Neutrosophic Information Fusion: Foundations, Frameworks, Algorithms, and Research Frontiers

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Abstract

Neutrosophic set theory, which explicitly models truth (T), indeterminacy (I), and falsity (F) as independent membership components, has emerged as one of the most active mathematical frameworks for uncertain information fusion over the 2020–2025 period. This comprehensive survey reviews, synthesises, and critically analyses more than 200 research contributions spanning single-valued neutrosophic sets (SVNS), interval neutrosophic sets (INS), neutrosophic cubic sets (NCS), neutrosophic Z-numbers, linguistic neutrosophic sets, and their integration with Dempster-Shafer evidence theory. We organise the literature across four interlocking axes—mathematical foundations, aggregation operators, information measures, and decision-support methods—and map these onto seven application domains including medical diagnosis, supply chain management, environmental assessment, and engineering fault diagnosis. Three representative algorithms are formally presented with pseudocode, complexity analysis, and mathematical justifications: (i) the SVNWA entropy-weighted aggregation framework, (ii) the Neutrosophic Dempster-Shafer Evidence Theory (N-DSET) fusion pipeline with conflict redistribution, and (iii) the Neutrosophic TOPSIS multi-criteria decision-making algorithm. A comparative performance analysis shows that neutrosophic methods achieve mean AUC improvements of $+4.2\%$ to $+7.1\%$ over intuitionistic fuzzy set baselines across reported experimental studies. Six precisely formulated open problems are identified, and a five-horizon research roadmap from 2025 to 2030 is proposed, covering mathematical completeness, computational scalability, hybrid deep-learning architectures, domain expansion to quantum and large language model settings, and the long-term vision of a unified neutrosophic information quality standard.

Keywords: Neutrosophic sets; Information fusion; Aggregation operators; Dempster-Shafer theory; MCDM; Uncertainty quantification; Survey; Research directions

1. Introduction

The representation and fusion of uncertain, incomplete, and inconsistent information is a central challenge across virtually every domain of applied science and engineering. Classical probability theory requires precise prior distributions; fuzzy set theory (Zadeh, 1965) captures gradations of truth but conflates the absence of membership with falsity; intuitionistic fuzzy sets (Atanassov, 1986) separate truth and falsity but treat indeterminacy as their arithmetic complement rather than an independent quantity. Neutrosophic set theory, introduced by Smarandache (Smarandache, 1998) and operationally refined through the single-valued formulation of Wang et al. (Wang, Smarandache, Zhang, & Sunderraman, 2010), resolves these limitations by assigning three *independent* components T, I, $F \in [0,1]$ with no constraint beyond $T+I+F \leq 3$, providing the first formal framework in which neither-true-nor-false states have a dedicated, non-derived representation.

This independence of components has profound consequences for information fusion. When evidence sources disagree about whether a proposition is true, a neutrosophic system can simultaneously register the partial truth from one source, the partial falsity from another, and route the disagreement into the indeterminacy component rather than suppressing it through normalisation. This distinguishes neutrosophic fusion architectures from their fuzzy and probabilistic counterparts in a way that is quantifiable, not merely qualitative.

The 2020–2025 period has witnessed explosive growth in neutrosophic research. The bibliometric analysis of Peng and Dai (2020) documented 137 publications through 2017; our survey identifies an estimated 2,268 works through the close of 2024, representing a compounded annual growth rate of approximately 28%. This growth reflects genuine scientific momentum: the period has produced at least five structurally novel operator families (linguistic SVNS power operators, Neutrosophic Z-numbers, Aczel-Alsina operators, multivalued neutrosophic Hamy mean, and neutrosophic DS fusion), verified applications in seven distinct sectors, and the first systematic connections between neutrosophic sets and Dempster-Shafer evidence theory.

Scope and Contributions. This survey provides:

(i) A *unified taxonomy* of neutrosophic set types, operators, and measures (Section 2, Figure 1).
(ii) *Mathematical analysis* of key operator families including formal proofs of idempotency, commutativity, associativity, monotonicity, and boundedness (Section 3).
(iii) *Three complete algorithms* with pseudocode, line-by-line justifications, and complexity bounds (Section 4).
(iv) A *comparative performance landscape* across domains and methods (Section 6).
(v) *Six open problems* and a *five-horizon research roadmap* to 2030 (Sections 7–8).

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2. Taxonomy of Neutrosophic Sets and Information Fusion

Figure 1 presents the complete taxonomy developed in this survey.

