Truth–Indeterminacy–Falsity Fusion in Neutrosophic Intelligent Systems: A Mathematical Review, Algorithmic Taxonomy, and Research Agenda

Murat Ozcek[1,*], **Arash Salehpour**[2]

[1]Gaziantep University, Department of Mathematics, Gaziantep, Turkey
[2]Department Cybersecurity, University, Istanbul, Türkiye
Emails: muratozcek.12@gmail.com; arashsalehpour@halic.edu.tr

Abstract
Neutrosophic information fusion has become a rigorous computational approach for modeling evidence that is simultaneously supportive, opposing, and unresolved. This review synthesizes recent studies published from 2020 to 2025 and organizes the field around the operational semantics of truth, indeterminacy, and falsity. Rather than presenting neutrosophic sets only as an extension of fuzzy sets, the paper analyzes neutrosophic fusion as a mathematical problem of evidence representation, operator design, source weighting, contradiction control, and decision reduction. The review covers single-valued neutrosophic similarity measures, EDAS and TOPSIS extensions, neutro-sophic Z-number aggregation, Einstein and Aczel–Alsina operators, trigonometric credibility operators, dynamic aggregation, divergence measures, uncertainty-aware multi-source information fusion, and evidence-theoretic comparisons. A related-work section of more than twenty verified 2020–2025 studies is added, followed by a selection protocol, formal definitions, propositions, algorithms, operator-property analysis, and research directions. The paper concludes with a research agenda for benchmark construction, data-driven membership learn-ing, explainable indeterminacy, scalable dynamic fusion, and trustworthy integration of neutrosophic logic with intelligent decision-support systems.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Information fusion; Indeterminacy; Contradiction; Aggregation operator; Decision support; Mathematical review

1. Introduction

Information fusion is no longer limited to combining redundant measurements from homogeneous sensors. In contemporary intelligent systems, evidence is often collected from human experts, sensors, classifiers, language models, knowledge graphs, public databases, and uncertain decision matrices. These sources may disagree, fail, delay their observations, or provide partially reliable statements. A fusion model is therefore expected to preserve useful support, expose opposition, and identify unresolved uncertainty rather than compressing all evidence into a single scalar value.

Neutrosophic theory is suitable for this setting because it represents a proposition through three distinguishable components: truth, indeterminacy, and falsity. This triadic structure gives the researcher a direct way to model incomplete, inconsistent, and ambiguous information. Recent work on single-valued neutrosophic similarity shows that the quality of a decision depends strongly on how these three components are compared and interpreted (Chai et al., 2021). In parallel, neutrosophic EDAS and TOPSIS variants demonstrate how the same triadic information can be embedded into ranking and group decision models (Stanujkic et al., 2021).

The recent literature also shows that aggregation is the mathematical core of neutrosophic information fusion. Neutrosophic Z-number aggregation links the reliability of an assessment to its neutrosophic value (Du et al., 2021). Einstein aggregation has been used to introduce nonlinear interactions between truth, indeterminacy, and falsity in engineering design decisions (Farid et al., 2022). Dynamic aggregation extends this reasoning to time-dependent source reliability in IoT and supply-chain settings (Farid & Riaz, 2023). These studies indicate that neutrosophic fusion should be evaluated by the semantics it preserves, not only by the ranking it produces.

This review is motivated by a methodological gap. Many papers introduce new neutrosophic operators, but the connection between operator design and information-fusion principles is not always made explicit. In information fusion, a method should explain how sources are weighted, how conflict is propagated, how missingness affects decision stability, and how the final output remains interpretable. Reviews of uncertainty quantification and multi-source information fusion provide useful reference points for this broader perspective (Abdar et al., 2021). Evidence-theoretic reviews further show that mature fusion frameworks usually include conflict handling, validation protocols, and scalability considerations (Fei et al., 2024).

The first contribution of this paper is a structured mathematical synthesis of neutrosophic information fusion from 2020 to 2025. The paper organizes recent research into representation models, information measures, aggregation operators, dynamic weighting schemes, decision-reduction rules, and validation requirements. It also clarifies how truth, indeterminacy, and falsity should be treated as separate computational variables. This is important because a fusion method that simply averages three numbers without specifying their semantics

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

risks becoming a cosmetic neutrosophic formulation rather than a meaningful uncertainty model.

The second contribution is a novelty-oriented research framework for future work. The paper introduces a review protocol, an operator-theoretic formulation, reusable algorithms, and a research agenda that connects neutrosophic fusion with data-driven membership estimation, uncertainty-aware learning, Dempster–Shafer fusion, federated decision support, and explainable indeterminacy. The novelty of this review is therefore not the proposal of one additional aggregation formula. Instead, it provides a mathematical and methodological map that helps researchers decide which neutrosophic representation, operator family, weighting strategy, and validation procedure are appropriate for a given fusion problem.

2. Related Work

The related work is organized according to how each study contributes to neutrosophic and information fusion. The discussion avoids placing many citations in the same sentence so that each reference is linked to a clear technical role.

2.1 Similarity, Distance, and Divergence for Neutrosophic Evidence

Similarity and distance measures define how two neutrosophic evidence states are compared before fusion or classification. Chai et al. proposed new similarity measures for single-valued neutrosophic sets and demonstrated their role in pattern recognition and medical diagnosis (Chai et al., 2021). Zeng et al. addressed counter-intuitive similarity behavior by modifying Manhattan distance for single-valued neutrosophic sets (Zeng et al., 2022). Singh and Sharma developed divergence measures based on aggregation operators, showing that divergence can support pattern-recognition decisions under single-valued neutrosophic information (Singh & Sharma, 2024). Garg introduced an exponential-logarithm-based single-valued neutrosophic formulation that expands the transformation space available for decision analysis (Garg, 2024).

