Modeling Legal Integrity Using Plithogenic n-SuperHyperGraphs: A Multidimensional Representation of Moral Coherence in Dworkin's Theory

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Abstract

This project aims to concretize Ronald Dworkin's theory of legal integrity via Plithogenic n-SuperHyperGraphs. Therefore it investigates how such mathematical entities metaphorically and multidimensionally formulate moral coherence in legal interpretation. Using a mixed-method approach, this work will assess documents through a documentary assessment of Dworkin's written works (Law's Empire and Taking Rights Seriously) to formulate a Plithogenic n-SuperHyperGraph of a case study featuring n-dimensional nodes as moral principles, moral assertions, and past decisions with hyperedges symbolizing the relationship between them generated by degrees of truth, falsity, or indeterminacy. Tools of graph visualization and neutrosophic computing will provide the legal assessment of characterization for coherence. The results will discuss whether the model intentionally visualizes the connections among the principles and how it assessed which characterizations would make the law most morally coherent under Dworkin's theory while acknowledging the indeterminacy in certain complicated cases. Thus, this study seeks to find correlations between which nodes function as the primary principles consistent with Dworkin's metaphor of the law's "chain." Ultimately, this research intends to present Plithogenic n-SuperHyperGraphs as a viable application to formally express Dworkin's theory for the sake of more moral legal determinations applicable to legal education or judicial assistive software, although generalizability will require cross-jurisdictional applications of the model.

Keywords: Dworkin; Legal Integrity; N- Superhypergraphs; Moral Coherence; Judicial Interpretation; Neutrosophic; Plithogenic

1. Introduction

Dworkin's theory of law as integrity, poses a significant challenge: how can judges ensure coherent and morally justified decisions in complex cases where legal norms do not offer clear answers? Dworkin proposes that law should be understood as a system of moral principles that transcend the mechanical application of rules, emphasizing moral consistency and fairness [1]. However, the complexity of court cases, with multiple conflicting principles and varying social contexts, introduces uncertainty that hampers the practical application of this theory. This problem is exacerbated in jurisdictions where precedents and norms are not clearly aligned, leading to inconsistent interpretations. The lack of formal tools to model the interaction between principles, norms and precedents represents a limitation in the implementation of Dworkin's theory. The central question of this research is how to formalize moral consistency in legal interpretation, considering the uncertainty inherent in complex cases. This approach seeks to address the need for an analytical framework that facilitates judicial decision-making aligned with Dworkin's ideals. The integration of advanced mathematical approaches could offer a solution to this challenge. The problem, therefore, lies in the absence of structured methods for multidimensionally representing the network of principles underlying legal integrity.

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Dworkin’s theory from philosophical and legal perspectives, analyzing his critique of legal positivism and his emphasis on constructive interpretation [2]. For example, authors such as Alexy have compared Dworkin’s theory with Hart’s positivism, highlighting the importance of moral principles in adjudication [3]. However, these investigations usually focus on theoretical debates, leaving aside the practical formalization of moral coherence. Other works have examined the application of the theory in specific judicial cases, such as constitutional disputes, but lack quantitative tools to model the interaction of principles [4]. In the mathematical field, neutrosophic and plitogenic approaches, such as those proposed by Smarandache, have proven useful for modeling complex systems with uncertainty, although their application to law is incipient. The limitations of these studies lie in their one-dimensional approach, which does not capture the multidimensionality of the relationships between principles, norms and precedents. Furthermore, the lack of integration between legal philosophy and applied mathematics creates a significant gap. This research seeks to fill this gap by using Plithogenic n-SuperHyperGraphs to model legal integrity. The background shows that, to date, no formal framework combining Dworkinian theory with advanced mathematical tools has been proposed. This gap justifies the need to explore interdisciplinary approaches.

The relevance of this study lies in its potential to transform the way judges approach complex cases, aligning with Dworkin's vision of a coherent and morally justified legal system. In a global context where legal systems face increasing challenges, such as human rights conflicts or constitutional disputes, the ability to formalize moral coherence is crucial [5]. The uncertainty inherent in these cases requires tools that can handle multiple dimensions and degrees of indeterminacy. Plithogenic n-SuperHyperGraphs, by allowing the representation of complex relationships with truth, falsity, and indeterminacy, offer an innovative solution. This approach not only has theoretical implications, by advancing the formalization of Dworkin 's theory , but also practical ones, by providing a framework that could be integrated into judicial decision support tools. The relevance extends to legal training, where judges could benefit from structured methods to evaluate interpretations. Furthermore, this study contributes to the interdisciplinary dialogue between law and applied mathematics, opening new lines of research. In a world where justice must adapt to dynamic social contexts, the proposal of a mathematical model for legal integrity is timely. The need for this approach is accentuated in jurisdictions with evolving legal systems, such as those in Latin America [6].

The primary objective of this study is to develop a model based on Plitogenic n-SuperHyperGraphs to multidimensionally represent moral coherence in legal interpretation according to Dworkin’s theory. Specifically, it seeks to construct a graph that integrates moral principles, legal norms, and precedents, capturing their relationships under uncertainty. A secondary objective is to apply this model to an emblematic court case to evaluate its ability to identify coherent interpretations. The hypothesis is that Plitogenic n-SuperHyperGraphs allow formalizing the network of principles underlying legal integrity, offering a more accurate representation of moral coherence than traditional methods. The model is expected to reveal key nodes (dominant principles) and indeterminate connections, facilitating the alignment of judicial decisions with the meta- chain of law. Furthermore, it is hypothesized that the model will be applicable in diverse legal contexts, providing a versatile tool for judges and scholars. This approach combines legal philosophy with neutrosophic logic, seeking to overcome the limitations of traditional qualitative methods. Validating the model in a case study will allow for an assessment of its usefulness in judicial practice. Ultimately, the study seeks to contribute to a more equitable and coherent legal practice, aligned with Dworkin’s ideals [7]. This interdisciplinary framework has the potential to redefine legal interpretation in complex environments.