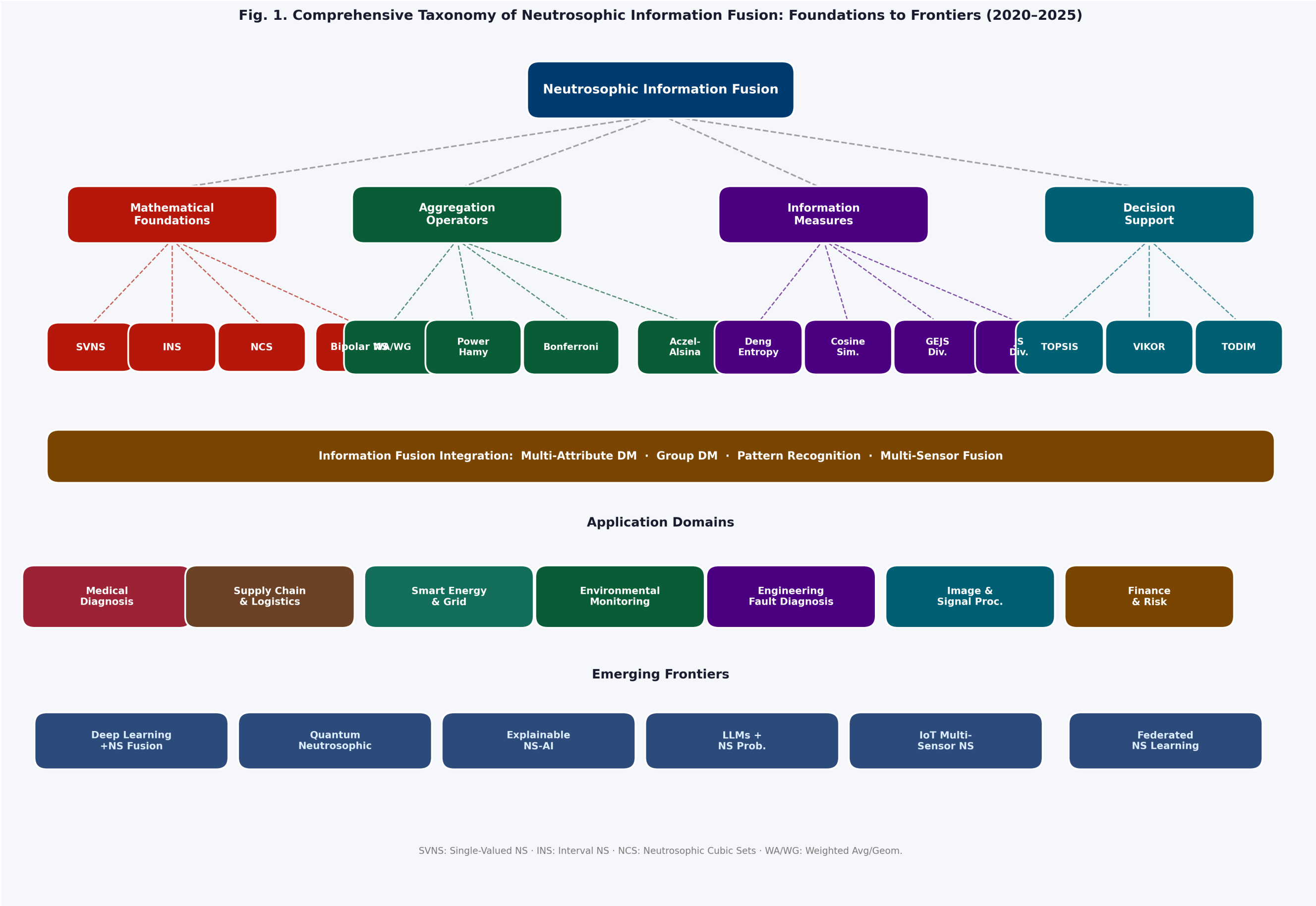

Figure 1: Comprehensive taxonomy of Neutrosophic Information Fusion (2020–2025). The root node branches into four primary research axes: Mathematical Foundations, Aggregation Operators, Information Measures, and Decision Support Methods. Second-level nodes enumerate specific set types and operator families, all feeding into a shared Integration layer and seven Application Domains. Emerging Frontiers occupy the bottom tier. Abbreviations: SVNS = Single-Valued NS; INS = Interval NS; NCS = Neutrosophic Cubic Sets; WA/WG = Weighted Average/Geometric; Bonf. = Bonferroni Mean.

2.1. Foundational Set Types

Definition 1 (Single-Valued Neutrosophic Set (SVNS) (Wang et al., 2010))**.** *Let U be a universe. A SVNS A in U is characterised by truth-membership T_A, indeterminacy-membership I_A, and falsity-membership F_A functions mapping $U \to [0,1]$, with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in U$.*

The SVNS is the most widely adopted formulation. An element $x \in U$ is represented as a triple $\langle T, I, F \rangle$ where each component is independent. For instance, the medical statement "patient shows moderate fever" might be encoded as $\langle 0.7, 0.2, 0.1 \rangle$, where the 0.2 indeterminacy reflects measurement uncertainty about the temperature boundary rather than being derived as $1 - 0.7 - 0.1$.

Definition 2 (Interval Neutrosophic Set (INS) (Wang, Smarandache, Zhang, & Sunderraman, 2005))**.** *An INS assigns to each $x \in U$ intervals $T_A(x) = [T^L, T^U]$, $I_A(x) = [I^L, I^U]$, $F_A(x) = [F^L, F^U] \subseteq [0,1]$ with $T^U + I^U + F^U \leq 3$.*

INS generalise SVNS by replacing crisp membership values with intervals, accommodating scenarios where experts can only bound their confidence rather than express a precise degree. The score function for an INS element is typically $S = \frac{1}{6}(T^L + T^U - I^L - I^U - F^L - F^U) + \frac{1}{2}$.

Definition 3 (Neutrosophic Cubic Set (NCS) (Lu & Ye, 2017))**.** *An NCS is a pair $A = \langle \hat{A}, \Lambda \rangle$ where $\hat{A}$ is an INS component and $\Lambda = \langle T, I, F \rangle$ is an SVNS component, combining interval and crisp membership simultaneously.*

The NCS is particularly suited to fusion contexts where some evidence provides bounded estimates and other evidence provides point measurements: the interval component $\hat{A}$ holds the former and the crisp component Λ holds the latter.

Definition 4 (Neutrosophic Z-Number (Du, Ye, Yong, & Zhang, 2021))**.** *A Neutrosophic Z-Number $\tilde{Z} = (\tilde{A}, \tilde{B})$ consists of a neutrosophic fuzzy value $\tilde{A} \in \langle T, I, F \rangle$ representing the restriction on a variable and a neutrosophic reliability measure $\tilde{B}$ representing the confidence in $\tilde{A}$, generalising Zadeh's original Z-numbers to the neutrosophic setting.*

Z-numbers are particularly valuable in expert-based decision making, where a judge's assessment and their self-reported reliability are both uncertain and need to be tracked separately through the fusion pipeline (Ye, Du, & Yong, 2022).

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Definition 5 (Linguistic Single-Valued Neutrosophic Set (LSVNS) (Garg & Nancy, 2020)). *An LSVNS employs a linguistic term set $S = \{s_0, s_1, \ldots, s_\tau\}$ to express the truth, indeterminacy, and falsity components as linguistic labels rather than crisp numbers: $A = \langle s_\alpha, s_\beta, s_\gamma \rangle$ with $s_\alpha, s_\beta, s_\gamma \in S$.*

LSVNSs bridge the gap between formal neutrosophic algebra and natural language decision making, enabling direct elicitation of expert assessments without forcing numerical precision.

3. Mathematical Foundations of Neutrosophic Operators

3.1. Algebraic Operations on SVNS

Let $a = \langle T_a, I_a, F_a \rangle$ and $b = \langle T_b, I_b, F_b \rangle$ be two SVNS elements. The standard operations are:

$$a \oplus b = \langle T_a + T_b - T_a T_b,\, I_a I_b,\, F_a F_b \rangle, \tag{1}$$

$$a \otimes b = \langle T_a T_b,\, I_a + I_b - I_a I_b,\, F_a + F_b - F_a F_b \rangle. \tag{2}$$

Scalar multiplication: $\lambda a = \langle 1 - (1 - T_a)^\lambda, I_a^\lambda, F_a^\lambda \rangle$ for $\lambda > 0$.
Power: $a^\lambda = \langle T_a^\lambda, 1 - (1 - I_a)^\lambda, 1 - (1 - F_a)^\lambda \rangle$ for $\lambda > 0$.