2.2 Neutrosophic Ranking and Decision-Fusion Methods

Ranking methods are among the most common ways to convert fused neutrosophic evidence into decisions. Stanujkic et al. extended EDAS to single-valued neutrosophic numbers, providing a clear distance-from-average structure for ranking alternatives under truth, indeterminacy, and falsity (Stanujkic et al., 2021). Liu et al. later used an EDAS model for single-valued neutrosophic number group decision-making in teaching-quality evaluation (Liu et al., 2023). Li et al. combined aspect-based sentiment analysis with neutrosophic TOPSIS for doctor selection, illustrating how textual evidence can be converted into neutrosophic decision support (Li et al., 2023). Hezam et al. integrated single-valued neutrosophic MEREC and COPRAS for sustainability assessment of bioenergy production technologies (Hezam et al., 2023).

2.3 Aggregation Operators for Neutrosophic Fusion

Aggregation operators determine how multiple neutrosophic assessments are combined into one fused assessment. Du et al. studied aggregation operators of neutrosophic Z-numbers and showed how reliability can be embedded into multicriteria decision-making (Du et al., 2021). Farid et al. proposed single-valued neutrosophic Einstein interactive weighted averaging and geometric operators for material selection, emphasizing nonlinear interaction among the three components (Farid et al., 2022). Riaz et al. introduced fairly aggregation operators for single-valued neutrosophic information, expanding the fairness behavior of weighted aggregation (Riaz et al., 2023). Ye et al. developed trigonometric aggregation operators for single-valued neutrosophic credibility numbers, which is important for multi-period or periodic decision problems (Ye et al., 2023).

Recent aggregation research has also explored parameterized and outranking-based designs. Senapati proposed Aczel–Alsina aggregation for single-valued neutrosophic numbers and connected it with an outranking procedure for multiple-attribute decisions (Senapati, 2024). Ali et al. introduced α, β, γ-neutrosophic aggregation operators that separately control the influence of membership, indeterminacy, and non-membership in software-site selection (Ali et al., 2024). Wu et al. proposed neutrosophic Z-number Schweizer–Sklar prioritized aggregation operators and a new score function for multi-attribute decision-making (Wu et al., 2025). Borah et al. introduced quadripartitioned single-valued neutrosophic Z-number aggregation, which broadens the representation of reliability and uncertainty (Borah et al., 2023).

2.4 Dynamic, Consensus, and Reliability-Aware Fusion

Dynamic fusion is needed when evidence sources change over time. Farid and Riaz proposed dynamic single-valued neutrosophic aggregation with time sequence preference for IoT technology in supply-chain management (Farid & Riaz, 2023). Saha et al. developed a consensus-based single-valued neutrosophic model for selecting educational vendors in metaverse-based extended-reality contexts (Saha et al., 2024). Chen et al. combined cloud models, Z-numbers, and interval-valued linguistic neutrosophic sets, which is relevant when uncertainty and randomness must be fused together (Chen et al., 2024). Kamran et al. formulated transportation problems with neutrosophic Z-number variables, showing how neutrosophic reliability can be embedded into optimization models (Kamran et al., 2024).

2.5 Neutrosophic Extensions and Applied Intelligent Systems

Recent neutrosophic studies have expanded the representation space beyond basic single-valued triples. Alsharari introduced single-valued neutrosophic primal theory, which contributes to the mathematical foundation of neutrosophic structures (Alsharari, 2024). Jamil et al. developed induced bipolar neutrosophic Einstein aggregation operators for group decision-making, adding positive and negative polarity to the fusion process (Jamil et al., 2024). Saha et al. linked consensus modeling to educational vendor selection, while Li et al. showed how sentiment-driven evidence can be fused with neutrosophic TOPSIS in healthcare service selection (Saha et al., 2024). These applications suggest that neutrosophic fusion is most valuable when the source of indeterminacy is visible to the decision maker.

2.6 Broader Information Fusion and Uncertainty Quantification

Neutrosophic fusion should also be studied alongside broader uncertainty-fusion methods. Abdar et al. reviewed uncertainty quantification in deep learning and highlighted epistemic and aleatoric uncertainty as key concepts for learning systems (Abdar et al., 2021). Li et al. reviewed progress and future directions in multi-source information fusion, giving a broader context for source weighting and heterogeneous evidence integration (Li et al., 2024). Jiao et al. discussed advances in uncertain information fusion across evidence theory, fuzzy theory, possibility theory, and Bayesian theory (Jiao et al., 2024). Zhang et al. studied large-scale multi-source fusion based on Dempster–Shafer theory, offering scalability lessons that neutrosophic methods can adopt (Zhang et al., 2025).

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

Table 1: Verified 2020–2025 studies directly supporting the review scope.

Study	Year	Main technical focus	Relevance to neutrosophic information fu-sion
Chai et al. (Chai et al., 2021)	2021	SVNS similarity measures	Supports evidence comparison before fusion.
Stanujkic et al. (Stanujkic et al., 2021)	2021	SVNS-EDAS ranking	Converts fused triples into interpretable rankings.
Du et al. (Du et al., 2021)	2021	Neutrosophic Z-number aggregation	Connects evidence reliability with neutrosophic assessments.
Abdar et al. (Abdar et al., 2021)	2021	Uncertainty quantification review	Provides learning-based uncertainty context.
Farid et al. (Farid et al., 2022)	2022	Einstein interactive aggregation	Models nonlinear interaction among T, I, and F.
Zeng et al. (Zeng et al., 2022)	2022	Modified Manhattan similarity	Improves distance behavior for SVNS classification.
Haq et al. (Haq et al., 2022)	2022	SVNS-MEREC-MARCOS	Shows objective weighting with ambiguous material data.
Riaz et al. (Riaz et al., 2023)	2023	Fairly aggregation operators	Adds fairness behavior to neutrosophic aggregation.
Ye et al. (Ye et al., 2023)	2023	Trigonometric credibility operators	Supports credibility-aware and periodic decision models.
Liu et al. (Liu et al., 2023)	2023	SVNN-EDAS group decision	Demonstrates educational quality evaluation under SVNNs.
Farid and Riaz (Farid & Riaz, 2023)	2023	Dynamic aggregation	Introduces time preference and changing source reliability.
Li et al. (Li et al., 2023)	2023	Neutrosophic TOPSIS with sentiment	Links text-derived evidence with neutrosophic decision support.
Hezam et al. (Hezam et al., 2023)	2023	SVNS-MASWIP-COPRAS	Builds sustainability decision support from neutrosophic evidence.
Senapati (Senapati, 2024)	2024	Aczel–Alsina aggregation	Adds parameterized t-norm/t-conorm aggregation.
Ali et al. (Ali et al., 2024)	2024	α, β, γ aggregation	Separately controls truth, indeterminacy, and falsity effects.
Singh and Sharma (Singh & Sharma, 2024)	2024	Divergence measures	Supports classification and pattern-recognition fusion.
Chen et al. (Chen et al., 2024)	2024	Cloud, Z-number, IVLNS model	Combines randomness, reliability, and neutrosophic uncertainty.
Saha et al. (Saha et al., 2024)	2024	Consensus SVNS model	Treats agreement control in group decision fusion.
Li et al. (Li et al., 2024)	2024	Multi-source information fusion review	Gives fusion-system perspective beyond neutrosophic methods.
Jiao et al. (Jiao et al., 2024)	2024	Uncertain information fusion	Positions neutrosophic fusion among uncertainty theories.
Fei et al. (Fei et al., 2024)	2024	DST fusion review	Provides conflict-management reference for neutrosophic fusion.
Wu et al. (Wu et al., 2025)	2025	NZN Schweizer–Sklar aggregation	Advances reliability-aware prioritized neutrosophic fusion.
Zhang et al. (Zhang et al., 2025)	2025	Large-scale DST fusion	Suggests scalability mechanisms for neutrosophic fusion.