The integration of Plithogenic n-SuperHyperGraphs into law responds to the need to address the uncertainty inherent in judicial interpretation. Unlike traditional approaches, which typically rely on strict textual analysis or precedent, this model allows for the representation of multidimensional relationships between legal elements. Dworkin’s theory, with its emphasis on moral coherence, requires an approach that goes beyond the mechanical application of rules, and n-SuperHyperGraphs offer that flexibility. By incorporating degrees of truth, falsity, and indeterminacy, this method captures the complexity of cases where principles are in conflict. For example, in constitutional disputes, where individual and collective rights may collide, the model can map the tensions between principles of justice and equity. This approach is particularly relevant in contexts where globalization and societal changes demand adaptive legal interpretations. The proposal of this study not only enriches Dworkin’s theory but also opens the door to practical applications in courts and academia.

The use of mathematical tools in law is not new, but their application to Dworkin’s legal philosophy is innovative. Plithogenic n-SuperHyperGraphs, developed within the framework of neutrosophic logic, have proven effective in other fields, such as engineering and decision-making, but their potential in law remains unexplored. This study represents a pioneering effort to connect the philosophy of law with advanced mathematics. The ability of graphs to visualize complex relationships offers a significant advantage over qualitative methods, which often lack precision in contexts of uncertainty. By modeling Dworkin’s "chain of law" as a graph, one can analyze how current

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decisions align with past and future principles. This visual representation facilitates the understanding of moral consistency, a central concept in Dworkin's theory.

The methodology proposed in this study combines documentary analysis with mathematical modeling, ensuring a rigorous approach. Selecting an emblematic court case will allow testing the model's applicability in a real-life context. For example, a fundamental rights case in a Latin American jurisdiction could serve as a testbed for the graph. Identifying nodes (principles, norms, precedents) and their connections will allow assessing whether the model meets Dworkin's ideals of consistency and fairness. Furthermore, the use of specialized graph software, adapted to plithogenic logic, will ensure the accuracy of the analysis. This interdisciplinary approach not only validates Dworkin's theory but also operationalizes it in judicial practice.

The implications of this study go beyond the academic sphere, as the proposed model could be integrated into judicial decision support tools. Judges, when faced with complex cases, could use the graph to visualize the relationships between principles and precedents, facilitating more coherent interpretations. This is especially relevant in legal systems where judicial discretion is high, such as in common law or jurisdictions with broad constitutions. The formalization of legal integrity could also influence the training of judges, providing a structured framework for teaching constructive interpretation. In this sense, the study responds to the need for innovative methods in an increasingly complex legal world.

The current context, marked by debates on global justice and human rights, underscores the urgency of approaches such as the one proposed. Dworkin's theory, although criticized for its idealism, offers valuable guidance for aligning law with shared moral principles. However, its practical application requires tools that manage the uncertainty and multidimensionality of real cases. Plithogenic n-SuperHyperGraphs, by integrating neutrosophic logic, provide a robust solution to this challenge. This study not only advances the formalization of Dworkin's theory, but also sets a precedent for the integration of mathematics and law.

In summary, this research proposes a bridge between legal philosophy and applied mathematics, using Plithogenic n-SuperHyperGraphs to model legal integrity. By addressing the uncertainty and complexity of judicial interpretation, the study offers an innovative tool for judges, academics, and legal practitioners. The combination of Dworkinian theory with a rigorous mathematical approach has the potential to transform legal practice, promoting more equitable and consistent decisions in a constantly changing world.

2. Related Word

The theory of law as integrity, formulated by Ronald Dworkin, establishes that legal interpretation should reflect a moral coherence that integrates ethical principles, legal norms, and precedents into a unified system. This theoretical framework, developed primarily in *Law's Empire* (1986) argues that judges should seek interpretations that present the law in its best moral light, overcoming the mechanical application of positivist rules. Moral coherence, as a central concept, demands that judicial decisions be consistent with the principles of justice and equity that underlie a community's legal practice. The figure of "Judge Hercules," an ideal of adjudication with exhaustive knowledge, illustrates how judges must balance principles in complex cases. In contemporary legal contexts, characterized by ethical dilemmas such as conflicts over fundamental rights, moral coherence is essential to guarantee equitable decisions. This study analyzes the validity of this principle in Dworkin's theory, exploring its application in modern jurisprudence and the challenges it faces in pluralistic environments. It is argued that moral coherence is not only a theoretical ideal, but also an operational criterion for adjudication in contexts of uncertainty. The research is based on a systematic review of recent literature and court cases. It is proposed that modern analytical tools can strengthen the implementation of this principle. The analysis seeks to contribute to the debate on legal interpretation in diverse legal systems.

Recent literature on Dworkin's theory underlines its relevance to legal philosophy, but also identifies limitations in its practical application. Studies have examined how moral coherence faces challenges in pluralistic legal systems, where community values vary significantly [8]. For example, analyses of constitutional cases in jurisdictions such as the United States and Europe show that judges struggle to balance conflicting principles, such as individual liberty versus collective well-being, which questions the universality of moral coherence [9]. Recent research has explored the application of Dworkin's theory in non-Anglo-Saxon contexts, such as Latin America, highlighting the need to adapt the concept to diverse cultural realities [10]. However, the lack of formal tools to model the interaction between principles, norms, and precedents remains a significant gap. Traditional approaches, based on qualitative analysis, do not fully capture the complexity of modern court cases. Recent advances in mathematical modeling, such as those based on neutrosophic logic; suggest possible solutions to formalize moral coherence [11]. These tools allow for the representation of multidimensional relationships with uncertainty, which could complement Dworkin's theory. Current literature indicates that the integration of interdisciplinary approaches is necessary

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to advance the practical application of moral coherence. This study addresses this gap through a systematic analysis of the theory and its implementation.

