Journal of Neutrosophic and Fuzzy Systems

ISSN: 2771-6449 Vol: 11, No. 01, 2026 PP. 09–16

Article DOI: https://doi.org/10.54216/JNFS.110102

FULL LENGTH ARTICLE

Robust Jensen–Shannon Consensus Geometry for Single-Valued Neutrosophic Information

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Received: September 01, 2025 Revised: November 04, 2025 Accepted: January 03, 2026 ⋆ Corresponding author

ABSTRACT

Let $x=(T,I,F)\in[0,1]^3$ denote a single-valued neutrosophic assessment and let $w_T+w_I+w_F=1$ with $w_c>0$. This paper introduces the probability-completed embedding

$$\Phi_w(x)=\big(w_T T, w_T(1-T), w_I I, w_I(1-I), w_F F, w_F(1-F)\big)\in\Delta_5.$$

and the pullback distance

$$d_w(x,y)=\left[\mathscr{J}(\Phi_w(x),\Phi_w(y))/\log 2\right]^{1/2},$$

where $\mathscr{J}$ is Jensen–Shannon divergence. The construction yields a bounded metric, separates into three weighted Bernoulli Jensen–Shannon terms, is invariant under the neutrosophic complement $x^c=(F,1-I,T)$ when $w_T=w_F$, and has the local information metric

$$d_w^2(x,x+\delta)=\frac{1}{8\log 2}\sum_{c\in\{T,I,F\}}\frac{w_c\delta_c^2}{x_c(1-x_c)}+O(\|\delta\|^3).$$

On this geometry, robust consensus is posed as the bounded M-estimation problem

$$\widehat{x}_\tau=\arg\min_{x\in(0,1)^3}\sum_{r=1}^{m}a_r\{1-e^{-\tau d_w^2(x,x_r)}\}.$$

A majorization–minimization iteration reduces each step to three one-dimensional weighted Jensen–Shannon barycenters and decreases the objective monotonically. The induced expert weight is proportional to $e^{-\tau d_w^2}$, so strongly conflicting assessments are downweighted without a hard rejection threshold. In a reproducible contamination study with 15 experts, 800 replications at each of five contamination levels, and a fixed $\tau=60$, the proposed estimator has mean normalized Jensen–Shannon error 0.0149 at 40% oppositional contamination; the coordinate median, ordinary Jensen–Shannon barycenter, and arithmetic mean obtain 0.0380, 0.1448, and 0.1489, respectively. The contribution is therefore a metric and optimization framework for consensus itself, rather than another ranking operator: neutrosophic disagreement is represented on a common information-geometric scale and robust aggregation follows from a bounded variational principle.

Keywords: Single-valued neutrosophic set ▪ Jensen–Shannon divergence ▪ Robust consensus ▪ Information geometry ▪ Group decision making ▪ Majorization–minimization

1. INTRODUCTION

A single-valued neutrosophic assessment retains three coordinates—truth, indeterminacy and falsity—without forcing them to sum to one [1]. This freedom is precisely what makes the representation useful for incomplete or internally conflicting judgments, but it also makes consensus less straightforward than averaging a probability vector. Recent work has developed distances, similarities, entropies, score functions and aggregation rules for single-valued neutrosophic information [2, 3, 4, 5, 6]. Other contributions embed the representation into multi-criteria procedures, including MABAC, TOPSIS, QUALIFLEX, MULTIMOORA, hybrid neutrosophic Delphi/AHP, and consensus goal-programming variants [7, 8, 9, 10, 11, 12, 13, 14]. These developments demonstrate the breadth of neutrosophic decision models, yet they leave a more elementary question open: before alternatives are ranked, how should mutually inconsistent expert triples be combined when a minority of assessments may be severely discordant?

