Holling Type-III Functional Response Action of Predator-Prey System with Harvested Predator Stability and Their Neutrosophic Generalized Versions

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Abstract

This paper focuses on the stability of Descriptor Predator-Prey economic system and its related neutrosophic system of Holling type-III functional action response with harvested predator under classical real environment and neutrosophic environment. Where the solvability and dimensionless forms have been presented along with the necessary mathematical justifications and proofs with some qualitative properties have been proposed and developed with systematic illustration.

Keywords: Ecosystem epidemiological; Predator-Prey model; Harvesting Predator-Prey; Holling type-III, Descriptor system and Stability; Neutrosophic Predator-Prey model; Neutrosophic Harvesting Predator-Prey; Neutrosophic coefficient

1. Introduction

Many studies and research were presented for Lotka-Volterra functional response about the Predator-Prey System, and those studies were for the ordinary differential equations[1], [2], as well as there are studies of differential-algebraic Prey- Predator System with Harvested Predator with Lotka-Volterra Functional Response with stability and bifurcation [3]–[5].

Understanding the intricacy of nature is best achieved by simulating the dynamics of biological ecology models. Particularly researching the interactions between biological species or population expansion. There are numerous mathematical models that represent competition and population dynamics [6], following the well-known Lotka-Volterra prey-predator model [3], [7]. The impact of harvest effort on the ecology is examined from an economic standpoint using the economic theory of a common-property resource. An equation to examine the economic interest of harvest effort yield [8] is proposed in [5].

The authors studied the differential equations and dynamical systems [8]–[10], a harvested differential-algebraic allelopathic phytoplankton model's dynamical behavior [8], the bioeconomic modeling of a prey predator system using differential-algebraic equations[9], [11], a single biological economic model's dynamics and stability[4], [12]. Lotka, Volteraa's model [13], the structure and interaction of prey and predators [2], and the mathematical modeling of ecological networks Semi-explicit differential-algebraic systems are important sub classes of nonlinear DAS [14], [15] due to its applications in bifurcation and differential equations with economic interest, rich dynamics in a basic predator-prey population model [1], [2]. Additionally, a predator-prey model using modified Leslie-Gower and Holling-type II schemes was mathematically modeled [1], [3].

Neutrosophic numbers and variables [19] can be applied for generalizing all classical mathematical models and systems [20-22]. The structure of neutrosophic number $a + bI$ with real coefficients a, b and $I^2 = I$.

In this paper will design a new mathematical model Descriptor Predator-Prey System with Harvested Predator in the present of economic interest with Holling type-III Functional Response with dimensionless forms along with

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its neutrosophic generalization. We will also study the solvability and equilibrium of the dynamic system and stability with the necessary assumptions and proofs with the algebraic solutions of the system and the systematic illustration.

2. Ecosystem Epidemiological

The environmental models of epidemiological can represent their temporal evolution through two independent continuous-time ordinary differential equations. As natural pandemic ecosystems contain all the necessary components that are non-linear, high dimensional, etc. Able our discussion to support induction, predator harvesting and stability [1], [3], [16].

economic equation in predator prey model with harvested predator have been examined a variety of analytical methods used to study the stability, regularity and continuity of it with the two types of ecosystems epidemiological which have [16]–[18].

3. Predator-Prey Model

The mathematical model in its general form that describes the dynamics between the types of interaction which are predator and prey, as the following structure [16]–[18].

$$\frac{dx}{dt} = g(x)x - f(x.y)y \qquad (1a)$$

$$\frac{dy}{dt} = \sigma\, f(x.y)y - d(y)y \qquad (1b)$$

Where: x: is the prey density, y: is the predator density, $g(x)$: is the growth rate of prey when there is no predator present, σ : is the conversion rate of eaten prey into new predator, $d(y)$: is the natural death rate of predator when there is no prey present, $f(x, y)$: is call the functional response in the prey equation.

4. Mathematical Modeling of Predator-Prey Model

There is a lot of interest in mathematical modeling of predator-prey interactions. Ever since Lutka and Volterra's work began in the 1920s, with functional response equations, such as the Holing family [9, 14], they predominate in literatures,

in this work implementation of the Holling Type-III functional response, this occurs when a predator's rate of consumption first increases and then decreases as it approaches satiation [3], [11] [1], [18]. According to [1], [2]:

$$f(x) = \frac{Ax^2}{1 + Ah\, x^2} = \frac{ax^2}{b + x^2}, b + x^2 \neq 0$$

Which will enter into the system and affect it.

***Remark 1*:** A pair of differential equations modeling the interaction between predator and prey with Holling Type-III functional response action can be considered as a special

$$\begin{Bmatrix} \frac{d\bar{N}}{d\bar{t}} = r\bar{N}\ \left(1 - \frac{\bar{N}}{K}\right) - \frac{a\bar{N}^2}{b+\bar{N}^2}\ \bar{P} \\ \frac{d\bar{P}}{d\bar{t}} = \bar{\sigma}\ \frac{a\bar{N}^2}{b+\bar{N}^2}\ \bar{P} - \bar{d}\bar{P} - \bar{E}\ \bar{P} \end{Bmatrix} \qquad (2)$$

Where: r, k, a, $\bar{\sigma}$ and $\bar{d}$ are positive constants, $\bar{P}$ and $\bar{N}$ stand for predator and prey, respectively. The prey grows logistically with an intrinsic growth rate (r) and carrying capacity (K) when there is no predation. Prey species N declines in the presence of a predator at a rate proportional to the functional response, where (a) is the predation rate.

There is no evaluation of predation, and the number of predatory species declines dramatically with mortality ($\bar{d}$). Predation efficiency is indicated by the factor ($\bar{\sigma}$), which is calculated by dividing the maximum birth rate per capita from predators by the maximum consumption rate per person without prey.

5. Harvesting and Economic of Prey-Predator System

We must know about the prey and predator at a new harvest rate. The ideas of the study provide a rate of harvest derived from the ability of virus therapies studied in dynamic models [2], [16], [18].

Assumption: Using the two dimensions normal differential equation system (2), and assume that there is place of harvesting, but in this predatory segment it is under harvesting and the introduction of the predator virus harvest function of $\bar{E}\bar{P}$ into the predatory-prey system (2).

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According to the equation proposal to investigate the economic benefit of the harvest effort crop, we have $\bar{E}(\bar{t})$ is the harvest effort, Prey and predator collected population densities are denoted by $\bar{N}(\bar{t})$ and $\bar{P}(\bar{t})$, respectively. So, the total revenue of predator is:

$$TR = \bar{w}\ \bar{E}(\bar{t})\ \bar{P}(\bar{t}) \qquad (3)$$

Where, the harvest's unit price is $(\bar{w})$, while the whole cost is

$$TC = \bar{c}\bar{E}(\bar{t}) \qquad (4)$$

the unit cost of harvested effort (constant) is denoted by $\bar{c}$.

Combined with (3), the algebraic equation that takes into account the economic benefit $\bar{m}$ from the harvest effort $\bar{E}(\bar{t})$, and can be expressed in relation to the predator as follows:

$$\bar{E}(\bar{t})[w\ \bar{P}(\bar{t}) - \bar{c}] = \bar{m} \qquad (5)$$

6. Mathematical Modeling of Neutrosophic Predator-Prey Model

We substitute the real coefficients and variables by neutrosophic ones, to get the neutrosophic formulas for equations:

$$f(x+yI) = \frac{(A+BI)(x+yI)^2}{1+(A+BI)h\,(x+yI)^2} = \frac{(a+dI)(x+yI)^2}{b+cI+(x+yI)^2}, b+cI+(x+yI)^2 \neq 0$$

$$\left\{\begin{array}{l} \frac{d\bar{N}}{d\bar{t}} = r\bar{N}\ \left(1-\frac{\bar{N}}{K}\right) - \frac{a\bar{N}^2}{b+\bar{N}^2}\ \bar{P} \\ \frac{d\bar{P}}{d\bar{t}} = \bar{\sigma}\ \frac{a\bar{N}^2}{b+\bar{N}^2}\ \bar{P} - \bar{d}\bar{P} - \bar{E}\ \bar{P} \end{array}\right\} \text{With the condition:} \qquad (2N)$$

$r, k, a, \bar{\sigma}, \bar{d} \in R(I)$,

7. Harvesting and Economic of Neutrosophic Prey-Predator System:

Assumption: Using the two dimensions normal neutrosophic differential equation system (2), and assume that there is place of harvesting, but in this predatory segment it is under harvesting and the introduction of the predator virus harvest function of $\bar{E}\bar{P}$ into the predatory-prey system (2).

We get:

$$TR = \overline{w+uI}\ \bar{E}(\overline{t+nI})\ \bar{P}(\overline{t+nI}) \qquad (3N)$$

Where, the harvest's unit price is $(\bar{w}+uI)$, while the whole cost is

$$TC = \overline{cI}\bar{E}(\overline{t+nI}) \qquad (4N)$$

the unit cost of harvested effort (constant) is denoted by $\overline{cI}$.

Combined with (3N), the algebraic equation that takes into account the neutrosophic economic benefit $\bar{m}+lI$ from the harvest effort $\bar{E}(\overline{t+nI})$, and can be expressed in relation to the predator as follows:

$$\bar{E}(\overline{t+nI})[(w+uI)\ \bar{P}(\overline{t+nI}) - \overline{cI}] = \overline{m+lI} \qquad (5N)$$

8. Descriptor Predator-Prey Model

The general form of two- dimensional system (2) is ordinary differential equations to transform it to the Descriptor system, then, the following assumption is very important:

Assumption: the transform of the ordinary differential equations (2) to the Descriptor system, which is possible by the affecting of harvesting effort only, with adding the economic interest equation as an algebraic (constraint) equation (5), then the system will become as:

$$\begin{aligned} \frac{d\bar{N}}{d\bar{t}} &= r\bar{N}\left(1-\frac{\bar{N}}{K}\right) - \frac{a\bar{N}^2}{b+\bar{N}^2}\bar{P} \\ \frac{d\bar{P}}{d\bar{t}} &= \sigma\frac{a\bar{N}^2\bar{P}}{b+\bar{N}^2} - \bar{d}\bar{P} - \bar{E}\bar{P} \\ 0 &= \bar{E}(\bar{w}\bar{P} - \bar{c}) - \bar{m} \end{aligned} \qquad (6)$$

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The above system is called Descriptor Predator-Prey economic model with harvesting of predator and Holing type-III functional response (DP-PS).

9. Non-Dimensional Transformation

The plan will start with determining the non-dimensional form of the Descriptor Predator-Prey system, then, the non-dimensional transformation (6) can be determining by the following lemma:

Lemma 1**:** Consider the Descriptor Predator-Prey system (6), let the linear

$t = r\bar{t}\,, N = \frac{\bar{N}}{K}, \alpha = \frac{a}{r}, \beta = \frac{b}{K^2}, P = \frac{\bar{P}}{K}, d = \frac{\bar{d}}{r}, E = \frac{\bar{E}}{r}, c = r\bar{c}, w = r\bar{w}K, \mu = \bar{m},$

$K \neq 0, r \neq 0$, then, the non-dimensional form of the DP-PS (6) is

$$\frac{dN}{dt} = N\left(1 - N - \frac{\alpha NP}{\beta + N^2}\right)$$
$$\frac{dP}{dt} = P\left(\sigma \frac{\alpha N^2}{\beta + N^2} - d - E\right) \qquad (7)$$
$$0 = E(wP - c) - \mu$$

Where N, P represents respectively the harvested population density of prey and predator, σ is the conversion rate of eaten prey into new predator, d is the predator's normal death rate when its prey is not present. α is maximum attack rate, β is a half of saturation level, E stands for harvest effort, w for harvest price, c for harvest effort cost, and μ for harvest effort economic benefit [15], [18].

10. Descriptor Predator-Prey neutrosophic Model:

The general form of two- dimensional system (2N) is ordinary differential equations to transform it to the Descriptor system, then, the following assumption is very important:

Assumption**:** the transform of the ordinary differential equations (2N) to the Descriptor system, which is possible by the affecting of harvesting effort only, with adding the economic interest equation as an algebraic (constraint) equation (5N), then the system will become as:

$$\frac{d\bar{N}}{d\bar{t}} = r\bar{N}\left(1 - \frac{\bar{N}}{K}\right) - \frac{a\bar{N}^2}{b + \bar{N}^2}\bar{P}$$
$$\frac{d\bar{P}}{d\bar{t}} = \sigma \frac{a\bar{N}^2\bar{P}}{b + \bar{N}^2} - \bar{d}\bar{P} - \bar{E}\bar{P} \qquad (6N)$$
$$0 = \bar{E}\left(\overline{(w + uI)}\bar{P} - \bar{c}I\right) - \overline{m + lI}$$

11. Non-Dimensional neutrosophic Transformation

Lemma 2**:** Consider the Descriptor Predator-Prey system (6N), let the linear

$t + nI = r(\overline{t + nI})\,, N = \frac{\bar{N}}{K}, \alpha = \frac{a}{r}, \beta = \frac{b}{K^2}, P = \frac{\bar{P}}{K}, d = \frac{\bar{d}}{r}, E = \frac{\bar{E}}{r}, c = r\bar{c}I, w + uI = r(\overline{w + uI})K, \mu = \overline{m + lI},$

$K \neq 0, r \neq 0$, then, the non-dimensional form of the DP-PS (6N) is

$$\frac{dN}{dt + nI} = N\left(1 - N - \frac{\alpha NP}{\beta + N^2}\right)$$
$$\frac{dP}{dt + nI} = P\left(\sigma \frac{\alpha N^2}{\beta + N^2} - d - E\right) \qquad (7N)$$
$$0 = E((w + uI)P - cI) - \mu$$

With all coefficients are considered as elements of R(I).

12. The Solvability of DP-PS

To study the solvability of DP-PS (7), consider a system described by the semi-explicit description with set $N_1 = (N, P)^T \in R^{n_1}$ as:

$$\dot{N} = F_1(N_1, E; \mu) = \begin{pmatrix} f_1(N, P, E; \mu) \\ f_2(N, P, E; \mu) \end{pmatrix}$$
$$0 = F_2(N, E; \mu)$$

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Assume that: $x_1 = N_1 = (N, P)^T \in R^{n_1}, x_2 = E \in R^{n_2}$ with the parameter μ. Then, by problem formulations in [13, 17], the above system becomes:

$$\dot{N} = F_1(N_1, E; \mu)$$
$$0 = F_2(N, E; \mu)$$

where, $F_1(N_1, E; \mu) \in C^1(D \times R^{n_2}: R^{n_1})$, $F_2(N, E; \mu) \in C^2(D \times R^{n_2}: R^{n_2})$,

$(N_1; \mu) \in D \subset R^{n_1+1}$, D is an open subset, $N \in R^{n_1}, E \in R^{n_2}$ and $\mu \in R$ with $n_1 + n_2 = n$. Therefore, the system (7) is solvable as well as it has a unique solution locally.

The linearization of DPPS (7) about an equilibrium point of the class of equilibrium points in general form is determined by Taylor expansion with $rank(\bar{E}) = 2 < n = 3$. Because system (7) of two ordinary differential equations and one algebraic equation [1], [18].

13. The Stability of DP-PS

Consider the non-dimension form of the Harvesting Descriptor Predator-Prey economic system with Holing type-III functional response (7) with the same region of system (6) as well as the location of the equilibrium points. Then, in the case of $\mu = 0$ at the system will consider. Then the system will become:

$$\begin{aligned} \dot{N} &= N(1 - N - \frac{\alpha PN}{\beta + N^2}) \\ \dot{P} &= P(\sigma \frac{\alpha N^2}{\beta + N^2} - d - E) \qquad (8) \\ 0 &= E(wP - c) \end{aligned}$$

which has the equilibrium points: $P_1(0,0,0), P_2(1,0,0)$ and general positive equilibrium point $P_3^*\left(1 - \frac{\alpha N^*}{\beta + N^{*2}} P^*, \frac{c}{w}, \sigma \frac{\alpha N^{*2}}{\beta + N^{*2}}\right)$

To determine the stability and regularity we depend on the Linearization system, then, the Jacobian matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, A = J$$

Where

$$a_{11} = \frac{df_1}{dN} = 1 - 2N - \left(\frac{2\alpha NP\,(\beta + N^2) - 2\alpha N^3 P}{(\beta + N^2)^2}\right) = 1 - 2N - \left(\frac{2\alpha NP\,(\beta + N^2) - 2\alpha N^3 P}{(\beta + N^2)^2}\right)$$

$$a_{12} = \frac{df_1}{dP} = \frac{\alpha N^2}{\beta + N^2}, a_{13} = \frac{df_1}{dE} = 0, a_{21} = \frac{df_2}{dN} = \frac{\sigma \alpha N^2 (\beta + N^2) - 2\sigma \alpha N^3 P}{(\beta + N^2)^2}$$

$$a_{22} = \frac{df_2}{dP} = \frac{\sigma \alpha N^2}{\beta + N^2} - d - E, a_{23} = -P, a_{31} = \frac{df_3}{dN} = 0$$

$$a_{32} = \frac{df_3}{dP} = wE, a_{33} = \frac{df_3}{dE} = wP - c$$

$$\therefore \bar{A} = J = \begin{bmatrix} 1 - 2N - \frac{2\alpha NP(\beta + N^2) - 2\alpha N^3 \beta P}{(\beta + N^2)^2} & \frac{\alpha N^2}{\beta + N^2} & 0 \\ \frac{\sigma \alpha N^2 (\beta + N^2) - 2\sigma \alpha N^2 P}{(\beta + N^2)^2} & \frac{\sigma \alpha N^2}{\beta + N^2} - d - E & -P \\ 0 & wE & wP - c \end{bmatrix} \qquad (9)$$

Now for stabilization analysis the following theorems are important.

Theorem 1: Consider the system (8) with $\mu = 0$ then, the equilibrium point P_1 is locally unstable (saddle).

Proof: The Jacobian matrix (9) of the DP-PS (8) at the equilibrium point $P_1(0,0,0)$ is

$$\bar{A}(P_1) = J_{P_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -d & 0 \\ 0 & 0 & -c \end{bmatrix}$$

The characteristic polynomial is: $det\,(\lambda \bar{E} - \bar{A}(P_1)) = c(\lambda - 1)(\lambda + d)$, were, $\bar{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The eigenvalues are $\lambda_1 = 1 > 0, \lambda_2 = -d < 0$, hence the proof.

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Theorem 2: Consider the DP-PS (8) with $\mu = 0$ then, the equilibrium point

(1) If $\frac{\sigma\alpha}{\beta+1} < d$ then, the equilibrium point P_2 is a stable;

(2) If $\frac{\sigma\alpha}{\beta+1} > d$ then, the equilibrium point P_2 is a saddle node (unstable).

Proof: The Jacobian matrix (9) of the DP-PS (8) at the equilibrium point $P_2(1,0,0)$ is:

$$\bar{A}(P_2) = J_{P_2} = \begin{bmatrix} -1 & \frac{\alpha}{\beta+1} & 0 \\ \frac{\sigma\alpha\beta}{(\beta+1)^2} & \frac{\sigma\alpha}{\beta+1} - d & 0 \\ 0 & 0 & -c \end{bmatrix}$$

Then, the characteristic polynomial is:

$$\det(\lambda\bar{E} - \bar{A}(P_2)) = c(\lambda+1)\left(\lambda - \left(\frac{\sigma\alpha}{\beta+1} - d\right)\right)$$

The eigenvalues are: $\lambda_1 = -1$ and $\lambda_2 = \frac{\sigma\alpha}{\beta+1} - d$

If $\frac{\sigma\alpha}{\beta+1} < d$ implies that $\lambda_2 < 0$, then P_2 is a stable;

If $\frac{\sigma\alpha}{\beta+1} > d$ implies that $\lambda_2 > 0$, then P_2 is a saddle.

14. The Solvability of neutrosophic DP-PS

To study the solvability of DP-PS (7N), consider a system described by the semi-explicit description with set $N_1 = (N+I, P+I)^T \in R^{n_1}(I)$ as:

$$N \dot{+} I = F_1(N_1 + I, E + I; \mu) = \begin{pmatrix} f_1(N+I, P+I, E+I; \mu) \\ f_2(N+I, P+I, E+I; \mu) \end{pmatrix}$$

$$0 = F_2(N+I, E+I; \mu)$$

Assume that: $x_1 = N_1 + I = (N+I, P+I)^T \in R^{n_1}(I), x_2 = E + I \in R^{n_2}(I)$ with the parameter μ. Then:

$$N \dot{+} I = F_1(N_1 + I, E + I; \mu)$$

$$0 = F_2(N+I, E+I; \mu)$$

where, $F_1(N_1 + I, E + I; \mu) \in C^1(D \times R^{n_2}(I): R^{n_1}(I)), F_2(N+I, E+I; \mu) \in C^2(D \times R^{n_2}(I): R^{n_2}(I))$, $(N_1 + I; \mu) \in D \subset R^{n_1+1}(I)$, D is an open subset, $N + I \in R^{n_1}(I), E + I \in R^{n_2}(I)$ and $\mu \in R$ with $n_1 + n_2 = n$. Therefore, the system (7N) is solvable as well as it has a unique solution locally.

Illustration:

Consider the Descriptor Predator-Prey economic system with Holing type-III functional response with the same region of system (7) in the case of $\mu = 0$ locally with parameter and constants values $\sigma = 5.75, \propto = 4, \beta = 5$ and d according to its changing value, is:

$$\begin{aligned} \dot{x} &= x(1 - x - \tfrac{4xy}{5+x^2}) \\ \dot{y} &= y(\tfrac{(5.75)(4)x^2}{5+x^2} - d) \\ 0 &= z(wy - c) \end{aligned} \tag{10}$$

Then, the equilibrium points of (10) are {(0,0), (1,0), (10/21, 1265/1764)}.

The matrix of linearized system is $\bar{A} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

The eigenvalues are (-2, 1). When the value of $d = -0.2$, the relation between the predator-prey together and separated with time as in Figure (1)

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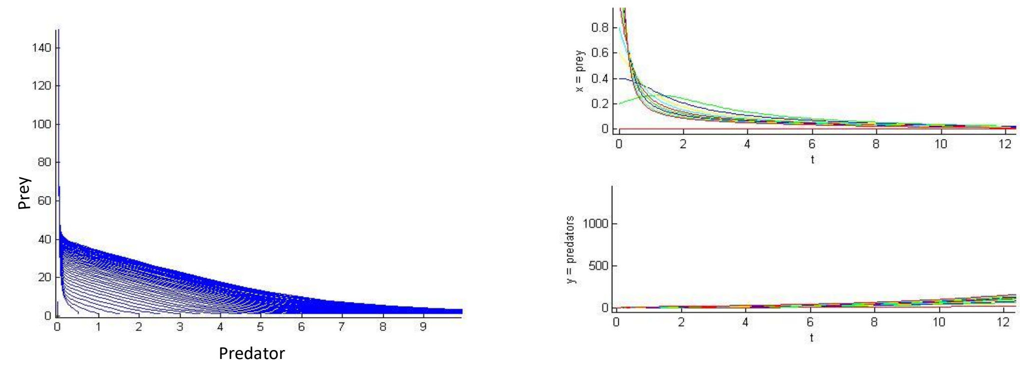
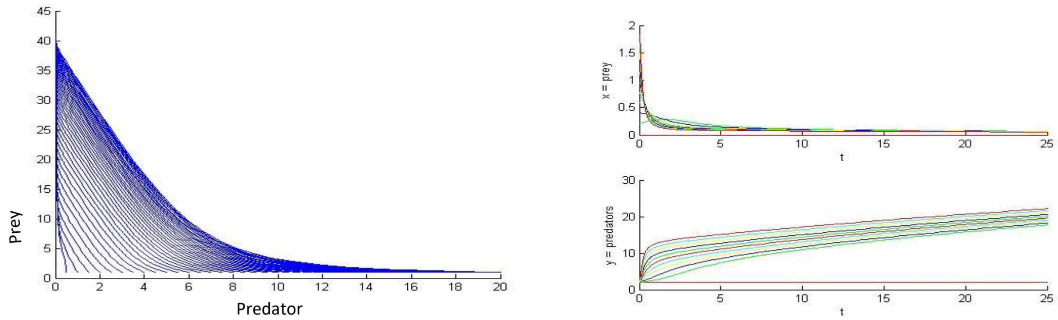


(a) Between the prey and predator **(b) separated with time**

Figure 1. The relation between the predator-prey together and separated with time at value of $d = -0.2$

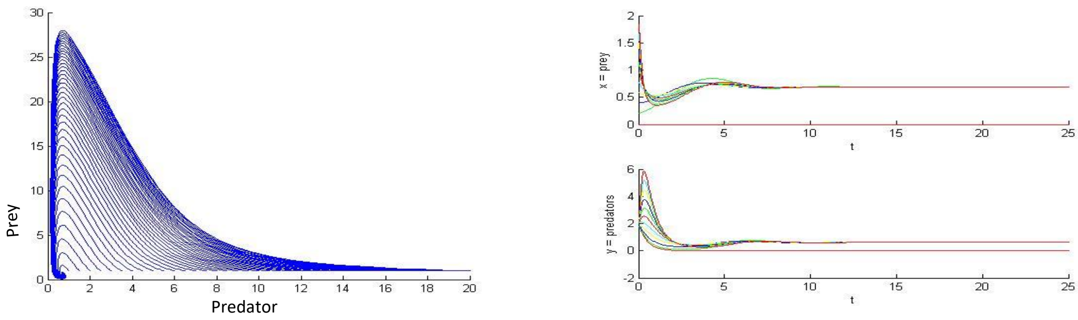
And at the value of $d = 0$, the relation between the predator-prey together and separated with time as in Figure (2)



(a) Between the prey and predator **(b) separated with time**

Figure 2. The relation between the predator-prey together and separated with time at the value of $d = 0$

And at the value of $d = 2$, the relation between the predator-prey together and separated with time as in Figure (3)



(a) Between the prey and predator **(b) separated with time**

Figure 3. The relation between the predator-prey together and separated with time at the value of $d = 2$

15. Conclusion

This work has investigated some dynamical behaviors of Descriptor Predator-Prey economic system with harvested predator with Holling type-III functional response with its neutrosophic-generalized version. The solvability and dimensionless forms have been presented. The necessary mathematical justifications and proofs with some qualitative properties have been proposed and developed with systematic illustration has been shown via numerical simulation results for the classical case and neutrosophic case.

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