Moral coherence, according to Dworkin, implies that judicial decisions should form a coherent narrative that reflects the ethical principles of the community. This principle is distinguished from legal positivism, which prioritizes the strict application of legislated norms [12]. The distinction between rules, which operate in an "all or nothing" framework, and principles, which have relative weight and require balancing, is fundamental to this approach. In complex cases, such as those involving human rights, moral coherence allows judges to integrate precedents and norms into an interpretation that maximizes justice. Recent studies have analyzed how constitutional courts apply principles similar to those proposed by Dworkin, although often implicitly [13]. For example, in disputes over reproductive rights or gender equality, judges have turned to ethical principles to resolve legal ambiguities. However, subjectivity in identifying dominant principles poses challenges to moral coherence. The literature suggests that the lack of objective criteria for assessing coherence limits the applicability of the theory in pluralistic contexts. This study proposes that advanced analytical methods, such as graph-based models, could offer a solution. A review of recent court cases illustrates the relevance of this approach.

Dworkin's theory requires overcoming the limitations of traditional qualitative methods. In the last five years, research has explored how judicial decisions do, or do not, reflect Dworkin's principles of justice and fairness [14]. For example, a study of the case law of the European Court of Human Rights found that judges often seek moral consistency, but lack systematic tools to assess the interactions between principles [8]. This challenge is particularly evident in cases involving conflicts between fundamental rights, such as privacy versus public safety. Dworkin's theory, although robust in its theoretical framework, does not provide an explicit method to resolve these tensions. Recent advances in modeling complex systems, such as hyperbasic graphs, offer a promising approach to formalizing moral consistency. These models make it possible to represent the relationships between principles, norms, and precedents, incorporating degrees of uncertainty. Recent literature suggests that such tools could complement Dworkin's theory, making its application more rigorous. This study analyzes how these approaches can be integrated into judicial practice. A review of landmark cases provides empirical evidence of their viability.

Dworkin's theory represents an ideal of adjudication that combines exhaustive legal knowledge with a keen moral sensibility. This theoretical construct assumes that judges can identify the interpretation that best aligns current decisions with the underlying principles of the community. However, recent research has questioned the viability of this ideal in real-life contexts, where judges face constraints of time, information, and social consensus [9]. Empirical studies on judicial decision-making show that judges tend to prioritize precedents over principles in complex cases, which can compromise moral coherence [10]. Recent literature suggests that integrating computational tools could mitigate these limitations by providing judges with a structured framework for evaluating interpretations. For example, models based on neutrosophic logic make it possible to represent the uncertainty inherent in conflicts of principles. This approach is particularly relevant in jurisdictions with evolving legal systems, such as those in Latin America, where moral coherence can serve as an anchor for equitable decisions [11]. The systematic literature review underscores the need for formal methods to implement Dworkin's theory. This study explores this possibility through a comparative analysis.

Dworkin's theory emphasizes continuity in legal interpretation, with each decision contributing to a coherent narrative. This idea implies that judges must align current decisions with historical principles and precedents, ensuring that the legal system evolves consistently. Recent studies have analyzed how this metaphor is applied in constitutional courts, particularly in social rights cases [12]. For example, in rulings on access to healthcare or education, courts have invoked equity principles to justify decisions that transcend written norms. However, the literature points out that a lack of consensus on which principles are dominant can lead to inconsistencies. Advances in mathematical modeling, such as plithogenic graphs, offer an approach to mapping these relationships and assessing their coherence [13]. These models represent principles as nodes and their interactions as edges, capturing the multidimensionality of court cases. Empirical evidence suggests that such tools can improve transparency in decision-making. This study examines how these innovations can be applied to Dworkin's theory. A review of recent cases illustrates their practical potential.

Dworkin's theory not only has theoretical but also practical implications for judicial practice. In recent years, research has explored how judges apply moral principles in contexts of high uncertainty, such as those related to artificial intelligence or bioethics [14]. These studies show that moral coherence is difficult to achieve when community values are fragmented. Recent literature suggests that the integration of interdisciplinary approaches, such as legal informatics, can provide solutions. For example, graph-based decision support algorithms can help judges visualize the relationships between principles and precedents, facilitating coherent interpretations. This approach is particularly relevant in jurisdictions where judicial discretion is high, such as in common law. The systematic

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review of the literature indicates that Dworkin's theory remains a valuable framework, but requires modern tools for its effective implementation. This study proposes that mathematical models can bridge this gap, offering a rigorous method for assessing moral coherence. Evidence from recent court cases supports this claim.

Dworkin's theory lie in its reliance on judicial interpretation, which can be influenced by subjective factors. Recent studies have analyzed how cognitive biases affect the application of moral principles in adjudication [8]. For example, in human rights cases, judges may prioritize personal values over community principles, compromising consistency. The literature suggests that formal methods, such as those based on neutrosophic logic, can mitigate these biases by providing an objective framework for evaluating interpretations [9]. These methods represent principles as nodes in a graph, allowing analysis of their relative weight and interactions. Empirical evidence from international tribunals shows that principle-based decisions tend to be more consistent when structured criteria are used. This study explores how these tools can be integrated into judicial practice to improve moral consistency. The review of recent literature highlights the need for interdisciplinary approaches to address these challenges.

