Fuzzy-Soft Modeling to Determine the Best Fertilizer for *Lactuca sativa L.* Crop Considering Three Agronomic Variables

Himera Hamburguer[1], **Vicente Vergara-Flórez**[1], **Kandy Ferrer Sotelo**[2], **Osmin Ferrer Villar**[3], **José Sanabria**[3,*]
[1]Universidad de Sucre, Facultad de Ingeniería, Sincelejo, Colombia
[2]Universidad Pontificia Bolivariana, Facultad de Ingenierías y Arquitectura, Montería, Colombia
[3]Universidad de Sucre, Facultad de Educación y Ciencias, Sincelejo, Colombia

Emails: himerahamsa@gmail.com; viceunisucre@yahoo.com; kandy.ferrer@upb.edu.co; osmin.ferrer@unisucre.edu.co; jose.sanabria@unisucre.edu.co

Abstract

Set-based theories have become key tools to address uncertainty and imprecision in complex systems. Fuzzy sets model gradual membership, soft sets add flexibility through parameterization, and neutrosophic sets generalize both by incorporating truth, indeterminacy, and falsity degrees. In this manuscript, a fuzzy-soft expert system is described to determine the efficiency of different fertilizations in lettuce (*Lactuca sativa L.*) crops considering agronomic variables such as fresh weight (FW), number of leaves (NL), and crown diameter (CD). The model, based on fuzzy membership functions and soft set operations, effectively manages the uncertainty inherent in agricultural data and provides a novel decision-support tool. Although this work focuses on fuzzy and soft sets, its extension to the neutrosophic framework could further enrich the analysis by explicitly modeling indeterminacy and inconsistency, offering a more comprehensive approach to agricultural decision-making.

Keywords: Fuzzy set; Soft set; Biomass; Agriculture; Lattuce

1 Introduction

Lettuce (*Lactuca sativa L.*) is one of the vegetables with the highest production worldwide, due to its wide consumption, high production volume and nutritional contribution, which positions it as the most relevant among vegetables.[16] According to data from SkyQuest (2024),[10] the global market value of lettuce reached USD 3.7 billion in 2023, and it is estimated that between 2025 and 2032 it will experience a Compound Annual Growth Rate (CAGR) of 4.2%. Against this backdrop, it is essential to have lettuce varieties with good commercial performance and suitable growth characteristics to meet consumer demand.

The yield or production of usable biomass in crops such as lettuce has decreased in recent years, mainly due to soil degradation resulting from the inadequate use of synthetic fertilizers.[8,12] Added to this is the high cost of these inputs, which in many cases becomes an economic barrier for farmers. Soil structure is a key component in agricultural productivity; however, intensive tillage and the continuous use of agrochemicals progressively deteriorate it. On the other hand, the use of organic fertilizers such as manures, plant remains and compost improves this structure, increases the levels of organic matter, nutrients and cation exchange capacity,[17] as well as favoring aeration and porosity, in addition to reducing the costs associated with agricultural production.[20]

In agriculture, decision-making is deeply influenced by uncertain factors such as climate, soil quality, the presence of pests, and yield variability. To address this uncertainty, the theories of fuzzy sets, soft sets, and neutrosophic sets offer powerful mathematical tools.

Fuzzy sets allow concepts such as “optimal moisture level” or “crop maturity” to be modeled gradually, assigning degrees of membership in the interval $[0, 1]$. Soft sets extend this framework by introducing multiple parameters, such as fertilizer type, geographic region, or time of year, making it easier to evaluate options under different criteria. Neutrosophic sets, meanwhile, incorporate an additional dimension by simultaneously considering degrees of truth (T), indeterminacy (I), and falsehood (F), which is particularly useful in complex situations such as pest prediction or the analysis of the impact of sustainable agricultural practices.

While fuzzy and soft sets represent information primarily through degrees of membership, which means that uncertainty can be modeled in a single dimension, neutrosophic sets offer a more general and comprehensive framework. This extension allows for the description of data that cannot be classified solely by membership, such as data derived from ambiguous measurements, contradictory reports, or scenarios where uncertainty prevails.

Consequently, the incorporation of neutrosophic sets into models based on fuzzy sets and soft sets provides more robust tools for optimizing decision-making, as they capture not only the inherent vagueness of information, but also its levels of indeterminacy and contradiction. This capability is essential in practical applications in areas such as medicine,[6] environmental engineering,[26] and social systems analysis,[7] among others.

1.1 Use of organic fertilizers in the production of lettuce

Currently, the use of organic fertilizers from different sources is common in lettuce cultivation.[21] Composts are stabilized bio-based fertilizers (BBF) derived from composting, which is a process of aerobic biological decomposition of organic waste, including biowaste, green waste and manure.[9] Among the wastes used as fertilizers, the association of manures with carbon-rich materials can be commonly found. According to Martínez et al.,[13] this technique involves mixing a carbon-rich source with another rich in nitrogen (such as urine and animal excreta), thus allowing the decomposition of organic matter through adequate aeration. The *Compost Bedded Pack Barn* or *Compost Barn* is a confinement system where animals (cows) are kept in a large area covered with carbon-rich organic material (commonly sawdust) where composting occurs; this area generates a residue composed of the material used for animal bedding and excrement that can be a potential source of nutrients to be used in agriculture once it matures.[18]

Likewise, dairy farming generates manure as its main waste, which, if not managed correctly, becomes a significant source of soil contamination, groundwater bodies and greenhouse gas emissions.[1] It has been proven that with proper management the use of cattle manure offers a variety of advantages such as favoring the chemical and physical properties of the soil, providing nutrient availability and optimizing soil structure producing a milder environmental effect.[20]

On the other hand, crop pruning and harvesting generate large quantities of plant residues. Without proper management, these materials are often discarded without control, resulting in wasted resources and pollution problems.[30]

Food production with sustainable practices is a challenge, since access to chemical fertilizers is easy and their early effect represents an advantage for the producer, in addition to the fact that they are capable of increasing the presence of nutrients that plants use for their development.[15]