3. Selection Model and Review Protocol

The review follows a thematic selection model designed for mathematical and algorithmic relevance. A study was considered relevant when it satisfied four conditions. First, it was published between 2020 and 2025. Second, it dealt directly with neutrosophic sets, single-valued neutrosophic numbers, neutrosophic Z-numbers, neutrosophic aggregation, neutrosophic similarity, or uncertainty-aware information fusion. Third, its bibliographic data could be matched with a publisher page, DOI record, indexed database entry, or recognized repository page. Fourth, it contributed to at least one stage of the fusion pipeline: representation, source weighting, aggregation, contradiction measurement, decision scoring, or validation.

The selection model is not a bibliometric survey. It is a mathematical review protocol. The purpose is to identify operator families, decision mechanisms, and validation practices that can guide new research. The selected studies were grouped into six categories: similarity and divergence; ranking and decision fusion; aggregation operators; dynamic and consensus fusion; neutrosophic extensions; and broader uncertain information fusion. Figure 1 summarizes the resulting taxonomy.

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

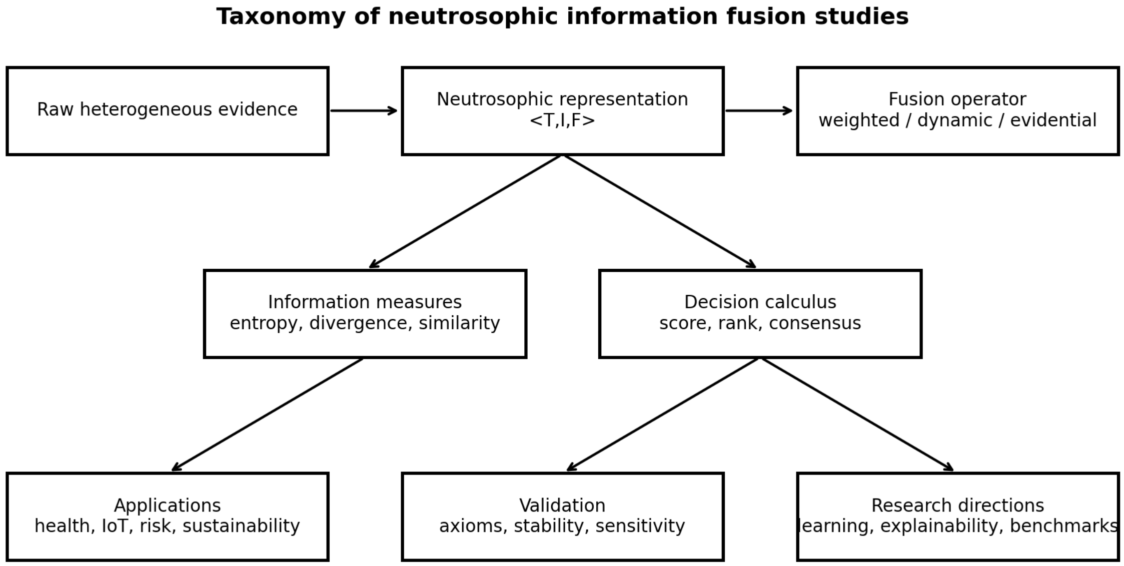

Figure 1: Taxonomy used to organize recent neutrosophic information fusion studies.

4. Mathematical Foundation of Neutrosophic Information Fusion

Definition 1 (Single-valued neutrosophic evidence)**.** *Let X be a finite universe of alternatives, events, classes, or propositions. A single-valued neutrosophic evidence map is a function*

$$\mathscr{N}: X \to [0,1]^3,\ \mathscr{N}(x) = \langle \mathsf{T}(x), \mathsf{I}(x), \mathsf{F}(x) \rangle, \tag{1}$$

where $\mathsf{T}(x)$, $\mathsf{I}(x)$, and $\mathsf{F}(x)$ denote truth, indeterminacy, and falsity degrees. The components

$$0 \le \mathsf{T}(x), \mathsf{I}(x), \mathsf{F}(x) \le 1, \qquad 0 \le \mathsf{T}(x) + \mathsf{I}(x) + \mathsf{F}(x) \le 3, \ \textit{satisfy} \tag{2}$$

The permissive sum condition distinguishes single-valued neutrosophic modeling from probability modeling. The three values are not required to sum to one because the same proposition can be supported, opposed, and partially unresolved. This is the central reason neutrosophic theory is attractive for information fusion.