For m experts, write $x_r = (T_r, I_r, F_r)$ and consider a consensus x. A useful consensus geometry should simultaneously satisfy

$$0 \le d(x,y) \le 1, \qquad d(x,y) = 0 \Leftrightarrow x = y, \tag{1}$$

respect symmetry and the triangle inequality, preserve the distinction among T, I, F, and remain interpretable under neutrosophic complementation. Euclidean or Manhattan distances satisfy some of these requirements, and sophisticated SVN distances have been proposed [3, 4, 5]; however, a coordinate displacement near 0 or 1 need not carry the same information as an equal displacement near $1/2$. This suggests replacing flat coordinate geometry by an information geometry whose local curvature increases near the boundaries.

Let $h(p) = -p\log p - (1-p)\log(1-p)$ and define the Bernoulli Jensen–Shannon divergence

$$j(a,b) = h\left(\frac{a+b}{2}\right) - \frac{h(a)+h(b)}{2}. \tag{2}$$

The Jensen–Shannon divergence is bounded and symmetric, and its square root defines a metric on finite probability simplices [15]. The construction below uses it without normalizing the neutrosophic triple itself. Each coordinate $c \in \{T, I, F\}$ is completed to a two-state distribution $(x_c, 1-x_c)$ and allocated a fixed mass w_c. The result is an injective map to a probability simplex. Consequently, metric properties are inherited from the Jensen–Shannon geometry while the original independence of truth, indeterminacy and falsity is retained.

Geometry alone does not solve the aggregation problem. The ordinary Fréchet barycenter minimizes $\sum_r a_r d^2(x, x_r)$ and therefore assigns every assessment a linear contribution to the estimating criterion. Under gross disagreement, a bounded loss is preferable. We use

$$\rho_\tau(s) = 1 - e^{-\tau s}, \qquad \tau > 0, \tag{3}$$

so that a distant assessment has derivative $\rho'_\tau(s) = \tau e^{-\tau s}$ rather than a constant derivative. The resulting estimator can be computed by a monotone majorization–minimization (MM) scheme in which the expert weights are updated exponentially from their current information-geometric conflict.

The paper makes four contributions. First, it derives a normalized Jensen–Shannon metric for single-valued neutrosophic triples through an explicit probability-completed embedding. Second, it proves complement invariance and obtains the local quadratic metric, revealing the boundary-sensitive weighting hidden in the divergence. Third, it formulates robust consensus as bounded M-estimation and derives a separable MM algorithm with guaranteed objective descent. Fourth, it evaluates the estimator under a controlled oppositional-contamination model and reports replicate-level errors, failure frequencies, paired bootstrap contrasts and parameter sensitivity. The formulation is intentionally prior to any particular MCDM ranking rule: once a group consensus has been obtained, it can be passed to a downstream decision method if required.

2. RELATED WORK

The recent SVN literature can be separated into three lines relevant here. The first concerns information measures. Qin and Wang [2] proposed new similarity and entropy measures, while Chai et al. [3] developed similarity constructions for pattern recognition and medical diagnosis. Matrix-norm and modified-Manhattan approaches were subsequently developed by Luo et al. [4] and Zeng et al. [5]. Ye [16] studied entropy for simplified neutrosophic sets. These papers establish that the choice of information measure can change both mathematical behavior and downstream decisions. The present work differs in that the distance is obtained by a pull-back from a probability simplex and is used as the geometry of a robust consensus optimization problem.

The second line concerns algebraic aggregation and ranking. Partitioned Heronian means [17], type-2 SVN TOPSIS [9], trigonometric aggregation of neutrosophic credibility numbers [18], Aczel–Alsina aggregation [19], exponential–logarithm operations [20], and new score functions [6] enlarge the range of permissible neutrosophic decision operators. Dynamic extensions also address time-varying expert preferences [21]. Applications include uncertain scheduling [22] and social-IoT trust [23]. Our objective is different: no alternative-ranking axiom is imposed, and no score function collapses (T, I, F) before consensus is found.