1.2 Modeling decision systems using set theory

Various mathematical frameworks have recently been developed to represent and manage uncertainty in complex systems, with fuzzy sets standing out for their ability to assign degrees of membership between 0 and 1, which allows for the construction of inference systems that simulate human reasoning and are successfully applied in areas such as medical diagnosis, intelligent control, and financial prediction. Soft sets extend this approach by introducing parametric structures that do not require membership functions, making them particularly useful when decision criteria are uncertain or variable. More recently, neutrosophic sets have extended this framework by also considering indeterminacy and rejection. This ability to explicitly represent different aspects of uncertainty has made these models key tools for the development of flexible, robust, and adaptive decision-making and prediction systems, as demonstrated by recent studies.[3,11,28] These theories allow knowledge to be modeled in domains where the boundaries between categories are not clearly defined, as is often the case in medicine, the environment, economics, and artificial intelligence. Their application in decision-making and prediction systems is constantly expanding, thanks to their compatibility with current computational approaches such as machine learning and optimization. In this paper, we use a hybridization of fuzzy set theory and soft set theory to develop a prediction system based on agronomic knowledge. For this, we use agronomic variables such as fresh weight, number of leaves, and crown diameter to estimate the commercial attractiveness of lettuce. In this way, we explore the potential of extended set theories in constructing a more flexible, interpretable, and adaptive model in light of the complexity of the data obtained in the evaluation of certain fertilizers applied to lettuce cultivation.

2 Preliminaries

2.1 Agronomic variables associated with lettuce cultivation

The efficiency of fertilization treatments applied to the lettuce crop is evaluated by determining different agronomic variables. In this study, we considered Fresh Weight (FW) in grams, Number of Leaves (NL) and Crown Diameter (CD) in centimeters. The reference values established for FW indicate that large lettuces have a weight ranging from 271 to 400 grams.[5] In the works,[5,19,22,27,31] it was reported that the FW of middle lettuces ranges from 136 to 270 grams. For small lettuces their weight is between 0 and 135 grams.[22] With respect to NL, the values considered as large are in the range of 26 to 39, the middle values from 13 to 26, and the small values are between 0 and 13.[4,14,19,22,31] Values for CD between 39 and 50 are classified as large, values from 27 to 38 as middle and values from 15 to 26 as small.[25,29]

2.2 Fuzzy sets and soft sets

Fuzzy sets are a mathematical concept that extends classical set theory to handle situations where elements do not neatly belong or not belong to a set. Instead, they can have degrees of membership, represented by values between 0 and 1, indicating the extent to which an element belongs to the set. This allows for the representation of vagueness and uncertainty, making it useful in various fields like control systems, decision making, and artificial intelligence. Next, we will present some of the notions of fuzzy set theory that will be used in the development of this manuscript.

Definition 2.1. A *fuzzy set* A over a universe of discourse X is a function (called a membership function)

$$\mu_A : X \to [0,1],$$

which establishes the degree of membership of x to A.

A fuzzy set A over X is represented by ordered pairs as follows:

$$A = \{(x, \mu_A(x)) : x \in X\} \subseteq X \times [0,1].$$

Remark 2.2. The family of all fuzzy sets over X is denoted by $\mathcal{F}(X)$, i.e.

$$\mathcal{F}(X) = \{A : A \text{ is a fuzzy set over } X\}.$$

Remark 2.3. Every crisp set can be viewed as a fuzzy set. To see this, we consider the characteristic function

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \end{cases}$$

as the membership function.

Definition 2.4. For each $A \in \mathcal{F}(X)$ we define:

1. The *α-cut* de A, denoted by A_α, as the crisp set containing all elements of the fuzzy set A that have degree of membership greater than or equal to the value $\alpha \in [0, 1]$, i.e.
$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\} \subseteq X.$$
2. The *strong α-cut* de A, denoted by $A_{\overline{\alpha}}$, as the crisp set containing all elements of the fuzzy set A that have degree of membership greater than the value $\alpha \in [0, 1]$, i.e.
$$A_{\overline{\alpha}} = \{x \in X : \mu_A(x) > \alpha\} \subseteq X.$$

Soft set theory is a mathematical tool that is used to model and analyze situations where there is uncertainty or vagueness in the information. Unlike classical set theory, which assigns to each element the membership or non-membership of a set, soft set theory uses parameters to define the membership of elements. Next, we present some notions of soft set theory that will be useful in this manuscript.

Definition 2.5. Let X be a non-empty set, $\mathcal{P}(X)$ be the power set of X and Λ be a non-empty set of parameters. A soft set over X is any pair (Ω, Λ), where $\Omega : \Lambda \to \mathcal{P}(X)$ is a function.

In this way, we can view a soft set as
$$(\Omega, \Lambda) := \{(\lambda, \Omega(\lambda)) : \ \lambda \in \Lambda, \Omega(\lambda) \in \mathcal{P}(X)\}.$$

Example 2.6. Imagine that X is the set of houses available for sale and Λ is the set of attributes such as "cheap", "large", "with garden", etc. A soft set could define, for each attribute, the set of houses that meet it. For example, Ω("cheap") could be the set of houses priced below a certain value, and Ω("large") could be the set of houses with more than x square meters.

Definition 2.7. If (Ω_1, Λ_1) and (Ω_2, Λ_2) are two soft sets over X, then the operation (Ω_1, Λ_1) AND (Ω_2, Λ_2), denoted by $(\Omega_1, \Lambda_1) \wedge (\Omega_2, \Lambda_2)$, is defined as $(\Omega_1, \Lambda_1) \wedge (\Omega_2, \Lambda_2) = (\Omega, \Lambda_1 \times \Lambda_2)$, where $\Omega(\lambda_1, \lambda_2) = \Omega_1(\lambda_1) \cap \Omega_2(\lambda_2)$, for each $(\lambda_1, \lambda_2) \in \Lambda_1 \times \Lambda_2$.