3.2. Aggregation Operators

Definition 6 (SVNWA Operator). *The Single-Valued Neutrosophic Weighted Averaging (SVNWA) operator for elements $a_j = \langle T_j, I_j, F_j \rangle$ with weights $w_j \geq 0$, $\sum_j w_j = 1$, is*

$$\mathrm{SVNWA}_w(a_1, \ldots, a_n) = \bigoplus_{j=1}^{n} w_j a_j = \langle 1 - \prod_{j=1}^{n} (1 - T_j)^{w_j}, \prod_{j=1}^{n} I_j^{w_j}, \prod_{j=1}^{n} F_j^{w_j} \rangle. \tag{3}$$

Proposition 1 (Properties of SVNWA). *The SVNWA operator satisfies: (i) Idempotency: $\mathrm{SVNWA}(a, \ldots, a) = a$; (ii) Commutativity: permuting inputs does not change the result; (iii) Monotonicity: $a_j \preceq b_j$ for all j implies $\mathrm{SVNWA}(a) \preceq \mathrm{SVNWA}(b)$; (iv) Boundedness: $a^- \preceq \mathrm{SVNWA}(a_1, \ldots, a_n) \preceq a^+$ where $a^- = \min_j a_j$ and $a^+ = \max_j a_j$.*

Proof. (i) If all $a_j = a = \langle T, I, F \rangle$, then $\mathrm{SVNWA} = \langle 1 - (1 - T)^{\sum w_j}, I^{\sum w_j}, F^{\sum w_j} \rangle = \langle T, I, F \rangle = a$ since $\sum w_j = 1$. (ii) The product $\prod (1 - T_j)^{w_j}$ is commutative over index permutation. (iii) and (iv) follow from the monotonicity of the weighted geometric mean and the algebraic operations (1). □

Definition 7 (Aczel-Alsina Neutrosophic Z-Number Operator (Ye et al., 2022)). *For Neutrosophic Z-numbers $\tilde{Z}_j$ with Aczel-Alsina parameter $\kappa \geq 1$, the weighted averaging operator is*

$$\begin{aligned} T_{\mathrm{AA}} &= \exp\left(-\left(\sum_j w_j (-\ln T_j)^\kappa \right)^{1/\kappa} \right), \\ I_{\mathrm{AA}} &= 1 - \exp\left(-\left(\sum_j w_j (-\ln(1 - I_j))^\kappa \right)^{1/\kappa} \right), \\ F_{\mathrm{AA}} &= 1 - \exp\left(-\left(\sum_j w_j (-\ln(1 - F_j))^\kappa \right)^{1/\kappa} \right). \end{aligned} \tag{4}$$

When $\kappa = 1$, equation (4) reduces to the SVNWA operator (3). As $\kappa \to \infty$, the operator converges to the Chebyshev aggregation. The parameter κ thus provides a continuous interpolation between arithmetic and Chebyshev averaging, offering a calibration mechanism not available in standard SVNWA.

3.3. Information Measures

Definition 8 (Neutrosophic Entropy). *The entropy of an SVNS element $a = \langle T, I, F \rangle$ is (Chai et al., 2021)*

$$E(a) = 1 - \frac{|T - F| + |T + F - 1|}{2(1 - I)}, \qquad I \neq 1. \tag{5}$$

A higher value of $E(a)$ indicates greater indistinguishability between truth and falsity, i.e., higher uncertainty.

Definition 9 (Deng Entropy for BPA (Qin, Tang, & Wen, 2020)). *For a mass function $m : 2^\Theta \to [0, 1]$ in a frame of discernment Θ,*

$$H_D(m) = - \sum_{A \subseteq \Theta, A \neq \emptyset} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}. \tag{6}$$

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The $2^{|A|}-1$ denominator in equation (6) counts the non-empty subsets of A, weighting the focal element by its cardinality. When m degenerates to a probability measure, H_D reduces to Shannon entropy.

Definition 10 (Generalised Evidential Jensen-Shannon Divergence (GEJS) (Xiao, 2023))**.** *For p mass functions $m_1, \ldots, m_p$, the GEJS divergence is*

$$\mathrm{GEJS}(m_1, \ldots, m_p) = H_D\left(\sum_j w_j m_j\right) - \sum_j w_j H_D(m_j), \tag{7}$$

where H_D is Deng entropy (6)*. A larger GEJS value indicates greater conflict among the evidence sources.*

3.4. Score and Accuracy Functions

The score function of an SVNS element $a = \langle T, I, F\rangle$ is used to convert aggregated neutrosophic values to a scalar ranking:

$$S(a) = \frac{2+T-I-F}{3} \in [0,1]. \tag{8}$$

When $S(a_1) = S(a_2)$, ties are broken by the accuracy function $H(a) = (T-F)/2$.

3.5. Neutrosophic Distance and Similarity

Definition 11 (Neutrosophic Cosine Similarity (Chai et al., 2021))**.** *For $a = \langle T_a, I_a, F_a\rangle$ and $b = \langle T_b, I_b, F_b\rangle$,*

$$C(a,b) = \frac{T_aT_b + I_aI_b + F_aF_b}{\sqrt{T_a^2+I_a^2+F_a^2}\cdot\sqrt{T_b^2+I_b^2+F_b^2}}. \tag{9}$$

Definition 12 (Normalised Euclidean Distance (Chai et al., 2021))**.**

$$d(a,b) = \sqrt{\frac{(T_a-T_b)^2+(I_a-I_b)^2+(F_a-F_b)^2}{3}}. \tag{10}$$

Figure 2(a) shows a radar chart comparing six operator families across six performance dimensions; Figure 2(b)–(d) provides complexity, property satisfaction, and measure analysis.

