A note on single valued neutrosophic sets in ordered groupoids

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Abstract

The aim of this paper is to combine the notions of ordered algebraic structures and neutrosophy. In this regard, we define for the first time single valued neutrosophic sets in ordered groupoids. More precisely, we study single valued neutrosophic subgroupoids of groupoids, single valued neutrosophic ideals of groupoids, and single valued neutrosophic filters of groupoids. And we present some remarks on single valued neutrosophic subgroups (ideals) of groups.

Keywords: SVNS, (α, β, γ)-level set, ordered groupoid, single valued neutrosophic subgroupoid, single valued neutrosophic ideal, single valued neutrosophic filter.

1 Introduction

Neutrosophy,[9] a new branch of science that deals with indeterminacy, was launched by Smarandache in 1998. The theory of neutrosophy considers every notion or idea $< A >$ together with its opposite or negation $< antiA >$ and with their spectrum of neutralities $< neutA >$ in between them (i.e. notions or ideas supporting neither $< A >$ nor $< antiA >$). The $< neutA >$ and $< antiA >$ ideas together are referred to as $< nonA >$. Smarandach[10] defined neutrosophic sets as a generalization of the fuzzy sets introduced by Zadeh[12] in 1965 and as a generalization of intuitionistic fuzzy sets introduced by Atanassov[4] in 1986. Fuzzy sets allow gradual membership of an element in a set by assigning each element a degree of membership between 0 and 1 that are both inclusive. Whereas intutionistic fuzzy sets allow gradual membership as well as gradual non-membership of an element in a set by assigning each element a degree of membership and a degree of non-membership in a way that their sum is a real number in the unit interval $[0, 1]$. Single valued neutrosophic sets (SVNS)[14] generalize these two concepts so that each element has a truth value, indeterminacy value, and a falsity value and each of these values is a number in the unit interval $[0, 1]$. Neutrosophy has many applications in different fields of Science. Many researchers,[1],[2],[11] worked on the connection between neutrosophy and algebraic structures.

Our paper introduces a new link between algebraic structures and neutrosophy. In particular, it is concerned about single valued neutrosophic sets in ordered groupoids and it is constructed as follows: after an Introduction, in Section 2, we present some definitions related to neutrosophy that are used throughout the paper. In Section 3, we present some definitions about ordered groupoids (groups) and elaborate some examples that are used in Section 4 and Section 5. In Section 4, we define single valued neutrosophic subgroupoids (ideals) as well as single valued neutrosophic filters of ordered groupoids, present many non-trivial examples about the new defined concepts, and study some of their properties. Finally in Section 5, we apply the definition of SVNS in ordered groupoids to ordered groups, present some remarks and results.

2 Single valued neutrosophic sets

In this section, we present some definitions about neutrosophy that are used throughout the paper.

Definition 2.1. [14] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A on X is characterized by truth-membership T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Definition 2.2. Let X be a non-empty set, $\alpha, \beta, \gamma \in [0,1]$, and A a SVNS over X. Then the (α,β,γ)-level set of A is defined as follows:

$$L_{(\alpha,\beta,\gamma)} = \{x \in X : T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}.$$

Definition 2.3. [14] Let X be a non-empty set and A, B be single valued neutrosophic sets over X defined as follows.

$$A = \{\frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in X\}, B = \{\frac{x}{(T_B(x), I_B(x), F_B(x))} : x \in X\}$$

Then

- A is called a single valued neutrosophic subset of B and denoted as $A \subseteq B$ if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, and $F_A(x) \geq F_B(x)$ for all $x \in X$.
  If $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- The union of A and B is defined to be the SVNS over X:

  $$A \cup B = \{\frac{x}{(T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x))} : x \in X\}.$$

  Where $T_{A\cup B}(x) = T_A(x) \vee T_B(x)$, $I_{A\cup B}(x) = I_A(x) \vee I_B(x)$, and $F_{A\cup B}(x) = F_A(x) \wedge F_B(x)$ for all $x \in X$.
- The intersection of A and B is defined to be the SVNS over X:

  $$S_{A\cap B} = \{\frac{x}{(T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x))} : x \in X\}.$$

  Where $T_{A\cap B}(x) = T_A(x) \wedge T_B(x)$, $I_{A\cap B}(x) = I_A(x) \wedge I_B(x)$, and $F_{A\cap B}(x) = F_A(x) \vee F_B(x)$ for all $x \in X$.