Example 2.8. Suppose that $X = \{h_1, h_2, h_3, h_4, h_5\}$ represents a set of houses. Let $\Lambda_1 = \{\text{cheap}, \text{beautiful}\}$ and $\Lambda_2 = \{\text{comfortable}, \text{beautiful}\}$ be two sets of parameters describing the characteristics of the houses under consideration. Two soft sets (Ω_1, Λ_1) and (Ω_2, Λ_2) over X are defined to represent the houses liked by Mr. A and Mr. B respectively. Thus,
$$\begin{aligned}
\Omega_1(\text{cheap}) =& \{h_1, h_3, h_5\} \text{ (cheap houses)},\\
\Omega_1(\text{beautiful}) =& \{h_1, h_2, h_4\} \text{ (beautiful houses)},\\
\Omega_2(\text{comfortable}) =& \{h_2, h_5\} \text{ (comfortable houses)},\\
\Omega_2(\text{beautiful}) =& \{h_2, h_4\} \text{ (beautiful houses)}.
\end{aligned}$$

Applying the AND operation between (Ω_1, Λ_1) and (Ω_2, Λ_2), we obtain the soft set $(\Omega_1, \Lambda_1) \wedge (\Omega_2, \Lambda_2) = (\Omega, \Lambda_1 \times \Lambda_2)$ as follows:
$$\begin{aligned}
\Omega(\text{cheap}, \text{comfortable}) =& \Omega_1(\text{cheap}) \cap \Omega_2(\text{comfortable}) = \{h_5\},\\
\Omega(\text{cheap}, \text{beautiful}) =& \Omega_1(\text{cheap}) \cap \Omega_2(\text{beautiful}) = \emptyset,\\
\Omega(\text{beautiful}, \text{comfortable}) =& \Omega_1(\text{beautiful}) \cap \Omega_2(\text{comfortable}) = \{h_2\},\\
\Omega(\text{beautiful}, \text{beautiful}) =& \Omega_1(\text{beautiful}) \cap \Omega_2(\text{beautiful}) = \{h_2, h_4\}.
\end{aligned}$$

Note that $(\Omega_1, \Lambda_1) \wedge (\Omega_2, \Lambda_2)$ is a new soft set that represents the characteristics that both misters consider desirable in the houses.

3 Lettuce growth modeling using fuzzy sets and soft sets

Taking into account the environmental damage caused by chemical fertilizers, in recent years different alternatives have been evaluated for the fertilization of agricultural crops.[23]In this work, four types of fertilizers

T_1: Compost Barn bedding, T_2: compost from vegetable waste, T_3: mature bovine manure and T_4: mineral fertilizer were evaluated in lettuce (Lactuca Sativa L. var. Vera). In order to determine which generates the largest product, taking into account the agronomic variables FW, NL, and CD. To determine which of these fertilizers generates the largest product size, taking into account the agronomic variables mentioned above, classical statistical analyzes have been used in previous studies. In this study, fuzzy set theory and soft set theory tools were used to determine which treatment applied to a lettuce crop is the most efficient. Initially, the ranges of classification of values in the variables FW, NL and CD were established, taking into account other studies carried out. Subsequently, we proceeded to fuzzify the data through the membership functions, using the values taken in the field. The fuzzified data were transformed into sets that allow a better interpretation of the information obtained in the field. Our purpose is to design a soft model using the data obtained on these variables as input values, while the output value will be the lettuce size.

Field delineation. The experiment was carried out with a randomized complete block design (RCBD), where four treatments and five replications were used, totaling twenty experimental parcels of $1m \times 2.1m$. Each parcel had 28 plantlets with spacings of $0.3m \times 0.3m$.

Method of evaluation of agronomic variables. Vera variety lettuce (Lactuca Sativa L. var. Vera) was used. After 30 days from germination, indirect sowing was performed. Data collection was carried out 40 days after sowing, where the following was evaluated: the fresh weight of the aerial part using an analytical balance with an uncertainty of $0.0001g$ (extracting the plant carefully with gardening tools), counting the number of leaves of each plant from the base to the apical part considering the true leaves and the diameter of the head; measuring the most extended leaves of the head of each plant evaluated using a tape measure (in cm).

Table 1 shows the values of the effect of different fertilization treatments on the agronomic variables evaluated: fresh weight, number of leaves and crowm diameter in lettuce cultivation. The data were obtained from a sample of twenty lettuces. In each treatment, five replicates of evaluation were performed for each variable. The distribution of treatments is presented as follows: T_1: $x_1, \dots, x_5$, T_2: $x_6, \dots, x_{10}$, T_3: $x_{11}, \dots, x_{15}$ and T_4: $x_{16}, \dots, x_{20}$.

Table 1: Input values for agronomic variables in lettuce under different fertilization treatments.

	$\boldsymbol{x}_1$	$\boldsymbol{x}_2$	$\boldsymbol{x}_3$	$\boldsymbol{x}_4$	$\boldsymbol{x}_5$	$\boldsymbol{x}_6$	$\boldsymbol{x}_7$	$\boldsymbol{x}_8$	$\boldsymbol{x}_9$	$\boldsymbol{x}_{10}$	$\boldsymbol{x}_{11}$	$\boldsymbol{x}_{12}$	$\boldsymbol{x}_{13}$	$\boldsymbol{x}_{14}$	$\boldsymbol{x}_{15}$	$\boldsymbol{x}_{16}$	$\boldsymbol{x}_{17}$	$\boldsymbol{x}_{18}$	$\boldsymbol{x}_{19}$	$\boldsymbol{x}_{20}$
PF	375.21	299.9	252.11	279.28	234.19	166.6	209.16	181.74	185.14	123.44	104.22	158.39	134.58	134.85	151.29	251.11	169.09	162.70	208.91	377.33
NH	31.5	26.0	25.0	25.0	26.0	24.5	27.0	23.5	22.0	21.0	17.0	20.0	21.5	21.5	21.0	27.0	23.0	22.5	25.5	32.0
DC	42.95	38.70	32.25	36.25	36.75	30.60	33.25	29.75	34.75	31.50	29.00	29.25	27.90	30.00	31.50	35.65	31.25	34.00	38.50	47.00

3.1 Transformation of the data set by fuzzyfication

Fuzzification is a fundamental concept within the framework of fuzzy logic, aimed at transforming real-world data into fuzzy sets, thereby enabling such data to be processed by systems based on fuzzy inference. This process involves mapping precise numerical values to degrees of membership in linguistic categories such as “small”, “middle”, or “big”, which are represented through membership functions. The significance of fuzzification lies in its ability to bridge the gap between precise information and fuzzy logic, facilitating the incorporation of imprecise, vague, or uncertain data into automated decision-making models.

Since its inception, fuzzification has been the subject of extensive theoretical development; however, its relevance goes beyond theory, finding practical applications in various fields such as control engineering, computer science, medical diagnostics, and quantitative finance. For instance, in clinical decision support systems, fuzzification enables the modeling of subjective symptoms like “moderate pain” or “slightly elevated blood pressure,” which are difficult to address using classical methods. Recent research, such as that by Sanabria et al.,[24] highlights how fuzzy logic and fuzzification-based models enhance the robustness and adaptability of intelligent systems operating in uncertain and complex environments, making them key tools for real-world decision-making.