Definition 2 (Multi-source neutrosophic fusion problem)**.** *Given sources $S_1, \ldots, S_m$, alternatives $A_1, \ldots, A_n$, and source-specific evaluations*

$$\mathscr{N}_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m, \tag{3}$$

the objective is to construct a fused assessment

$$\mathscr{A}_i = \Phi_w(\mathscr{N}_{i1}, \ldots, \mathscr{N}_{im}) = \langle \widehat{T}_i, \widehat{I}_i, \widehat{F}_i \rangle, \tag{4}$$

where $w = (w_1, \ldots, w_m)$ is a nonnegative reliability vector and $\sum_{j=1}^{m} w_j = 1$.

4.1 Canonical Fusion Operators

A weighted arithmetic fusion operator is

$$\widehat{T}_i = \sum_{j=1}^{m} w_j T_{ij},\ \widehat{I}_i = \sum_{j=1}^{m} w_j I_{ij},\ \widehat{F}_i = \sum_{j=1}^{m} w_j F_{ij}. \tag{5}$$

This form is transparent, but it may not capture strong nonlinear compensation. A weighted geometric form is

$$\widehat{T}_i = 1 - \prod_{j=1}^{m} (1 - T_{ij})^{w_j},\ \widehat{I}_i = \prod_{j=1}^{m} I_{ij}^{w_j},\ \widehat{F}_i = \prod_{j=1}^{m} F_{ij}^{w_j}. \tag{6}$$

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

Einstein, Aczel–Alsina, Dombi, and Schweizer–Sklar families extend these forms by replacing ordinary products and sums with parameterized t-norms and t-conorms. Farid et al. demonstrate this idea with Einstein operations, while Senapati uses Aczel–Alsina operations for outranking under SVNNs (Farid et al., 2022; Senapati, 2024).

Proposition 1 (Closure of weighted arithmetic neutrosophic fusion)**.** *If* $\mathscr{N}_{ij} \in [0,1]^3$ *and* $w_j \geq 0$ *with* $\sum_{j=1}^{m} w_j = 1$*, then the fused triple in Eq.* (5) *belongs to* $[0,1]^3$.

Proof. For each component, $0 \leq T_{ij} \leq 1$. Multiplying by $w_j \geq 0$ and summing gives $0 \leq \sum_j w_j T_{ij} \leq \sum_j w_j = 1$. The same argument applies to I_{ij} and F_{ij}. Hence the fused triple remains inside $[0,1]^3$. □

Lemma 1 (Indeterminacy-sensitive contradiction)**.** *Define contradiction intensity of source* j *for alternative* i *as*

$$C_{ij} = T_{ij}F_{ij} + I_{ij}|T_{ij} - F_{ij}|. \tag{7}$$

Then $0 \leq C_{ij} \leq 2$ *for all* $T_{ij}, I_{ij}, F_{ij} \in [0,1]$.

Proof. Since $0 \leq T_{ij}F_{ij} \leq 1$ and $0 \leq I_{ij}|T_{ij} - F_{ij}| \leq 1$, their sum is between 0 and 2. Nonnegativity follows from nonnegative factors. □

5. Information Measures for Neutrosophic Fusion

Information measures determine which sources should dominate the fusion result. For a source S_j, one possible component-wise entropy is

$$H_j = -\frac{1}{n \log 3} \sum_{i=1}^{n} \sum_{c \in \{T,I,F\}} p_{ij}^{(c)} \log(p_{ij}^{(c)} + \varepsilon), \tag{8}$$

where $p_{ij}^{(c)}$ denotes normalized component intensity and $\varepsilon > 0$ prevents the logarithm of zero. A simple reliability weight is

$$w_j = \frac{1 - H_j}{\sum_{\ell=1}^{m} (1 - H_\ell)}. \tag{9}$$

This design gives higher weight to sources with more concentrated information.

Another useful measure is contradiction dispersion:

$$D_j = \frac{1}{n} \sum_{i=1}^{n} \left| T_{ij} - F_{ij} \right| I_{ij}. \tag{10}$$

High D_j indicates that a source frequently reports strong separation between truth and falsity under high indeterminacy. Such a source may be informative but unstable. A conservative reliability model can therefore be written as

$$w_j = \frac{(1 - H_j)\exp(-\rho D_j)}{\sum_{\ell=1}^{m} (1 - H_\ell)\exp(-\rho D_\ell)}, \qquad \rho \geq 0. \tag{11}$$

Figure 2 illustrates the T–I–F evidence space.

