A New Approach Of Neutrosophic Topological space

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Abstract

In this paper, a new approach of neutrosophic topological space (NA-NTS) is going to be introduced which is more general than neutrosophic topological space. Moreover, a new kind of neutrosophic sets and neutrosophic concepts in this new space is going to be created, which may makes us created a new kind in neutrosophic topology. We prove that a new approach of neutrosophic topological space is not a classical topological space. Also, a new approach of neutrosophic topological space is neither neutrosophic topological space nor neutrosophic crisp topological space. Many examples and theories are presented..

Keywords: New approach of neutrosophic topological spaces, new approach of neutrosophic open sets, new approach of neutrosophic closed sets.

1. Introduction

Recently, as a generalization of fuzzy set was defined by Zadeh [1] and intuitionistic fuzzy set was defined by K. Atanassov[2], the concept of the neutrosophic set was first given by F. Smarandache [3,4]. A.A. Salama and S.A. Alblowi [5] presented neutrosophic topological space via neutrosophic sets. In recent years, the theory of neutrosophic theory becomes very widespread among scientists around the world. For more details about neutrosophic topological space and applications of neutrosophic set theory, the readers should see [6–13].

Recently, Agboola et al. in[14,15], presented the concept of neutrosophic ring and neutrosophic group. Then, in 2015, Agboola in[16], presented the concept of refined neutrosophic algebraic structures. Also, he introduced refined neutrosophic groups. Recently, several works have been done to generalize the neutrosophic algebraic structures to refined neutrosophic algebraic structures. In 2020, Adeleke et al. In [17,18] studied several refined concepts such as refined neutrosophic rings and introduced their basic properties, refined neutrosophic ideals and refined neutrosophic homomorphisms in details. Many researchers had many contributions to neutrosophic ring [19] and neutrosophic topology as [20], [21] and [22]. Also, F. Smarandache extended the neutrosophic set to refined [n-valued] neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability, See[23].

This paper is devoted to the study of a new approach of neutrosophic topology and new approach of neutrosophic topological space, and investigate its basic properties. Also, we prove that a new approach of neutrosophic

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topological space is not a classical neutrosophic topological space. We provide many new definitions, important results, some theories and examples.

2. Definition

In this section, we recall some basic definitions such as neutrosophic group, refined neutrosophic group and neutrosophic ring which are useful in the sequel.

Definition 2.1. [17] Let (G, *) be any group, the neutrosophic group is generated by I and G under * denoted by

$N(G)=\{<G\cup I>,*\}$.

Definition 2.2: [19]
Let R be any ring. The neutrosophic ring $\langle R \cup I\rangle$ is also a ring generated by R and I under the operations of R.
Example 2.3: [19]
Let Z be the ring of integers; $\langle Z \cup I\rangle = \{a + bI : a, b \in Z\}$. $\langle Z \cup I\rangle$ is a ring called the neutrosophic ring of integers. Also $Z\neq \subseteq \langle Z\cup I\rangle$.

Definition 2.4: [5]

A neutrosophic topology (NT) on a set $X\neq\varnothing$ is a family Γ of neutrosophic subsets in X satisfying the following axioms.

1. 1_N , $0_N \in\Gamma$.
2. Γ is closed finite intersection.
3. Γ is closed under arbitrary union.

the pair (X, Γ) is called neutrosophic topological space (NTS) in X. Moreover, elements of Γ are known as neutrosophic open sets (NOS) and their complements are neutrosophic closed sets(NCS).
For a neutrosophic set A over X, the neutrosophic interior and the neutrosophic closure of A are defined as: $Nint(A)=\cup\{G : G \subseteq A , G \in \Gamma \}$ and $Ncl (A)=\cap\{F : A \subseteq F , F^c\in \Gamma \}$.

3. A new Approach Of Neutrosophic Topological Space:

In this section, we study a new approach of neutrosophic topological space, and investigate its basic properties. We denote the indeterminacy by (I). The indeterminacy I is taken to have the properties $I.I = I^2= I$.

Definition 3.1:

Let $\chi\neq\varnothing$ be any set, then we define $(\chi)_N$ as following $(\chi)_N=\{ a\oplus bI : a\in \chi ,b\in \chi\cup \{0 \} \}$ (the set $N(\chi)$ is generated by I and G), also, bI is indeterminacy and bI = I.

- The power set of χ is denoted by $P(\chi)$.
- The power set of $(\chi)_N$ is denoted by $P[(\chi)_N]$.

Definition 3.2:

1. If $A \in P(\chi)$ and $A\neq\varnothing$ then $(A)_N \in P[(\chi)_N]$; $(A)_N=\{ a\oplus bI : a\in \chi ,b\in \chi\cup \{0 \} \}$, also, bI is indeterminacy and bI = I. But if $A=\varnothing$ then $(A)_N =\varnothing\oplus I$.
2. If $(A)_N \in P[(\chi)_N]$, then $(\varnothing\oplus I) \cup (A)_N = (A)_N$ and $(\varnothing\oplus I) \cap (A)_N = \varnothing\oplus I$.

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Example 3.3:

Let X={1,2,3} then $(\chi)_N$={ a⊕bI : a∈ χ ,b∈ χ∪ {0 } }={1, 2, 3, 1⊕I, 2⊕I, 3⊕I}

We remove many elements in N(χ) such as 1⊕2I, 1⊕3I, 2⊕2I, 2⊕3I, 3⊕2I, 3⊕3I because, every one of them equal to member in {1⊕I, 2⊕I, 3⊕I}.

If A={1,2}then, $(A)_N$={ a⊕bI : a∈ χ ,b∈ χ∪ {0 } }={1, 2, 1⊕I, 2⊕I}.

Example 3.4:

Let χ={x, y} then $(\chi)_N$={ a⊕bI : a∈ χ ,b∈ χ∪ {0 } }={ x,y, x⊕I, y⊕I}

We remove many elements in N(χ) as x⊕xI, x⊕yI, y⊕xI, y⊕yI. because, every one of them equals to member in { x⊕I, y⊕I}.

If B={x}then, $(B)_N$={ a⊕bI : a∈ χ ,b∈ χ∪ {0 } }={x, x⊕I}.

Definition 3.5:

Let χ≠∅, if T={A_i}$_{i\in\Delta}$ is topology on χ, then a new approach of neutrosophic topology (N_A-NT) on χ is a family Ţ={$(A_i)_N$}$_{i\in\Delta}$ of $(\chi)_N$.

The pair (χ,Ţ) is called a new approach of neutrosophic topological space (N_A-NTS) in χ. Moreover, members of Ţ are known as a new approach of neutrosophic open sets (N_A-NOS) and their complements are a new approach of neutrosophic closed sets (N_A-NCS), members of P[$(\chi)_N$] are known as a new approach of neutrosophic sets (N_A-NS).

Remark 3.6:

- N_A-NOS(χ) means the family of the new approach of neutrosophic open sets on χ.
- N_A-NCS(χ) means the family of the new approach of neutrosophic closed sets on χ.

Example 3.7:

Let χ={e , f , g}. $T = \{\emptyset, A, B, C, X\}$,

$A = \{e, f\}, B = \{e, g\}, C = \{e\}$. Ţ $= \{\emptyset\oplus I, (A)_N, (B)_N, (C)_N, (X)_N\}$

$(A)_N = \{e, f, e\oplus I, f\oplus I\}, (B)_N = \{e, g, e\oplus I , g\oplus I\}, (C)_N = \{e, e\oplus I\}$.

Then(χ, Ţ) is a new approach of neutrosophic space.

Remark 3.8:

New approach of neutrosophic topological space is not a classical topological space.

Proof:

Since ∅∉Ţ, then new approach of neutrosophic topological space is not a classical topological space.

Theorem 3.9:

If I=0 then the new approach of neutrosophic topological space is a classical topological space.

Proof:

If I=0, then, Ţ $= T$ therefore, new approach of neutrosophic topological space is a classical topological space.

Remark 3.10: Let $(A_i)_N$∈ Ţ, for all i∈Δ, then:

1. $\cup_{i\in\Delta}(A_i)_N = (\cup_{i\in\Delta}A_i)_N$.
2. $(A_1)_N \cap (A_2)_N = (A_1\cap A_2)_N$.

Theorem 3.11: Let χ≠∅ then if Ţ={$(A_i)_N$}$_{i\in\Delta}$ is a new approach of neutrosophic topology (N_A-NT), then:

τ=Ţ∪{∅,X} is N-topology on $(\chi)_N$, and (χ, τ) is N-topological space.

(N= neutrosophic, but we saied N-topology in this Theorem is for neutrosophic topology because in [4], is defined,

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but they don't have the same concepts).

Proof:

- It is clear that $\emptyset, X \in \tau$.
- Let $(A_1)_N$, $(A_2)_N \in \tau$ then $A_1, A_2 \in T$, but T is topology, therefore, $A_1 \cap A_2 \in T$, hence $(A_1 \cap A_2)_N \in \mathcal{T}$.

  Therefore $(A_1)_N \cap (A_2)_N = (A_1 \cap A_2)_N \in \tau$ by remark 3.10.
- For every $i \in \Delta$, let $(A_i)_N \in \tau$ then $A_i \in T$, but T is topology, therefore, $\cup_{i\in\Delta} A_i \in T$, hence $(\cup_{i\in\Delta} A_i)_N \in \mathcal{T}$.

  Therefore, $\cup_{i\in\Delta}(A_i)_N = (\cup_{i\in\Delta} A_i)_N \in \tau$ by remark 3.10.

Therefore τ is N-topology on $(\chi)_N$, and (χ, τ) is N-topological space.

Remark 3.12:

New approach of neutrosophic topological space is not a neutrosophic topological space.

Remark 3.13:

New approach of neutrosophic topological space is not a neutrosophic crisp topological space.

4. **The interior and closure operations in a new approach of neutrosophic topological space:**

In this part, we define the closure and interior via new approach of neutrosophic open (closed) set.

Definition 4.1: Let $(\chi, \mathcal{T})$ be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NS) then :
The union of any N_A-NOS, contained in A is called the a new approach of neutrosophic interior of A (N_A-int(A)).

$N_A\text{-int}(A) = \cup\{B \;;\; B \subseteq A \;;\; B \in N_A\text{-NOS}\}$.

Theorem 4.2:
Let $(\chi, \mathcal{T})$ be an N_A-NTS, A, B are a new approach of the neutrosophic set (N_A-NS) then :

1. $N_A\text{-int}(A) \subseteq A$.
2. N_A-int(A) is N_A-NOS.
3. $A \subseteq B \Rightarrow N_A\text{-int}(A) \subseteq N_A\text{-int}(B)$.

Proof :

1. Follows from the definition of N_A-int(A) as a union of any N_A-NOS, contains in A.
2. Since the union of any N_A-NOS, is N_A-NOS, then $N_A\text{-int}(A) = \cup\{B \;;\; B \subseteq A \;;\; B \in N_A\text{-NOS}(X)\}$ is N_A-NOS.
3. Proof is obvious.

Definition 4.3:
Let $(\chi, \mathcal{T})$ be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NS) then :
The intersection of any N_A-NCS, including A is called a new approach of neutrosophic closure of A (N_A-cl(A)).

$N_A\text{-cl}(A) = \cap\{B \;;\; B \supseteq A \;;\; B \in N_A\text{-NCS}(\chi)\}$.

Theorem 4.4:
Let $(\chi, \mathcal{T})$ be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NCS) then :

1. $A \subseteq N_A\text{-cl}(A)$.
2. N_A-cl(A) is N_A-NCS.

Proof :

1. Follow from the definition of N_A-cl(A) as an intersection of any N_A-NCS contained in A.

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2. Prof is obvious.

Theorem 4.5:

Let (χ, Ƭ) be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NCS) then :

1. N_A-cl(χ-A)= χ - (N_A-int(A)).
2. N_A-int(χ-A)= χ - (N_A-cl(A)).
3. N_A-int(A)= χ - N_A-cl(χ -A)).
4. N_A-cl(A)= χ - (N_A-int(χ -A)).

Proof :

1. χ - (N_A-int(A))= χ -[∪{B ; B⊆A ; B∈ N_A-NOS }]
   =∩{ χ -B ; χ -B⊇ χ -A ; χ -B∈ N_A-NOS(χ) }= N_A-cl(χ -A)
   =∪{ χ -B ; χ -B⊆ A ; X-B∈ N_A-NOS(X) }= χ - N_A-int(A).
2. X- N_A-c(A)= χ -[∩{B ; B⊇A ; B∈ N_A-NCS(χ) }]
   =∪{ χ -B ; χ -B⊆ χ -A ; χ -B∈ N_A-NCS(χ) }= N_A-int(χ -A).
3. Follows from (2) by put χ -A in place of A.
4. Follows from (1) by put χ -A in place of A.

Theorem 4.6:

Let (χ, Ƭ) be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NCS) then :

1. A is N_A-NCS, iff N_A-cl(A) = A.
2. A is N_A-NOS, iff N_A-N-int(A)= A.

Proof :

1. Follow from the definition of N_A-cl(A) and Theorem 3.4.
2. Follow from the definition of N_A-int(A) and Theorem 3.2**.**

Theorem 4.7:

Let (χ, Ƭ) be an N_A-NTS, and A is a new approach of neutrosophic set (N_A-NCS) then :

1. N_A-cl[N_A-cl(A)]= N_A-cl(A).
2. N_A-int[N_A-int(A)]= N_A-int(A).

Proof :

Prof is Obvious.

Remark 4.8:

Let (χ, Ƭ) be an N_A-NTS, A, B are a new approach of neutrosophic set (N_A-NS) then :

1. N_A-int(A∩B) ⊆ N_A-int(A)∩N_A-int(B).
2. N_A-cl(A∩B) ⊆ N_A-cl(A)∩N_A-cl(B).
3. N_A-int(A∪B) ⊇ N_A-int(A)∪N_A-int(B).
4. N_A-cl(A∪B) ⊇ N_A-cl(A)∪N_A-cl(B).

Proof:

1. Since A∩B ⊆ A, A∩B ⊆ B then N_A-int(A∩B)⊆ N_A-int (A) and N_A-int(A∩B)⊆ N_A-int(B), hence N_A-int(A∩B)⊆ N_A-int(A)∩ N_A-int(A).
2. Since A∩B ⊆ A, A∩B ⊆ B then N_A-cl(A∩B)⊆ N_A-cl(A) and N_A-cl(A∩B)⊆ N_A-cl(B), hence N_A-cl(A∩B)⊆ N_A-cl(A)∩N_A-cl(A).
3. Since A⊆A∪B, B⊆A∪B then N_A-int(A)⊆ N_A-int(A∪B) and N_A-int(B)⊆N_A-int(A∪B), hence N_A-int(A)∪N_A-int(B)⊆ N_A-int(A∩B).
4. Since A⊆A∪B, B⊆A∪B then N_A-cl(A)⊆ N_A-cl(A∪B) and N_A-cl(B)⊆N_A-cl(A∪B), hence N_A-cl(A)∪ N_A-cl(B)⊆N_A-cl(A∩B).

Conclusion

In this work, we have introduced a new approach of neutrosophic topology and a new approach of neutrosophic topological space. Then, we have introduced a new approach of the neutrosophic open (closed) sets in a new approach of the neutrosophic topological space. Also, we studied some of their basic properties. Finally, This paper

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is just a beginning of a new structure and we have studied a few ideas only. It will be necessary to carry out more theoretical research to establish a general framework for the practical application. In the future, using these notions, various classes of mappings and separation axioms on the new approach of neutrosophic topological space can be studied.

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