The first step in this modeling consists of fuzzyfying the values of the agronomic variables fresh weight (FW), number of leaves (NL), and crown diameter (CD) using membership functions, which are constructed using the following linguistic variables.

- Fresh weight (FW): Small (FWS) with range 0-135, middle (FWM) with range 136-270, big (FWB) with range 271-400.
- Number of leaves (NL): Small (NLS) with range 0-13, middle (NLM) with range 13-26, big (NLB) with range 26-39.
- Crown diameter (DC): Small (DCS) with range 15-26, middle (DCM) with range 27-38, big (DCB) with range 39-50.

To implement the fuzzy system, the three agronomic variables were fuzzified. This process defines the membership functions that assign each data point a degree within the range [1-0]. Figure 1 shows the membership functions for fresh weight, where the relationship between the values for this variable (g) and its degree of membership in the defined fuzzy sets is evident. Similarly, Figure 2 represents the number of leaves, and finally, Figure 3 shows the membership functions for crown diameter, completing the fuzzyfying of all output variables.

The following relationship provides the fuzzyfication of the values of fresh weight (FW).

$$R_{FW}(x) = \begin{cases} \frac{1}{70}x & \text{if } x < 70 \\ \frac{140-x}{70} & \text{if } 70 \leq x \leq 140 \\ \frac{2x-260}{145} & \text{if } 130 < x < 202.5 \\ \frac{530-2x}{145} & \text{if } 202.5 \leq x \leq 275 \\ \frac{2x-530}{135} & \text{if } 265 \leq x \leq 332.5 \\ 1 & \text{if } 332.5 \leq x \end{cases}$$

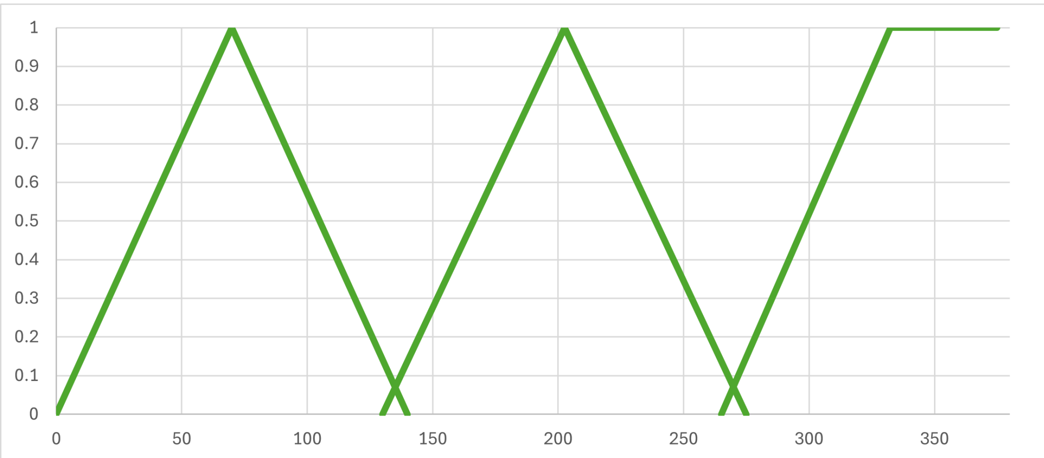

Figure 1: Membership functions for fresh weight.

The following relationship provides the fuzzyfication of the values of number of leaves (NL).

$$R_{NL}(x) = \begin{cases} \frac{1}{7}x & \text{if } 0 < x \leq 7 \\ 2 - \frac{1}{7}x & \text{if } 7 < x \leq 14 \\ \frac{2x-24}{15} & \text{if } 12 < x \leq 19.5 \\ \frac{54-2x}{15} & \text{if } 19.5 < x \leq 27 \\ \frac{x-25}{7} & \text{if } 25 < x \leq 32 \\ \frac{39-x}{7} & \text{if } 32 < x < 39. \end{cases}$$

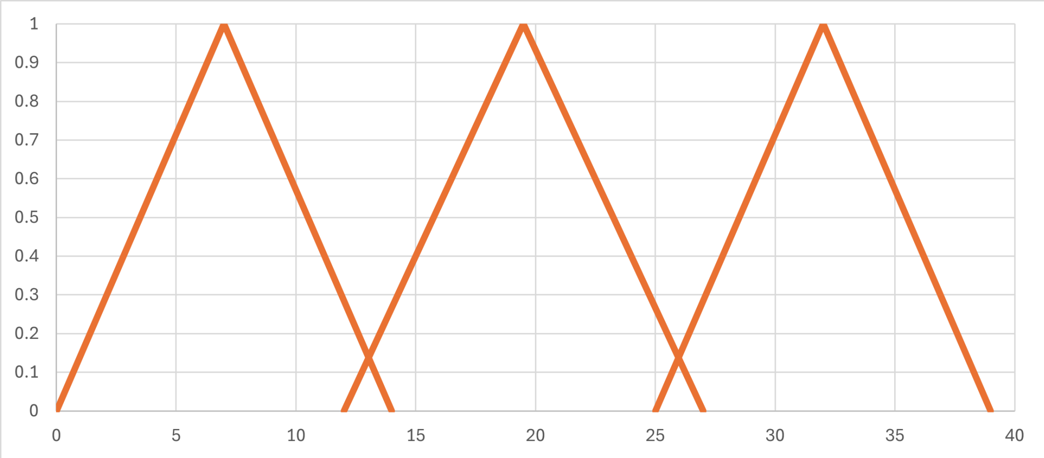

Figure 2: Membership functions for number of leaves.