Example 2.4. Le $X = \{s, a, m\}$ and A, B be SVNS over X defined as follows.

$$A = \{\frac{s}{(0.7, 0.6, 0.5)}, \frac{a}{(0.8, 0.4, 0.2)}, \frac{m}{(0.1, 0.6, 1)}\},$$

$$B = \{\frac{s}{(0.9, 0.1, 0.7)}, \frac{a}{(1, 0, 0.6)}, \frac{m}{(0.9, 0.3, 0.2)}\}.$$

Then the SVNS $A \cap B$ and $A \cup B$ over X are as follows.

$$A \cap B = \{\frac{s}{(0.7, 0.1, 0.7)}, \frac{a}{(0.8, 0, 0.6)}, \frac{m}{(0.1, 0.3, 1)}\},$$

$$A \cup B = \{\frac{s}{(0.9, 0.6, 0.5)}, \frac{a}{(1, 0.4, 0.2)}, \frac{m}{(0.9, 0.6, 0.2)}\}.$$

3 Ordered groupoids and ordered groups

In this section, we present some examples on ordered groupoids and ordered groups that are used in Section 4 and Section 5. For more details about algebraic structures, we refer to[5] and.[6]

Definition 3.1. [5] Let $(G, \cdot)$ be a groupoid (group) and "$\leq$" be a partial order on G. Then $(G, \cdot, \leq)$ is an ordered groupoid (ordered group) if the following condition holds for all $z \in G$.

$$\text{If } x \leq y \text{ then } z \cdot x \leq z \cdot y \text{ and } x \cdot z \leq y \cdot z.$$

Definition 3.2. Let $(G, \cdot, \leq)$ be an ordered groupoid (group). Then G is called a *total ordered groupoid (group)* if $x \leq y$ or $y \leq x$ for all $x, y \in G$.

An ordered groupoid $(G, \cdot, \leq)$ is said to be *commutative* if $x \cdot y - y \cdot x$ for all $x, y \in G$. And an element e in an ordered groupoid $(G, \cdot, \leq)$ is called an *identity* if $e \cdot x = x \cdot e = x$ for all $x \in G$. If such an element exists then it is unique.

Remark 3.3. Let $(G, \cdot)$ be any groupoid. Then by defining "$\leq$" on G as follows: For all $x, y \in G$,

$$x \leq y \text{ if and only if } x = y.$$

Then $G, \cdot, \leq)$ is an ordered groupoid.
Such an order is called the **trivial order**.

Ordered groups are a special case of ordered groupoids. We present some examples on infinite ordered groups.

Example 3.4. The groups of integers, rational numbers, real numbers under standard addition and usual order are ordered groups.

Example 3.5. Let $\mathbb{Q}^+$ be the set of positive rational numbers. Then $(\mathbb{Q}^+, \cdot, \leq)$ is an ordered group. Where "$\leq$" is defined as follows: For all $x, y \in \mathbb{Q}^+$,

$$x \leq y \quad \text{if and only if} \quad \frac{y}{x} \in \mathbb{N}.$$

We show that the partial order "$\leq$" defines an order on $\mathbb{Q}^+$. Let $x \leq y$ and $z \in \mathbb{Q}^+$. Having $\frac{y}{x} \in \mathbb{N}$ and $z > 0$ implies that $\frac{yz}{xz} \in \mathbb{N}$. Thus, $xz \leq yz$.

We present some examples on ordered groupoids that are not ordered groups.

Example 3.6. Let G be any non-empty set with $a \in G$ and "$\leq$" a partial order on G. Then by setting $x \cdot y = a$ for all $x, y \in G$, we get that $(G, \cdot, \leq)$ is an ordered groupoid.

Example 3.7. Let $\mathbb{N} = \{1, 2, \ldots\}$ be the set of natural numbers and define "$\leq_p$" in $\mathbb{N}$ as follows: For all $x, y \in \mathbb{N}$,

$$x \leq_p y \text{ if and only if } x \geq y.$$

Then $(\mathbb{N}, +, \leq_p)$ is a commutative ordered groupoid. This is easily seen as $\leq_p$ is a partial order and if $x \leq_p y$ and $z \in \mathbb{N}$ then $x + z \geq y + z$ and hence $x + z \leq_p y + z$.

Finite groupoids can be presented by means of Cayley's table.

Example 3.8. Let $(G_1, \cdot_1)$ be the groupoid defined by Table 1.

Table 1: The groupoid $(G_1, \cdot_1)$

$\cdot_1$	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

By setting $\leq_1 = \{(a, a), (a, b), (a, c), (b, b), (c, c)\}$, we get that $(G_1, \cdot_1, \leq_1)$ is a commutative ordered groupoid.

Example 3.9. Let $(G_1, \star)$ be the groupoid defined by Table 2.

Table 2: The groupoid $(G_1, \star)$

$\cdot_1$	a	b	c
a	a	a	a
b	a	a	c
c	a	c	a

By setting $\leq_1 = \{(a, a), (a, b), (a, c), (b, b), (c, c)\}$, we get that $(G_1, \star, \leq_1)$ is an ordered groupoid.

Example 3.10. Let $(G_2, \cdot_2)$ be the groupoid defined by Table 3.
By setting $\leq_2 = \{(1, 1), (2, 2), (2, 3), (3, 3), (4, 4)\}$, we get that $(G_2, \cdot_2, \leq_2)$ is an ordered groupoid.

Example 3.11. Let $(G_3, \cdot_3)$ be the groupoid defined by Table 4.
By setting $\leq_3 = \{(e, e), (c, e), (c, d), (d, e), (d, d)\}$, we get that $(G_3, \cdot_3, \leq_3)$ is a total ordered groupoid with an identity "e".

