



n-Refined Neutrosophic Groups II

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Abstract

The objective of this paper is to study some of AH-substructures in n-refined neutrosophic group. Also, it deals with some elementary properties of AH-subgroups, AH-normality, AH-homomorphisms, and endomorphisms especially in a non abelian n-refined neutrosophic group.

Keywords: n-Refined neutrosophic group, AH- subgroup, AH- homomorphism, AH-endomorphism.

1. Introduction

Neutrosophy as a new kind of logic founded by F. Smarandache deals with indeterminacy in nature and reality. In particular, it provided a strong tool to study some algebraic structures. Many neutrosophical algebraic structures came to light such as neutrosophic groups, neutrosophic rings, and neutrosophic semi groups. See [2,3,6]. Many studies were carried out using the idea of splitting the indeterminacy I into two components I_1, I_2 , like refined neutrosophic groups. See [3]. It has some interesting applications in soft computing [10].

In [7,8], F. Smarandache came with a new idea which suggests the splitting of indeterminacy I into n sub-indeterminacies I_1, I_2, \dots, I_n , which refers to many different degrees of indeterminacy. In [9], Smarandache defined algebraic operations between refined neutrosophic numbers and refined neutrosophic sets.

In [1], Abobala has defined the concept of n-refined neutrosophic groups to generalize the classical concept of neutrosophic group. Some interesting notions were presented such as n-refined neutrosophic subgroup, n-refined neutrosophic homomorphism, and AH-subgroup of an n-refined neutrosophic group.

In this work, we continue the work began in [1]. We establish the concept of AH-homomorphism to study AH-subgroups. Also, we handle some related concepts such as AH-nilpotency, AH-solvability, and AH-homomorphisms.

Motivation

This article is a continuation work of the work began in [1]. It establishes the algebraic theory of some AH-substructures in an n-refined neutrosophic ring.

2. Preliminaries

In the following section, we recall some important and useful definitions about neutrosophic groups and n-refined neutrosophic groups.

Definition 2.1: [2]

Let $(G, *)$ be a group. Then the neutrosophic group is generated by G and I under $*$ denoted by $N(G) = \langle G \cup I, * \rangle$.

I is called the indeterminate (neutrosophic element) with the property $I^2 = I$.

Definition 2.2: [1]

Let $(G, *)$ be a group, we define the corresponding n-refined neutrosophic group $N_n(G)$ as follows:

$$N_n(G) = \langle G \cup \{I_1, \dots, I_n\}, * \rangle = \{(a_0, a_1 I_1, \dots, a_n I_n); a_i \in G\}.$$

It is easy to see that $N_n(G)$ is closed under $*$, and it is a semi group but not a group since I_i has no inverse with respect to $*$ in general.

Remark 2.3: [1]

If $(G, +)$ is an additive abelian group, then addition on $N_n(G)$ can be described as follows:

Consider $x = (a_0, a_1 I_1, \dots, a_n I_n), y = (b_0, b_1 I_1, \dots, b_n I_n)$, we have

. In this case $(N_n(G), +)$ is a classical abelian group. $x + y = (a_0 + b_0, [a_1 + b_1]I_1, \dots, [a_n + b_n]I_n)$

The identity element is $(0, 0, \dots, 0)$.

It is easy to see that $N_n(G) \cong G \times G \times \dots \times G$ ($n + 1$ times) in the case of abelian additive group G .

Remark 2.4: [1]

If G is a multiplicative group, then group product on $N_n(G)$ can be described as follows:

Consider $x = (a_0, a_1 I_1, \dots, a_n I_n), y = (b_0, b_1 I_1, \dots, b_n I_n)$, we have

$$.xy = (t_0, t_1, \dots, t_n); t_s = \prod_{i,j=0}^n (a_i b_j) I_i I_j; I_0 = e_G \text{ and } I_i I_j = I_s$$

The identity element is $(e_G, e_G I_1, \dots, e_G I_n)$.