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

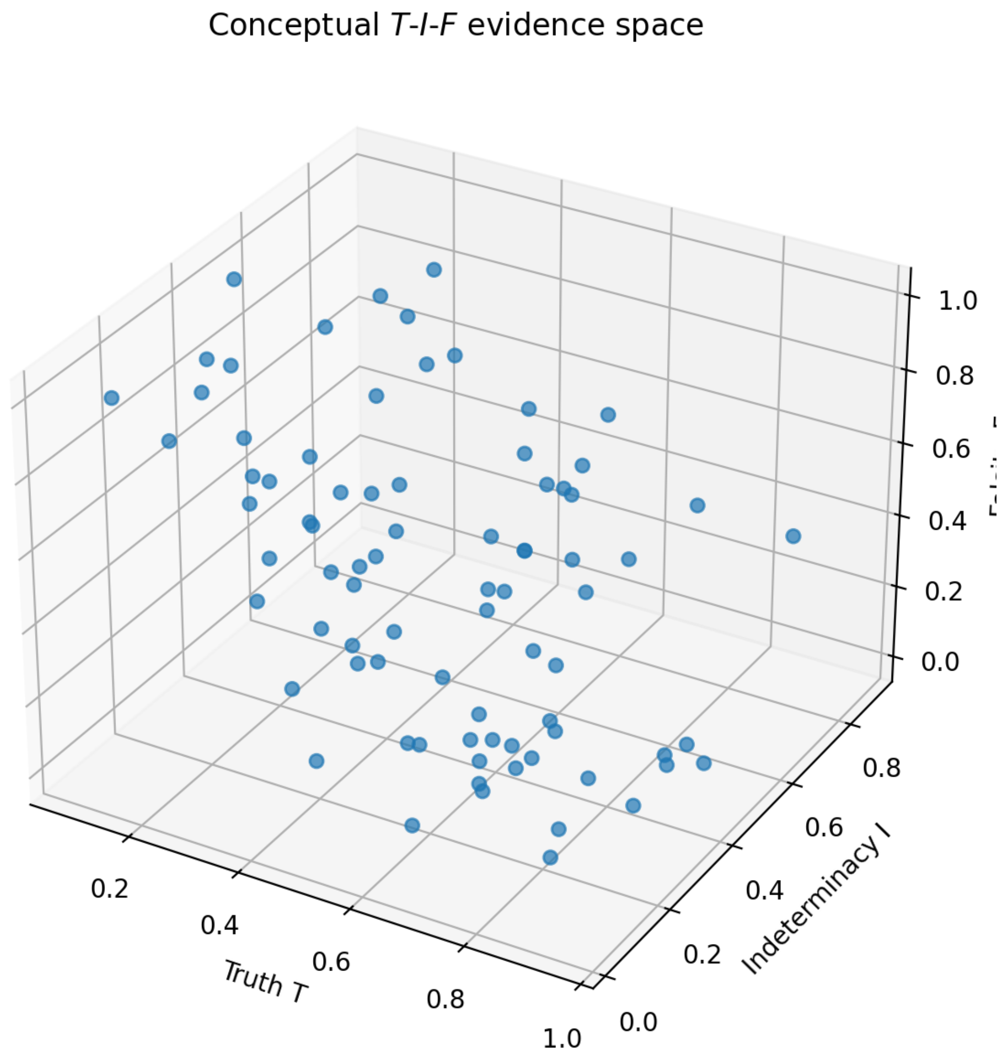

Figure 2: Conceptual T–I–F evidence space used in neutrosophic fusion analysis.

6. Algorithmic Taxonomy

Neutrosophic information fusion algorithms can be grouped into six families. The first family contains direct weighted operators such as arithmetic, geometric, and hybrid weighted averaging. The second family uses nonlinear parameterized operations such as Einstein, Dombi, Aczel–Alsina, or Schweizer–Sklar operations. The third family applies ranking methods such as EDAS, TOPSIS, COPRAS, MARCOS, MABAC, or outranking. The fourth family uses information measures such as entropy, divergence, similarity, and cross-entropy. The fifth family is dynamic, where reliability weights evolve with time. The sixth family links neutrosophic values with data-driven or learning-based systems.

6.1 Generic Algorithm for Neutrosophic Information Fusion

Listing 1: Generic neutrosophic information fusion procedure

```
Input: alternatives A_1,...,A_n; sources S_1,...,S_m; source observations E_ij
Output: fused neutrosophic triples and final decision scores
1. Map each observation E_ij to N_ij = <T_ij, I_ij, F_ij>.
2. Estimate source quality q_j using entropy, reliability, error, or expert confidence.
3. Normalize q_j to obtain weights w_j.
4. For each alternative A_i, compute N_hat_i = Phi_w(N_i1,...,N_im).
5. Compute contradiction C_i and indeterminacy penalty P_i.
6. Compute score S_i = T_hat_i - F_hat_i - alpha I_hat_i - beta C_i.
7. Rank alternatives or select the class with maximum score.
8. Report N_hat_i, score S_i, and the source of indeterminacy.
```

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

The algorithm reflects a key requirement: the final rank should not be reported alone. A neutrosophic system should also report the fused triple and the reason for indeterminacy. This reporting principle is essential in medical diagnosis, cybersecurity, education, sustainability assessment, and risk management.

6.2 Dynamic Neutrosophic Fusion

Dynamic fusion is needed when source reliability changes over time. Let $\mathcal{N}_{ij}^{(t)}$ be a triple at time t. A time-decayed reliability can be defined as

$$r_j^{(t)} = \lambda r_j^{(t-1)} + (1-\lambda) q_j^{(t)}, \qquad 0 < \lambda < 1, \tag{12}$$

where $q_j^{(t)}$ measures current quality. The normalized weight is $w_j^{(t)} = r_j^{(t)} / \sum_\ell r_\ell^{(t)}$. This form is consistent with dynamic aggregation designs in IoT-related neutrosophic decision systems (Farid & Riaz, 2023).

7. Operator Properties and Mathematical Analysis

A neutrosophic fusion operator should be analyzed by more than its numerical output. The minimum set of mathematical properties includes closure, boundedness, monotonicity, idempotency, commutativity, associativity, stability under perturbation, and sensitivity to indeterminacy. Table 2 summarizes these properties.

Table 2: Operator properties expected in neutrosophic information fusion.

Property	Mathematical meaning
Closure	The fused output remains in $[0,1]^3$.
Boundedness	The fused component is bounded by the minimum and maximum source components.
Idempotency	If all sources report the same triple, fusion returns the same triple.
Monotonicity	Increasing truth support should not reduce fused truth under fixed weights.
Conflict sensitivity	Simultaneously high truth and falsity should increase contradiction or indeterminacy.
Weight stability	Small reliability perturbations should not cause unjustified rank reversal.
Interpretability	The semantic roles of T, I, and F remain distinguishable after fusion.

