)
Optimal Control Theory (proposed method)	96
SVM	93
RF	90
KNN	86
LR	84

When compared to other models, the suggested Optimal Control Theory approach achieves better accuracy than conventional methods. The suggested approach outperforms the competition in accurately modelling and predicting adoption behaviours and pricing approaches, as shown by its 96% accuracy rate. Optimum control-based methods still outperform Support Vector Machine (SVM), which comes in second with a respectable 93% accuracy. With accuracy levels of 90% and 86%, accordingly, Random Forest (RF) and K-Nearest Neighbours (KNN) perform rather poorly, indicating that both of them would have trouble representing the complicated nature of pricing's effect on consumer uptake. With an accuracy of only 84%, Logistic Regression (LR) is clearly not up to the task of dealing with non-linear correlations that arise during adoption. When it comes to forecasting ideal pricing and optimising product acceptance, the suggested strategy stands far above the competition.

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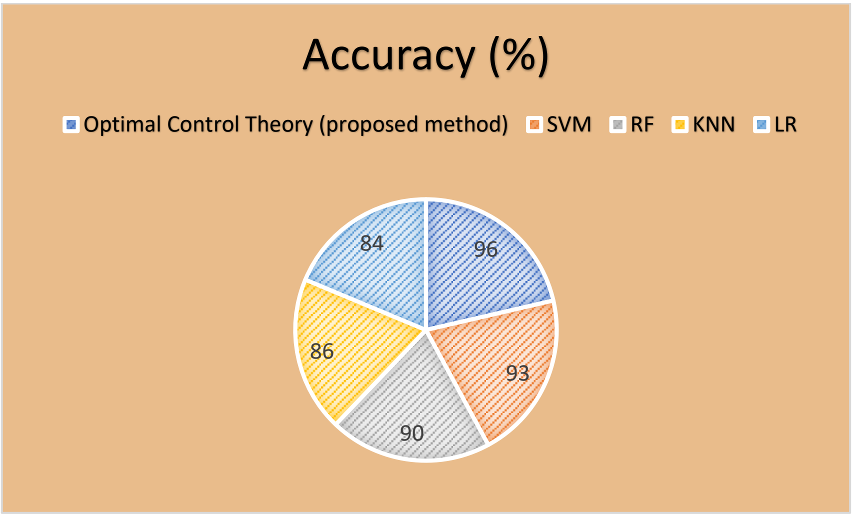

Figure 3. Evaluation of ML models in comparison to approaches that are more traditional.

Table 2: Results of F1-Score Statistical Measures

Models	F1-Score (%)
Optimal Control Theory (proposed method)	94
SVM	91
RF	88
KNN	84
LR	82

In terms of forecasting adoption of the product, the suggested strategy according to Optimal Control Theory has the greatest F1-Score of 94% when compared to other models. This is a reflection of its outstanding equilibrium between recall and accuracy. The next best approach, SVM, with an F1-Score of 91%, shows good results but can't quite match the suggested method when it comes to accurately capturing actual positives as well as actual negatives. Although both RF and KNN performed well, their lower F1-Scores of 88% and 84%, accordingly, indicate that their models could fail to strike the ideal proportion of recall and accuracy in this particular setting. With an F1-Score of 82%, LR is the least effective, showing that it struggles with complicated dynamics, especially in the face of unbalanced or non-linear information. The suggested approach is effective in adopting forecasting, as it beats all different models because of F1-Score

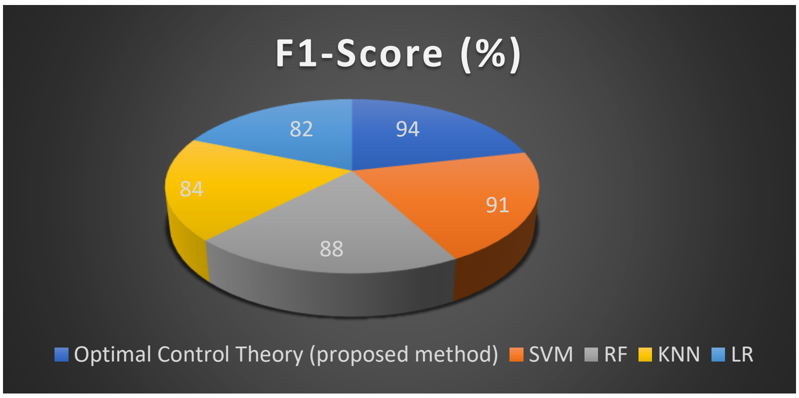

Figure 4. Comparing ML models to more conventional methods for evaluation.

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Table 3: Statistical Measures for Recall Outcomes

Models	Recall (%)
Optimal Control Theory (proposed method)	95
SVM	92
RF	89
KNN	85
LR	83

For accurate predictions for effective adoption of a product, the suggested Optimal Control Theory-based technique has an outstanding recall of 95%, showing that it can precisely recognise a majority of real adopters. SVM comes in second with a recall of 92%, indicating that it also does a good job of detecting real positives, although it is still not as good as the suggested technique. The recall values of RF and KNN are 89% and 85%, accordingly, which indicates that they are not as good at capturing adopters. This might be because it is difficult to accurately describe the adoption patterns. With a recall of only 83%, LR clearly has the worst time finding adopters when when compared with the remaining models. When compared to competing methods, the suggested method's recall performance is far higher, which makes it a more precise predictor of who would actually buy the products.