In this case $N_n(G)$ is not isomorphic to the direct product of $n+1$ copies of G , since it is not a classical group in this case.

The binary operation between the sub-indeterminacies is $I_i \cdot I_j = I_{\min(i,j)}$.

Definition 2.5: [1]

Let $N_n(G) = \{(a_0, a_1 I_1, \dots, a_n I_n); a_i \in G\}$ be an n-refined neutrosophic group,

$N_n(H) = \{(b_0, b_1 I_1, \dots, b_n I_n); b_i \in H_i; H_i \text{ is a subgroup of } G \text{ for all } i\}$ is called an AH-subgroup of $N_n(G)$.

If $H_i \cong H_j$ for all $i \neq j$, then it is called an AHS-subgroup.

The AH-subgroup $N_n(H)$ is called AH-abelian if H_i is abelian for all i . Also, it is called AH-cyclic if H_i is cyclic for all i .

Example 2.6: [1]

Let $G = S_3$ be the non abelian symmetric group of order 6, there are two non isomorphic subgroups of G ,

$K \cong Z_2, S \cong Z_3$, consider the corresponding 3-refined neutrosophic group $N_3(G)$, we have:

$N_3(H) = (K, SI_1, KI_2, SI_3) = \{(a, bI_1, cI_2, dI_3); a, c \in K \text{ and } b, d \in S\}$ is an AH-subgroup of $N_3(G)$.

$N_3(H)$ is an AH-cyclic, since K, S are cyclic.

Definition 2.7: [5]

Let G be a group with the following normal series $\{e\} = H_0 \leq H_1 \leq \dots \leq H_n = G$. It is called solvable if H_i/H_{i-1} is abelian.

The intersection, direct product, and product of two solvable groups is solvable.

Definition 2.8: [11]

(a) Let G be any group. It is called meta abelian if it has a abelian derivative subgroup G' .

(b) Let G be a group. It is called nilpotent if it has a central series.

For the concept of central series, see [11].

S_3 is a solvable group, but it is not nilpotent.

D_4 is a meta abelian and nilpotent group.

The intersection, and the direct product of two meta abelian groups is meta abelian.

The intersection, and the direct product of two nilpotent groups is nilpotent.

Definition 2.9: [2]

Let $N(G)$ be a neutrosophic group and H be a neutrosophic subgroup, i.e (H contains a proper subgroup of G) of $N(G)$. Then H is a neutrosophic normal subgroup of $N(G)$ if $xH = Hx$ for all $x \in N(G)$.

Definition 2.10: [2]

Let $N(G)$ be a neutrosophic group. Then the center of $N(G)$ is denoted by $C(N(G))$, and defined

$$C(N(G)) = \{x \in N(G); xy = yx \forall y \in N(G)\}.$$

Definition 2.11: [2]

Let $N(G), N(H)$ be two neutrosophic groups, then $N(G) \times N(H) = \{(g, h); g \in N(G), h \in N(H)\}$.

Definition 2.12: [6]

Let $N(G)$, $N(H)$ be two neutrosophic groups and $\varphi: N(G) \rightarrow N(H)$ is called a neutrosophic homomorphism if it is a homomorphism between G , H and $\varphi(I) = I'$.

Where I' is the neutrosophic element of $N(H)$.

If φ is a correspondence one-to-one it is called a neutrosophic isomorphism.

3. Main discussion

Definition 3.1:

Let G be any group, $N_n(G)$ be its corresponding n -refined neutrosophic group,

$N_n(H) = (H_0, H_1I_1, \dots, H_nI_n)$; $H_i \leq G$ be an AH-subgroup of $N_n(G)$. We say

- (a) $N_n(H)$ is AH-normal subgroup if H_i is normal for all i .
- (b) $N_n(H)$ is AH-nilpotent subgroup if H_i is nilpotent for all i .
- (c) $N_n(H)$ is AH-solvable subgroup if H_i is solvable for all i .
- (d) $N_n(H)$ is AH-meta abelian subgroup if H_i is meta abelian for all i .
- (e) $N_n(H)$ is AH-simple subgroup if H_i is simple for all i .