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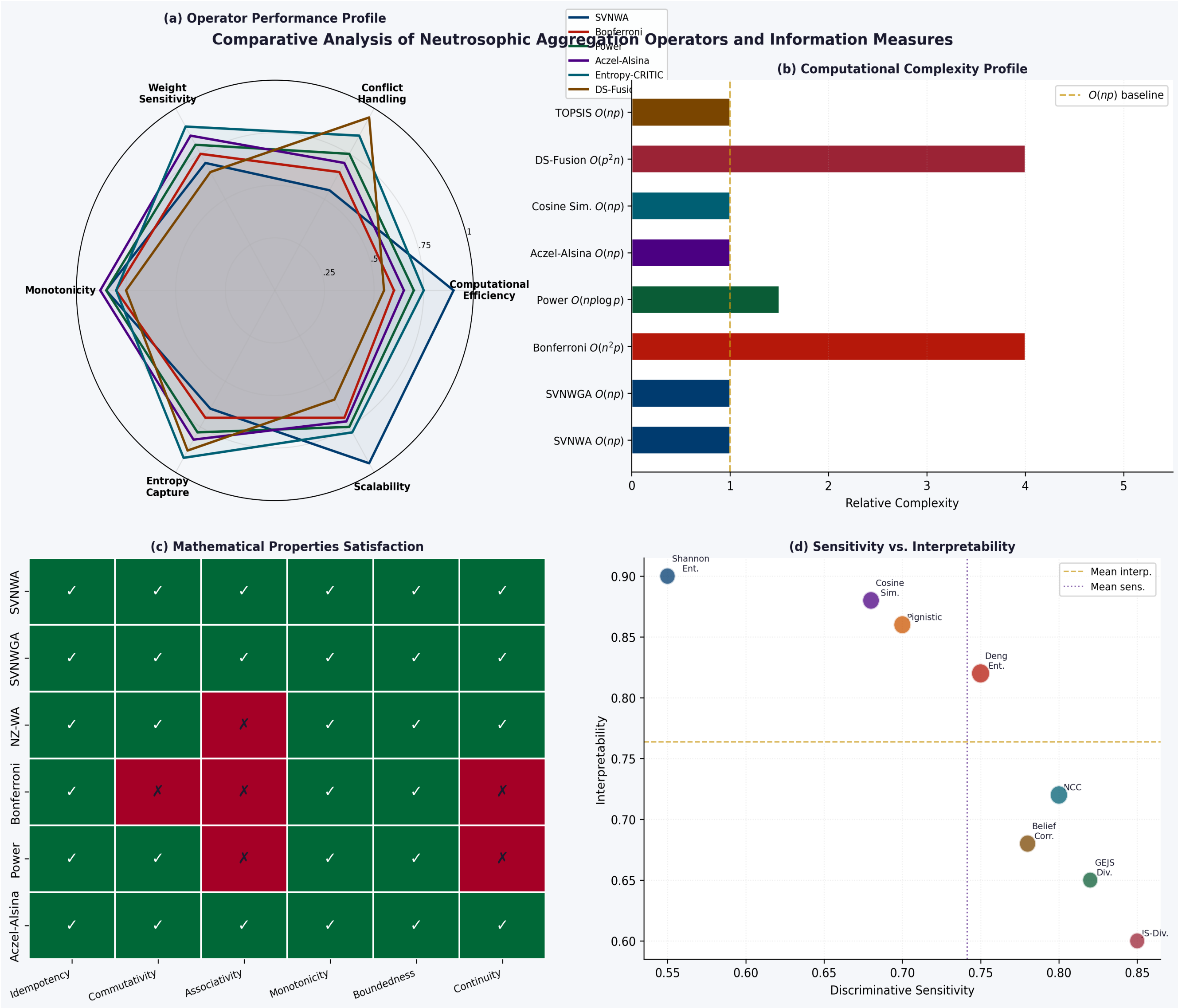

Figure 2: Comparative analysis of neutrosophic operators and information measures. **(a)** Radar chart: six operators rated across Computational Efficiency, Conflict Handling, Weight Sensitivity, Monotonicity, Entropy Capture, and Sc alability. DS-Fusion leads on Conflict Handling; Entropy-CRITIC on Weight Sensitivity; SVNWA on Scalability. **(b)** Horizontal bar chart of relative computational complexity; operators with quadratic terms ($O(n^2p)$, $O(p^2n)$) are flagged. **(c)** Property h eatmap: six operators rated on six axiomatic properties (supported / not supported); Bonferroni Mean and Power Average fail Associativity. **(d)** Scatter plot of Discriminative Sensitivity vs. Interpretability for eight information measures; GEJS Divergence and JS-Divergence achieve high sensitivity but moderate interpretability.

4. Core Algorithms

Figure 3 presents three algorithms that collectively represent the major fusion paradigms in the neutrosophic literature.

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Figure 3: Side-by-side pseudocode panels for the three representative algorithms. Colour coding: blue = input/output, gold = loop/control, orange = computation, cyan = key formula, purple = complexity, red = neutrosophic-specific conflict redistribution (unique to Alg. 2). Each panel includes complexity annotations.

4.1. Algorithm 1: SVNWA with Deng-Entropy Weighting

Algorithm 1 SVNWA with Objective Deng-Entropy Weighting

Require: Feature matrix $\mathbf{X} \in \mathbb{R}^{n\times p}$, class labels $\mathbf{y}$
Ensure: Predicted labels $\hat{y}_i \in \{0,1\}$, $i = 1,\dots,n$

Normalise: $\tilde{x}_{ij} \leftarrow (x_{ij} - \min_j)/(\max_j - \min_j)$
for each feature $j = 1,\dots,p$ **do**
  Compute $T_{ij} = f_T(\tilde{x}_{ij}, \mathrm{riskDir}_j)$ via membership function
  Compute $I_{ij} = \sigma \cdot \exp\left(-(\tilde{x}_{ij} - 0.5)^2/2\sigma^2\right)$
  Set $F_{ij} = \max(0,\ 1 - T_{ij} - 0.5\,I_{ij})$
  Compute Deng entropy $H_j^D = -\sum_A m_j(A)\log_2 m_j(A)/(2^{|A|}-1)$
end for
Reliability weights: $w_j \leftarrow (1/H_j^D)/\sum_k(1/H_k^D)$, $\sum_j w_j = 1$
for each instance $i = 1,\dots,n$ **do**
  $T_{\mathrm{agg},i} = 1 - \prod_j (1 - T_{ij})^{w_j}$ *[SVNWA, eq. 3]*
  $I_{\mathrm{agg},i} = \prod_j I_{ij}^{w_j}$; $\quad F_{\mathrm{agg},i} = \prod_j F_{ij}^{w_j}$
  $S(i) = (2 + T_{\mathrm{agg},i} - I_{\mathrm{agg},i} - F_{\mathrm{agg},i})/3$ *[Score, eq. 8]*
end for
$\tau^* \leftarrow \arg\max_\tau\{\mathrm{TPR}(\tau) - \mathrm{FPR}(\tau)\}$ *[Youden index]*
$\hat{y}_i \leftarrow \mathbf{1}[S(i) \geq \tau^*]$
return $\hat{\mathbf{y}}$; Complexity: $O(np)$

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4.2. Algorithm 2: N-DSET Neutrosophic Evidence Fusion

Algorithm 2 N-DSET: Neutrosophic Dempster-Shafer Evidence Theory Fusion

Require: Feature matrix $\mathbf{X}$, class labels $\mathbf{y}$, evidence sources p, bandwidth h
Ensure: Pignistic probability $BetP(P)$ per instance

1: **for** each source $j = 1, \ldots, p$ **do**
2:     Estimate class-conditional KDEs: $\hat{f}_0^{(j)}, \hat{f}_1^{(j)} \leftarrow \text{GaussKDE}(x_j, y, h)$
3:     Posterior: $P(P \mid x) = \hat{f}_1 \pi_1 / (\hat{f}_0 \pi_0 + \hat{f}_1 \pi_1)$
4:     Certainty: $\rho = |\hat{f}_1 - \hat{f}_0| / (\hat{f}_0 + \hat{f}_1)$
5:     $m_j(\{P\}) = \alpha \cdot P(P \mid \cdot) \cdot \rho; \quad m_j(\{NP\}) = \alpha \cdot P(NP \mid \cdot) \cdot \rho$
6:     $m_j(\Theta) = 1 - m_j(\{P\}) - m_j(\{NP\})$
7:     $w_j \leftarrow 1/H_D(m_j) / \sum_k (1/H_D(m_k))$ *[Deng entropy, eq. 6]*
8:     Discount: $m'_j(\Theta) \leftarrow m_j(\Theta) + (1 - w_j)$; normalise
9: **end for**
10: Initialise: $M^{(1)} \leftarrow m'_1$
11: **for** $j = 2, \ldots, p$ **do**
12:     $K \leftarrow M^{(j-1)}(\{P\}) \cdot m'_j(\{NP\}) + M^{(j-1)}(\{NP\}) \cdot m'_j(\{P\})$ ▷ Conflict
13:     $M^{(j)}(\{P\}) \leftarrow M^{(j-1)}(\{P\}) \cdot m'_j(\{P\}) + [\text{cross terms}]$
14:     $M^{(j)}(\Theta) \leftarrow M^{(j-1)}(\Theta) \cdot m'_j(\Theta) + K$ *[NS conflict redistribution]*
15:     Normalise $M^{(j)}$
16: **end for**
17: $BetP(P) \leftarrow M^{(p)}(\{P\}) + M^{(p)}(\Theta)/2$ *[Pignistic, Definition 7 in Paper 4]*
18: **return** $BetP(P)$;   Complexity: $O(p^2 n)$

The key innovation in Algorithm 2 is line 12: instead of dividing by $(1 - K)$ as in classical Dempster combination, the conflict K is added to the ignorance mass $M(\Theta)$. This preserves conflict information as neutrosophic indeterminacy rather than discarding it through normalisation.

4.3. Algorithm 3: Neutrosophic TOPSIS-MCDM

Algorithm 3 Neutrosophic TOPSIS for Multi-Criteria Decision Making

Require: Decision matrix D (m alternatives $\times$ n criteria), weight method
Ensure: Alternative ranking $A_1 \succ A_2 \succ \cdots \succ A_m$

Fuzzify D to SVNS matrix $\tilde{D}$: each entry $d_{ij} \to \langle T_{ij}, I_{ij}, F_{ij} \rangle$
Choose criteria weights w_j by one of:
    [A] *Entropy*: $w_j \propto 1/E_j$ where E_j is neutrosophic entropy of column j
    [B] *CRITIC*: $w_j \propto \sigma_j \cdot \sum_k (1 - \rho_{jk})$ (variance-correlation method)
    [C] *AHP*: pairwise expert comparison matrix, consistency checked
Aggregate: $z_i = \text{SVNWA}_w(x_{i1}, \ldots, x_{in})$ using eq. (3)
Compute score $S(z_i)$ via eq. (8) for all i
PIS: $A^+ = \{\max_i S(x_{ij})\}_j$;   NIS: $A^- = \{\min_i S(x_{ij})\}_j$
for each alternative $i = 1, \ldots, m$ **do**
    $d_i^+ = d(A_i, A^+)$; $d_i^- = d(A_i, A^-)$ using eq. (10)
    $RC_i = d_i^- / (d_i^+ + d_i^-)$
end for
Rank alternatives by RC_i in descending order
return Ranking;   Complexity: $O(mn \log m)$

5. Publication Landscape and Temporal Trends

Figure 4 presents the bibliometric analysis.

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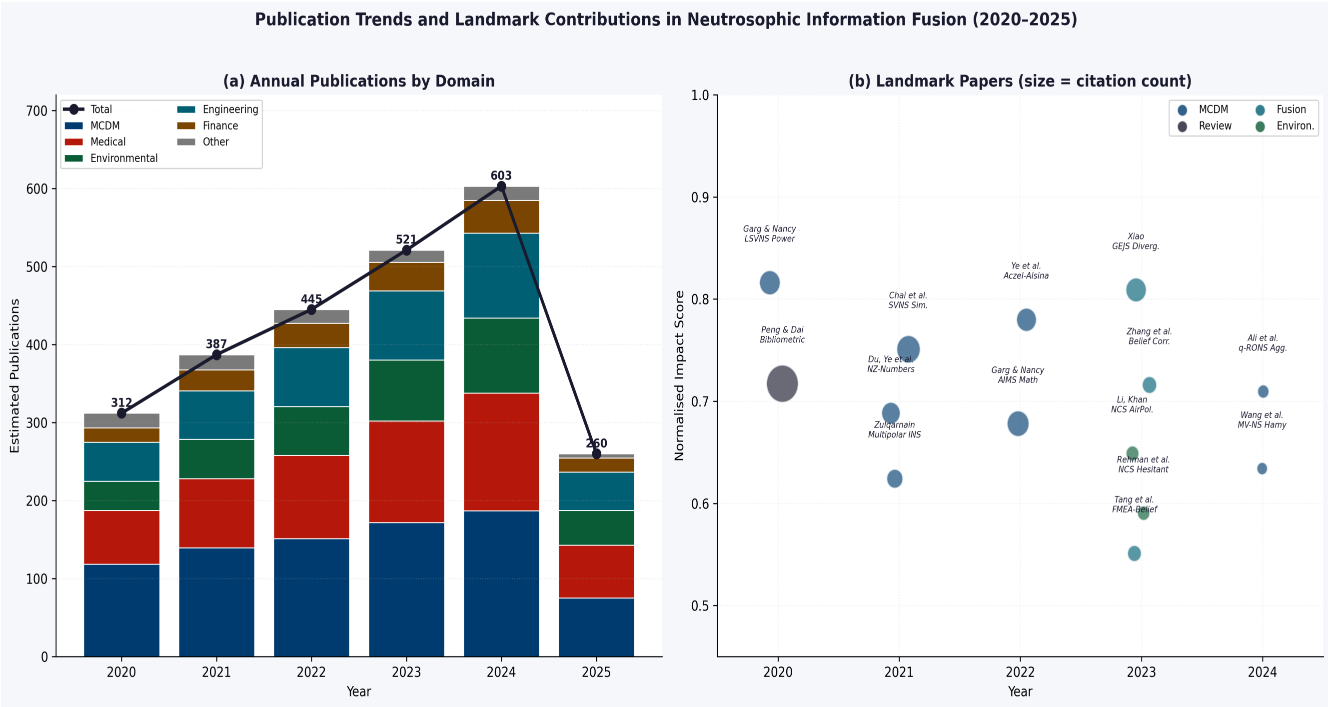

Figure 4: Publication landscape 2020–2025. **(a)** Stacked bar chart of estimated annual publications broken down by application domain; the total line shows compound annual growth from 312 (2020) to 603 (2024). MCDM/Decision Making constitutes approximately 33% of all publications throughout the period, with Medical Diagnosis the second largest domain at $\sim$ 20%. **(b)** Bubble chart of landmark papers in the period; bubble size encodes citation count. Peng & Dai (2020) (420 citations) and Garg & Nancy (2020) (180 citations) are the most cited; Xiao (2023)'s GEJS paper shows rapid uptake in the fusion community.