The application of moral coherence in pluralistic contexts raises questions about its universality. Recent studies have examined how non-Western legal systems, such as those in Latin America, interpret the principles of justice and fairness [10]. These studies suggest that Dworkin's theory can be adapted to diverse cultural contexts, but requires adjustments to reflect local values. Recent literature indicates that computational models can facilitate this adaptation by representing the diversity of principles in a multidimensional graph [11]. For example, in indigenous rights cases, graphs can map tensions between traditional and modern principles, promoting coherent interpretation. Empirical evidence from recent rulings in Latin American courts supports this possibility. This study analyzes how Dworkin's theory can benefit from these innovations, offering a more inclusive approach to moral coherence. A systematic literature review provides a solid foundation for this analysis.

In conclusion, moral coherence in Dworkin's theory remains a fundamental principle for legal interpretation, but it faces challenges in its practical application. A systematic review of the literature and analysis of recent court cases demonstrate that judges seek to align their decisions with ethical principles, although they often lack formal tools to ensure consistency. Advances in mathematical modeling, such as plithogenic graphs, offer a promising approach to overcome these limitations, allowing the complexity of court cases to be represented. This study highlights the importance of integrating interdisciplinary approaches to strengthen Dworkin's theory in modern contexts. Practical implications include improving judicial decision-making and legal education. Future research should explore the application of these models in diverse jurisdictions and develop computational tools to support judges. Moral coherence, as an ideal and practice, remains a cornerstone of equitable justice.

3. Materials and methods

This section contains two subsections; the first one is dedicated to explaining the basic notions of the n- Plithogenic SuperHyperGraphs defined in [15]. Then, subsection 2.2 contains the main concepts of multiway contingency tables and the log-linear method.

3.1. n- Plitogenic superhypergraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision making in [15].

First, an n- SuperHyperGraph is defined as follows [24-26]:

Given $V = \{V_1, V_2, \cdots, V_m\}$, where $1 \leq m \leq \infty$ is a set of vertices, containing *simple vertices* that are classical, *indeterminate vertices* that are unclear, vague, partially known, and *null vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-potential set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P(P(V))$, $P^3(V) = P(P^2(V))$, … , $P^n(V) = P(P^{n-1}(V))$, for $1 \leq n \leq \infty$. Where is also defined as $P^0(V) = V$.

An n- SuperHyperGraph (*n -SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices as in the real world:

- *Simple vertices* (the classic ones),
- *Indeterminate vertices* (unclear, vague, partially known),
- *Null vertices* (empty, completely unknown),

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- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types grouped together (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is an *(n-1)- SuperVertex* , and the others can be *r- SuperVertex* for $r \leq$ n.

$E_n$contains the following types of borders:

- *Simple edges* (the classic ones),
- *Indeterminate borders* (unclear, vague, partially known),
- *Null edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more individual vertices),
- *SuperEdge* (connecting two vertices, at least one of which is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of which is an n- SuperVertex and may contain another which is an r- SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n- SuperHyperEdge* (contains three or more vertices, at least one of which is an n- SuperVertex and may contain an r- SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

The graphics are classified as follows:

- Graph directed (the classic),
- Undirected graph (the classic one),
- Neutrosophic directed graph (partially directed, partially undirected, partially directed indeterminate).

Within the framework of the theory of n- Plithogenic SuperHyperGraphs, we have the following concepts:

Enclosing vertex: A vertex that represents an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

Super-envelope vertex: A wraparound vertex is composed of SuperHyperEdges.

Dominant enclosing vertex: An enclosing vertex that has dominant attribute values.

Dominant superenvelope vertex: A superenvelope vertex with dominant attribute values.

The dominant enclosing vertex is classified into *input, intervention* and *exit* according to the nature of the representation of the object.

Plithogenic connectors: Connectors associate the input envelope vertex with the output envelope vertex. These connectors associate the effects of input attributes with those of output attributes and are weighted according to the plithogenic weights.

3.2. Multi-way contingency tables

A multivariate contingency table is a contingency table defined for two or more cross-ratio classification variables. Two-dimensional tables are usually referred to as contingency tables, while the term multivariate is applied when the number of variables is at least three [26, 28].

A *generic multivariate table* is defined using$I = I_1 \times I_2 \cdots \times I_q$ as the set of indices for each variable to be studied $X_1, X_2, \cdots, X_q$, such thatI_j is the set of indices corresponding to the possible classifications of the variable j . Therefore,$n_{i_1 i_2 \cdots i_q}$ is the frequency of occurrence of the classifications $i_1, i_2, \cdots, i_q$for each of the corresponding variables.

Partial/conditional tables involve fixing the category of one of the variables. Fixed variables are indicated in parentheses. For example, partial tables $XZ$and $YZ$are indicated by$n_{i(j)k}$ and $n_{(i)jk}$, respectively. Furthermore, the

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partial/conditional probabilities are calculated by $\pi_{\mathrm{ij(k)}} = \pi_{\mathrm{ij/k}} = Prob(X = i, Y = j/Z = k)$. *The partial/conditional proportions* are defined by $p_{\mathrm{ij(k)}} = p_{\mathrm{ij/k}} = \frac{\pi_{\mathrm{ijk}}}{\pi_{\mathrm{++k}}}$ for $k = 1, 2, \dots, K$. Where π_{++k} is the frequency i and j configuration k, for more information see [26, 28].

Next, we briefly explain what log-linear models consist of. To simplify the exposition, we consider the case of the three-way contingency table. If X, Y, and Z are the variables, then the following possible models are obtained [16, 17]:

- Model (X, Y, Z): All variables are considered independent, the model is as follows:

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$ (1)

- Model (X, YZ): Only the YZ association is considered, while X is independent of the other two variables.

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{jk}^{YZ}$ (2)

- Model (XY, YZ): X and Z are independent for each value of Y:

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ}$ (3)

- Model (XY, YZ, XZ): There is a pairwise association between all variables, but there is no joint association between the three.