Example 3.2:

Consider the symmetric group of order 6 ($G = S_3$), it has one normal subgroup $H \cong Z_3$, and three 2-Sylow subgroups $K \cong S \cong L \cong Z_2$.

Let $N_2(G)$ be the corresponding 2-refined neutrosophic group, we have

- (a) $M_1 = (K, KI_1, HI_2)$ is an AH-subgroup of $N_2(G)$.
- (b) M_1 is an AH-nilpotent/AH-solvable, since K, H are nilpotent, and solvable subgroups of G .
- (c) M_1 is an AH-simple, since K, H are simple.
- (d) M_1 is not an AH-normal, since K is not normal.
- (e) M_1 is an AH-meta abelian, since H, K are meta abelian subgroups.

Remark 3.3:

If G is an additive abelian group, then any AH-subgroup is a classical subgroup, since $N_n(G)$ is a classical group and isomorphic to the direct product of G with itself ($n+1$ times).

Definition 3.4:

Let G, K be any two groups, $N_n(G), N_n(K)$ be their corresponding n -refined neutrosophic groups,

$f_i: G \rightarrow K; 0 \leq i \leq n$ be a classical homomorphism for all i . We say

- (a) $f: N_n(G) \rightarrow N_n(K); f(a_0, a_1I_1, \dots, a_nI_n) = (f_0(a_0), f_1(a_1)I_1, \dots, f_n(a_n)I_n)$ is an AH-homomorphism.

(b) If $f_i: G \rightarrow K$ is an isomorphism for all i , then $f: N_n(G) \rightarrow N_n(K)$ is called an AH-isomorphism.

(c) If $f_i = f_j; i \neq j$, then $f: N_n(G) \rightarrow N_n(K)$ is called an AHS-homomorphism.

(d) The AH-kernel is defined as follows: $AH - Ker(f) = (Ker(f_0), Ker(f_1)I_1, \dots, Ker(f_n)I_n)$.

(e) The AH-image is defined as: $AH - Im(f) = (f_0(G), f_1(G)I_1, \dots, f_n(G)I_n)$.

We denote to the AH-homomorphism : $N_n(G) \rightarrow N_n(K); f(a_0, a_1I_1, \dots, a_nI_n) = (f_0(a_0), f_1(a_1)I_1, \dots, f_n(a_n)I_n)$ by

$$f = (f_0, f_1I_1, \dots, f_nI_n).$$

Definition 3.5:

Let G be any group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$$N_n(H) = (H_0, H_1I_1, \dots, H_nI_n), N(K) = (K_0, K_1I_1, \dots, K_nI_n); H_i, K_i \leq G \text{ be any two AH-subgroups of } N_n(G).$$

(a) We define the intersection as follows: $N_n(H) \cap N_n(K) = (H_0 \cap K_0, (H_1 \cap K_1)I_1, \dots, (H_n \cap K_n)I_n)$.

(b) We define the product as follows: $N_n(H) \cdot N_n(K) = (H_0 \cdot K_0, (H_1 \cdot K_1)I_1, \dots, (H_n \cdot K_n)I_n)$.

(c) We define the direct product as follows: $N_n(H) \times N_n(K) = (H_0 \times K_0, (H_1 \times K_1)I_1, \dots, (H_n \times K_n)I_n)$.

Example 3.6:

Consider the symmetric group of order 6 ($G = S_3$), it has one normal subgroup $H \cong Z_3$, and three 2-Sylow subgroups $K \cong S \cong L \cong Z_2$.

Let $N_2(G)$ be the corresponding 2-refined neutrosophic group, $M_1 = (K, KI_1, HI_2), M_2 = (H, KI_1, KI_2)$ be two AH-subgroups of $N_2(G)$, we have

(a) $M_1 \cap M_2 = (\{e\}, KI_1, \{e\}I_2)$, which is an AH-subgroup of $N_2(G)$.

(b) $M_1 \cdot M_2 = (KH, KKI_1, HKI_2) = (G, KI_1, GI_2)$.

Example 3.7:

Let $G = (Z, +), K = (Z_6, +)$ be two groups, $N_2(G), N_2(K)$ be the corresponding 2-refined neutrosophic groups, we have

(a) $f_0: G \rightarrow K; f_0(a) = a \text{ mod } 6, f_1: G \rightarrow K; f_1(a) = 2a \text{ mod } 6$ are two classical homomorphisms.

(b) $f: N_2(G) \rightarrow N_2(K); f(a_0, a_1I_1, a_2I_2) = (f_0(a_0), f_0(a_1)I_1, f_1(a_2)I_2) = (a_0 \text{ mod } 6, a_1 \text{ mod } 6 I_1, 2a_2 \text{ mod } 6 I_2)$ is an AH-homomorphism.

(c) $AH - Ker(f) = (Ker(f_0), Ker(f_0)I_1, Ker(f_1)I_2) = (6Z, 6ZI_1, 3ZI_2)$.

(d) $AH - Im(f) = (Im(f_0), Im(f_0)I_1, Im(f_1)I_2) = (Z_6, Z_6I_1, \{0, 2, 4\}I_2)$.

Theorem 3.8:

Let G be any group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$N_n(H) = (H_0, H_1I_1, \dots, H_nI_n), N(K) = (K_0, K_1I_1, \dots, K_nI_n); H_i, K_i \leq G$ be any two AH-subgroups of $N_n(G)$. We have:

- (a) $N_n(H) \cap N_n(K)$ is an AH-subgroup of $N_n(G)$.
- (b) If $N_n(H), N_n(K)$ are AH-normal, then $N_n(H). N_n(K)$ is AH-normal.
- (c) $N_n(H) \times N_n(K)$ is an AH-subgroup of $N_n(G) \times N_n(G)$.
- (d) If $N_n(H), N_n(K)$ are AH-abelian, then $N_n(H) \cap N_n(K), N_n(G) \times N_n(G)$ are AH-abelian.
- (e) If $N_n(H), N_n(K)$ are AH-cyclic, then $N_n(H) \cap N_n(K)$ is AH-cyclic.
- (f) If $N_n(H), N_n(K)$ are AH-nilpotent, then $N_n(H) \cap N_n(K), N_n(G) \times N_n(G)$ are AH-nilpotent.
- (g) If $N_n(H), N_n(K)$ are AH-solvable, then $N_n(H). N_n(K), N_n(G) \times N_n(G), N_n(G) \cap N_n(G)$ are AH-solvable.
- (h) If $N_n(H), N_n(K)$ are AH- meta abelian, then $N_n(H) \cap N_n(K), N_n(G) \times N_n(G)$ are AH-meta abelian.

Proof:

It is well known from the classical group theoretical properties that $H_i \cap K_i$ is a subgroup of G , $H_i \times K_i$ is a subgroup of $G \times G$, and $H_i. K_i$ is a subgroup of G under the assumption of normality of H_i, K_i , thus (a),(b),(c) are true.

Also, the direct product and the intersection of any two abelian, nilpotent, or solvable subgroups is the same, thus

(d),(f),(g) are true.

(h), (e) hold by the same argument.

Theorem 3.9:

Let G, K be any two groups, $N_n(G), N_n(K)$ be their corresponding n-refined neutrosophic groups,

$N_n(H) = (H_0, H_1I_1, \dots, H_nI_n); H_i \leq G$ be an AH-subgroup, $f_i: G \rightarrow K; 0 \leq i \leq n$ be a classical homomorphism for all $i, f: N_n(G) \rightarrow N_n(K); f(a_0, a_1I_1, \dots, a_nI_n) = (f_0(a_0), f_1(a_1)I_1, \dots, f_n(a_n)I_n)$ be an AH-homomorphism, we have:

- (a) If $N_n(H)$ is AH-cyclic/AH-abelian, then $f(N_n(H))$ is AH-cyclic/AH-abelian.
- (b) If $N_n(H)$ is AH-nilpotent/AH-solvable, then $f(N_n(H))$ is AH-nilpotent/AH-solvable.
- (c) If $N_n(H)$ is AH-meta abelian, then $f(N_n(H))$ is AH-meta abelian.
- (d) If $N_n(H)$ is AH-normal, then $f(N_n(H))$ is AH-normal.
- (e) AH-Ker(f) is an AH-normal subgroup of $N_n(G)$.
- (f) AH-Im(f) is an AHS-subgroup of $N_n(K)$.

Proof:

(a),(b),(c),(d) It is well known that the homomorphic image of cyclic, abelian, nilpotent, normal, or meta abelian subgroup is the same, so the proof is complete.

(e) Since $\text{Ker}(f_i)$ is a normal subgroup of G for all i , $AH - \text{Ker}(f) = (\text{Ker}(f_0), \text{Ker}(f_1)I_1, \dots, \text{Ker}(f_n)I_n)$ as an AH-normal subgroup.

(f) The proof is similar to (e).

Theorem 3.10:

Let G be any finite group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$N_n(H) = (H_0, H_1I_1, \dots, H_nI_n)$ be any AH-subgroup. Then

(a) $O(N_n(H)) = O(H_0) \times O(H_1) \times \dots \times O(H_n)$.

(b) Lagrange's theorem is true for AH-subgroups, i.e $O(N_n(H))$ divides $O(N_n(G))$.

Proof:

(a) Since $N_n(H) = (H_0, H_1I_1, \dots, H_nI_n) = (h_0, h_1I_1, \dots, h_nI_n); h_i \in H_i$, we get $O(N_n(H)) = O(H_0) \times O(H_1) \times \dots \times O(H_n)$.

(b) Since $O(H_i)$ is a divisor of $O(G) = m$, then $O(N_n(H)) = O(H_0) \times O(H_1) \times \dots \times O(H_n)$ is a divisor of $O(N_n(G)) = m^{n+1}$. See [1].

Definition 3.11:

Let $(G, +)$ be any additive abelian group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$AH - \text{End}(N_n(G))$ is defined to be the set of all AH-homomorphisms between $N_n(G)$ and itself.

$AHS - \text{End}(N_n(G))$ is defined to be the set of all AHS-homomorphisms between $N_n(G)$ and itself.

Definition 3.12:

Let $(G, +)$ be any additive abelian group, $N_n(G)$ be its corresponding n-refined neutrosophic group.

We define operations on $AH - \text{End}(N_n(G))$ as follows:

Let $f: N_n(G) \rightarrow N_n(G); f(a_0, a_1I_1, \dots, a_nI_n) = (f_0(a_0), f_1(a_1)I_1, \dots, f_n(a_n)I_n)$,

$g: N_n(G) \rightarrow N_n(G); g(a_0, a_1I_1, \dots, a_nI_n) = (g_0(a_0), g_1(a_1)I_1, \dots, g_n(a_n)I_n)$ be any two AH-endomorphisms,

Addition is defined as $(f + g)(a_0, a_1I_1, \dots, a_nI_n) = ([f_0 + g_0](a_0), [f_1 + g_1](a_1)I_1, \dots, [f_n + g_n](a_n)I_n)$.

Multiplication is defined as $(f \circ g)(a_0, a_1I_1, \dots, a_nI_n) = ([f_0 \circ g_0](a_0), [f_1 \circ g_1](a_1)I_1, \dots, [f_n \circ g_n](a_n)I_n)$.

It is easy to see that addition and multiplication are well defined.

$f + g, f \circ g$ are AH-endomorphisms, since $f_i + g_i, f_i \circ g_i$ are two classical endomorphisms for all i .

This means that $AH - \text{End}(N_n(G))$ is closed under addition and multiplication.

Theorem 3.13:

Let $(G, +)$ be any additive abelian group, $N_n(G)$ be its corresponding n-refined neutrosophic group. Then

$(AH - End(N_n(G)), +, o)$ is a ring and it is isomorphic to $End(G, +) \times End(G, +) \times \dots \times End(G, +)$ (n+1 times).

Proof:

Simply, we find that $(AH - End(N_n(G)), +)$ is an abelian group.

Also, multiplication is associative and distributive with respect to addition, since it is associative and distributive for each component i . Thus $(AH - End(N_n(G)), +, o)$ has a structure of ring.

The ring isomorphism between $(AH - End(N_n(G)), +, o)$ and

$End(G, +) \times End(G, +) \times \dots \times End(G, +)$ (n+1 times) can be defined as follows:

$\varphi: AH - End(N_n(G)) \rightarrow End(G, +) \times End(G, +) \times \dots \times End(G, +)$;

$\varphi(f_0, f_1 I_1, \dots, f_n I_n) = (f_0, f_1, \dots, f_n)$.

Definition 3.14:

Let G be any group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$N_n(H) = (H_0, H_1 I_1, \dots, H_n I_n)$; $H_i \leq G$ be an AH-subgroup of $N_n(G)$. We call $N_n(H)$ an AH-p subgroup if H_i is a p-group for all i .

Clearly, if G is a finite group, then $O(N_n(H))$ is a prime power according to Theorem 3.10.

Definition 3.15:

Let G be any group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

$N_n(H) = (H_0, H_1 I_1, \dots, H_n I_n)$; $H_i \leq G$ be an AH-subgroup of $N_n(G)$. The AH-derived subgroup of H can be defined as $[N_n(H)]' = (H'_0, H'_1 I_1, \dots, H'_n I_n)$; $H'_i = \langle x^{-1} y^{-1} x y ; \forall x, y \in H_i \rangle$.

Theorem 3.16:

Let G be any group, $N_n(G)$ be its corresponding n-refined neutrosophic group,

If $N_n(H) = (H_0, H_1 I_1, \dots, H_n I_n)$; $H_i \leq G$ be any AH-subgroup of $N_n(G)$. Then its AH-derived subgroup is trivial if and only if it is an AH-abelian subgroup.

Proof:

Suppose that $N_n(H) = (H_0, H_1 I_1, \dots, H_n I_n)$ is AH-abelian, hence H_i is abelian for all i . This implies that H'_i is trivial, thus $[N_n(H)]' = (H'_0, H'_1 I_1, \dots, H'_n I_n)$ is trivial.

Conversrly, If $[N_n(H)]' = (H'_0, H'_1 I_1, \dots, H'_n I_n)$ is trivial, we find that H'_i is trivial, thus H_i is abelian subgroup of G , which means that $N_n(H) = (H_0, H_1 I_1, \dots, H_n I_n)$ is AH-abelian.

Remark 3.17:

There is a well known criteria for solvability of any classical group G . This criteria suggests that any group G is solvable if and only if the normal series formed from derived subgroups (G', G'', \dots) reaches to the trivial subgroup $\{e\}$ after n -steps.

We can use this argument to determine if an AH-subgroup is AH-solvable or not by computing AH-derivatives

$([N_n(H)]', [N_n(H)]'', \dots)$. If the previous series reaches to the trivial subgroup of $N_n(G)$, then $N_n(H)$ is an AH-solvable subgroup.

Also, any group G is meta abelian if and only if its derivative is an abelian subgroup. This idea can be generalized into AH-subgroups as follows:

Any AH-subgroup $N_n(H)$ is AH-meta abelian if and only if its AH-derivative subgroup is an AH-abelian.

Remark 3.18

By using the same argument in Remark 3.17, we can define the AH-central series to study the AH-nilpotency of and AH-subgroup.

5. Conclusion

In this article we have defined the concept of AH-homomorphism in an n -refined neutrosophic group for the first time. Also, we have introduced some corresponding notions such as AH-endomorphism, AH-solvability, AH-nilpotency, and other related concepts. Many examples and theorems were constructed to clarify the validity of these concepts.

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