The following relationship provides the fuzzyfication of the values of Crown diameter (CD).

$$R_{DC}(x) = \begin{cases} 0 & \text{if } x < 15 \\ \frac{x-15}{6} & \text{if } 15 \leq x < 21 \\ \frac{27-x}{6} & \text{if } 21 \leq x < 27 \\ \frac{2x-50}{15} & \text{if } 25 < x < 32.5 \\ \frac{80-2x}{15} & \text{if } 32.5 < x < 40 \\ \frac{x-38}{6} & \text{if } 38 < x < 44 \\ 1 & \text{if } x \geq 44. \end{cases}$$

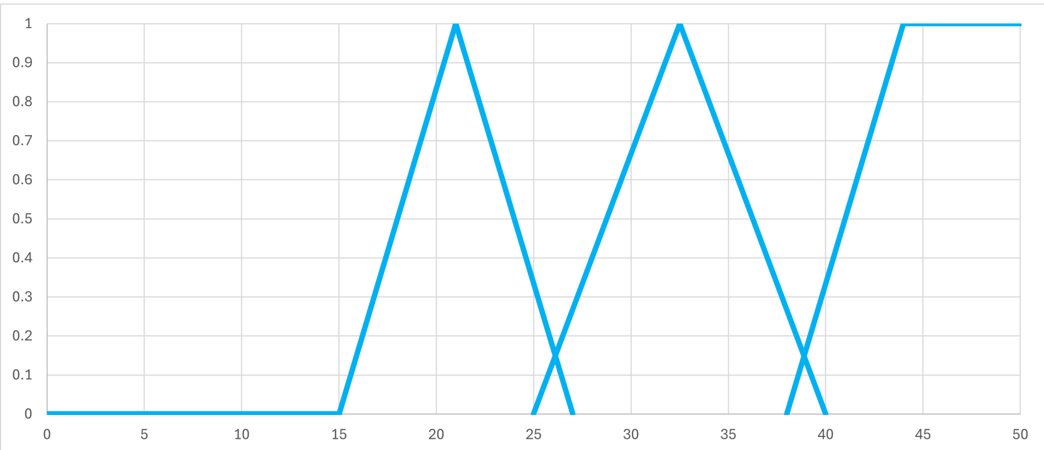

Figure 3: The following relationship provides the fuzzyfication of crown diameter.

Table 2 presents the results of the fuzzyfying process applied to the initial dataset. For each input data item, the table shows its degree of membership in linguistic sets classified as “Small,” “Medium,” or “Big,” applying the relationships established above.

Table 2: Input values obtained by fuzzyfication.

Lettuce	FW	NL	CD
x_1	0 FWM, 1 FWB	0 NLM, 0.929 NLB	0 CDM, 0.825 CDB
x_2	0 FWM, 0.517 FWB	0 NLM, 0.133 NLB	0.173 CDM, 0.117 CDB
x_3	0 FWS, 0.178 FWM	0 NLS, 0.267 NLM	0 CDS, 0.967 CDM
x_4	0 FWM, 0.211 FWB	0 NLS, 0.267 NLM	0 CDS, 0.500 CDM
x_5	0 FWS, 0.425 FWM	0.133 NLM, 0.143 NLB	0 CDS, 0.433 CDM
x_6	0 FWS, 0.505 FWM	0 NLS, 0.333 NLM	0 CDS, 0.747 CDM
x_7	0 FWS, 0.770 FWM	0 NLM, 0.286 NLB	0 CDS, 0.900 CDM
x_8	0 FWS, 0.714 FWM	0 NLS, 0.467 NLM	0 CDS, 0.633 CDM
x_9	0 FWS, 0.761 FWM	0 NLS, 0.667 NLM	0 CDS, 0.700 CDM
x_{10}	0.237 FWS, 0 FWM	0 NLS, 0.800 NLM	0 CDS, 0.867 CDM
x_{11}	0.511 FWS, 0 FWM	0 NLS, 0.667 NLM	0 CDS, 0.533 CDM
x_{12}	0 FWS, 0.392 FWM	0 NLS, 0.933 NLM	0 CDS, 0.567 CDM
x_{13}	0.077 FWS, 0.063 FWM	0 NLS, 0.733 NLM	0 CDS, 0.387 CDM
x_{14}	0.074 FWS, 0.067 FWM	0 NLS, 0.733 NLM	0 CDS, 0.667 CDM
x_{15}	0 FWS, 0.294 FWM	0 NLS, 0.800 NLM	0 CDS, 0.867 CDM
x_{16}	0 FWS, 0.192 FWM	0 NLM, 0.286 NLB	0 CDS, 0.580 CDM
x_{17}	0 FWS, 0.539 FWM	0 NLS, 0.533 NLM	0 CDS, 0.833 CDM
x_{18}	0 FWS, 0.451 FWM	0 NLS, 0.600 NLM	0 CDS, 0.800 CDM
x_{19}	0 FWS, 0.774 FWM	0.200 NLM, 0.071 NLB	0.200 CDM, 0.083 CDB
x_{20}	0 FWM, 1 FWB	0 NLB, 0.100 NLB	0 CDM, 1 CDB

3.2 Soft sets induced by α-cuts and soft rules

Since each fuzzy set can be considered as a soft set, we will now transform the fuzzy sets obtained in the previous step into soft sets. The process consists of taking the fuzzy values to construct parameterized sets using α-cuts. The soft sets obtained by selecting parameter sets based on the membership functions are presented below.
(1). For the soft set fresh weight small, we have:

$$X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.074, 0.237\}$$
$$(\Omega_{FWS}, \Lambda) = \{0.074 = \{x_{10}, x_{11}, x_{13}, x_{14}\}, 0.237 = \{x_{10}, x_{11}\}\}.$$

(2). For the soft set fresh weight middle, we have:

$$\begin{aligned} &X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.063, 0.294, 0.505, 0.761\} \\ &(\Omega_{FWM}, \Lambda) = \{0.063 = \{x_3, x_5, x_6, x_7, x_8, x_9, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}\}, \\ &\quad 0.294 = \{x_5, x_6, x_7, x_8, x_9, x_{12}, x_{15}, x_{17}, x_{18}, x_{19}\}, 0.505 = \{x_6, x_7, x_8, x_9, x_{17}, x_{19}\}, \\ &\quad 0.761 = \{x_7, x_9, x_{19}\}\}. \end{aligned}$$

(3) For the soft set fresh weight big, we have:

$$X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.211, 0.367, 1\}$$
$$(\Omega_{FWB}, \Lambda) = \{0.211 = \{x_1, x_2, x_4, x_{20},\}, 0.367 = \{x_1, x_2, x_{20}\}, 1 = \{x_1, x_{20}\}\}.$$

(4). For the soft set number of leaves middle, we have:

$$\begin{aligned} &X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.133, 0.267, 0.600, 0.800\} \\ &(\Omega_{NLM}, \Lambda) = \{0.133 = \{x_2, x_3, x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{17}, x_{18}, x_{19}\}, \\ &\quad 0.267 = \{x_3, x_4, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{17}, x_{18}\}, \\ &\quad 0.600 = \{x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{18}\}, 0.800 = \{x_{10}, x_{12}, x_{15}\}\}. \end{aligned}$$

(5). For the soft set number of leaves big, we have:

$$X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.071, 0.286, 0.929\}$$
$$(\Omega_{NLB}, \Lambda) = \{0.071 = \{x_1, x_2, x_5, x_7, x_{16}, x_{19}, x_{20}\}, 0.286 = \{x_1, x_7, x_{16}, x_{20}\}, 0.929 = \{x_1, x_{20}\}\}.$$

(6). For the soft set crown diameter middle, we have:

$$X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.173, 0.500, 0.667, 0.800\}$$
$$(\Omega_{CDM}, \Lambda) = \{0.173 = \{x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}\},$$
$$0.500 = \{x_3, x_4, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}\},$$
$$0.667 = \{x_3, x_6, x_7, x_9, x_{10}, x_{14}, x_{15}, x_{17}, x_{18}\},$$
$$0.800 = \{x_3, x_7, x_9, x_{10}, x_{17}, x_{18}\}, 1 = \{x_{20}\}\}.$$

(7). For the soft set crown diameter big, we have:

$$X = \{x_1, x_2, x_3, \ldots, x_{20}\}, \Lambda = \{0.083, 0.500\}$$
$$(\Omega_{CDB}, \Lambda) = \{0.083 = \{x_1, x_2, x_{19}, x_{20}\}, 0.500 = \{x_1, x_{20}\}\}$$

Table 3 shows a representative subset of the fuzzy rules generated by applying the AND operator to the fuzzy sets defined above.
Each rule arises from patterns in the data for individual lettuces. The table includes the identifiers of the lettuces from which each rule was defined.

Table 3: Soft rules obtained by AND operation.

01) $\Omega_{FWS}(0.074) \wedge S_{NLM}(0.133) \wedge \Omega_{CDM}(0.173) = \{x_{10}, x_{11}, x_{13}, x_{14}\}$
14) $\Omega_{FWS}(0.074) \wedge \Omega_{NLM}(0.600) \wedge \Omega_{CDM}(0.500) = \{x_{10}, x_{11}, x_{14}\}$
15) $\Omega_{FWS}(0.074) \wedge \Omega_{NLM}(0.600) \wedge \Omega_{CDM}(0.667) = \{x_{10}, x_{14}\}$
43) $\Omega_{FWS}(0.237) \wedge \Omega_{NLM}(0.133) \wedge \Omega_{CDM}(0.173) = \{x_{10}, x_{11}\}$
85) $\Omega_{FWM}(0.063) \wedge \Omega_{NLM}(0.133) \wedge \Omega_{CDM}(0.173) = \{x_3, x_5, x_6, x_8, x_9, x_{12}, x_{13}, x_{14}, x_{15}, x_{17}, x_{18}, x_{19}\}$
86) $\Omega_{FWM}(0.063) \wedge \Omega_{NLM}(0.133) \wedge \Omega_{CDM}(0.500) = \{x_3, x_6, x_8, x_9, x_{12}, x_{14}, x_{15}, x_{17}, x_{18}\}$
91) $\Omega_{FWM}(0.063) \wedge \Omega_{NLM}(0.267) \wedge \Omega_{CDM}(0.173) = \{x_3, x_6, x_8, x_9, x_{12}, x_{13}, x_{14}, x_{15}, x_{17}, x_{18}\}$
93) $\Omega_{FWM}(0.063) \wedge \Omega_{NLM}(0.267) \wedge \Omega_{CDM}(0.667) = \{x_3, x_6, x_9, x_{14}, x_{15}, x_{17}, x_{18}\}$
94) $\Omega_{FWM}(0.063) \wedge \Omega_{NLM}(0.267) \wedge \Omega_{CDM}(0.800) = \{x_3, x_9, x_{17}, x_{18}\}$
127) $\Omega_{FWM}(0.294) \wedge \Omega_{NLM}(0.133) \wedge \Omega_{CDM}(0.173) = \{x_5, x_6, x_8, x_9, x_{12}, x_{15}, x_{17}, x_{18}, x_{19}\}$
134) $\Omega_{FWM}(0.294) \wedge \Omega_{NLM}(0.267) \wedge \Omega_{CDM}(0.500) = \{x_6, x_8, x_9, x_{12}, x_{15}, x_{17}, x_{18}\}$
151) $\Omega_{FWM}(0.294) \wedge \Omega_{NLB}(0.071) \wedge \Omega_{CDM}(0.173) = \{x_5, x_7, x_{19}\}$
192) $\Omega_{FWM}(0.505) \wedge \Omega_{NLM}(0.800) \wedge \Omega_{CDB}(0.500) = \{\}$
253) $\Omega_{FWB}(0.211) \wedge \Omega_{NLM}(0.133) \wedge \Omega_{CDM}(0.173) = \{x_2, x_4\}$
281) $\Omega_{FWB}(0.211) \wedge \Omega_{NLB}(0.071) \wedge \Omega_{CDB}(0.083) = \{x_1, x_2, x_{20}\}$
282) $\Omega_{FWB}(0.211) \wedge \Omega_{NLB}(0.071) \wedge \Omega_{CDB}(0.500) = \{x_1, x_{20}\}$

4 Analysis of results

According to the information provided in Table 2, there are three lettuces belonging to treatment T_1 whose fuzzified values of the agronomic variable FW (0.211, 0.517, and 1) placed them in the linguistic variable FWB. Likewise, compared to other treatments, in treatment T_1 there are lettuces whose fuzzified values of the agronomic variables NL (0.929, 0.133, and 0.143) and CD (0.825 and 0.117) classified them in the linguistic variables NLB and CDB, respectively. In the case of these last two agronomic variables, treatment T_4 also had the same amount of lettuce in the Big characteristic (x_{19} and x_{20}), being the only treatment similar to T_1, but only for those two variables (NL and CD). This shows that Compost Barn (T_1) is capable of providing the optimal conditions for lettuce growth, being the best treatment to generate high levels of biomass, even above mineral fertilizer (T_4). The Compost Barn system bed composition includes wood waste (sawdust), which, when used as organic matter, provides non-fossil carbon. This component enriches the biomass and supplies residual nitrate to plants, providing additional benefits as a fertilizer.[2] In the case of lettuces x_6, x_7, x_8 and x_9, belonging to treatment T_2, a prevalence in belonging to the linguistic variable FWM is observed. Lettuce x_{10} was the only one in treatment T_2 that was classified within the linguistic variable FWS. This particular lettuce showed high degrees of membership in the linguistic variables NLM and CDM, indicating that it has a low weight despite its medium size. However, its commercial value would be low, because the parameter most commonly used worldwide for marketing is weight, rather than size. Treatment T_3 shows characteristics similar to those of treatment T_2. However, lettuces in treatment T_2 have a higher degree of membership in the linguistic variable FWM. Regarding the agronomic variable NL, lettuces in treatment T_3 stood out for having a higher degree of membership in the linguistic variable NLM, while the opposite occurred with the linguistic variable CDM, where lettuces in treatment T_2 obtained a higher degree of membership. It is also evident that lettuces x_{13} and x_{14}, belonging to treatment T_3, are located in the diffuse zone, as they show simultaneous integration in the linguistic variables FWS and FWM. However, the model indicates that they have median characteristics in the agronomic variables NL and CD, which allows us to interpret that these two lettuces tend more towards a medium size classification than a small one.

From Table 3, it can be inferred that there are three lettuces that meet the conditions established by Rule 281. These lettuces have the highest commercial appeal, as they simultaneously exhibit a high fresh weight, a high number of leaves, and a large crown diameter. Therefore, the percentage of compliance with Rule 281 is $(3 \div 3) \times 100 = 100\%$. Consequently, lettuces whose values of FW, NL, and CD comply with Rule 281 can be stated to have commercial attractiveness 100%. A similar percentage is observed for lettuces that meet the conditions of Rules 253 and 282. In contrast, Rule 85 groups twelve lettuces, of which only two have the characteristics that are desirable from a commercial point of view. Therefore, the commercial attractiveness associated with this rule is 16%. However, although lettuce x_5 meets the criteria of several rules, when comparing the corresponding attractiveness percentages, it can be seen that Rule 151 has the highest value, with a percentage of 33%. It is worth noting that no lettuce exhibited values consistent with Rule 192. Moreover, lettuces whose characteristics matched Rule 281 were fertilized using treatment T_1 for the agronomic variables considered. The use of soft model theory for this study is based on overcoming the limitations of traditional methods, since an ANOVA-type analysis could have concluded that there are significant differences between the data provided for each treatment. However, it would not have been able to model the individual development of each plant around the variables evaluated, nor would it have identified the membership values that the model used did detected. Therefore, the main advantage of the model lies in its diagnostic and predictive capacity, not only offering the answer to "which treatment is best?", but also a broader view of how and why superior development occurs in certain lettuce(s).

This outcome is made possible thanks to the power of distributed computing and the innovative data processing method based on fuzzy expert knowledge.[24] This type of decision-making system can help extract information from the data obtained in agronomic evaluations of different crops and provide a clear picture of the most suitable fertilization.

5 Conclusion

Fuzzy and soft set theories are effective tools for modeling in the study of lettuce crops, as they allow us to address the uncertainty and variability inherent in agricultural systems. In addition, they offer a more realistic interpretation than that provided by conventional statistical methods, which favors more informed and accurate decision-making in crop management. Treatment with Compost Barn bedding proved effective in optimizing lettuce growth based on evaluated agronomic variables, particularly in terms of fresh weight, where the largest number of lettuces remained in the "Big" category. It was evident that the use of organic amendments as fertilizers for vegetables, in this case lettuce, provides favorable conditions for their development, allowing the plants to show better agronomic responses compared to other treatments, including mineral fertilizer. The results of this study represent a significant contribution to environmental management in agricultural production, given that Compost Barn is generated from waste that, if not properly managed, can contaminate water bodies and other components of the environment. Likewise, this type of organic fertilizer is emerging as an excellent alternative to chemical fertilizers, which often have adverse effects on the environment and climate. In this context, the study contributes to the fulfillment of several Sustainable Development Goals (SDGs), in particular: Zero Hunger, Clean Water and Sanitation, Responsible Production and Consumption, and Climate Action. Furthermore, future research could integrate neutrosophic set theory as a complement to fuzzy and soft approaches, as it offers the possibility of representing not only the degree of membership of elements, but also levels of indeterminacy and falsity. This expanded framework would facilitate more realistic modeling of the complexity of agricultural systems, especially when analyzing agronomic variables such as fresh weight, number of leaves, and canopy diameter in lettuce crops. Such measurements are often influenced by heterogeneous conditions and, as a result, generate ambiguous or contradictory results; hence, the neutrosophic perspective provides a more accurate treatment of uncertainty and inconsistency. This would strengthen decision-making processes in crop management and environmental sustainability, while broadening the applicability and relevance of the findings of this study.

References

[1] L. A. Achucarro, *Dinámica de degradación de estiércol y digerido bovino en dos suelos del sudoeste bonaerense*, Tesis de Grado, Universidad Nacional del Sur, Argentina, 2022.

[2] M. Aeberhard, J. Corace, P. Martina, E. García, G. Tortosa, A. Ventin, Procesos de mejoramiento en el rendimiento del aserrín utilizado como materia orgánica en un biodigestor, *Avances en Energías Renovables y Medio Ambiente* **11** (2007), 11-15.

[3] J. C. R. Alcantud, A. Z. Khameneh, G. Santos-García, M. Akram, A systematic literature review of soft set theory, *Neural Computing and Applications* **36** (2024), 8951-8975.

[4] G. N. Al-Karaki, Y. Othman, Effect of foliar application of amino acid biostimulants on growth, macronutrient, total phenol contents and antioxidant activity of soilless grown lettuce cultivars, *South African Journal of Botany* **154** (2023), 225-231.

[5] A. Christou, M. Stylianou, C. Georgiadou, S. Gedeon, A. Ioannou, C. Michael, P. Papanastasiou, V. Fotopoulos, D. Fatta-Kassinos, Effects of biochar derived from the pyrolysis of either biosolids, manure or spent coffee grounds on the growth, physiology and quality attributes of field-grown lettuce plants, *Environmental Technology & Innovation* **26** (2022), 102263.

[6] A. K. Essa1, R. Sabbagh, A. A. Salama, H. E. Khalid, A.-A. A. Aziz, A. A. Mohammed, An overview of neutrosophic theory in medicine and healthcare, *Neutrosophic Sets and Systems* **61**(1) (2023), 196-208.

[7] H. T. Hinostroza Robles, M. T. Cabanillas López, F. D. Huasco Espinoza, Analysis of the evolution of Social competence in students through research methods based on neutrosophic sets, *Neutrosophic Sets and Systems* **74**(1) (2024), 24-36.

[8] M. Hungria, M. A. Nogueira, *Fixação biológica de nitrogênio*, Em: C. Seixas, N. Neumaier, A. Junior, F. Krzyzanowski, R. Bôas, Tecnologias de produção de soja, Sistemas de Produção **17** (2020), 185-195.

[9] L. F. Kebalo, P. Garnier, L. Vieublé Gonod, S. Houot, Using bio-based fertilizer derived from peri-urban wastes affects soil properties and lettuce yield and quality, *Scientia Horticulturae* **324** (2024), 112599.

[10] Lettuce Market Analysis, Size, Share, Growth & Trends 2032. *SkyQuest* (2024), 157 pages.

[11] Y. Li, M. Zhang, An extended power geometric technique for multiple-attribute decision-making under single-valued neutrosophic sets and applications to embedded computers' performance evaluation, *Soft Computing* **28** (2024), 10301–10316.

[12] G. M. Martínez, V. H. Suárez, Bienestar de las vacas lecheras en los sistemas de compost barn, *Ciencia Veterinaria* **24**(2) (2022), 131-149.

[13] N. V. Martínez Esteban, *Efecto de los abonos orgánicos en el rendimiento de la lechuga (Lactuca sativa L.) en condiciones del CIFO–UNHEVAL, Huánuco 2020*, Tesis de Grado, Universidad Nacional Hermilio Valdizán, Perú, 2022

[14] T. Meskelu, A. F. Senbeta, Y. G. Keneni, S. Getachew, Growth and marketable yield of lettuce (Lactuca sativa L.) as affected by bio-slurry and chemical fertilizer application, *Heliyon* **10**(1) (2024), e23600.

[15] G. Navarro García, S. Navarro García, *Fertilizantes: Química y acción*, Segunda Edición, MundiPrensa, España, 2023.

[16] A. M. Ocón Gómez, *Comportamiento agronómico de cuatro variedades de lechuga (Lactuca sativa L) en sistema Biointensivo, Centro experimental, El Plantel 2023*, Tesis de Grado, Universidad Nacional Agraria, Nicaragua, 2024.

[17] C. R. Ortiz Argueta, *Prácticas para la mejora en el proceso de compostaje de abonos orgánicos elaborados a base de estiércol y su efecto en el suelo: Revisión de literatura*, Tesis de Grado, Escuela Agrícola Panamericana, Honduras, 2020.

[18] M. Ortiz Mackinson, B. Bonel, R. Grasso, R. Rotondo, D. M. Balaban, E. Vita Larrieu, Utilización de compost de cama profunda porcina como abono orgánico en un sistema productivo de lechuga (Lactuca sativa L.) a campo, *Revista Ciencias Agronómicas* **40** (2022), e023.

[19] R. Ortiz, G. Gascó, A. Méndez, A. Obrador, D. González, P. Almendros, Zinc biofortification of lettuce using environmentally friendly zinc sources in an acidic soil, *Scientia Horticulturae* **338** (2024), 113620.

[20] A. V. Paredes Peralta, Z. R. Guzmán Brito, Revisión bibliográfica del efecto de la adición de estiércol bovino en la producción agrícola, *Conciencia Digital* **7**(4) (2024), 87-102.

[21] H. Paterlini, M. V. González, L. I. Picone, Producción de lechuga en un suelo con aplicación de compost de cama de pollo, *Ciencias del Suelo* **37**(1) (2019), 38-50.

[22] P. Rani Das, D. S. Del Moro, S. R. Givens, S. P. Armstrong, K. J. Walters, Propagation light intensity influences yield, morphology, and phytochemistry of purple-leaf butterhead lettuce (Lactuca sativa), *Journal of Agriculture and Food Research* **16** (2024), 101210.

[23] B. Qiang, Z. Yan, X. Zhang, M. Cheng, Y. Wu, N. Tang, B. C. Timbang, T. Cai, E. Liu, X. Zhao, X. Ren, X. Chen, 2025. Elevating crop productivity and sustainability: The role of organic fertilizers in shaping wheat's nitrogen economy and photosynthetic response, *European Journal of Agronomy* **170** (2025), 127757.

[24] J. Sanabria, M. Álvarez, O. Ferrer, Fuzzy set and soft set theories as tools for vocal risk diagnosis, *Applied Computational Intelligence and Soft Computing* **2023**(1) (2023), 5525978, 12 pages.

[25] O. Santos, D. Vaz, F. Sebastião, H. Sousa, J. Vieira, Wastewater as a nutrient source for hydroponic production of lettuce: Summer and winter growth, *Agricultural Water Management* **301** (2024), 108966.

[26] A. Savitha Mary, D. Sarukasan, C. Kayelvizhi, L. Jethruth Emelda Mary, F. Josephine Daisy, K. Pitchaimani, An application of site selection for solid waste management system using neutrosophic set, *Neutrosophic Sets and Systems* **85**(1) (2025), 748-765.

[27] S. Smoleń, I. Kowalska, L. Skoczylas, M. Tabaszewska, J. Pitala, J. Mrożek, P. Kováčik, Effectiveness of enriching lettuce with iodine using 5-iodosalicylic and 3,5-diiodosalicylic acids and the chemical composition of plants depending on the type of soil in a pot experiment, *Food Chemistry* **382** (2022), 132347.

[28] A. K. Varshney and V. Torra, Literature review of the recent trends and applications in various fuzzy rule-based systems, *International Journal of Fuzzy Systems* **25** (2023), 2163–2186.

[29] Y. Wang, N. Akdeniz, Utilizing co-composted biochar as a growing medium for buttercrunch lettuce, *Environmental Challenges* **13** (2023), 100748.

[30] R. Wang, Y. Zhao, X. Xie, T. Mohamed, L. Zhu, Y. Tang, Y. Chen, Z. Wei, Role of NH3 recycling on nitrogen fractions during sludge composting, *Bioresource Technology* **295** (2020), 122175.

[31] C. Zhu, Z. Lin, W. Fen, W. Jiajia, Z. Xiang, C. Kai, Z. Yu, Z. Kelai, J. Yelin, K. R, Salin, Suitability of coconut bran and biochar as a composite substrate for lettuce cultivation in aquaponic systems, *Heliyon* **10**(15) (2024), e35515.