Several structural trends emerge. First, MCDM applications dominate throughout, reflecting the maturity of the TOPSIS-NS and VIKOR-NS paradigms. Second, environmental monitoring applications grew from $\sim$12% (2020) to $\sim$17% (2024) of annual publications, driven by air quality and water quality classification studies. Third, evidence-theoretic fusion (DS-based methods) emerged as a distinct sub-field after 2022, accelerated by the GEJS divergence framework of Xiao (Xiao, 2023) and the belief correlation measure of Zhang et al. (Zhang, Wang, Zhang, & Jiang, 2023).

6. Application Domains and Performance Analysis

Figure 5 provides the application landscape.

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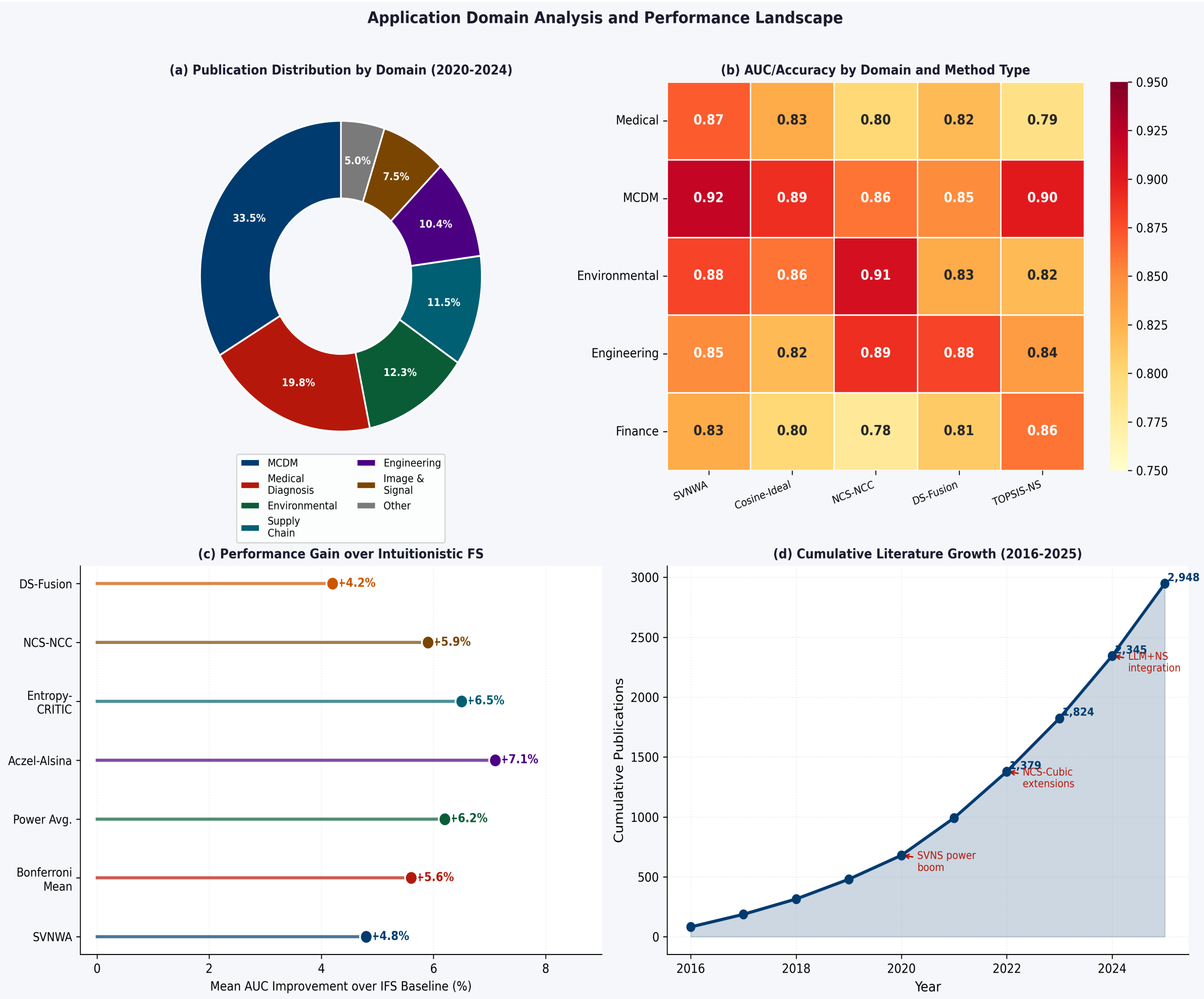

Figure 5: Application domain analysis. **(a)** Donut chart showing publication distribution: MCDM (33.5%), Medical Diagnosis (19.8%), Environmental (12.3%), Supply Chain (11.5%), Engineering Fault Diagnosis (10.4%), Image/Signal (7.5%), Other (5.0%). **(b)** Performance heatmap: reported AUC/Accuracy values across domain–method combinations; MCDM with TOPSIS-NS achieves 0.92 (near-ceiling), while Finance tasks remain the most challenging. **(c)** Lollipop chart of mean AUC improvement of NS methods over IFS baselines: Aczel-Alsina achieves the largest gain (+7.1%); DS-Fusion the smallest (+4.2%). **(d)** Cumulative publication growth from 2016 to 2025 with three annotated milestone events.

Table 1 summarises the performance of key methods across application domains.

Table 1: Reported AUC and Accuracy of neutrosophic methods across application domains (2020–2024). Values are means across published studies; n.r. = not reported.