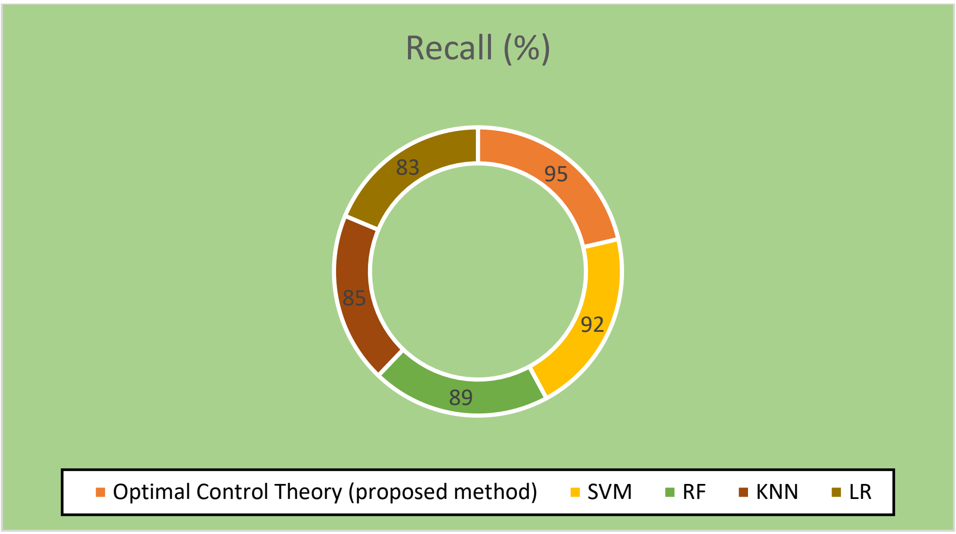

Figure 5. Efficacy of different systems

Table 5: Exploratory DL models in relation to proposed methods

Models	Precision (%)
Optimal Control Theory (proposed method)	93
SVM	90
RF	87
KNN	84
LR	82

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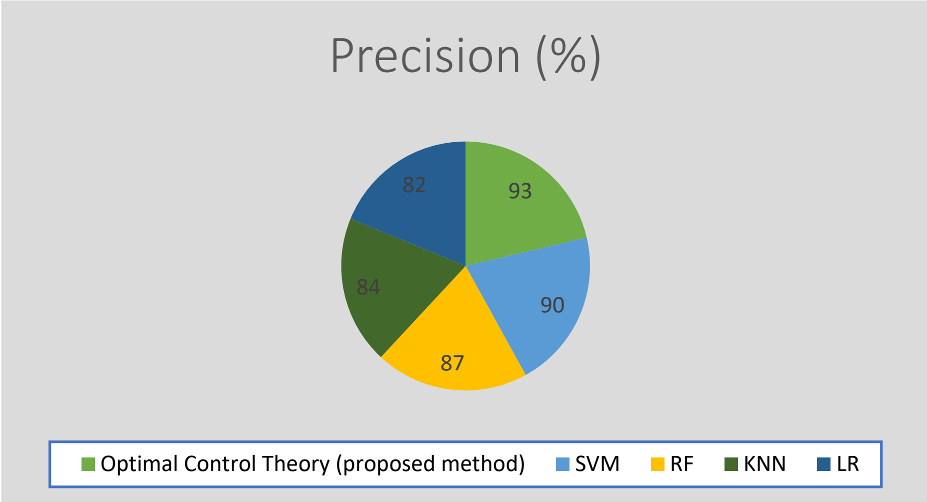

Figure 6. Effectiveness of different models

With a maximum accuracy of 93%, the suggested Optimal Control Theory-based approach shows that it can effectively detect genuine adopters with few false positives. SVM comes in second with a 90% accuracy rate, which is excellent but not quite up to the suggested technique's accuracy rate for adopter prediction. The accuracy ratings of both RF and KNN are 87% and 84 %, respectively which shows that RF and KNN makes more number of false positive predictions when compared with SVM and the proposed approach. This is the lowest for any model at 82 % suggesting that it is not good at identifying who would actually adopt a product in this environment. When it comes to making accurate and reliable, probabilistic predictions about who will adopt, the proposed method is generally most accurate.

7. Conclusion and Future Scope

In order to outline the maxima of the control techniques for the dynamic networks, which change their configuration and functionality with time, Optimal Control Theory brings a conceptually sound fundament to rely on and understand that one is dealing with a dynamical decision-making environment. It assists an individual to reduce cost, increase on productivity or enhance the system performance through the addition of state parameters, control factors and a preferred function. This hypothesis may apply to virtually all areas of aeronautical engineering, robotics, economists, medical care, and financing. Today's control systems are based on its principles including a programming concept in dynamic and the concept of the optimum fundamental which offers both theoretical support and real-life example. Difficulties appear when trying to use Optimal Control Theory for rather complex, highly dimensional, and stochastic systems no matter how useful the theory is in general. The theory is gradually transforming into the situation requiring its practical application due to the advancement of mathematical approaches and the connection with such growing fields as machine learning and artificial intelligence. It is thus feasible in many fields such as autonomy, the improvement of solar energy conversion, and especially, health improvement through tailored approaches. Last but not the least Optimal Control Theory hence completes the theory line in both study and practice and offers a unifying efficient schemes framework for optimisation of complex systems whilst being optimally flexible/robust. The fact that it can easily adapt to the variation in technical specifications provides evidence for the continual relevance and applicability of the solution to pressing worldwide concerns.

However, it is important to remember that Optimal Control Theory has developed a great deal as regards the issues of dynamic optimisation but it has its potential for further development. Further research in this area might focus on what multinode and nonlinear computing is in an area that strives to model higher dimensional networks. Open loop, issues that originate with an unclear or a partially understood system nature may be resolved by combining Optimal Control Theory with the machine reinforcement type of learning.

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