Table 3: The groupoid $(G_2, \cdot_2)$

$\cdot_2$	1	2	3	4
1	4	4	4	4
2	4	4	4	4
3	4	4	4	4
4	4	4	4	1

Table 4: The groupoid $(G_3, \cdot_3)$

$\cdot_1$	e	c	d
e	e	c	d
c	c	c	c
d	d	c	d

Table 5: The groupoid $(G_4, \cdot_4)$

$\cdot_4$	1	2	3
1	1	1	1
2	1	1	1
3	1	1	3

Example 3.12. Let $(G_4, \cdot_4)$ be the groupoid defined by Table 5.
By setting $\leq_4 = \{(1,1),(1,3),(2,2),(2,1),(2,3),(3,3)\}$, we get that $(G_4, \cdot_4, \leq_4)$ is a commutative total ordered groupoid.

Definition 3.13. Let $(G, \cdot, \leq)$ be an ordered groupoid and $S \subseteq G$. Then

$$(S] = \{x \in G : x \leq s \text{ for some } s \in S\}.$$

Remark 3.14. Let $(G, \cdot, \leq)$ be an ordered groupoid and $S \subseteq G$. Then $S \subseteq (S]$.

Definition 3.15. Let $(G, \cdot, \leq)$ be an ordered groupoid and $S \subseteq G$. Then

- S is a *subgroupoid* of G if $(S, \cdot)$ is a groupoid and $(S] \subseteq S$.
- S is a *left ideal* of G if $G \cdot S \subseteq S$ and $(S] \subseteq S$.
- S is a *right ideal* of G if $S \cdot G \subseteq S$ and $(S] \subseteq S$.
- S is an *ideal* of G if it is a left ideal of G and a right ideal of G.

Example 3.16. In Example 3.10, $\{1,4\}$, $\{1,2,4\}$, and $\{1,2,3,4\}$ are ideals of $(G_2, \cdot_2, \leq_2)$.

Definition 3.17. Let $(G, \cdot, \leq)$ be an ordered groupoid and $S \subseteq G$. Then S is a *filter* of G if the following conditions are satisfied.

1. $x \cdot y \in S$ for all $x, y \in S$;
2. If $x \cdot y \in S$ then $x \in S$ and $y \in S$ for all $x, y \in G$;
3. If $x \in S, y \in G$ and $x \leq y$ then $y \in S$.

Example 3.18. In Example 3.12, $\{3\}$ is the only filter of $(G_4, \cdot_4, \leq_4)$.

4 SVNS in ordered groupoids

In this section and inspired by the definition of fuzzy sets in ordered groupoids,[7] we define for the first time single valued neutrosophic subgroupoids (ideals) (in Subsection 4.1) as well as single valued neutrosophic filters (in Subsection 4.2) of ordered groupoids and study some of their properties such as finding a relationship between subgroupoids/ideals/filters of ordered groupoids and single valued neutrosophic subgroupoids/ideals/filters of these ordered groupoids. Moreover, we construct many non-trivial examples about them.

4.1 Single valued neutrosophic subgroupoids (ideals) of groupoids

Definition 4.1. Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over G. Then A is single valued neutrosophic subgroupoid of G if for all $x, y \in G$, the following conditions hold:

- $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$;
- $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$;
- $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$;
- If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$, and $F_A(x) \leq F_A(y)$.

Definition 4.2. Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over G. Then A is single valued neutrosophic left ideal of G if for all $x, y \in G$, the following conditions hold:

- $T_A(x \cdot y) \geq T_A(y)$;
- $I_A(x \cdot y) \geq I_A(y)$;
- $F_A(x \cdot y) \leq F_A(y)$;
- If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$, and $F_A(x) \leq F_A(y)$.

Definition 4.3. Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over G. Then A is single valued neutrosophic right ideal of G if for all $x, y \in G$, the following conditions hold:

- $T_A(x \cdot y) \geq T_A(x)$;
- $I_A(x \cdot y) \geq I_A(x)$;
- $F_A(x \cdot y) \leq F_A(x)$;
- If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$, and $F_A(x) \leq F_A(y)$.

Definition 4.4. Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over G. Then A is a single valued neutrosophic ideal of G if it is both: a single valued neutrosophic left ideal of G and a single valued neutrosophic right ideal of G.

Remark 4.5. Let $(G, \cdot, \leq)$ be a commutative ordered groupoid and A a SVNS over G. If A is a single valued neutrosophic right (or left) ideal of G then A is a single valued neutrosophic ideal of G.

Remark 4.6. Let $(G, \cdot, \leq)$ be an ordered groupoid and $\alpha, \beta, \gamma \in [0, 1]$. Then $A = \{\frac{x}{(\alpha,\beta,\gamma)} : x \in G\}$ is single valued neutrosophic ideal of G. Moreover, it is called the **trivial single valued neutrosophic ideal**.