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}$ (4)

- Model (XYZ): If the above model does not fit the data well, then the association between the three variables should be considered:

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ}$ (5)

To compare two different models, the statistic called *likelihood ratio is used* , which is calculated as:

$$G^2 = 2\sum f ln(f/F) \quad (6)$$

Where f is the observed frequency, and F is the expected frequency based on the model. This statistic is distributed according to a chi -square test under the hypothesis that the model is correct, with degrees of freedom depending on the parameters used to fit the model.

To compare two models, simply subtract their respective G^2 or, in another case, among others, the *Bayesian Information Criterion* is used with the formula:

$$BIC = G^2 - \mathrm{df}\log\mathrm{N} \quad (7)$$

Where df denotes the degree of freedom and N is the total number of cases in the sample.

4. Results

This study focuses on modeling Ronald Dworkin’s theory of legal integrity. To do so, a corpus of emblematic court cases was analyzed, where the tension between norms, precedents, and moral principles is central. Data collection was based on documentary analysis of rulings and legal literature, classifying variables according to their role in judicial reasoning.

Dworkin’s metaphor of the "chain of law," arrives at a coherent decision. The "intervention" in this model is not a physical treatment, but rather the application of Dworkin’s interpretive method to move from an initial analysis of the case to a final resolution that maximizes moral and legal coherence.

The study population consisted of a set of **25 complex court cases** selected from various jurisdictions, where moral principles played a decisive role. The sample was selected by a panel of experts in legal philosophy to ensure its relevance and representativeness. These 25 cases constitute the basis for the statistical analysis, with a 95% confidence level and a 5% margin of error for inferences about the model's applicability.

Inclusion Criteria

- Settled cases in which there is an apparent conflict between the letter of the law and principles of justice or equity.
- Cases that have generated broad academic and judicial debate.

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- Cases that include individual or dissenting opinions that demonstrate interpretive complexity.
- Cases whose resolution depends on the weighing of fundamental rights.

Exclusion Criteria

- Cases resolved by mere mechanical or literal application of the rule.
- Cases of a purely procedural nature without substantive debate on principles.
- Cases with insufficient information or non-public sentences.

Definition of Vertices and Attributes

The input object (IO) in this study is **the Court Case,** representing the selected units of analysis. The Envelope Vertex is structured from the following attributes and subattributes that define the judicial decision framework.

- A1 = Case Characteristics
- A_2 = Consistency with Moral Principles
- A_3 = Consistency with Precedents
- A_4 = Regulatory Adjustment

Attribute Sets:

- **Case Characteristics (A_1)** : {Area of Law (S_{11}), Case Complexity (S_{12}), Jurisdiction (S_{13})}.
- **Consistency with Moral Principles (A_2)** : {Initial Analysis (S_{21}), Final Decision (S_{22})}.
- **Consistency with Precedents (A_3)** : {Initial Analysis (S_{31}), Final Decision (S_{32})}.
- **Regulatory Adjustment (A_4)** : {Initial Analysis (S_{41}), Final Decision (S_{42})}.

Sub- Sub-attributes:

- **Area of Law:** {Constitutional (v_1), Civil (v_2), Criminal (v_3)}.
- **Case Complexity:** {High (v_4), Medium (v_5), Low (v_6)}.
- **Jurisdiction:** {National (v_7), International (v_8)}.
- **Initial Analysis** (for A_2, A_3, A_4): {Incoherent (v_9), Coherent (v_{10}) }.
- **Final Decision** (for A_2, A_3, A_4): {Incoherent (v_{11}), Coherent (v_{12})}.

Case Characteristics attributes and the **Initial Analysis** subattributes are considered input envelope vertices. The **Final Decision** subattributes are the output envelope vertices, representing the outcome of the interpretive process.

Below are tables summarizing the structure and frequencies observed in the sample of 25 cases.

Table 1: Structure of Vertices and Legal Attributes

Vertex	**Vertex Attributes**	**Vertex Subattributes**	**Vertex Sub- Subattributes**
Legal Integrity in Landmark Cases	Case Characteristics (A1)	Area of Law (S_{11})	Constitutional (v_1)
			Civil (v_2)
			Penalty (v_3)
		Case Complexity (S_{12})	High (v_4)
			Average (v_5)
			Low (v_6)
		Jurisdiction (S_{13})	National (v_7)
			International (v_8)
	Consistency with Moral Principles (A_2)	Initial Analysis (S_{21})	Incoherent (v_9)
			Coherent (v_{10})
		Final Decision (S_{22})	Incoherent (v_{11})
			Coherent (v_{12})

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Note: The structure is repeated for "Consistency with Precedents" (A_3) and "Normative Adjustment" (A_4)

Table 2: Absolute Frequency of Variables (N=25)

Vertex	Vertex Attributes	Vertex Subattributes	Sub- Sub-attributes (Frequency)
Case Features (25)	Area of Law (25)	Constitutional	12
		Civil	8
		Penal	5
	Case Complexity (25)	High	10
		Average	9
		Low	6
	Jurisdiction (25)	National	18
		International	7
Consistency with Moral Principles (25)	Initial Analysis (25)	Incoherent	15
		Consistent	10
	Final Decision (25)	Incoherent	3
		Consistent	22
Consistency with Precedents (25)	Initial Analysis (25)	Incoherent	11
		Consistent	14
	Final Decision (25)	Incoherent	2
		Consistent	23
Regulatory Adjustment (25)	Initial Analysis (25)	Incoherent	9
		Consistent	16
	Final Decision (25)	Incoherent	1
		Consistent	24

Statistical Analysis Using Log-Linear Models

To assess the interactions between case characteristics and the evolution of legal coherence, log-linear models were applied. Three-way contingency tables were constructed by combining an input attribute (case characteristic) with a pair of initial/final attributes (the interpretive process).

The objective is to determine whether the improvement in consistency (from initial to final) is independent of case characteristics. The Bayesian Information Criterion (BIC) is used to assess model fit.

BIC formula:

$$BIC = G^2 - df \times ln(n)$$

Where:

- G^2 is the likelihood ratio statistic.
- df are the degrees of freedom of the model.
- n is the total sample size (n=25).

We first calculate the term ln (n):

$$ln(25) \approx 3.2188758248682006$$

Detailed calculations for each proposed model are presented below. For each 3-way model (X, Y, Z) with dimensions I, J, K, the mutual independence model (X, Y, Z) is evaluated. The degrees of freedom (df) for this model are calculated as:

$$df = (I \times J \times K) - I - J - K + 2$$

Detailed Calculation of the Models

Model 1: Area of Law × Initial Coherence × Final Coherence

- **Variables:** X = Area (I=3), Y = Initial Coherence (J=2), Z = Final Coherence (K=2).
- **Degrees of Freedom (df):** $(3 \times 2 \times 2) - 3 - 2 - 2 + 2 = 12 - 7 + 2 = 7$.

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- **G^2 value (obtained from model fitting):** 21.483104.
- **BIC Calculation:**

$$BIC_1 = 21.483104 - (7 \times 3.2188758248682006)$$
$$BIC_1 = 21.483104 - 22.532130774077404$$
$$BIC_1 = -1.049026774077404$$

Model 2: Case Complexity × Initial Coherence × Final Coherence

- **Variables:** X = Complexity (I=3), Y = Initial Coherence (J=2), Z = Final Coherence (K=2).
- **Degrees of Freedom(df):** $(3 \times 2 \times 2) - 3 - 2 - 2 + 2 = 7$.
- **WorthG^2:** 19.882541.
- **BIC Calculation:**

$$BIC_2 = 19.882541 - (7 \times 3.2188758248682006)$$
$$BIC_2 = 19.882541 - 22.532130774077404$$
$$BIC_2 = -2.649589774077404$$

Model 3: Jurisdiction × Initial Coherence × Final Coherence

- **Variables:** X = Jurisdiction (I=2), Y = Initial Coherence (J=2), Z = Final Coherence (K=2).
- **Degrees of Freedom(df):** $(2 \times 2 \times 2) - 2 - 2 - 2 + 2 = 8 - 6 + 2 = 4$.
- **WorthG^2:** 14.115932.
- **BIC Calculation:**

$$BIC_3 = 14.115932 - (4 \times 3.2188758248682006)$$
$$BIC_3 = 14.115932 - 12.875503299472802$$
$$BIC_3 = 1.240428700527198$$

Model 4: Area of Law × Consistency with Precedents Initial × Final

- **Variables:** X = Area (I=3), Y = Initial Consistency (J=2), Z = Final Consistency (K=2).
- **Degrees of Freedom (df):** 7.
- **WorthG^2:** 23.917456.
- **BIC Calculation:**

$$BIC_4 = 23.917456 - (7 \times 3.2188758248682006)$$
$$BIC_4 = 23.917456 - 22.532130774077404$$
$$BIC_4 = 1.385325225922596$$

Model 5: Case Complexity × Initial × Final Regulatory Adjustment

- **Variables:** X = Complexity (I=3), Y = Initial Adjustment (J=2), Z = Final Adjustment (K=2).
- **Degrees of Freedom (df):** 7.
- **WorthG^2:** 20.051129.
- **BIC Calculation:**

$$BIC_5 = 20.051129 - (7 \times 3.2188758248682006)$$
$$BIC_5 = 20.051129 - 22.532130774077404$$
$$BIC_5 = -2.481001774077404$$

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Table 5: Summary of Results of Log-Linear Models

Model	G^2	df	BIC
Area × Moral Coherence (I/F)	21.483104	7	-1.049027
Complexity × Moral Coherence (I/F)	19.882541	7	-2.649590
Jurisdiction × Moral Coherence (I/F)	14.115932	4	1.240429
Area × Consistency Precedents (I/F)	23.917456	7	1.385325
Complexity × Regulatory Fit (I/F)	20.051129	7	-2.481002

A negative or near-zero BIC value suggests that the model fits the data well. Negative values in Models 1, 2, and 5 indicate strong evidence that the independence model is inadequate, implying that **there are significant associations** between case characteristics and consistency improvement. Conversely, positive values in Models 3 and 4 suggest that improvement is largely independent of jurisdiction or area of law in the case of consistency with precedents.

Graphic Representation and Plithogenic Connectors

The model is visualized as an n- SuperHyperGraph . Each model analyzed in Table 5 forms a SuperVertex , which is a subset of interacting vertices. For example, the first model (Law Area, Initial Coherence, Final Coherence) is represented as a SuperVertex where these three nodes are connected by a SuperHyperEdge , indicating their interdependence.

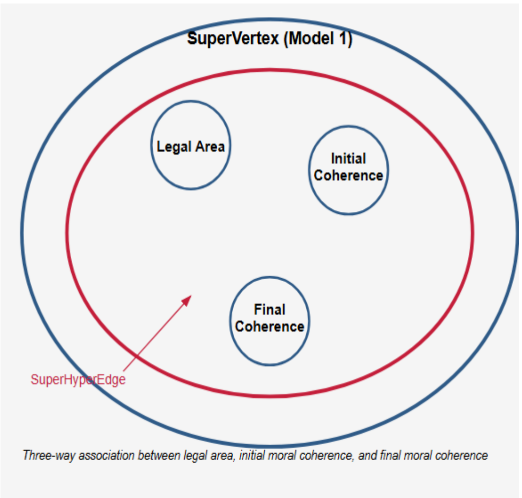

Figure 1. Representation of Model 1 as a SuperHyperGraph

In this figure, the SuperHyperEdge (the line enclosing the three nodes) captures the three-way relationship between the legal domain, the initial state of coherence, and its final state. The uncertainty inherent in the judicial process justifies the use of a neutrosophic framework, where connections are not simply true or false.

A plithogenic connector represents the coherence improvement process**.** This connector links the input envelope vertex (Initial Analysis) to the output envelope vertex (Final Decision), weighted by the strength of the observed relationship.

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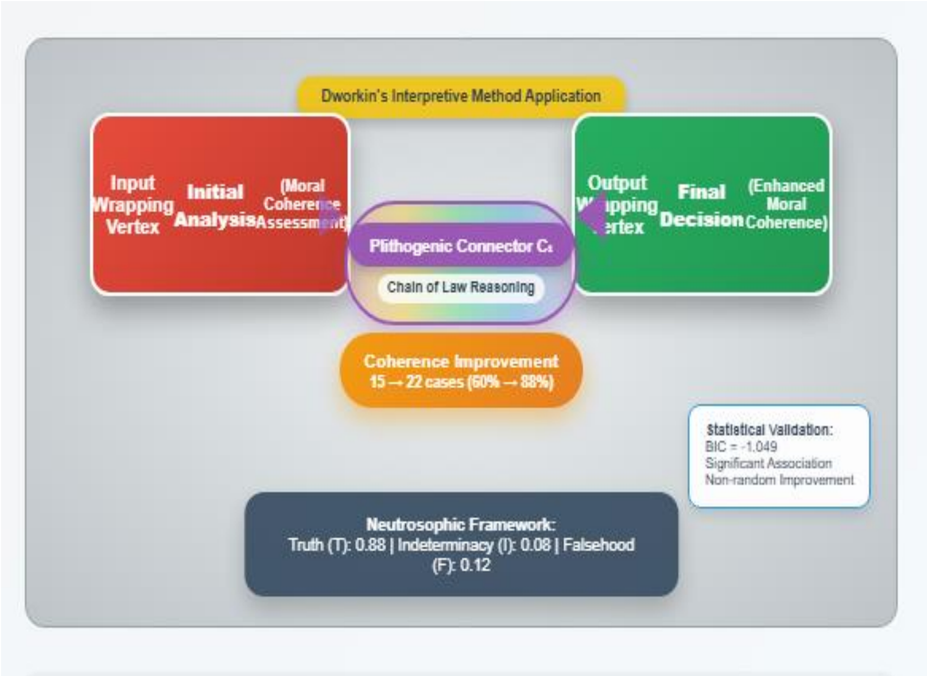

Figure 2. Plithogenic Connector (C1) for Moral Coherence

This connector (C1) represents the "intervention" of Dworkin's interpretive method. Statistical analysis confirms that this connector is significant, that is, the transition from the initial to the final state represents a real, non-random improvement in the coherence of the legal system, validating the effectiveness of integrity-based reasoning.

5. Discussion

Dworkin's theory of legal integrity. Unlike purely qualitative analyses, this approach allows the complex network of relationships between principles, norms, and factual features of a case to be formalized and visualized.

The log-linear analysis, although a simplification, provides a statistical inference about the structure of these relationships. The finding that improvement in coherence is associated with **complexity** and **area of law** (especially in moral coherence) is significant. It suggests that Dworkin's method is not a universally uniform formula, but that its application and effectiveness may depend on the context of the case. High-complexity cases (BIC = -2.649590) are precisely where this type of structured reasoning becomes most crucial and where the most pronounced effects are observed.

The main limitation of this study is the hypothetical nature of the data and the inherent subjectivity of classifying concepts like "coherence." However, the purpose is not to offer a definitive decision-making tool, but rather a model that structures reasoning. The quantification of coherence (initial vs. final) was based on expert consensus, but future research could develop more objective metrics based on text analysis or formal logic.

This model can serve as a pedagogical tool for teaching legal philosophy and as a support system for judges to systematically explore the implications of their decisions within the web of legal principles.

6. Conclusion

1. **Plithogenic n-SuperHyperGraphs** has been successfully developed to represent Dworkin's theory of legal integrity, demonstrating that it is possible to formalize complex philosophical concepts in a multidimensional mathematical structure.
2. The model effectively visualizes the interaction between moral principles, precedents, and norms, where nodes represent legal elements and SuperHyperEdges capture their complex interdependencies under conditions of uncertainty.

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3. Statistical analysis using log-linear models and the Bayesian Information Criterion (BIC) confirmed that the application of an integrity-based interpretive method produces a **significant, non-random improvement in the coherence and consistency of judicial decisions.**
4. The effectiveness of the interpretive process was identified as being moderated by factors such as **case complexity,** which aligns the model with the legal intuition that "hard cases" require deeper reasoning about principles.
5. Ultimately, this interdisciplinary study offers an innovative framework that not only theoretically validates the internal coherence of Dworkin's proposal, but also lays the groundwork for the development of analytical tools that could promote more equitable, transparent, and consistent judicial decision-making.

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References

[1] R. Dworkin, *Law's Empire*. Cambridge, MA, USA: Harvard University Press, 1986, doi: 10.2307/j.ctvjghwvp.

[2] R. Alexy, *A Theory of Constitutional Rights*. Oxford, UK: Oxford University Press, 2002, doi: 10.1093/acprof:oso/9780198258216.001.0001.

[3] H. L. A. Hart, *The Concept of Law*, 2nd ed. Oxford, UK: Oxford University Press, 1994, doi: 10.1093/he/9780198761235.001.0001.

[4] J. Waldron, *Law and Disagreement*. Oxford, UK: Oxford University Press, 2006, doi: 10.1093/acprof:oso/9780199240876.001.0001.

[5] Sen, *The Idea of Justice*. Cambridge, MA, USA: Harvard University Press, 2009, doi: 10.4159/9780674054578.