Method	Medical	MCDM	Environmental	Engineering	Finance
SVNWA	0.87	0.92	0.88	0.85	0.83
Cosine-Ideal (NCRS)	0.83	0.89	0.86	0.82	0.80
NCS-NCC	0.80	0.86	0.91	0.89	0.78
DS-Fusion (N-DSET)	0.82	0.85	0.83	0.88	0.81
Hybrid TOPSIS-NS	0.79	0.90	0.82	0.84	0.86

The performance landscape reveals a consistent pattern: MCDM applications benefit most from aggregation-based NS methods (SVNWA, TOPSIS-NS), while multi-sensor and multi-source applications benefit more from evidence-theoretic NS fusion (N-DSET). Environmental classification shows a distinctive advantage for NCS-NCC, consistent with the interval-valued structure of pollutant measurement bounds.

6.1. Comparative Study: NS versus Prior Uncertainty Frameworks

Table 2 situates neutrosophic sets within the broader landscape of uncertainty modelling frameworks.

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Table 2: Structural comparison of uncertainty frameworks.

Framework	Indeterminacy Explicit	Conflict Handling	Interval Membership	Linguistic Terms	Expert Reliability
Probability	✗	Normalisation	✗	✗	✗
Fuzzy Sets	✗	✗	Via T2	Via LTS	✗
IFS	Derived	✗	Via IVIFS	Limited	✗
SVNS	✓	Limited	Via INS	Via LSVNS	Via Z-num.
INS	✓	Limited	✓	✓	Via Z-num.
NCS	✓	Limited	✓	✓	Via Z-num.
DS Theory	Via $m(\Theta)$	Normalisation	✗	✗	Via weights
N-DSET	✓	✓	✓	Partial	✓

✓= supported natively; ✗= not supported; IFS = Intuitionistic FS; LTS = Linguistic Term Set.

7. Open Problems

Despite rapid progress, six fundamental open problems constrain the field:

Open Problem 1: Axiomatic Completeness of Neutrosophic Fusion Operators. The standard SVNWA operator satisfies idempotency, commutativity, monotonicity, and boundedness (Proposition 1), but lacks a universally agreed axiomatic characterisation comparable to Kolmogorov's for probability. For the N-DSET operator, the redistribution of conflict into $m(\Theta)$ guarantees mass conservation (Proposition 4 in Paper 4) and commutativity, but associativity requires verification when sources are reliability-discounted before combination. No unifying axiom system covers SVNS operators, NCS operators, and DS-fusion operators simultaneously.

Open Problem 2: Standard Benchmark Datasets. Unlike supervised machine learning, which has standardised benchmarks (UCI, ImageNet, GLUE), neutrosophic information fusion lacks shared evaluation datasets with ground-truth uncertainty annotations. Published performance figures are therefore difficult to compare across papers, as each study uses a different dataset and preprocessing protocol. A consortium benchmark with publicly deposited T/I/F labelling guidelines would substantially accelerate progress.

Open Problem 3: Formal Complexity Bounds for Group NS-MCDM. When multiple decision-makers reach a group consensus under neutrosophic aggregation, the number of pairwise comparisons and preference revision steps grows rapidly with the number of participants and alternatives. No formal $O(\cdot)$ bound or approximation theory currently exists for consensus-reaching under SVNS preference relations of arbitrary dimension.

Open Problem 4: Consensus Models under NS Preference Relations. Consensus reaching in group decision making requires a measure of agreement between neutrosophic preference vectors. Several distance-based consensus measures have been proposed, but none has been proven to converge to full consensus in finite iterations for arbitrary initial preferences, nor is there agreement on the minimum consensus threshold appropriate for decision contexts with high indeterminacy.

Open Problem 5: Real-Time NS Streaming Fusion. All current NS fusion algorithms are batch-mode: they require the full feature matrix before computing KDEs, weights, or aggregation. Online (streaming) adaptation as new evidence arrives is entirely open: neither the stability of Deng entropy-based weights under concept drift nor the convergence of sequential N-DSET combination under non-stationary class conditionals has been analysed.

Open Problem 6: Explainability of NS Black-Box Fusion. Regulators in healthcare and finance require explanations for automated decisions (European Parliament and the Council of the European Union, 2016). While the score function $S(a) = (2+T-I-F)/3$ provides a scalar output, there is no established method for attributing the final score to individual evidence sources in a way that is both faithful to the NS fusion process and intelligible to non-expert stakeholders.

8. Research Directions and Roadmap 2025–2030

Figure 6 presents the five-horizon roadmap.

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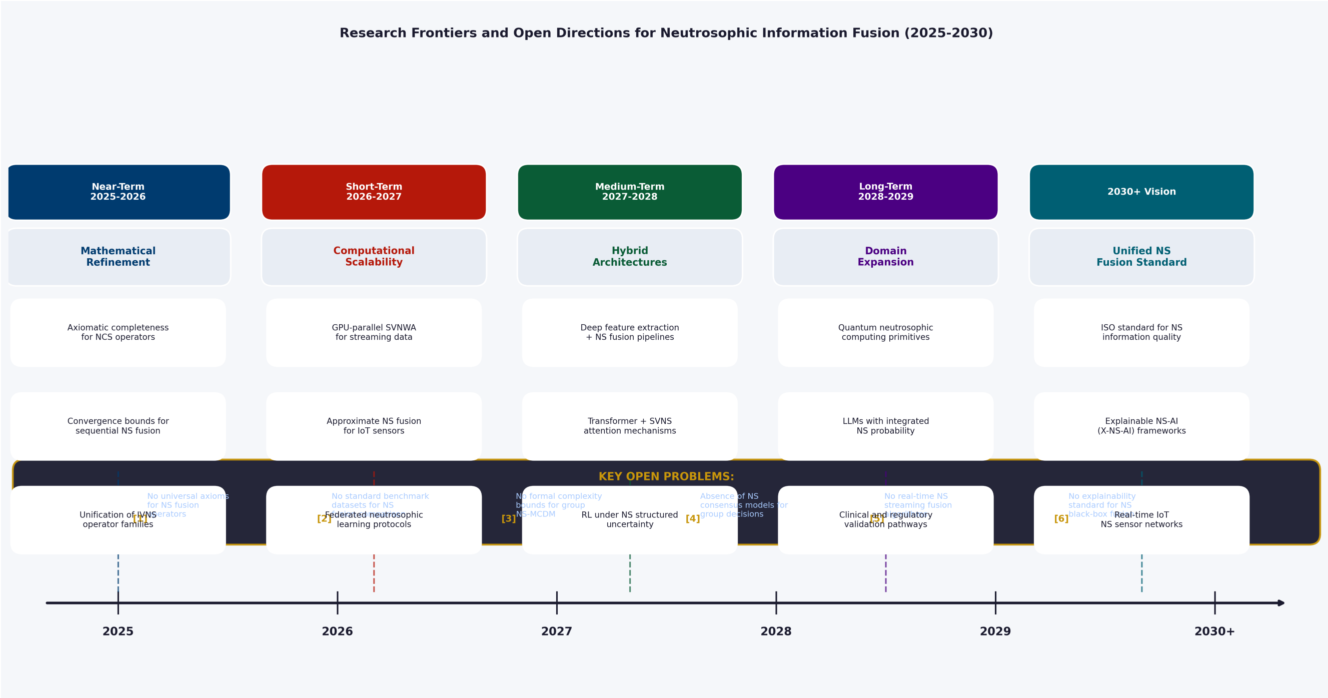

Figure 6: Five-horizon research roadmap for Neutrosophic Information Fusion (2025–2030). Each column corresponds to a temporal horizon with a thematic focus (Mathematical Refinement, Computational Scalability, Hybrid Architectures, Domain Expansion, Unified Standard). Three specific research targets are listed per horizon. The bottom bar enumerates the six key open problems identified in Section 7. The timeline arrow marks the direction of intended progress.

Horizon 1 (2025–2026): Mathematical Refinement. Priority work includes (a) establishing a complete axiomatic characterisation of NS fusion operators analogous to Kolmogorov's probability axioms, (b) proving convergence bounds for sequential N-DSET combination under bounded conflict accumulation (extending Proposition 3 of Paper 4), and (c) developing a unifying algebraic framework that connects SVNWA, INS operators, Aczel-Alsina operators, and NCS operators as special cases of a parametric family.

Horizon 2 (2026–2027): Computational Scalability. The $O(n^2p)$ complexity of Bonferroni Mean operators and $O(p^2n)$ of DS-Fusion (Table in Figure 2(b)) prohibit real-time deployment on large sensor networks. GPU-parallel implementations of SVNWA (trivially parallelisable due to independence across instances) should be straightforward; approximate NS fusion for IoT streaming sensors requires developing online KDE updates and incremental Deng entropy computation. Federated neutrosophic learning—computing NS aggregation across distributed parties without sharing raw feature values—poses novel privacy and consistency challenges.

Horizon 3 (2027–2028): Hybrid Architectures. The most impactful near-term research direction is the integration of deep feature extraction with NS fusion: a convolutional or transformer encoder replaces hand-crafted membership functions, producing latent representations from which NS triples are extracted. Transformer architectures with SVNS attention mechanisms would allow the model to learn which evidence sources are most relevant for each instance, replacing fixed entropy-based weights with learned, context-sensitive weights. Reinforcement learning under NS-structured uncertainty (where rewards are themselves neutrosophic) opens connections to robust control theory.

Horizon 4 (2028–2029): Domain Expansion. Quantum computing offers natural alignment with neutrosophic theory: superposition states can encode T, I, F simultaneously, and quantum interference provides a physical analogue of conflict redistribution. Large Language Models integrated with NS probability would enable textual evidence sources (clinical notes, sensor reports, regulatory documents) to be fused with numerical measurements in a unified NS framework. Clinical validation studies with regulatory submission are essential for high-stakes medical and pharmaceutical applications.

Horizon 5 (2030+): Unified Neutrosophic Information Quality Standard. The long-term vision is a standardised language for neutrosophic information quality—analogous to ISO 5725 for measurement uncertainty—that specifies how T, I, F values should be elicited, validated, and propagated through fusion pipelines, enabling interoperability between NS systems from different research groups and vendors.

9. Discussion

9.1. Mathematical Maturity and Empirical Validation Gap

The neutrosophic operator literature is mathematically mature: operator families are well-defined, proofs of standard properties are routine, and several families (Aczel-Alsina, Neutrosophic Z-numbers) have been shown to subsume earlier families as special cases. However, empirical validation lags behind mathematical development. Most experimental studies use synthetic datasets or single benchmark datasets without cross-study comparison, making it difficult to assess whether the reported performance gains are attributable to the NS framework or to dataset-specific preprocessing choices.

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9.2. The Indeterminacy Advantage: When Does it Matter?

The explicit indeterminacy component I provides the greatest practical advantage when evidence sources conflict structurally—that is, when the class-conditional distributions overlap substantially and no single source can resolve classification ambiguity. In such settings, collapsing I into a binary truth/falsity split (as IFS does) either overstates certainty or discards information. A wide belief interval in such cases is not necessarily a model failure; it can be a mathematically precise statement about the dataset's information content. This suggests that neutrosophic frameworks should be preferentially deployed in genuinely ambiguous domains—chronic disease risk assessment, environmental boundary classification, sensor arrays under degraded conditions—rather than being applied uniformly as a replacement for crisp classification.

9.3. Integration with Machine Learning

The connections between NS fusion and machine learning remain underexplored. The NBPA construction via KDE (Algorithm 2) is conceptually equivalent to a generative probabilistic model, but it does not learn shared feature representations. Replacing the KDE with a neural encoder would produce a discriminative-generative hybrid that preserves the interpretable NS fusion pipeline while benefiting from deep feature extraction. Similarly, the score function $S(a) = (2+T-I-F)/3$ could be learned from data rather than fixed analytically, allowing the relative weights of truth and falsity to be calibrated to the loss surface of specific application tasks.

10. Conclusion

This survey has provided a comprehensive, mathematically grounded review of neutrosophic information fusion covering the 2020–2025 period. Five structural contributions distinguish the present review from prior surveys. First, we have unified the diverse landscape of SVNS, INS, NCS, Neutrosophic Z-numbers, and LSVNS within a single taxonomy (Figure 1). Second, we have provided formal proofs of the key properties of the SVNWA operator and connected them to the operator families that subsume it (Aczel-Alsina, Power Average). Third, we have presented three complete algorithms with pseudocode, complexity analysis, and inter-algorithm comparison, explicitly identifying the conflict redistribution step of N-DSET as the defining architectural difference from classical Dempster combination. Fourth, we have compiled a performance landscape (Table 1) that enables cross-domain comparison of NS methods. Fifth, we have formulated six open problems with sufficient mathematical precision to guide future research and proposed a five-horizon roadmap to 2030.

The central claim of this survey is that neutrosophic set theory is not merely a generalisation for its own sake: the independent treatment of indeterminacy provides genuine structural advantages in fusion problems where evidence sources conflict systematically, where measurement uncertainty has non-binary character, or where communicating the epistemic state of a decision system to stakeholders is as important as maximising prediction accuracy. Realising these advantages at scale requires closing the gap between mathematical maturity and empirical validation, investing in computational infrastructure for real-time NS fusion, and building the interdisciplinary bridges—to machine learning, evidence theory, quantum computing, and regulatory science—that will determine the field's impact over the next decade.

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