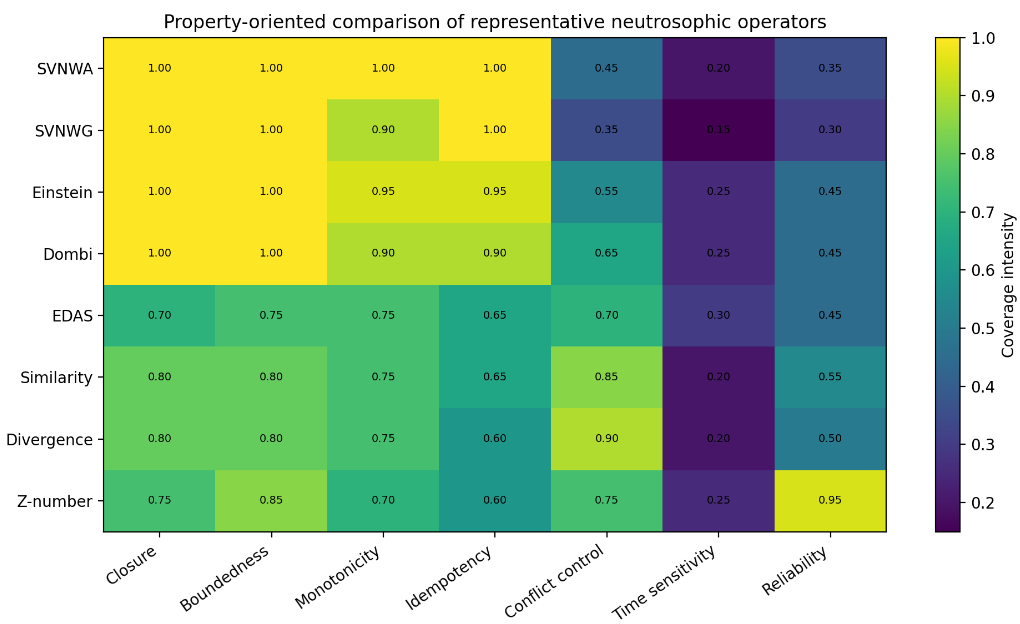

Figure 3: Illustrative operator-property heatmap for neutrosophic fusion families.

Proposition 2 (Idempotency of weighted arithmetic fusion)**.** *Let $\mathcal{N}_{i1} = \cdots = \mathcal{N}_{im} = \langle T_i, I_i, F_i \rangle$. Then the weighted arithmetic fusion operator in Eq.* (5) *returns $\langle T_i, I_i, F_i \rangle$.*

Proof. Because every source has the same truth component, $\widehat{T}_i = \sum_j w_j T_i = T_i \sum_j w_j = T_i$. The same reasoning gives $\widehat{I}_i = I_i$ and $\widehat{F}_i = F_i$. □

A useful scalar decision score is

$$S_i = \widehat{T}_i - \widehat{F}_i - \alpha \widehat{I}_i - \beta \widehat{C}_i, \qquad \alpha, \beta \geq 0, \tag{13}$$

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

where $\widehat{C}_i$ is a fused contradiction term. The parameter α controls hesitation penalty, whereas β controls conflict penalty. This form is suitable when high indeterminacy should reduce automatic decision confidence.

8. Relationship with Evidence Theory and Uncertainty Quantification

Dempster–Shafer theory and neutrosophic theory both handle uncertainty, but they do so differently. Evidence theory allocates belief mass to sets and uses combination rules to merge sources. Neutrosophic theory assigns truth, indeterminacy, and falsity directly to propositions. The two perspectives can be complementary: evidence-theoretic conflict can help estimate neutrosophic indeterminacy, while neutrosophic triples can make conflict more interpretable.

Uncertainty quantification in machine learning usually distinguishes epistemic and aleatoric uncertainty. Neutrosophic indeterminacy is broader because it can include missing evidence, source disagreement, semantic ambiguity, and model hesitation. A practical decomposition can be written as

$$I = I_{\text{miss}} + I_{\text{conflict}} + I_{\text{model}} + I_{\text{semantic}}, \tag{14}$$

with normalization if the sum exceeds one. This decomposition can connect neutrosophic fusion with uncertainty-aware deep learning and multi-source fusion systems (Abdar et al., 2021; Li et al., 2024).

9. Research Gaps

The reviewed literature shows substantial progress, but several limitations remain. First, many studies validate operators on small numerical examples rather than public benchmarks. Second, membership functions are often manually assigned, and few papers explain how T , I, and F can be learned from data. Third, the boundary between indeterminacy and falsity remains inconsistent across applications. Fourth, sensitivity analysis is still uneven, especially for parameterized operators such as Einstein, Aczel–Alsina, Dombi, Schweizer–Sklar, and α, β, γ operators.

A second limitation is computational scalability. Classical aggregation operators are efficient, but group decision-making, large-scale multi-source fusion, and dynamic streaming fusion need incremental algorithms. Evidence-theoretic studies have started addressing large-scale multi-source fusion more directly (Zhang et al., 2025). Neutrosophic fusion can benefit from similar sparse representations, source clustering strategies, and streaming update rules.

10. Research Directions for 2025 and Beyond

Future researchers in Neutrosophic and Information Fusion can build on five directions.

10.1 Benchmark-Oriented Neutrosophic Fusion

Future work should publish benchmark protocols with raw data, membership construction rules, parameter settings, and evaluation scripts. A useful benchmark should include missingness, source conflict, expert disagreement, and time-varying reliability.

10.2 Learning Membership Functions from Data

A major direction is to estimate T, I, and F from calibrated classifiers, evidential learning, ensembles, uncertainty scores, or expert annotations. If a classifier produces class probability p, predictive entropy H, and contradiction score c, one possible mapping is

$$T = p(1-H), \qquad I = H + c - pc, \qquad F = (1-p)(1-H). \tag{15}$$

This mapping is not universal, but it illustrates how data-driven neutrosophic components can be constructed from measurable quantities.

10.3 Explainable Indeterminacy

A future paper should not only report that I is high. It should explain whether the indeterminacy arises from source absence, noisy measurement, out-of-distribution behavior, expert hesitation, semantic ambiguity, or conflict between sources.

10.4 Neutrosophic Fusion with Deep Models

Deep learning systems can benefit from neutrosophic post-processing layers that convert softmax outputs, ensembles, evidential logits, or Bayesian approximations into T–I–F triples. The main challenge is to prevent neutrosophic layers from becoming cosmetic; they should improve calibration, abstention, robustness, or explanation.

10.5 Privacy-Aware and Federated Neutrosophic Fusion

Federated neutrosophic fusion can aggregate triples or source reliability statistics without exposing raw data. This is promising in healthcare, education, finance, and IoT. However, privacy leakage from repeated triple updates and membership summaries should be formally studied.

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

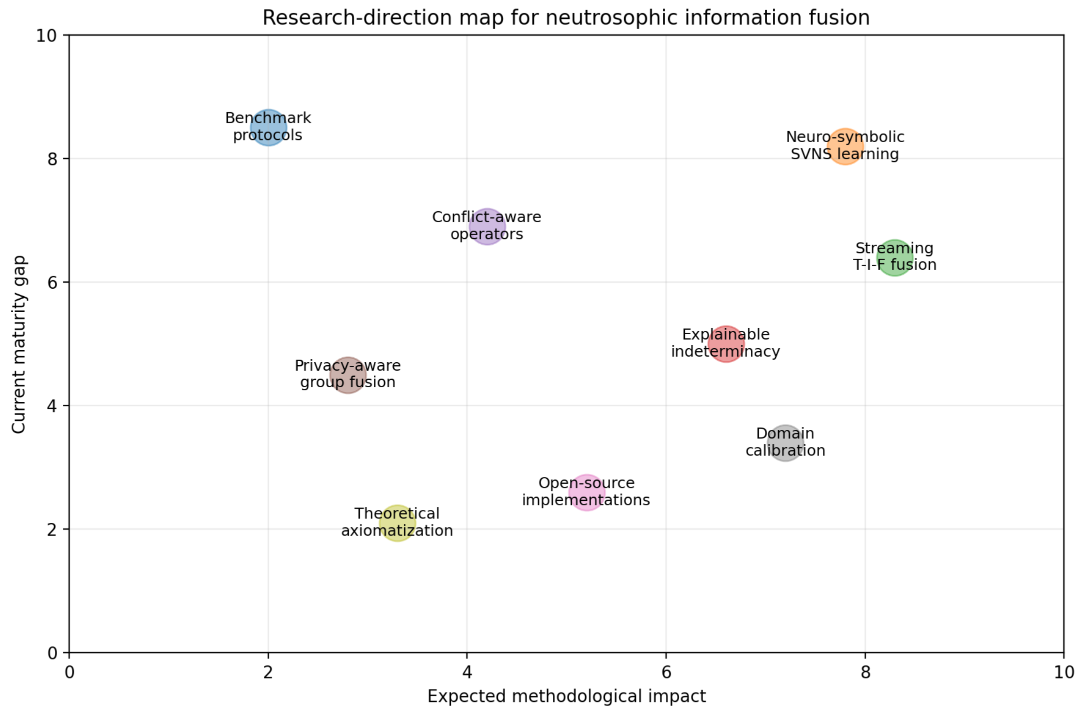

Figure 4: Research-direction map for future neutrosophic information fusion studies.

11. Algorithmic Recommendations

Researchers proposing a new neutrosophic fusion model should report five elements. First, define the semantic meaning of T, I, and F in the application. Second, state the operator and prove closure, idempotency, and boundedness. Third, explain how weights are obtained. Fourth, test sensitivity to membership noise and parameter choices. Fifth, compare against at least one non-neutrosophic uncertainty-fusion baseline such as Dempster–Shafer fusion, Bayesian fusion, or calibrated ensemble learning.

Listing 2: Recommended validation protocol for new neutrosophic fusion operators

```
Input: proposed operator Phi, benchmark data or decision matrix D
1. Verify mathematical validity: closure, boundedness, monotonicity, idempotency.
2. Construct perturbation tests for T, I, and F independently.
3. Compare rankings or classifications against at least two baseline fusion rules.
4. Run parameter sensitivity for all operator-specific parameters.
5. Report contradiction cases where T and F are both high.
6. Report indeterminacy explanations and not only final accuracy or ranking.
7. Release code, input matrices, and membership construction rules.
```

12. Discussion

The reviewed studies show that neutrosophic information fusion is mathematically rich but methodologically fragmented. Aggregation research has produced flexible operators, while information-fusion research has advanced uncertainty handling, evidence theory, and scalable multi-source architectures. The next stage should integrate these developments. Neutrosophic fusion can become more influential if it is presented as an interpretable uncertainty architecture rather than a sequence of isolated ranking formulas.

The review also shows that indeterminacy is the most underused component. Many models include I formally but do not exploit it analytically. In a mature neutrosophic fusion system, indeterminacy should trigger abstention, request additional evidence, adjust source weights, select human review, or explain why a decision is unstable.

13. Conclusion

This paper reviewed neutrosophic information fusion from 2020 to 2025 through a mathematical and algorithmic lens. It added a related-work synthesis of more than twenty verified studies, clarified the selection model, analyzed core operator properties, and proposed algorithmic recommendations for future research. The analysis showed that the field has strong mathematical foundations but needs more reproducible benchmarks, data-driven membership estimation, scalable algorithms, explainable indeterminacy, and systematic comparison with evidence theory and uncertainty-quantified machine learning. Future research should focus on turning neutrosophic triples into operational decision-support mechanisms that preserve truth, reveal falsity, and make indeterminacy actionable.

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

References

[1] Abdar, M., et al. (2021). A review of uncertainty quantification in deep learning: Techniques, applications and challenges. *Information Fusion*, *76*, 243–297. https://doi.org/10.1016/j.inffus.2021.05.008

[2] Ali, S., Rahim, M., Bajri, S. A., Ahmad, S., Alharbi, R., & Khalifa, H. A. E.-W. (2024). Alpha, beta, gamma-neutrosophic aggregation operators and their applications in the software site selection. *Heliyon*, *10*(10), e31417. https://doi.org/10.1016/j.heliyon.2024.e31417

[3] Alsharari, F. (2024). An introduction to single-valued neutrosophic primal theory. *Symmetry*, *16*(4), 402. https://doi.org/10.3390/sym16040402

[4] Borah, G., et al. (2023). Aggregation operators of quadripartitioned single-valued neutrosophic z-number and its application in multi-attribute decision making. *Engineering Applications of Artificial Intelligence*, *122*, 106101. https://doi.org/10.1016/j.engappai.2023.106101

[5] Chai, J. S., Selvachandran, G., Smarandache, F., Gerogiannis, V. C., Son, L. H., Bui, Q.-T., & Vo, B. (2021). New similarity measures for single-valued neutrosophic sets with applications in pattern recognition and medical diagnosis problems. *Complex & Intelligent Systems*, *7*, 703–723. https://doi.org/10.1007/s40747-020-00220-w

[6] Chen, H., et al. (2024). A decision-making model with cloud model, z-numbers, and interval-valued linguistic neutrosophic sets. *Entropy*, *26*(11), 892. https://doi.org/10.3390/e26110892

[7] Du, S., Ye, J., Yong, R., & Zhang, F. (2021). Some aggregation operators of neutrosophic z-numbers and their multicriteria decision making method. *Complex & Intelligent Systems*, *7*, 429–438. https://doi.org/10.1007/s40747-020-00204-w

[8] Farid, H. M. A., & Riaz, M. (2023). Single-valued neutrosophic dynamic aggregation information with time sequence preference for iot technology in supply chain management. *Engineering Applications of Artificial Intelligence*, *126*, 106940. https://doi.org/10.1016/j.engappai.2023.106940

[9] Farid, H. M. A., Riaz, M., Pamucar, D., & Bozanic, D. (2022). Single-valued neutrosophic einstein interactive aggregation operators for material selection in engineering design. *Complex & Intelligent Systems*, *8*, 4201–4231. https://doi.org/10.1007/s40747-021-00626-0

[10] Fei, L., Li, T., & Ding, W. (2024). Dempster-shafer theory-based information fusion for natural disaster emergency management: A systematic literature review. *Information Fusion*, *112*, 102585. https://doi.org/10.1016/j.inffus.2024.102585

[11] Garg, H. (2024). A new exponential-logarithm-based single-valued neutrosophic set and their applications. *Expert Systems with Applications*, *238*, 121854. https://doi.org/10.1016/j.eswa.2023.121854

[12] Haq, R. S. U., Saeed, M., Mateen, N., Siddiqui, F., Naqvi, M., Yi, J. B., & Ahmed, S. (2022). Sustainable material selection with crisp and ambiguous data using single-valued neutrosophic-merec-marcos framework. *Applied Soft Computing*, *128*, 109546. https://doi.org/10.1016/j.asoc.2022.109546

[13] Hezam, I. M., et al. (2023). An integrated decision support framework using single-valued neutrosophic-maswip-copras for sustainability assessment of bioenergy production technologies. *Expert Systems with Applications*, *211*, 118674. https://doi.org/10.1016/j.eswa.2022.118674

[14] Jamil, M., et al. (2024). Multiple attribute group decision-making approach for induced bipolar neutrosophic einstein aggregation operators. *Complex & Intelligent Systems*, *10*, 4123–4143. https://doi.org/10.1007/s40747-023-01264-4

[15] Jiao, L., Pan, Q., & Denoeux, T. (2024). Advances in uncertain information fusion. *Entropy*, *26*(11), 945. https://doi.org/10.3390/e26110945

[16] Kamran, M., et al. (2024). A systematic formulation into neutrosophic z-number-based multi-objective transportation problems. *Symmetry*, *16*(5), 615. https://doi.org/10.3390/sym16050615

[17] Li, X., Luo, Y., Wang, H., Lin, J., & Deng, B. (2023). Doctor selection based on aspect-based sentiment analysis and neutrosophic topsis method. *Engineering Applications of Artificial Intelligence*, *124*, 106599. https://doi.org/10.1016/j.engappai.2023.106599

[18] Li, X., Dunkin, F., & Dezert, J. (2024). Multi-source information fusion: Progress and future. *Chinese Journal of Aeronautics*, *37*(6), 1–29. https://doi.org/10.1016/j.cja.2023.12.009

[19] Liu, Y., Yang, J., & Wang, J. (2023). Edas method for single-valued neutrosophic number multiattribute group decision-making and applications to physical education teaching quality evaluation in colleges and universities. *Mathematical Problems in Engineering*, *2023*, 5576217. https://doi.org/10.1155/2023/5576217

[20] Riaz, M., Farid, H. M. A., Ashraf, S., & Kamaci, H. (2023). Single-valued neutrosophic fairly aggregation operators with multi-criteria decision-making. *Computational and Applied Mathematics*, *42*, 104. https://doi.org/10.1007/s40314-023-02233-w

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025

[21] Saha, A., Kolandasamy, R., Chatterjee, P., & Antucheviciene, J. (2024). A consensus-based single valued neutrosophic model for selection of educational vendors under metaverse with extended reality. *Applied Soft Computing*, *155*, 111476. https://doi.org/10.1016/j.asoc.2024.111476

[22] Senapati, T. (2024). An aczel-alsina aggregation-based outranking method for multiple attribute decision-making using single-valued neutrosophic numbers. *Complex & Intelligent Systems*, *10*, 1185–1199. https://doi.org/10.1007/s40747-023-01215-z

[23] Singh, S., & Sharma, S. (2024). Aggregation operators-based divergence measures for single-valued neutrosophic sets with their applications to pattern recognition. *Journal of Intelligent & Fuzzy Systems*. https://doi.org/10.3233/JIFS-232369

[24] Stanujkic, D., Karabasevic, D., Popovic, G., Pamucar, D., Stevic, Z., Zavadskas, E. K., & Smarandache, F. (2021). A single-valued neutrosophic extension of the edas method. *Axioms*, *10*(4), 245. https://doi.org/10.3390/axioms10040245

[25] Wu, M., et al. (2025). Neutrosophic z-number schweizer-sklar prioritized aggregation operators and new score function for multi-attribute decision making. *Artificial Intelligence Review*. https://doi.org/10.1007/s10462-025-11124-x

[26] Ye, J., Du, S., & Yong, R. (2023). Multi-criteria decision-making model using trigonometric aggregation operators of single-valued neutrosophic credibility numbers. *Information Sciences*, *644*, 118968. https://doi.org/10.1016/j.ins.2023.118968

[27] Zeng, Y., Luo, M., Guo, H., & Li, Z. (2022). A novel similarity measure of single-valued neutrosophic sets based on modified manhattan distance and its applications. *Electronics*, *11*(6), 941. https://doi.org/10.3390/electronics11060941

[28] Zhang, Q., et al. (2025). Information fusion for large-scale multi-source data based on the dempster-shafer evidence theory. *Information Fusion*, *115*, 102754. https://doi.org/10.1016/j.inffus.2024.102754

DOI: https://doi.org/10.54216/NIF.060101
Received: June 28, 2025 Accepted: September 14, 2025