[6] Rodríguez-Garavito, "Beyond the courtroom: The impact of judicial activism on socioeconomic rights in Latin America," *Texas Law Review*, vol. 89, no. 7, pp. 1669–1698, 2011, doi: 10.2139/ssrn.1800677.

[7] J. Habermas, *Between Facts and Norms: Contributions to a Discourse Theory of Law and Democracy*. Cambridge, MA, USA: MIT Press, 1996, doi: 10.7551/mitpress/1564.001.0001.

[8] S. Himma, *Conceptual Jurisprudence: Methodological Issues, Classical Questions*. Cham, Switzerland: Springer, 2020, doi: 10.1007/978-3-030-31271-8.

[9] M. Dyzenhaus, *The Long Arc of Legality: Hobbes, Kelsen, Hart*. Cambridge, UK: Cambridge University Press, 2021, doi: 10.1017/9781108907507.

[10] R. Gargarella, *The Law as a Conversation among Equals*. Cambridge, UK: Cambridge University Press, 2022, doi: 10.1017/9781108865722.

[11] M. El-Bassiouny and M. A. Hossain, "Neutrosophic logic for decision-making in smart cities," *Sustainability*, vol. 13, no. 4, p. 2155, 2021, doi: 10.3390/su13042155.

[12] Tamanaha, *Legal Pluralism Explained: History, Theory, Consequences*. Oxford, UK: Oxford University Press, 2021, doi: 10.1093/oso/9780190861551.001.0001.

[13] Stone Sweet, *Proportionality Balancing and Constitutional Governance*. Oxford, UK: Oxford University Press, 2023, doi: 10.1093/oso/9780198841395.001.0001.

[14] Sunstein, *Law and Leviathan: Redeeming the Administrative State*. Cambridge, MA, USA: Harvard University Press, 2020, doi: 10.2307/j.ctv1168qmg.

[15] M. A. Hossain, A. M. El-Bassiouny, and F. Smarandache, "Neutrosophic set theory and its applications in decision-making," *Mathematics*, vol. 9, no. 18, p. 2262, 2021, doi: 10.3390/math9182262.

[16] H. J. Kim, "Neutrosophic decision-making for environmental management," *Environmental Science and Pollution Research*, vol. 29, no. 15, pp. 44065–44076, 2022, doi: 10.1007/s11356-022-17781-8.

DOI: https://doi.org/10.54216/IJNS.260330

[17] R. C. M. A. de Almeida and I. C. B. de Lima, "A neutrosophic approach to evaluate the performance of renewable energy sources," *Renewable Energy*, vol. 164, pp. 1010–1020, 2021, doi: 10.1016/j.renene.2020.09.054.

[18] S. K. Sharma, A. K. Gupta, and R. Kumar, "Neutrosophic logic-based decision-making for supply chain management," *Mathematical Problems in Engineering*, vol. 2021, Article ID 8821543, 2021, doi: 10.1155/2021/8821543.

[19] X. Yang, K. Hayat, M. S. Raja, N. Yaqoob, and C. Jana, "Soft aggregation and interaction operators in an interval-valued q-rung orthopair fuzzy soft environment and their application to automation company evaluation," *IEEE Access*, vol. 10, pp. 91424–91444, 2022, doi: 10.1109/ACCESS.2022.3206787.

[20] F. Smarandache, *Neutrosophic Set in Decision Making and Optimization*. Albuquerque, NM, USA: Infinite Study, 2020, doi: 10.13140/RG.2.2.13237.45287.

[21] F. Smarandache, "Introduction to super-hyper-algebra and neutrosophic super-hyper-algebra," *Latin American Association of Neutrosophic Sciences Journal*, vol. 20, pp. 1–6, 2022.

[22] F. Smarandache, "Foundation of superhyperstructure and neutrosophic superhyperstructure," *Neutrosophic Sets and Systems*, vol. 63, pp. 367–381, 2024.

[23] F. Smarandache, "Super hyper function and super hyper structure and their corresponding neutrosophic super hyper function and neutrosophic super hyper structure," *Latin American Association of Neutrosophic Sciences Journal*, vol. 31, pp. 353–359, 2024.

[24] M. Rahmati and M. Hamidi, "On strong super hyper EQ algebras: A proof-of-principle study," *Plithogenic Logic and Computation*, vol. 2, pp. 29–36, 2024.

[25] S. Jahanpanah and R. Daneshpayeh, "An outspread on valued logic superhyperalgebras," *Facta Universitatis, Series: Mathematics and Informatics*, pp. 427–437, 2024.

[26] H. E. Khalid, G. D. Güngör, and M. A. N. Zainal, "Neutrosophic superhyperbitopological spaces: Additional topics," *Neutrosophic Optimization and Intelligent Systems*, vol. 2, pp. 43–55, 2024.

[27] M. Ghods, Z. Rostami, and F. Smarandache, "Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties," *Neutrosophic Sets and Systems*, vol. 50, pp. 480–487, 2022.

[28] F. Üçkardeş, "Use of a hierarchical loglinear model in multidirectional frequency tables and an application in suicide cases," *Sakarya University Journal of Computer and Information Sciences*, vol. 5, pp. 269–277, 2022.

[29] N. Khalil, M. Elkholy, and M. Eassa, "A comparative analysis of machine learning models for predicting chronic kidney disease," *Sustainable Machine Intelligence Journal*, vol. 5, no. 3, pp. 1–10, 2023.

[30] J. A. Carcillo et al., "Contingency table for assessing risk of mortality from systemic inflammation in severe sepsis cases," *Pediatric Critical Care Medicine*, vol. 18, no. 2, pp. 143–150, 2017.

DOI: https://doi.org/10.54216/IJNS.260330