Example 4.7. Let $(\mathbb{N}, +, \leq_p)$ be the ordered groupoid defined in Example 3.7 and A be a SVNS over $\mathbb{N}$ defined as follows: For all $k \in \mathbb{N}$,

$$N_A(k) = (1 - \frac{1}{k}, 1 - \frac{1}{k}, \frac{1}{k}).$$

Then A is a single valued neutrosophic ideal of $\mathbb{N}$. To prove that and by means of Remark 4.5, it suffices to show that A is a single valued neutrosophic right ideal of G. Let $k, m \in \mathbb{N}$. Then $k + m \geq k$ and thus, $T_A(k + m) = I_A(k + m) = 1 - \frac{1}{k+m} \geq 1 - \frac{1}{k} = T_A(k) = I_A(k)$ and $F_A(k + m) = \frac{1}{k+m} \leq \frac{1}{k} = F_A(k)$. Let $k \leq_p m$. Then $k \geq m$ and hence, $T_A(k) = I_A(k) = 1 - \frac{1}{k} \geq 1 - \frac{1}{m} = T_A(k) = I_A(m)$ and $F_A(k) = \frac{1}{k} \leq \frac{1}{m} = F_A(m)$.

Proposition 4.8. *Let $(G, \cdot, \leq)$ be an ordered groupoid with identity and A a SVNS over G. Then A is a single valued neutrosophic left/right ideal of G if and only if A is the trivial single valued neutrosophic ideal of G.*

Proof. We prove the case when A is a single valued neutrosophic right ideal of G and the case when A is a single valued neutrosophic left ideal of G is done similarly. Let A be a single valued neutrosophic right ideal of G. Then for all $x \in G$, we have:

$$T_A(x) = T_A(e \cdot x) \geq T_A(e), I_A(x) = I_A(e \cdot x) \geq I_A(e), \text{ and } F_A(x) = F_A(e \cdot x) \leq F_A(e);$$

$$T_A(x) = T_A(x \cdot e) \geq T_A(x), I_A(x) = I_A(x \cdot e) \geq I_A(x), \text{ and } F_A(x) = F_A(x \cdot e) \leq F_A(x).$$

The latter implies that

$$T_A(x) = T_A(e), I_A(x) = I_A(e), \text{ and } F_A(x) = F_A(e).$$

Therefore, A is the trivial single valued neutrosophic ideal of G. □

Example 4.9. Proposition 4.8 asserts that the ordered groupoid $(G_3, \cdot_3, \leq_3)$ in Example 3.11 has no non-trivial left (right) single valued neutrosophic ideals.

Definition 4.10. Let $(G, \cdot)$ be a groupoid and $A \subseteq G$. Then the characteristic function $N_A : G \to [0,1] \times [0,1] \times [0,1]$ is defined as follows.

$$N_A(x) = \begin{cases} (1,1,0) & \text{if } x \in A; \\ (0,0,1) & \text{otherwise.} \end{cases}$$

Lemma 4.11. *Let $(G, \cdot, \leq)$ be an ordered groupoid and $A \subseteq G$. Then A is a subgroupoid of G if and only if the single valued neutrosophic set corresponding to N_A is a single valued neutrosophic subgroupoid of G.*

Proof. Let A be a subgroupoid of G and let $x, y \in G$. We consider the two cases for $x \cdot y$: Case $x \cdot y \in A$ and Case $x \cdot y \notin A$.
Case $x \cdot y \in A$**.** Having $N_A(x \cdot y) = (1,1,0)$ implies that $T_A(x \cdot y) = 1 \geq T_A(x) \wedge T_A(y)$, $I_A(x \cdot y) = 1 \geq I_A(x) \wedge I_A(y)$, and $F_A(x \cdot y) = 0 \leq F_A(x) \vee F_A(y)$.
Case $x \cdot y \notin A$**.** Having $x \cdot y \notin G$ implies that either $x \notin G$ or $y \notin G$. Without loss of generality, we suppose that $x \notin G$. Then $N_A(x \cdot y) = N_A(x) = (0,0,1)$ and hence, $T_A(x \cdot y) = 0 \geq T_A(x) \wedge T_A(y)$, $I_A(x \cdot y) = 0 \geq I_A(x) \wedge I_A(y)$, and $F_A(x \cdot y) = 1 \leq F_A(x) \wedge F_A(y)$. Let $x \leq y$. We consider the two cases for x: Case $x \in A$ and Case $x \notin A$.
Case $x \in A$**.** Having $y \in A$ implies that $N_A(y) = (1,1,0)$ and hence $1 = T_A(y) \geq T_A(x)$, $1 = I_A(y) \geq I_A(x)$, and $0 = F_A(y) \leq F_A(x)$.
Case $x \notin A$**.** Having $x \notin A$ implies that $N_A(x) = (0,0,1)$ and hence $T_A(y) \geq T_A(x) = 0$, $I_A(y) \geq I_A(x) = 0$, and $F_A(y) \leq F_A(x) = 1$.

Conversely, let the SVNS corresponding to N_A be a single valued neutrosophic subgroupoid of G and $x, y \in A$. Then $N_A(x) = N_A(y) = (1,1,0)$. Since $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$, $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$, and $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$, it follows that $N_A(x \cdot y) = (1,1,0)$ and hence, $x \cdot y \in A$. Let $x \in A$ and $y \leq x$. Then $T_A(y) \geq T_A(x) = 1$, $I_A(y) \geq I_A(x) = 1$, and $F_A(y) \leq F_A(x) = 0$. Thus, $N_A(y) = (1,1,0)$ and hence, $y \in A$. □

Lemma 4.12. *Let $(G, \cdot, \leq)$ be an ordered groupoid and $A \subseteq G$. Then A is a left((right) ideal of G if and only if the single valued neutrosophic set corresponding to N_A is a single valued neutrosophic left (right) ideal of G.*

Proof. The proof is similar to that of Lemma 4.11. □

Lemma 4.13. *Let $(G, \cdot, \leq)$ be an ordered groupoid and $A \subseteq G$. Then A is an ideal of G if and only if the single valued neutrosophic set corresponding to N_A is a single valued neutrosophic ideal of G.*

Proof. The proof from Lemma 4.12 and having an ideal of an ordered groupoid is a left ideal and right ideal of it. □

Example 4.14. Let $(G_1, \cdot_1, \leq_1)$ be the ordered groupoid defined in Example 3.8. Let A, B be the SVNS on G defined by N_A, N_B respectively as follows.

$$N_A(a) = (0.9, 0.8, 0.1), N_A(b) = N_A(c) = (0.7, 0.6, 0.2);$$

$$N_B(a) = (0.9, 0.8, 0.1), N_B(b) = (0.8, 0.5, 0.4), N_B(c) = (0.7, 0.6, 0.2).$$

Then A is a single valued neutrosophic ideal of G_1 and B is a single valued neutrosophic right ideal of G_1. Moreover, B is not a single valued neutrosophic left ideal of G_1 as $T_B(bc) = T_B(c) \ngeq T_B(b)$.

Example 4.15. Let $(G_1, \star, \leq_1)$ be the ordered groupoid defined in Example 3.9 and B be the SVNS on G defined by N_B as follows.

$$N_B(a) = (0.9, 0.8, 0.1), N_B(b) = (0.8, 0.5, 0.4), N_B(c) = (0.7, 0.6, 0.2).$$

Then B is a single valued neutrosophic subgroupoid of G_1 that is neither a single valued neutrosophic left ideal of G_1 nor a single valued neutrosophic right ideal of G_1.

Lemma 4.16. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic subgroupoid of G. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic subgroupoid of G.*

Proof. Let $x, y \in G$. Then $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(x) \wedge T_{A_\alpha}(y)$, $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(x) \wedge I_{A_\alpha}(y)$, and $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(x) \vee F_{A_\alpha}(y)$ for all α. The latter implies that

$$T_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha T_{A_\alpha}(x \cdot y) \geq \inf_\alpha \{T_{A_\alpha}(x) \wedge T_{A_\alpha}(y)\} = \inf_\alpha T_{A_\alpha}(x) \wedge \inf_\alpha T_{A_\alpha}(y) = T_{\bigcap_\alpha A_\alpha}(x) \wedge T_{\bigcap_\alpha A_\alpha}(y);$$

$$I_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha I_{A_\alpha}(x \cdot y) \geq \inf_\alpha \{I_{A_\alpha}(x) \wedge I_{A_\alpha}(y)\} = \inf_\alpha I_{A_\alpha}(x) \wedge \inf_\alpha I_{A_\alpha}(y) = I_{\bigcap_\alpha A_\alpha}(x) \wedge I_{\bigcap_\alpha A_\alpha}(y);$$

$$F_{\bigcap_\alpha A_\alpha}(x \cdot y) = \sup_\alpha F_{A_\alpha}(x \cdot y) \leq \sup_\alpha \{F_{A_\alpha}(x) \wedge F_{A_\alpha}(y)\} = \sup_\alpha F_{A_\alpha}(x) \vee \sup_\alpha F_{A_\alpha}(y) = F_{\bigcap_\alpha A_\alpha}(x) \vee F_{\bigcap_\alpha A_\alpha}(y).$$

Let $y \leq x$. Then $T_{A_\alpha}(y) \geq T_{A_\alpha}(x)$, $I_{A_\alpha}(y) \geq I_{A_\alpha}(x)$, and $F_{A_\alpha}(y) \leq F_{A_\alpha}(x)$ for all α. One can easily see that $T_{\bigcap_\alpha A_\alpha}(y) \geq T_{\bigcap_\alpha A_\alpha}(x)$, $I_{\bigcap_\alpha A_\alpha}(y) \geq I_{\bigcap_\alpha A_\alpha}(x)$, and $F_{\bigcap_\alpha A_\alpha}(y) \leq F_{\bigcap_\alpha A_\alpha}(x)$. Therefore, $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic subgroupoid of G. □

Remark 4.17. Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic subgroupoid of G. Then $\bigcup_\alpha A_\alpha$ may not be a single valued neutrosophic subgroupoid of G.

We illustrate Remark 4.17 by the following example.

Example 4.18. Let $(\mathbb{N}, +, \leq)$ be the ordered groupoid of natural numbers under standard addition and trivial order. Define the SVNS A, B on $\mathbb{N}$ as follows.

$$N_A(x) = \begin{cases} (1,1,0) & \text{if } x \text{ is a multiple of 2;} \\ (0,0,1) & \text{otherwise.} \end{cases}$$

$$N_B(x) = \begin{cases} (1,1,0) & \text{if } x \text{ is a multiple of 3;} \\ (0,0,1) & \text{otherwise.} \end{cases}$$

It is clear that A and B are single valued neutrosophic subgroupoids of $\mathbb{N}$. But $A \cup B$ is not a single valued neutrosophic subgroupoid of $\mathbb{N}$ as $N_{A\cup B}(2+3) = N_{A\cup B}(5) = (0,0,1)$ so $T_{A\cup B}(2+3) = T_{A\cup B}(5) = 0 \ngeq 1 = T_{A\cup B}(2) \wedge T_{A\cup B}(3)$.

Lemma 4.19. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic left (right) ideal of G. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic left (right) ideal of G.*

Proof. The proof is similar to that of Lemma 4.16. □

Lemma 4.20. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic ideal of G. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic ideal of G.*

Proof. The proof from Lemma 4.19 and having an ideal of an ordered groupoid is a left ideal and right ideal of it. □

Lemma 4.21. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic ideal of G. Then $\bigcup_\alpha A_\alpha$ is a single valued neutrosophic ideal of G.*

Proof. Let $x, y \in G$. Having A_α a single valued neutrosophic right ideal of G implies that $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(x)$, $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(x)$, and $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(x)$ for all α. The latter implies that

$$T_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha T_{A_\alpha}(x \cdot y) \geq \sup_\alpha T_{A_\alpha}(x) = \sup_\alpha T_{A_\alpha}(x) = T_{\bigcup_\alpha A_\alpha}(x);$$

$$I_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha I_{A_\alpha}(x \cdot y) \geq \sup_\alpha I_{A_\alpha}(x) = \sup_\alpha I_{A_\alpha}(x) = I_{\bigcup_\alpha A_\alpha}(x);$$

$$F_{\bigcup_\alpha A_\alpha}(x \cdot y) = \inf_\alpha F_{A_\alpha}(x \cdot y) \leq \inf_\alpha F_{A_\alpha}(x) = \inf_\alpha F_{A_\alpha}(x) = F_{\bigcup_\alpha A_\alpha}(x).$$

Similarly, having A_α a single valued neutrosophic left ideal of G implies that $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(y)$, $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(y)$, and $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(y)$ for all α. The latter implies that

$$T_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha T_{A_\alpha}(x \cdot y) \geq \sup_\alpha T_{A_\alpha}(y) = \sup_\alpha T_{A_\alpha}(y) = T_{\bigcup_\alpha A_\alpha}(y);$$

$$I_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha I_{A_\alpha}(x \cdot y) \geq \sup_\alpha I_{A_\alpha}(y) = \sup_\alpha I_{A_\alpha}(y) = I_{\bigcup_\alpha A_\alpha}(y);$$

$$F_{\bigcup_\alpha A_\alpha}(x \cdot y) = \inf_\alpha F_{A_\alpha}(x \cdot y) \leq \inf_\alpha F_{A_\alpha}(y) = \inf_\alpha F_{A_\alpha}(y) = F_{\bigcup_\alpha A_\alpha}(y).$$

Let $y \leq x$. Then $T_{A_\alpha}(y) \geq T_{A_\alpha}(x)$, $I_{A_\alpha}(y) \geq I_{A_\alpha}(x)$, and $F_{A_\alpha}(y) \leq F_{A_\alpha}(x)$ for all α. One can easily see that $T_{\bigcup_\alpha A_\alpha}(y) \geq T_{\bigcup_\alpha A_\alpha}(x)$, $I_{\bigcup_\alpha A_\alpha}(y) \geq I_{\bigcup_\alpha A_\alpha}(x)$, and $F_{\bigcup_\alpha A_\alpha}(y) \leq F_{\bigcup_\alpha A_\alpha}(x)$. Therefore, $\bigcup_\alpha A_\alpha$ is a single valued neutrosophic ideal of G. □

Theorem 4.22. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over X. Then A is a single valued neutrosophic subgroupoid of G if and only if $L_{(\alpha,\beta,\gamma)}$ is either the empty set or a subgroupoid of G for all $0 \leq \alpha, \beta, \gamma \leq 1$.*

Proof. Let A be a single valued neutrosophic subgroupoid of G and $x, y \in L_{(\alpha,\beta,\gamma)} \neq \emptyset$. Then $T_A(x), T_A(y) \geq \alpha$, $I_A(x), I_A(y) \geq \beta$, and $F_A(x), F_A(y) \leq \gamma$. Since A is a single valued neutrosophic subgroupoid of G, it follows that $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y) \geq \alpha$, $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y) \geq \beta$, and $F_A(x \cdot y) \leq F_A(x) \vee F_A(y) \leq \gamma$. Thus, $x \cdot y \in L_{(\alpha,\beta,\gamma)}$. Let $y \leq x$ and $x \in L_{(\alpha,\beta,\gamma)}$. Then $T_A(y) \geq T_A(x) \geq \alpha$, $I_A(y) \geq I_A(x) \geq \beta$, and $F_A(y) \leq F_A(x) \leq \gamma$. Thus, $y \in L_{(\alpha,\beta,\gamma)}$ and hence, $L_{(\alpha,\beta,\gamma)}$ is a subgroupoid of G.

Conversely, let $L_{(\alpha,\beta,\gamma)} \neq \emptyset$ be a subgroupoid of G for all $0 \leq \alpha, \beta, \gamma \leq 1$ and $x, y, z \in G$ with $N_A(x) = (\alpha_1, \beta_1, \gamma_1)$ and $N_A(y) = (\alpha_2, \beta_2, \gamma_2)$. By setting $(\alpha, \beta, \gamma) = (\alpha_1 \wedge \alpha_2, \beta_1 \wedge \beta_2, \gamma_1 \vee \gamma_2)$, we get that $x, y \in L_{(\alpha,\beta,\gamma)}$. Having $L_{(\alpha,\beta,\gamma)} \neq \emptyset$ a subgroupoid of G implies that $x \cdot y \in L_{(\alpha,\beta,\gamma)}$. The latter implies that $T_A(x \cdot y) \geq \alpha = T_A(x) \wedge T_A(y)$, $I_A(x \cdot y) \geq \beta = I_A(x) \wedge I_A(y)$, and $F_A(x \cdot y) \leq \gamma = F_A(x) \vee F_A(y)$. Let $y \leq x$ with $N_A(x) = (\alpha, \beta, \gamma)$. Then $y \in L_{(\alpha,\beta,\gamma)}$ and hence, $T_A(y) \geq \alpha = T_A(x)$, $I_A(y) \geq \beta = I_A(x)$, and $F_A(y) \leq \gamma = F_A(x)$. Thus, A is a single valued neutrosophic subgroupoid of G. □

Theorem 4.23. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over X. Then A is a single valued neutrosophic left (right) ideal of G if and only if $L_{(\alpha,\beta,\gamma)}$ is either the empty set or a left (right) ideal of G for all $0 \leq \alpha, \beta, \gamma \leq 1$.*

Proof. The proof is similar to that of Theorem 4.22. □

Theorem 4.24. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over X. Then A is a single valued neutrosophic ideal of G if and only if $L_{(\alpha,\beta,\gamma)}$ is either the empty set or an ideal of G for all $0 \leq \alpha, \beta, \gamma \leq 1$.*

Proof. The proof from Theorem 4.23 and having an ideal of an ordered groupoid is a left ideal and right ideal of it. □

Corollary 4.25. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over X. If A is a single valued neutrosophic left (right) ideal of G then A is a single valued neutrosophic subgroupoid of G.*

Proof. The proof follows from Theorem 4.22 and Theorem 4.23. □

Remark 4.26. The converse of Corollary 4.25 may not hold. See Example 4.15.

4.2 Single valued neutrosophic filters of groupoids

Definition 4.27. Let $(G, \cdot, \leq)$ be an ordered groupoid and A a SVNS over G. Then A is single valued neutrosophic filter of G if for all $x, y \in G$, the following conditions hold:

1. $T_A(x \cdot y) = T_A(x) \wedge T_A(y)$;
2. $I_A(x \cdot y) = I_A(x) \wedge I_A(y)$;
3. $F_A(x \cdot y) = F_A(x) \vee F_A(y)$;
4. If $x \leq y$ then $T_A(x) \leq T_A(y)$, $I_A(x) \leq I_A(y)$, and $F_A(x) \geq F_A(y)$.

Example 4.28. Let $(G_4, \cdot_4, \leq_4)$ be the ordered groupoid defined in Example 3.12. Then $A = \{\frac{1}{(0.1,0.6,1)}, \frac{2}{(0.1,0.6,1)}, \frac{3}{(0.9,0.8,0)}\}$ is a single valued neutrosophic filter of G_4.

Remark 4.29. Let $(G, \cdot, \leq)$ be an ordered groupoid and $\alpha, \beta, \gamma \in [0, 1]$. Then $A = \{\frac{x}{(\alpha,\beta,\gamma)} : x \in G\}$ is single valued neutrosophic filter of G. Moreover, it is called the **trivial single valued neutrosophic filter**.

Lemma 4.30. *Let $(G, \cdot, \leq)$ be an ordered groupoid and $A \subseteq G$. Then A is a filter of G if and only if the single valued neutrosophic set corresponding to N_A is a single valued neutrosophic filter of G.*

Proof. Let A be a filter of G and $x, y \in G$. We consider two cases for $x \cdot y$: $x \cdot y \in A$ and $x \cdot y \notin A$.
Case $x \cdot y \in A$. Having A a filter of G implies that $x, y \in A$ and hence, $N_A(x) = N_A(y) = N_A(x \cdot y) = (1, 1, 0)$. Thus, Conditions 1., 2., and 3. of Definition 4.27 are satisfied.
Case $x \cdot y \notin A$. Having A a filter of G implies that $x \notin A$ or $y \notin A$. Having $N_A(x \cdot y) = (0, 0, 1)$ and either $N_A(x) = (0, 0, 1)$ or $N_A(y) = (0, 0, 1)$ implies that Conditions 1., 2., and 3. of Definition 4.27 are satisfied.
Let $x \leq y$. If $x \in A$ then $y \in A$ and hence, $N_A(x) = N_A(y)$ which implies the Condition 4. in Definition 4.27 is satisfied. If $x \notin A$ then $N_A(x) = (0, 0, 1)$. Having $N_A(y)$ is either equal to $(0, 0, 1)$ (if $y \notin A$) or

$(1, 1, 0)$ (if $y \in A$) implies the Condition 4. in Definition 4.27 is satisfied. Consequently, the single valued neutrosophic set corresponding to N_A is a single valued neutrosophic filter of G.

Conversely, let the single valued neutrosophic set corresponding to N_A be a single valued neutrosophic filter of G. Let $x, y \in A$. It is easy to see that $x \cdot y \in A$ as $N_A(x \cdot y) = N_A(x) = N_A(y) = (1, 1, 0)$. Moreover, $x \cdot y \in A$. Then $N_A(x \cdot y) = (1, 1, 0)$. Conditions 1., 2., and 3. of Defintion 4.27 imply that $N_A(x) = N_A(y) = (1, 1, 0)$ and hence $x, y \in A$. Finally, let $x \leq y$ and $x \in A$. Having $N_A(x) = (1, 1, 0)$ and Condition 4. of Definition 4.27 imply that $N_A(y) = (1, 1, 0)$. Thus, $y \in A$. □

Lemma 4.31. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic filter of G. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic filter of G.*

Proof. The proof can be done in a similar way to that of Lemma 4.16. □

Remark 4.32. Let $(G, \cdot, \leq)$ be an ordered groupoid and A_α a single valued neutrosophic filter of G. Then $\bigcup_\alpha A_\alpha$ may not be a single valued neutrosophic filter of G.

We illustrate Remark 4.17 by the following example.

Example 4.33. Let $(G, \cdot)$ be the groupoid defined by Table 6.

Table 6: The groupoid $(G, \cdot)$

$\cdot$	1	2	3
1	1	2	2
2	2	2	2
3	2	2	3

By setting $\leq = \{(1, 1), (2, 2), (3, 3)\}$, we get that $(G, \cdot, \leq)$ is an ordered groupoid. By defining the SVNS A, B on G as follows.

$$N_A(1) = N_A(2) = (0.6, 0.8, 0.1), N_A(3) = (1, 0.8, 0.1);$$

$$N_B(1) = (1, 0.6, 0.4), N_B(2) = N_B(3) = (0.9, 0.6, 0.4),$$

we get that A, B are single valued neutrosophic filters of G. Since $T_{A\cup B}(1 \cdot 3) = T_{A\cup B}(2) = 0.9 \neq 1 = T_{A\cup B}(1) \wedge T_{A\cup B}(3)$, it follows that $A \cup B$ is not a single valued neutrosophic filter of G.

Lemma 4.34. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a single valued neutrosophic set over G. If A is is a single valued neutrosophic filter of G then $L_{(\alpha,\beta,\gamma)}$ is either the empty set or a filter of G for all $0 \leq \alpha, \beta, \gamma \leq 1$.*

Proof. Let A be a single valued neutrosophic filter of G and $x, y \in G$ such that $x \cdot y \in L_{(\alpha,\beta,\gamma)} \neq \emptyset$. Then $T_A(x \cdot y) \geq \alpha$, $I_A(x \cdot y) \geq \beta$, and $F_A(x \cdot y) \leq \gamma$. Since A is a single valued neutrosophic filter of G, it follows that $T_A(x \cdot y) = T_A(x) \wedge T_A(y) \geq \alpha$, $I_A(x \cdot y) = I_A(x) \wedge I_A(y) \geq \beta$, and $F_A(x \cdot y) = F_A(x) \vee F_A(y) \leq \gamma$. The latter implies that $T_A(x), T_A(y) \geq \alpha$, $I_A(x), I_A(y) \geq \beta$, and $F_A(x), F_A(y) \leq \gamma$ and hence, $x, y \in L_{(\alpha,\beta,\gamma)}$. Let $y \leq x$ and $y \in L_{(\alpha,\beta,\gamma)}$. Then $\alpha \leq T_A(y) \leq T_A(x)$, $\beta \leq I_A(y) \leq I_A(x)$, and $\gamma \geq F_A(y) \geq F_A(x)$. