Extending One-Way ANOVA to Neutrosophic Sets: A Method for Uncertainty-Based Decision Making

Sasiwimon Iwsakul[1,*], **Ronnason Chinram**[1]
[1]Division of Computational Science, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand
Emails: sasiwimon.i@psu.ac.th; ronnason.c@psu.ac.th

Abstract

Classical statistical methods assume that data are precise and free from uncertainty, which may not hold in many real-world applications. Neutrosophic statistics provides a flexible framework for handling indeterminacy, vagueness, and inconsistency in data. In this paper, we propose a new formulation of one-way analysis of variance (ANOVA) within the neutrosophic framework. The method treats membership, indeterminacy, and non-membership components separately, with explicit F-tests for each, and employs a maximum-based decision rule to determine significance. We also compare the proposed method with the classical one-way ANOVA. The results demonstrate that the neutrosophic ANOVA is more sensitive in detecting group differences, particularly in cases where the classical approach yields smaller F-values and may fail to reject the null hypothesis. These findings highlight the potential of neutrosophic ANOVA as a more robust alternative to classical ANOVA for analyzing data with inherent uncertainty and indeterminacy.

Keywords: Neutrosophic sets; One-way analysis of variance; Group means; Decision-making

1 Introduction

The concept of a neutrosophic set was first introduced by Smarandache in 1998.[6] A neutrosophic set is characterized by three functions: a membership function, an indeterminacy function, and a non-membership function. This framework can address vagueness, indeterminacy, and inconsistency in both data and reasoning processes. Neutrosophic set theory has been applied in various fields, including medical diagnosis,[2,10,11] database management,[8,9] and decision-making.[3,13] Statistical methods also play a crucial role in decision-making under uncertainty, which motivates the integration of neutrosophic concepts into statistical analysis. Neutrosophic statistics extends classical statistical methods by incorporating data uncertainty. While classical statistics assumes that data are precise and represented by crisp values, neutrosophic statistics allows for indeterminacy in the data.[7] Several classical statistical techniques have been extended to the neutrosophic setting, particularly for comparing group means. For example, a neutrosophic version of one-way analysis of variance (ANOVA) was proposed,[4] followed by the development of post hoc multiple comparison tests.[5] Additionally, the neutrosophic Kruskal–Wallis H test has been applied in the analysis of COVID-19 data.[12] However, in Aslam,[4] neutrosophic ANOVA was modeled as interval-based uncertainty, which does not account for the distinct roles of membership, indeterminacy, and non-membership.

In this study, we propose a new formulation of one-way ANOVA within the neutrosophic framework, which differs from previous approaches. Specifically, we treat the three neutrosophic components, membership, indeterminacy, non-membership, separately with explicit F-tests for each. Our overall decision rule is based on

$$F = \max\{F_{\mathcal{T}}, F_{\mathcal{I}}, F_{\mathcal{F}}\},$$

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which ensures that a difference can be detected if at least one component differs across groups. This formulation offers enhanced sensitivity compared to interval-based neutrosophic ANOVA, enabling the detection of differences that classical methods may overlook. The framework adopted here is consistent with the principles of neutrosophic statistics, extending classical inference to accommodate vagueness and uncertainty.

2 Neutrosophic Sets

This section presents the essential definition of neutrosophic sets, which serve as the theoretical foundation for the proposed analysis.

Definition 2.1. *Let S be a nonempty set. A* neutrosophic set *$\mathcal{A}$, defined on S, is given by:*

$$\mathcal{A} = \{\langle x, \mathcal{T}_A(x), I_A(x), F_A(x)\rangle \mid x \in S\}$$

where $\mathcal{T}_A : S \to [0,1]$, $I_A : S \to [0,1]$, and $F_A : S \to [0,1]$ are the membership, indeterminacy, *and* non-membership *functions, respectively, of the neutrosophic set $\mathcal{A}$.*

The functions $\mathcal{T}_A(x)$, $I_A(x)$, and $F_A(x)$ are collectively referred to as the neutrosophic components.

In single-valued neutrosophic sets, the sum of the three components depends on the degree of their interdependence and satisfies the following bounds:

- ***All components independent:***

$$1 \leq \mathcal{T}_A(x) + I_A(x) + F_A(x) \leq 3.$$

- ***Two components dependent, one independent:***

$$0 \leq \mathcal{T}_A(x) + I_A(x) + F_A(x) \leq 2.$$

- ***All components dependent:***

$$0 \leq \mathcal{T}_A(x) + I_A(x) + F_A(x) \leq 1.$$

3 Classical One-way Analysis of Variance

In this section, we introduce the classical one-way ANOVA, which is used to compare the means of two or more populations. We begin with hypothesis testing.

The null hypothesis H_0 is defined as

$$H_0 : \mu_1 = \mu_2 = ... = \mu_k$$

which states that all population means are equal.

The corresponding alternative hypothesis, H_A is defined as

$$H_A : \text{At least one } \mu_i \text{ differ from the others.}$$

The test statistics for evaluating the null hypotheses is based on the following sum of squares:

- The **total sum of squares** is defined as

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \overline{y})^2,$$

where $\overline{y}$ is the overall mean.

- The **sum of squares between groups** is defined as

$$SSB = \sum_{i=1}^{k} n_i(\overline{y_i} - \overline{y})^2,$$

where $\overline{y_i}$ is the sample mean of group i.

- The **sum of squares within groups** is defined as

$$SSW = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \overline{y_i})^2.$$

Alternatively, the within-group sum of squares can also be computed as:

$$SSW = SST - SSB.$$

Then the mean square between groups is defined as

$$MSB = \frac{SSB}{k-1}.$$

Additionally, the mean square within groups is defined as

$$MSW = \frac{SSW}{n-k},$$

Finally, these quantities are summarized in the ANOVA table:

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	$k-1$	SSB	MSB	$\frac{MSB}{MSW}$
Within groups	$n-k$	SSW	MSW	
Total	$n-1$	SST		

4 One-way Analysis of Variance within the neutrosophic framework

In this section, we extend the classical one-way ANOVA to the neutrosophic setting. This approach incorporates concepts from neutrosophic sets to handle uncertainty and indeterminacy in the data. Each observation is represented by three components: membership ($\mathcal{T}$), indeterminacy ($\mathcal{I}$), and non-membership ($\mathcal{F}$). We assume that these components are mutually independent and normally distributed with homogeneous variances across groups.

Definition 4.1. *Let $\{x_{i1}, x_{i2}, \dots, x_{in_i}\}$ be a random sample of size n_i drawn from a crisp set X_i. Each element x_{ij} is associated with the membership, indeterminacy and non-membership functions $\mathcal{T}_A(x_{ij})$, $\mathcal{I}_A(x_{ij})$ and $\mathcal{F}_A(x_{ij})$ respectively, defined with respect to the neutrosophic set $\mathcal{A}$.*
The sample means of the membership, indeterminacy and non-membership values for the i^{th} group are denoted by $\overline{\mathcal{T}}_A(X_i)$, $\overline{\mathcal{I}}_A(X_i)$ and $\overline{\mathcal{F}}_A(X_i)$, respectively, and are defined as follows:

$$\overline{\mathcal{T}}_A(X_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\mathcal{T}_A(x_{ij}), \quad \overline{\mathcal{I}}_A(X_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\mathcal{I}_A(x_{ij}), \quad \overline{\mathcal{F}}_A(X_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\mathcal{F}_A(x_{ij}).$$

Definition 4.2. *Let $\{x_{i1}, x_{i2}, \dots, x_{in_i}\}$ be a random sample of size n_i drawn from a crisp set X_i. Each element x_{ij} is associated with the membership, indeterminacy and non-membership functions $\mathcal{T}_A(x_{ij})$, $\mathcal{I}_A(x_{ij})$ and $\mathcal{F}_A(x_{ij})$ respectively, defined with respect to the neutrosophic set $\mathcal{A}$.*

The grand mean of the membership, indeterminacy and non-membership values across all groups are denoted by $\overline{\mathcal{T}}_A(X)$, $\overline{\mathcal{I}}_A(X)$ *and* $\overline{\mathcal{F}}_A(X)$*, respectively, and are defined as follows:*

$$\overline{\mathcal{T}}_A(X) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\mathcal{T}_A(x_{ij})}{n}, \quad \overline{\mathcal{I}}_A(X) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\mathcal{I}_A(x_{ij})}{n} \quad \textit{and} \quad \overline{\mathcal{F}}_A(X) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\mathcal{F}_A(x_{ij})}{n}.$$

where $n = \sum_{i=1}^{k} n_i$ *is the total number of observations across all groups.*

Next, we present the methodology for testing differences among neutrosophic population means.

4.1 Significance Testing for Differences Among Three or More Population Means Using Neutrosophic Set Concepts

Let X_i be the i^{th} crisp population, and let $\mathcal{A}$ be a neutrosophic set defined on X_i, where $i = 1, 2, \ldots, k$. Consider a random sample $\{x_{i1}, x_{i2}, \ldots, x_{in_i}\}$ of size n_i drawn from X_i, with corresponding membership, indeterminacy and non-membership values $\mathcal{T}_A(x_{ij}), \mathcal{I}_A(x_{ij})$, and $\mathcal{F}_A(x_{ij})$, respectively for each $j = 1, 2, \ldots, n_i$. We assume that for each group X_i, the membership values $\mathcal{T}_A(x_{ij})$, indeterminacy values $\mathcal{I}_A(x_{ij})$, and non-membership values $\mathcal{F}_A(x_{ij})$, where $j = 1, 2, \ldots, n_i$, are independently and normally distributed. Furthermore, it is assumed that the variances of each component are equal across all groups; that is, the variances of $\mathcal{T}_A(x_{ij})$, $\mathcal{I}_A(x_{ij})$, and $\mathcal{F}_A(x_{ij})$ are homogeneous for all $i = 1, 2, \ldots, k$. Based on these samples, the hypotheses to be tested are:

1. The mean membership values of population X_i with respect to $\mathcal{A}$, denoted by $\overline{\mathcal{T}}(A, X_i)$, are equal across all groups.
2. The mean indeterminacy values of population X_i with respect to $\mathcal{A}$, denoted by $\overline{\mathcal{I}}(A, X_i)$, are equal across all groups.
3. The mean non-membership values of population X_i with respect to $\mathcal{A}$, denoted by $\overline{\mathcal{F}}(A, X_i)$, are equal across all groups. .

The corresponding null hypotheses H_0 are defined as follows:

First, for the membership values:

$$H_0 : \overline{\mathcal{T}}(A, X_1) = \overline{\mathcal{T}}(A, X_2) = \ldots = \overline{\mathcal{T}}(A, X_k)$$

which states that the mean membership values of populations X_i with respect to the neutrosophic set $\mathcal{A}$ are equal across all groups, $i = 1, 2, \ldots, k$.

Second, for the indeterminacy values:

$$H_0 : \overline{\mathcal{I}}(A, X_1) = \overline{\mathcal{I}}(A, X_2) = \ldots = \overline{\mathcal{I}}(A, X_k)$$

which states that the mean indeterminacy values of populations X_i with respect to the neutrosophic set $\mathcal{A}$ are equal across all groups.

Third, for the non-membership values:

$$H_0 : \overline{\mathcal{F}}(A, X_1) = \overline{\mathcal{F}}(A, X_2) = \ldots = \overline{\mathcal{F}}(A, X_k)$$

which states that the mean non-membership values of populations X_i with respect to the neutrosophic set $\mathcal{A}$ are equal across all groups, The corresponding alternative hypothesis, H_A, are defined as follows:

1. At least one $\overline{\mathcal{T}}(A, X_i)$ differs across groups.

2. At least one $\overline{\mathcal{I}}(A, X_i)$ differs across groups.
3. At least one $\overline{\mathcal{F}}(A, X_i)$ differs across groups.

After defining the individual hypotheses for the membership, indeterminacy, and non-membership components, we now consider the overall hypotheses for the neutrosophic one-way ANOVA. The overall null hypothesis H_0 states that the mean membership, mean indeterminacy, and mean non-membership values of the populations X_i, for $i = 1, 2, \dots, k$, with respect to the neutrosophic set $\mathcal{A}$, are equal across all groups. That is,

$$\overline{\mathcal{T}}(A, X_1) = \overline{\mathcal{T}}(A, X_2) = \ldots = \overline{\mathcal{T}}(A, X_k),$$

$$\overline{\mathcal{I}}(A, X_1) = \overline{\mathcal{I}}(A, X_2) = \ldots = \overline{\mathcal{I}}(A, X_k),$$

$$\overline{\mathcal{F}}(A, X_1) = \overline{\mathcal{F}}(A, X_2) = \ldots = \overline{\mathcal{F}}(A, X_k).$$

The corresponding alternative hypothesis H_A asserts that at least one of the three components differs across the groups.

The test statistics for evaluating the null hypotheses can be calculated as follows:
Total sum of squares for the membership, indeterminacy and non-membership values, denoted by $SST_{\mathcal{T}}$, $SST_{\mathcal{I}}$, $SST_{\mathcal{F}}$ respectively, are defined as:

$$SST_{\mathcal{T}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{T}_A(x_{ij}) - \overline{\mathcal{T}}_A(X))^2,$$

$$SST_{\mathcal{I}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{I}_A(x_{ij}) - \overline{\mathcal{I}}_A(X))^2,$$

$$SST_{\mathcal{F}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{F}_A(x_{ij}) - \overline{\mathcal{F}}_A(X))^2.$$

Sum of squares between groups for the membership, indeterminacy and non-membership values, denoted by $SSB_{\mathcal{T}}$, $SSB_{\mathcal{I}}$, $SSB_{\mathcal{F}}$ respectively, are defined as:

$$SSB_{\mathcal{T}} = \sum_{i=1}^{k} n_i (\overline{\mathcal{T}}_A(X_i) - \overline{\mathcal{T}}_A(X))^2,$$

$$SSB_{\mathcal{I}} = \sum_{i=1}^{k} n_i (\overline{\mathcal{I}}_A(X_i) - \overline{\mathcal{I}}_A(X))^2,$$

$$SSB_{\mathcal{F}} = \sum_{i=1}^{k} n_i (\overline{\mathcal{F}}_A(X_i) - \overline{\mathcal{F}}_A(X))^2.$$

Sum of squares within groups for the membership, indeterminacy and non-membership values, denoted by $SSW_{\mathcal{T}}$, $SSW_{\mathcal{I}}$, $SSW_{\mathcal{F}}$ respectively, are defined as:

$$SSW_{\mathcal{T}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{T}_A(x_{ij}) - \overline{\mathcal{T}}_A(X_i))^2,$$

$$SSW_{\mathcal{I}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{I}_A(x_{ij}) - \overline{\mathcal{I}}_A(X_i))^2,$$

$$SSW_{\mathcal{F}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{F}_A(x_{ij}) - \overline{\mathcal{F}}_A(X_i))^2.$$

Alternatively, the within-group sum of squares can also be computed as:

$$SSW_{\mathcal{T}} = SST_{\mathcal{T}} - SSB_{\mathcal{T}},$$

$$SSW_{\mathcal{I}} = SST_{\mathcal{I}} - SSB_{\mathcal{I}},$$

$$SSW_{\mathcal{F}} = SST_{\mathcal{F}} - SSB_{\mathcal{F}}.$$

Then the mean square between groups for the the membership, indeterminacy and non-membership value, denoted by $MSB_{\mathcal{T}}$, $MSB_{\mathcal{I}}$, $MSB_{\mathcal{F}}$ respectively, are defined as:

$$MSB_{\mathcal{T}} = \frac{SSB_{\mathcal{T}}}{k-1},$$

$$MSB_{\mathcal{I}} = \frac{SSB_{\mathcal{I}}}{k-1},$$

$$MSB_{\mathcal{F}} = \frac{SSB_{\mathcal{F}}}{k-1}.$$

Additionally, the mean square within groups for the membership, indeterminacy and non-membership values, denoted by $MSW_{\mathcal{T}}, MSW_{\mathcal{I}}, MSW_{\mathcal{F}}$ respectively, are defined as:

$$MSW_{\mathcal{T}} = \frac{SSW_{\mathcal{T}}}{n-k},$$

$$MSW_{\mathcal{I}} = \frac{SSW_{\mathcal{I}}}{n-k},$$

$$MSW_{\mathcal{F}} = \frac{SSW_{\mathcal{F}}}{n-k}.$$

Accordingly, three separate ANOVA tables are constructed, one for each neutrosophic component. Finally, these quantities are summarized in the neutrosophic ANOVA tables for membership, indeterminacy, and non-membership, respectively.

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	$k-1$	$SSB_{\mathcal{T}}$	$MSB_{\mathcal{T}}$	$\frac{MSB_{\mathcal{T}}}{MSW_{\mathcal{T}}}$
Within groups	$n-k$	$SSW_{\mathcal{T}}$	$MSW_{\mathcal{T}}$	
Total	$n-1$	$SST_{\mathcal{T}}$		

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	$k-1$	$SSB_{\mathcal{I}}$	$MSB_{\mathcal{I}}$	$\frac{MSB_{\mathcal{I}}}{MSW_{\mathcal{I}}}$
Within groups	$n-k$	$SSW_{\mathcal{I}}$	$MSW_{\mathcal{I}}$	
Total	$n-1$	$SST_{\mathcal{I}}$		

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	$k-1$	$SSB_{\mathcal{F}}$	$MSB_{\mathcal{F}}$	$\frac{MSB_{\mathcal{F}}}{MSW_{\mathcal{F}}}$
Within groups	$n-k$	$SSW_{\mathcal{F}}$	$MSW_{\mathcal{F}}$	
Total	$n-1$	$SST_{\mathcal{F}}$		

Thus, three separate ANOVA tables are obtained, one for each neutrosophic component. The computed ANOVA tables for the membership, indeterminacy, and non-membership functions encapsulate the decomposition of total variability into between-group and within-group components. These summaries form the basis for assessing whether significant differences exist among the neutrosophic characteristics of the groups. While the three ANOVA tables provide separate insights into the membership, indeterminacy, and non-membership components, they do not by themselves yield an overall conclusion.

To address this, we introduce a maximum-based decision rule that captures the strongest evidence across the three components.

Specifically, we define the overall test statistic as

$$F = \max\{F_{\mathcal{T}}, F_{\mathcal{I}}, F_{\mathcal{F}}\},$$

where $F_{\mathcal{T}}, F_{\mathcal{I}}, F_{\mathcal{F}}$ represent the test statistics for the membership, indeterminacy, and non-membership components, respectively. This maximum-based approach ensures that the most significant deviation among the three neutrosophic dimensions is captured in the final decision rule.

Let $F_{\alpha,k-1,n-k}$ denote the critical value of the F-distribution with degrees of freedom $df_1 = k-1$ and $df_2 = n-k$, corresponding to a significance level α.

The null hypothesis H_0 is rejected if

$$F \geq F_{\alpha,k-1,n-k}.$$

If the computed value of the overall test statistic F exceeds the critical value $F_{\alpha,k-1,n-k}$, we conclude that there is statistically significant evidence to reject the null hypothesis H_0. This indicates that at least one of the three neutrosophic components, membership, indeterminacy, or non-membership, differs significantly among the groups.

We now present two examples to illustrate the application of the proposed method. In both cases, the membership, indeterminacy, and non-membership components are assumed to be independent. We also compare the results with those obtained from classical ANOVA. For comparison, the corresponding classical ANOVA hypotheses are also tested under the same framework, where the null hypothesis states that all population means are equal and the alternative asserts that at least one mean differs. Example 4.3 illustrates a case in which there is no significant difference among the neutrosophic population means, whereas Example 4.4 demonstrates a case in which significant differences are observed.

Example 4.3. Let X_1, X_2, X_3, and X_4 represent four populations, where each X_i denotes the set of all final-year high school students in the provinces of Songkhla, Pattani, Yala, and Narathiwat, respectively.

We define the neutrosophic set $\mathcal{A}$ on these populations as follows. The membership function $\mathcal{T}$ represents the degree to which students intend to apply for admission to the Statistics major within the Faculty of Science at Prince of Songkla University. The indeterminacy function $\mathcal{I}$ captures students' uncertainty about applying specifically to this major. The non-membership function $\mathcal{F}$ reflects the degree to which students do not intend to apply for admission to the program.

We assume that, within each group X_i, the membership values $\mathcal{T}_A(x_{ij})$, indeterminacy values $\mathcal{I}_A(x_{ij})$, and non-membership values $\mathcal{F}_A(x_{ij})$, where $j = 1, 2, \ldots, n_i$, are independently and normally distributed. In addition, it is assumed that the variances of these components are homogeneous across all groups.

We now examine whether the membership, indeterminacy, and non-membership functions, which capture the intentions and levels of uncertainty among the four populations, differ significantly across groups at significance level $\alpha = 0.05$.

Let $S_1 = \{x_{11}, x_{12}, x_{13}, x_{14}, x_{15}\}$ be a sample of size five drawn from the population of final-year high school students in Songkhla (population X_1). Let $S_2 = \{x_{21}, x_{22}, x_{23}, x_{24}, x_{25}\}$ be a sample of size five drawn from the population in Pattani (population X_2). Let $S_3 = \{x_{31}, x_{32}, x_{33}, x_{34}, x_{35}\}$ be a sample of size five drawn from the population in Yala (population X_3). Let $S_4 = \{x_{41}, x_{42}, x_{43}, x_{44}, x_{45}\}$ be a sample of size five drawn from the population in Narathiwat (population X_4).

Table 1: Neutrosophic values for five observations per province

Province	**Membership** ($\mathcal{T}$)	**Indeterminacy** ($\mathcal{I}$)	**Non-membership** ($\mathcal{F}$)
Songkhla	0.90	0.18	0.15
	0.92	0.16	0.14
	0.91	0.17	0.12
	0.93	0.16	0.13
	0.92	0.18	0.13
Pattani	0.90	0.18	0.15
	0.91	0.15	0.16
	0.91	0.17	0.12
	0.93	0.16	0.13
	0.92	0.18	0.13
Yala	0.90	0.18	0.15
	0.92	0.16	0.14
	0.90	0.17	0.12
	0.93	0.16	0.13
	0.92	0.19	0.15
Narathiwat	0.90	0.18	0.15
	0.92	0.16	0.14
	0.92	0.17	0.12
	0.93	0.18	0.13
	0.92	0.18	0.12

Since the neutrosophic components satisfy

$$1 \leq \mathcal{T}(x) + I(x) + F(x) \leq 3,$$

we assume that the membership, indeterminacy, and non-membership values in populations X_i are mutually independent.

In this example, the overall null hypothesis H_0 states that the mean membership, indeterminacy, and non-membership values for students from Songkhla, Pattani, Yala, and Narathiwat are equal. That is,

$$\overline{\mathcal{T}}(A, X_1) = \overline{\mathcal{T}}(A, X_2) = \overline{\mathcal{T}}(A, X_3) = \overline{\mathcal{T}}(A, X_4),$$

$$\overline{\mathcal{I}}(A, X_1) = \overline{\mathcal{I}}(A, X_2) = \overline{\mathcal{I}}(A, X_3) = \overline{\mathcal{I}}(A, X_4),$$

$$\overline{\mathcal{F}}(A, X_1) = \overline{\mathcal{F}}(A, X_2) = \overline{\mathcal{F}}(A, X_3) = \overline{\mathcal{F}}(A, X_4).$$

The corresponding alternative hypothesis H_A asserts that at least one of these components differs among the four provinces.

The sample means of the membership, indeterminacy, and non-membership values for the student populations from Songkhla, Pattani, Yala, and Narathiwat are as follows:

$$\overline{\mathcal{T}}_A(X_1) = 0.916,\ \overline{\mathcal{T}}_A(X_2) = 0.914,\ \overline{\mathcal{T}}_A(X_3) = 0.914,\ \overline{\mathcal{T}}_A(X_4) = 0.918$$

$$\overline{\mathcal{I}}_A(X_1) = 0.170,\ \overline{\mathcal{I}}_A(X_2) = 0.168,\ \overline{\mathcal{I}}_A(X_3) = 0.172,\ \overline{\mathcal{I}}_A(X_4) = 0.174$$

$$\overline{\mathcal{F}}_A(X_1) = 0.134,\ \overline{\mathcal{F}}_A(X_2) = 0.138,\ \overline{\mathcal{F}}_A(X_3) = 0.138,\ \overline{\mathcal{F}}_A(X_4) = 0.132.$$

Total sum of squares for the membership, indeterminacy and non-membership values are given as follows:

$$SST_{\mathcal{T}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathcal{T}_A(x_{ij}) - \overline{\mathcal{T}}_A(X))^2 = 0.002295,$$

$$SST_{\mathcal{I}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathcal{I}_A(x_{ij}) - \overline{\mathcal{I}}_A(X))^2 = 0.00218,$$

$$SST_{\mathcal{F}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathcal{F}_A(x_{ij}) - \overline{\mathcal{F}}_A(X))^2 = 0.003095.$$

Sum of squares between groups for the membership, indeterminacy and non-membership values are given as follows:

$$SSB_{\mathcal{T}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{T}}_A(X_i) - \overline{\mathcal{T}}_A(X))^2 = 0.000055,$$

$$SSB_{\mathcal{I}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{I}}_A(X_i) - \overline{\mathcal{I}}_A(X))^2 = 0.00010,$$

$$SSB_{\mathcal{F}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{F}}_A(X_i) - \overline{\mathcal{F}}_A(X))^2 = 0.000135.$$

Sum of square within groups for the membership, indeterminacy and non-membership values are given as follow:

$$SSW_{\mathcal{T}} = SST_{\mathcal{T}} - SSB_{\mathcal{T}} = 0.002240,$$

$$SSW_{\mathcal{I}} = SST_{\mathcal{I}} - SSB_{\mathcal{I}} = 0.00208,$$

$$SSW_{\mathcal{F}} = SST_{\mathcal{F}} - SSB_{\mathcal{F}} = 0.002960.$$

The mean square between groups for the the membership, indeterminacy and non-membership values are given as follows:

$$MSB_{\mathcal{T}} = \frac{SSB_{\mathcal{T}}}{k-1} = 1.833 \times 10^{-5},$$

$$MSB_{\mathcal{I}} = \frac{SSB_{\mathcal{I}}}{k-1} = 3.333 \times 10^{-5},$$

$$MSB_{\mathcal{F}} = \frac{SSB_{\mathcal{F}}}{k-1} = 0.000045.$$

Additionally, the mean square within groups for the membership, indeterminacy and non-membership values are given as follow:

$$MSW_{\mathcal{T}} = \frac{SSW_{\mathcal{T}}}{n-k} = 1.400 \times 10^{-4},$$

$$MSW_{\mathcal{I}} = \frac{SSW_{\mathcal{I}}}{n-k} = 1.300 \times 10^{-4},$$

$$MSW_{\mathcal{F}} = \frac{SSW_{\mathcal{F}}}{n-k} = 0.000185.$$

Three one-way ANOVA tables for the membership, indeterminacy, and non-membership values are presented below:

ANOVA Table: Membership Function

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.000055	1.833×10^{-5}	0.131
Within groups	16	0.002240	1.400×10^{-4}	
Total	19	0.002295		

ANOVA Table: Indeterminacy Function

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.000100	3.333×10^{-5}	0.256
Within groups	16	0.002080	1.300×10^{-4}	
Total	19	0.002180		

ANOVA Table: Non-membership Function

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	3	0.000135	0.000045	0.243
Within groups	16	0.002960	0.000185	
Total	19	0.003095		

Finally, the overall test statistic is given by

$$F = \max\{0.131, 0.256, 0.243\} = 0.256.$$

Since $F = 0.256$, which is less than the critical value $F_{0.05,\ 3,\ 16} = 3.238872$, we conclude that there is no statistically significant evidence to reject the null hypothesis H_0. This indicates that the mean membership, indeterminacy, and non-membership values for students from Songkhla, Pattani, Yala, and Narathiwat do not differ significantly.

For comparison, we also construct the classical ANOVA table. Because classical ANOVA can only handles crisp single responses, in this example, we restrict the analysis to the membership values $\mathcal{T}_A(x_{ij})$. The resulting classical one-way ANOVA table is presented below:

ANOVA Table (Membership Function)

Source of Variation	**df**	**Sum of Squares**	**Mean Square**	**F**
Between groups	3	0.000055	1.833×10^{-5}	0.131
Within groups	16	0.002240	1.400×10^{-4}	
Total	19	0.002295		

Since $F = 0.131$, which is less than the critical value $F_{0.05,\ 3,\ 16} = 3.238872$, the null hypothesis H_0 is not rejected. Thus, the classical ANOVA leads to the same conclusion as the neutrosophic ANOVA in this example: there is no significant difference among the population means. An important observation is that the F-value from the classical ANOVA is smaller than the corresponding maximum F from the neutrosophic framework, suggesting that in other situations the classical ANOVA may be less sensitive in detecting differences means.

Example 4.4. Let X_1, X_2, X_3, and X_4 represent four populations, where each X_i denotes the set of all final-year high school students in the provinces of Songkhla, Phuket, Krabi, and Trang, respectively.

We define the neutrosophic set $\mathcal{A}$ on these populations as follows. The membership function $\mathcal{T}$ represents the degree to which students intend to apply for admission to the Mathematics major within the Faculty of Science at Prince of Songkla University. The indeterminacy function $\mathcal{I}$ captures students' uncertainty about applying specifically to this major. The non-membership function $\mathcal{F}$ reflects the degree to which students do not intend to apply for admission to the program.

We assume that for each group X_i, the membership values $\mathcal{T}_A(x_{ij})$, indeterminacy values $\mathcal{I}_A(x_{ij})$, and non-membership values $\mathcal{F}_A(x_{ij})$, where $j = 1, 2, \ldots, n_i$, are independently and normally distributed. Furthermore, the variances of each component are assumed to be equal across all groups.

We now proceed to test whether the membership, indeterminacy, and non-membership functions, representing the intentions and uncertainties of the four populations, differ significantly across groups.

Let $S_1 = \{x_{11}, x_{12}, x_{13}, x_{14}, x_{15}\}$ be a sample of size five drawn from the population of final-year high school students in Songkhla (population X_1). Let $S_2 = \{x_{21}, x_{22}, x_{23}, x_{24}, x_{25}\}$ be a sample of size five drawn from the population in Phuket (population X_2). Let $S_3 = \{x_{31}, x_{32}, x_{33}, x_{34}, x_{35}\}$ be a sample of size five drawn from the population in Krabi (population X_3). Let $S_4 = \{x_{41}, x_{42}, x_{43}, x_{44}, x_{45}\}$ be a sample of size five drawn from the population in Trang (population X_4).

Table 2 presents the membership, indeterminacy, and non-membership values for the four samples, with respect to the neutrosophic set $\mathcal{A}$.

Table 2: Neutrosophic values for five observations per province

Province	**Membership** ($\mathcal{T}$)	**Indeterminacy** ($\mathcal{I}$)	**Non-membership** ($\mathcal{F}$)
Songkhla	0.90	0.05	0.05
	0.92	0.04	0.04
	0.91	0.08	0.02
	0.93	0.04	0.03
	0.92	0.09	0.03
Phuket	0.83	0.18	0.13
	0.83	0.16	0.12
	0.82	0.17	0.12
	0.81	0.16	0.13
	0.83	0.18	0.13
Krabi	0.81	0.17	0.13
	0.82	0.18	0.12
	0.83	0.17	0.14
	0.82	0.16	0.13
	0.84	0.18	0.12
Trang	0.83	0.17	0.13
	0.84	0.17	0.12
	0.81	0.16	0.11
	0.83	0.18	0.13
	0.80	0.18	0.13

Since the neutrosophic components satisfy

$$1 \leq \mathcal{T}(x) + I(x) + F(x) \leq 3,$$

we assume that the membership, indeterminacy, and non-membership values in populations X_i are mutually independent.

In this example, the overall null hypothesis H_0 states that the mean membership, indeterminacy, and non-membership values for students from Songkhla, Phuket, Krabi, and Trang are equal. That is,

$$\overline{\mathcal{T}}(A, X_1) = \overline{\mathcal{T}}(A, X_2) = \overline{\mathcal{T}}(A, X_3) = \overline{\mathcal{T}}(A, X_4),$$

$$\overline{\mathcal{I}}(A, X_1) = \overline{\mathcal{I}}(A, X_2) = \overline{\mathcal{I}}(A, X_3) = \overline{\mathcal{I}}(A, X_4),$$

$$\overline{\mathcal{F}}(A, X_1) = \overline{\mathcal{F}}(A, X_2) = \overline{\mathcal{F}}(A, X_3) = \overline{\mathcal{F}}(A, X_4).$$

The corresponding alternative hypothesis H_A asserts that at least one of these components differs among the four provinces.

The sample means of the membership, indeterminacy, and non-membership values for the student populations from Songkhla, Phuket, Krabi, and Trang are as follows:

$$\overline{\mathcal{T}}_A(X_1) = 0.916,\ \overline{\mathcal{T}}_A(X_2) = 0.824,\ \overline{\mathcal{T}}_A(X_3) = 0.824,\ \overline{\mathcal{T}}_A(X_4) = 0.822,$$

$$\overline{\mathcal{I}}_A(X_1) = 0.060,\ \overline{\mathcal{I}}_A(X_2) = 0.170,\ \overline{\mathcal{I}}_A(X_3) = 0.172,\ \overline{\mathcal{I}}_A(X_4) = 0.172,$$

$$\overline{F}_A(X_1) = 0.046,\ \overline{F}_A(X_2) = 0.126,\ \overline{F}_A(X_3) = 0.128,\ \overline{F}_A(X_4) = 0.124.$$

Total sum of squares for the membership, indeterminacy and non-membership values are given as follows:

$$SST_{\mathcal{T}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{T}_A(x_{ij}) - \overline{\mathcal{T}}_A(X))^2 = 0.03466,$$

$$SST_{\mathcal{I}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathcal{I}_A(x_{ij}) - \overline{\mathcal{I}}_A(X))^2 = 0.04966,$$

$$SST_{\mathcal{F}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathcal{F}_A(x_{ij}) - \overline{\mathcal{F}}_A(X))^2 = 0.02528.$$

Sum of squares between groups for the membership, indeterminacy and non-membership values are given as follows:

$$SSB_{\mathcal{T}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{T}}_A(X_i) - \overline{\mathcal{T}}_A(X))^2 = 0.03222,$$

$$SSB_{\mathcal{I}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{I}}_A(X_i) - \overline{\mathcal{I}}_A(X))^2 = 0.04650,$$

$$SSB_{\mathcal{F}} = \sum_{i=1}^{k} n_i(\overline{\mathcal{F}}_A(X_i) - \overline{\mathcal{F}}_A(X))^2 = 0.02404.$$

Sum of square within groups for the membership, indeterminacy and non-membership values are given as follow:

$$SSW_{\mathcal{T}} = SST_{\mathcal{T}} - SSB_{\mathcal{T}} = 0.00244,$$

$$SSW_{\mathcal{I}} = SST_{\mathcal{I}} - SSB_{\mathcal{I}} = 0.00316,$$

$$SSW_{\mathcal{F}} = SST_{\mathcal{F}} - SSB_{\mathcal{F}} = 0.00124.$$

The mean square between groups for the the membership, indeterminacy and non-membership value are given as follows:

$$MSB_{\mathcal{T}} = \frac{SSB_{\mathcal{T}}}{k-1} = 0.010738,$$

$$MSB_{\mathcal{I}} = \frac{SSB_{\mathcal{I}}}{k-1} = 0.000197,$$

$$MSB_{\mathcal{F}} = \frac{SSB_{\mathcal{F}}}{k-1} = 0.008013.$$

Additionally, the mean square within groups for the membership, indeterminacy and non-membership values are given as follow:

$$MSW_{\mathcal{T}} = \frac{SSW_{\mathcal{T}}}{n-k} = 0.000152,$$

$$MSW_{\mathcal{I}} = \frac{SSW_{\mathcal{I}}}{n-k} = 0.000067,$$

$$MSW_{\mathcal{F}} = \frac{SSW_{\mathcal{F}}}{n-k} = 0.000078.$$

Three one-way ANOVA tables for the membership, indeterminacy, and non-membership values are presented below:

ANOVA Table: Membership Function

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.03222	0.010738	70.42
Within groups	16	0.00244	0.000152	
Total	19	0.03466		

ANOVA Table: Indeterminacy Function

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.04650	0.015498	78.47
Within groups	16	0.00316	0.000197	
Total	19	0.04966		

ANOVA Table: Non-membership Function

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.02404	0.008013	103.4
Within groups	16	0.00124	0.000078	
Total	19	0.02528		

Finally, the overall test statistic is given by

$$F = \max\{70.42, 78.47, 103.4\} = 103.4,$$

Since $F = 103.4$, greater than the critical value $F_{0.05,\ 3,\ 16} = 3.238872$, we conclude that there is statistically significant evidence to reject the null hypothesis H_0. This indicates that the mean membership, indeterminacy, and non-membership values for students from Songkhla, Phuket, Krabi, and Trang differ significantly.

For comparison, we also construct the classical ANOVA table. Because classical ANOVA can only handles crisp single responses, in this example, we restrict the analysis to the membership values $\mathcal{T}_A(x_{ij})$. The resulting classical one-way ANOVA table is presented below:

ANOVA Table (Membership Function)

Source of Variation	df	Sum of Squares	Mean Square	F
Between groups	3	0.03222	0.010738	70.42
Within groups	16	0.00244	0.000152	
Total	19	0.03466		

Since $F = 70.42$, which is greater than the critical value $F_{0.05,\ 3,\ 16} = 3.238872$, the null hypothesis H_0 is rejected. Thus, the classical ANOVA leads to the same conclusion as the neutrosophic ANOVA in this example: there is a significant difference among the population means. An important observation is that the F-value from the classical ANOVA is smaller than the corresponding maximum F from the neutrosophic framework, suggesting that in other situations the classical ANOVA may be less sensitive in detecting differences. means.

Taken together, Examples 4.3 and 4.4 illustrate that the neutrosophic ANOVA is more sensitive than the classical ANOVA, as it considers all three components, membership, indeterminacy, and non-membership, rather than relying solely on crisp data.

5 Limitations

A key limitation of this study lies in the assumption of independence among the three neutrosophic components (membership, indeterminacy, and non-membership). While the bounds

$$1 \leq \mathcal{T}(x) + I(x) + F(x) \leq 3$$

are consistent with the case of independent components in single-valued neutrosophic sets, this assumption is not empirically tested in our examples. In real-world applications, the three components may exhibit dependence, which could alter the distributional properties and the validity of the proposed test. Future research should explore extensions of neutrosophic ANOVA under alternative dependence structures, such as partial correlation among components.

6 Conclusion

In this paper, we have proposed a one-way analysis of variance (ANOVA) framework tailored for neutrosophic sets, specifically addressing the case in which all three components, membership, indeterminacy, and non-membership, are considered independent. The proposed method fundamentally differs from conventional one-way ANOVA, as it is designed to handle data characterized by vagueness, indeterminacy, and inconsistency, which are inherent in neutrosophic representations.

We have also outlined the corresponding decision rules for hypothesis testing within this framework. The proposed tests offer a systematic and practical approach for decision-makers to draw valid conclusions in the presence of uncertainty, thereby extending the applicability of statistical inference to neutrosophic data contexts. The neutrosophic ANOVA is more sensitive in detecting group differences, particularly in cases where the classical approach yields smaller F-values and may fail to reject the null hypothesis. These findings highlight the potential of neutrosophic ANOVA as a more robust alternative to classical ANOVA for analyzing data with inherent uncertainty and indeterminacy.

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