The third line addresses group consensus more directly. Saha et al. [24] developed a consensus-based SVN model for educational-vendor selection, while recent frameworks address consistency, consensus, expert weighting, and hybrid neutrosophic MCDM through goal-programming and Delphi/AHP constructions [14, 13]. Multi-scale single-valued neutrosophic decision systems further show that indeterminate information remains an active modeling problem beyond conventional MCDM [25]. The unresolved issue motivating this paper is robustness at the aggregation stage. A consensus operator can be mathematically well behaved yet remain sensitive to a small group of far-away assessments. The bounded-loss formulation below addresses that point directly.

3. MATHEMATICAL PRELIMINARIES

3.1 Single-valued neutrosophic state space

Let

$$\mathcal{N} = [0,1]^3, \qquad x = (T_x, I_x, F_x). \tag{4}$$

No unit-sum constraint is imposed. Let $w = (w_T, w_I, w_F)$ satisfy

$$w_c > 0, \qquad w_T + w_I + w_F = 1. \tag{5}$$

The weights are geometric saliences, not expert weights. Define

$$\Phi_w(x) = \begin{pmatrix} w_T T_x,\ w_T(1-T_x),\ w_I I_x, \\ w_I(1-I_x),\ w_F F_x,\ w_F(1-F_x) \end{pmatrix}. \tag{6}$$

Every component is nonnegative and the six components sum to one, hence $\Phi_w(x) \in \Delta_5$. Because every w_c is positive, Φ_w is injective.

For probability vectors p, q, let

$$\mathscr{J}(p,q) = \frac{1}{2}\mathrm{KL}(p\|m) + \frac{1}{2}\mathrm{KL}(q\|m), \quad m = \frac{p+q}{2}. \tag{7}$$

Zero components are interpreted by continuity. Define the weighted neutrosophic Jensen–Shannon distance

$$d_w(x,y) = \sqrt{\frac{\mathscr{J}(\Phi_w(x), \Phi_w(y))}{\log 2}}. \tag{8}$$

Proposition 1 (Coordinate decomposition)**.** *For $x, y \in \mathscr{N}$,*

$$d_w^2(x,y) = \frac{1}{\log 2} \sum_{c \in \{T,I,F\}} w_c\, j(x_c, y_c), \tag{9}$$

with j defined in (2).

Proof. The six-dimensional distributions in (6) consist of three disjoint binary blocks having fixed masses w_T, w_I, w_F. In each block the common mass factors out of both Kullback–Leibler terms in (7). Summing the resulting Bernoulli divergences gives (9). □

Theorem 1 (Bounded metric)**.** *The function d_w is a metric on $\mathscr{N}$ and $0 \le d_w(x,y) \le 1$.*

Proof. The square root of Jensen–Shannon divergence is a metric on a probability simplex; division by the positive constant $\sqrt{\log 2}$ preserves metricity [15]. Pulling this metric back through the injective map Φ_w gives symmetry, the triangle inequality and identity of indiscernibles on $\mathscr{N}$. Since $\mathscr{J}(p,q) \le \log 2$, the normalized bound follows. □

3.2 Complement symmetry and local geometry

Use the common SVN complement

$$x^c = (F_x, 1 - I_x, T_x). \tag{10}$$

Corollary 1 (Complement isometry)**.** *If $w_T = w_F$, then $d_w(x^c, y^c) = d_w(x,y)$ for all $x, y \in \mathscr{N}$.*

Proof. The transformation swaps the T and F Bernoulli blocks and complements the indeterminacy block. Bernoulli Jensen–Shannon divergence satisfies $j(a,b) = j(1-a, 1-b)$. Equal T and F weights therefore leave (9) unchanged. □

The geometry is not locally Euclidean with a constant metric tensor.

Theorem 2 (Local information metric)**.** *For $x \in (0,1)^3$ and $\delta \to 0$ such that $x + \delta \in (0,1)^3$,*

$$d_w^2(x, x+\delta) = \frac{1}{8\log 2} \sum_c \frac{w_c \delta_c^2}{x_c(1-x_c)} + O(\|\delta\|^3). \tag{11}$$

Proof. For the binary entropy, $h''(p) = -[p(1-p)]^{-1}$. Taylor expansion of (2) around $b = a$ gives $j(a, a+u) = u^2/[8a(1-a)] + O(|u|^3)$. Substitution into (9) yields (11). □

Thus a coordinate perturbation close to 0 or 1 is geometrically larger than the same absolute perturbation close to $1/2$. This boundary sensitivity arises from the information geometry rather than from an externally chosen nonlinear score.

3.3 Metric tensor and intrinsic volume

Equation (11) identifies the diagonal local tensor

$$G_w(x) = \frac{1}{8\log 2}\,\mathrm{diag}\left(\frac{w_T}{T(1-T)}, \frac{w_I}{I(1-I)}, \frac{w_F}{F(1-F)}\right). \tag{12}$$

Hence

$$\det G_w(x) = \frac{w_T w_I w_F}{(8\log 2)^3 T(1-T) I(1-I) F(1-F)}. \tag{13}$$

The induced local volume factor, $\sqrt{\det G_w(x)}$, grows near the boundary of the cube. The geometry therefore allocates more resolution to highly committed assessments, where one component approaches 0 or 1, than to equally sized perturbations around an equivocal component value near $1/2$. This is a geometric property of the embedding and not a post-hoc weighting function.

4. PROPOSED METHOD

4.1 Ordinary Jensen–Shannon barycenter

Let $x_1, \ldots, x_m \in \mathscr{N}$ be expert assessments and let base expert weights $a_r > 0$ satisfy $\sum_r a_r = 1$. The ordinary squared-distance barycenter is

$$\bar{x}_{\mathrm{JS}} = \arg\min_{x \in (0,1)^3} \sum_{r=1}^{m} a_r d_w^2(x, x_r). \tag{14}$$

By (9), the minimization separates over T, I, F. For one coordinate, data $q_r \in (0,1)$ and normalized weights b_r, the scalar barycenter solves

$$\min_{0<z<1} \sum_{r=1}^{m} b_r j(z, q_r). \tag{15}$$

Its stationary equation is

$$\sum_r b_r \log \frac{1-(z+q_r)/2}{(z+q_r)/2} - \log \frac{1-z}{z} = 0. \tag{16}$$

A safeguarded Newton step solves this one-dimensional equation rapidly.

4.2 Bounded-loss robust consensus

Define $s_r(x) = d_w^2(x, x_r) \in [0,1]$ and, for $\tau > 0$,

$$Q_\tau(x) = \sum_{r=1}^{m} a_r \rho_\tau(s_r(x)), \qquad \rho_\tau(s) = 1 - e^{-\tau s}. \tag{17}$$

The proposed robust neutrosophic Jensen–Shannon consensus (RNJSC) is

$$\widehat{x}_\tau \in \arg\min_{x\in(0,1)^3} Q_\tau(x). \tag{18}$$

Unlike a trimming rule, (18) retains every assessment. Its marginal contribution is controlled by

$$\frac{d\rho_\tau}{ds} = \tau e^{-\tau s}, \tag{19}$$

which decays exponentially with squared disagreement.

4.3 Majorization–minimization algorithm

Because ρ_τ is concave in s, its tangent at s_0 majorizes it:

$$\rho_\tau(s) \le \rho_\tau(s_0) + \tau e^{-\tau s_0}(s-s_0). \tag{20}$$

At iterate $x^{(k)}$, set

$$\beta_r^{(k)} = \frac{a_r e^{-\tau d_w^2(x^{(k)},x_r)}}{\sum_{\ell=1}^m a_\ell e^{-\tau d_w^2(x^{(k)},x_\ell)}}. \tag{21}$$

Then update

$$x^{(k+1)} = \arg\min_x \sum_{r=1}^m \beta_r^{(k)} d_w^2(x,x_r). \tag{22}$$

Equation (22) is again three scalar problems of the form (15). The procedure therefore alternates between information-geometric conflict weights and a weighted Jensen–Shannon barycenter.

5. THEORETICAL ANALYSIS

Theorem 3 (Monotone descent)**.** *If $x^{(k+1)}$ minimizes (22), then*

$$Q_\tau(x^{(k+1)}) \le Q_\tau(x^{(k)}). \tag{23}$$

Proof. Apply (20) to each $s_r(x)$ at $s_r(x^{(k)})$, multiply by a_r, and sum. The terms independent of x form a constant; the remaining surrogate is proportional to $\sum_r a_r e^{-\tau s_r(x^{(k)})} s_r(x)$. Normalization of the positive coefficients does not change its minimizer, hence (22) minimizes the majorizer. Equality holds at $x = x^{(k)}$, which proves (23). □

Proposition 2 (Small-τ limit)**.** *Uniformly for $s \in [0,1]$,*

$$\frac{\rho_\tau(s)}{\tau} = s + O(\tau), \qquad \tau \downarrow 0. \tag{24}$$

Consequently, $Q_\tau(x)/\tau$ converges uniformly to the ordinary Fréchet criterion $\sum_r a_r d_w^2(x,x_r)$.

Proof. The expansion $1-e^{-\tau s} = \tau s - \tau^2 s^2/2 + O(\tau^3)$ is uniform on $[0,1]$. Weighted summation gives the result. □

Thus RNJSC connects continuously to the ordinary Jensen–Shannon barycenter; robustness is introduced by increasing τ, not by changing the underlying metric.

Proposition 3 (Relative suppression of discordant experts)**.** *For two experts r and ℓ with equal base weights, their MM-weight ratio at x is*

$$\frac{\beta_r}{\beta_\ell} = \exp\{-\tau[d_w^2(x,x_r) - d_w^2(x,x_\ell)]\}. \tag{25}$$

If expert r is farther from the current consensus by squared-distance gap $\Delta > 0$, its relative weight is multiplied by $e^{-\tau\Delta}$.

The result gives a direct interpretation of τ: it controls the exponential separation between mutually compatible and conflicting assessments.

Proposition 4 (Existence on a compact interior)**.** *For any $0 < \varepsilon < 1/2$, the restriction of Q_τ to $[\varepsilon, 1-\varepsilon]^3$ attains a minimum.*

Proof. The Bernoulli Jensen–Shannon divergence is continuous on the closed square $[0,1]^2$, hence Q_τ is continuous. The restricted domain is compact; the extreme-value theorem applies. □

In computation we use $\varepsilon = 10^{-8}$ only as a numerical safeguard; none of the reported solutions touches this boundary.

Proposition 5 (Fixed-point form)**.** *If $x^\star$ is an interior fixed point of the MM iteration, then*

$$x^\star = \mathrm{Bar}_{\mathscr{J}}\left(\{x_r\}_{r=1}^m; \left\{\frac{a_r e^{-\tau d_w^2(x^\star,x_r)}}{\sum_\ell a_\ell e^{-\tau d_w^2(x^\star,x_\ell)}}\right\}_{r=1}^m\right), \tag{26}$$

where $\mathrm{Bar}_{\mathscr{J}}$ denotes the weighted Jensen–Shannon barycenter. Conversely, any solution of (26) is an MM fixed point.

Proof. At a fixed point, the reweighting step (21) is evaluated at $x^\star$ and the subsequent minimization (22) returns the same point. This is exactly (26). The converse follows by reversing the argument. □

The fixed-point relation makes the estimator interpretable: RNJSC is an ordinary information-geometric barycenter under endogenous expert weights determined by the barycenter itself.

For m experts, one MM iteration requires $O(m)$ distance evaluations and three scalar barycenter solves. With a fixed number of safeguarded Newton steps, the arithmetic cost is $O(m)$ and the memory requirement is $O(m)$. No pairwise $m \times m$ expert-distance matrix is required.

6. EXPERIMENTAL DESIGN

The numerical experiment is a controlled robustness study rather than a benchmark ranking problem. Its purpose is to isolate the effect of conflicting expert information on consensus estimation.

For each replication, a latent state

$$x_0 \sim \mathrm{Unif}([0.12, 0.88]^3) \tag{27}$$

is generated. With $m = 15$ experts and nominal contamination setting η, $m_o = \mathrm{round}(15\eta)$ assessments are designated oppositional and the remainder are clean. Because m is finite, the realized contaminated fractions are 0, $2/15$, $3/15$,

$4/15$, and $6/15$ for $\eta = 0, 0.1, 0.2, 0.3, 0.4$, respectively. For a clean expert,

$$\mathrm{logit}(x_r) = \mathrm{logit}(x_0) + \varepsilon_r, \quad \varepsilon_r \sim N(0, 0.22^2 I_3). \tag{28}$$

For an oppositional expert, the center is $1 - x_0$ and

$$\mathrm{logit}(x_r) = \mathrm{logit}(1 - x_0) + \zeta_r, \quad \zeta_r \sim N(0, 0.35^2 I_3). \tag{29}$$

The transformation keeps all coordinates in $(0,1)$. The nominal contamination settings are $\eta \in \{0, 0.1, 0.2, 0.3, 0.4\}$, with 800 independent replications per setting. Equal expert weights are used. The geometric weights are fixed at

$$(w_T, w_I, w_F) = (0.4, 0.2, 0.4), \tag{30}$$

which gives complement symmetry while assigning greater joint emphasis to truth and falsity than to indeterminacy.

Four estimators are compared: coordinate arithmetic mean, coordinate median, the ordinary Jensen–Shannon barycenter (14), and RNJSC. The robustness parameter is fixed at $\tau = 60$ before the contamination comparison. Accuracy is measured by $d_w(\widehat{x}, x_0)$ and coordinate RMSE. A severe-error indicator is also reported:

$$\mathbf{1}\{d_w(\widehat{x}, x_0) > 0.20\}. \tag{31}$$

In the implementation this indicator is evaluated directly; no threshold is used by the estimator itself. Paired bootstrap intervals use 3000 resamples of replicate-wise error differences. A separate τ analysis uses 200 replications at $\eta = 0.3$. All pseudo-random generators use fixed seeds and the replicate-level output is retained with the source code.

7. RESULTS

7.1 Consensus error under contamination

Table 1 reports the central experiment. Without contamination, the arithmetic mean, ordinary JS barycenter and RNJSC are essentially indistinguishable; their mean normalized JS errors are all about 0.01. This is important because robustness is not purchased by a substantial efficiency loss in the clean setting. Once oppositional assessments enter, the behaviors separate sharply.

At $\eta = 0.4$, the ordinary JS barycenter has mean error 0.14475, close to the arithmetic mean at 0.14891. The coordinate median is substantially more resistant, with error 0.03797. RNJSC obtains 0.01489, approximately 61% below the median and 90% below the ordinary JS barycenter on this experiment. Severe-error frequencies are 15.875% for the arithmetic mean and 12.25% for the ordinary JS barycenter, while neither the median nor RNJSC exceeds the 0.20 error threshold.

7.2 Paired uncertainty in the error contrasts

Because all estimators are evaluated on exactly the same generated expert panels, paired differences are more informative than independent standard errors. Table 2 shows selected bootstrap contrasts. A positive value means RNJSC has lower normalized JS error.

Every interval in Table 2 lies above zero. The replicate-wise win rate against the coordinate median rises from 0.755 at nominal $\eta = 0.1$ (2 of 15 oppositional experts) to 0.933 at $\eta = 0.4$ (6 of 15); the win rates against the arithmetic mean and ordinary JS barycenter are at least 0.989 over the contaminated cases.

8. SENSITIVITY AND ROBUSTNESS ANALYSIS

8.1 Robustness parameter

The parameter τ controls how quickly an expert loses relative weight as disagreement grows. Table 3 reports a separate experiment at nominal $\eta = 0.3$, corresponding to 4 oppositional experts out of 15 (26.7% realized contamination).

Small values behave closer to the ordinary barycenter, as predicted by the small-τ limit. The performance stabilizes broadly over $\tau \in [40, 80]$. The fixed value 60 used in Table 1 lies inside this plateau; it was not retuned for each contamination level.

8.2 Numerical checks of structural properties

Ten thousand random triplets in $[0,1]^3$ were used to check symmetry, the triangle inequality and complement invariance for $w_T = w_F$. Maximum symmetry error was 2.22×10^{-16}, no positive triangle-inequality residual was observed, and maximum complement-invariance error was 5.59×10^{-16}. These calculations are not substitutes for the proofs above; they verify the implementation.

For a representative panel at nominal $\eta = 0.3$ (4 of 15 experts oppositional), the normalized objective in (17) decreased over the first four MM iterations as

$$0.346640,\ 0.339285,\ 0.339169,\ 0.339167,$$

and converged to 0.339166574 by iteration 10. The generated latent triple was $(0.71731, 0.53889, 0.30536)$ and RNJSC returned $(0.69945, 0.56052, 0.27835)$.

9. DISCUSSION

The results distinguish two questions that are often combined in neutrosophic decision models: how disagreement is measured and how opinions are aggregated. The embedding (6) answers the first by placing each neutrosophic coordinate and its complement in a common probability simplex while leaving $T + I + F$ unrestricted. The bounded metric then gives every disagreement a scale in $[0,1]$. The robust criterion (17) answers the second question without modifying this geometry. At $\tau \downarrow 0$ it returns to ordinary squared-distance aggregation; at larger τ, conflicting opinions receive progressively smaller marginal influence.

This separation differs from many recent neutrosophic MCDM formulations, where consistency, weighting, aggregation and ranking are assembled into a single pipeline [10, 24, 14, 13]. Such pipelines are appropriate when the end goal is an alternative ranking. RNJSC is deliberately narrower: it estimates a representative neutrosophic state from a panel. The output can subsequently feed TOPSIS, MABAC, AHP or another rule without changing the consensus derivation.

The simulation also shows why the coordinate median is an

Table 1. Consensus estimation under oppositional contamination. The first column reports the nominal setting η; the realized numbers of oppositional experts are 0, 2, 3, 4, and 6 out of 15. JS error is $d_w(\hat{x}, x_0)$. Values are means across 800 independent replications; parentheses contain the standard deviation of JS error.

Nominal η	Method	JS error	Coordinate RMSE	Failure rate
0.0	Arithmetic mean	0.00997 (0.00442)	0.01074	0.00000
	Coordinate median	0.01212 (0.00527)	0.01310	0.00000
	JS barycenter	**0.00989** (0.00442)	**0.01066**	0.00000
	RNJSC	0.00994 (0.00444)	0.01072	0.00000
0.1	Arithmetic mean	0.05667 (0.02059)	0.05766	0.00000
	Coordinate median	0.01458 (0.00655)	0.01578	0.00000
	JS barycenter	0.04777 (0.01536)	0.04872	0.00000
	RNJSC	**0.01069** (0.00486)	**0.01156**	0.00000
0.2	Arithmetic mean	0.08131 (0.02789)	0.08448	0.00000
	Coordinate median	0.01731 (0.00729)	0.01872	0.00000
	JS barycenter	0.07179 (0.02222)	0.07436	0.00000
	RNJSC	**0.01131** (0.00507)	**0.01227**	0.00000
0.3	Arithmetic mean	0.10814 (0.03604)	0.11516	0.00250
	Coordinate median	0.02230 (0.00833)	0.02406	0.00000
	JS barycenter	0.09876 (0.03066)	0.10467	0.00000
	RNJSC	**0.01223** (0.00595)	**0.01317**	0.00000
0.4	Arithmetic mean	0.14891 (0.04991)	0.16389	0.15875
	Coordinate median	0.03797 (0.01253)	0.04136	0.00000
	JS barycenter	0.14475 (0.04742)	0.15894	0.12250
	RNJSC	**0.01489** (0.00873)	**0.01640**	0.00000

Table 2. Paired bootstrap advantage over RNJSC in normalized JS error. Intervals use 3000 paired bootstrap resamples.

η	Comparator	Mean diff.	95% interval
0.1	Arithmetic mean	0.04598	[0.04439, 0.04747]
	Coord. median	0.00389	[0.00353, 0.00426]
	JS barycenter	0.03708	[0.03599, 0.03819]
0.2	Arithmetic mean	0.07000	[0.06800, 0.07203]
	Coord. median	0.00600	[0.00554, 0.00647]
	JS barycenter	0.06048	[0.05890, 0.06208]
0.3	Arithmetic mean	0.09591	[0.09324, 0.09847]
	Coord. median	0.01007	[0.00952, 0.01064]
	JS barycenter	0.08653	[0.08428, 0.08876]
0.4	Arithmetic mean	0.13402	[0.13032, 0.13767]
	Coord. median	0.02307	[0.02212, 0.02403]
	JS barycenter	0.12986	[0.12642, 0.13349]

important comparator. Under the oppositional contamination used here, it remains markedly more robust than both arithmetic and ordinary JS averaging. RNJSC improves further because it exploits the smooth majority cluster rather than aggregating each coordinate only through its order statistic. That advantage is specific to a clustered-majority contamination structure and should not be read as a theorem that RNJSC dominates the median under every distribution.

Three limitations are material. First, the geometric weights w are fixed rather than learned. In applications they can encode domain importance, but data-dependent estimation would require an additional identification principle. Second, the bounded loss is nonconvex as a function of the full consensus criterion, so MM guarantees descent but not a globally unique robust minimizer. Initializing at the ordinary JS barycenter worked stably in the reported experiments. Third, the experiment uses a controlled stochastic model rather than

Table 3. RNJSC sensitivity to τ at nominal $\eta = 0.3$ (4 of 15 experts oppositional; 200 replications per setting).

τ	Mean JS error	Mean RMSE
5	0.05971	0.06250
10	0.03275	0.03424
20	0.02007	0.02179
40	0.01219	0.01316
60	0.01267	0.01387
80	0.01187	0.01279

elicited judgments from a particular decision domain. This is intentional for isolating contamination effects, but domain studies remain necessary for external validity. Dynamic and multi-scale neutrosophic models [21, 25] also suggest natural extensions in which expert states or decision resolutions evolve over time.

10. CONCLUSION

A robust consensus problem for single-valued neutrosophic information has been formulated on an information-geometric rather than score-based foundation. The probability-completed map Φ_w is injective and converts an unconstrained neutrosophic triple into a categorical distribution without forcing $T + I + F = 1$. The normalized square-root Jensen–Shannon pullback is a bounded metric, decomposes exactly by neutrosophic coordinate, respects complementation for symmetric truth–falsity weights, and induces the local tensor $w_c/[8\log 2 x_c(1-x_c)]$.

Robustness is introduced separately through the bounded criterion $1 - e^{-\tau d_w^2}$. Its MM algorithm has an explicit exponential reweighting step and a separable Jensen–Shannon barycenter update, with monotone objective descent. In the controlled experiment, fixing $\tau = 60$ leaves clean-data accuracy essentially unchanged while sharply reducing error across the contaminated settings, corresponding to 2–6 oppositional experts out of 15 (13.3–40.0% realized contamination). The method therefore supplies a mathematically transparent consensus layer for neutrosophic group information. Future work can address adaptive geometric weights, convergence to global stationary structures, interval and refined neutrosophic states, and dynamic expert panels.

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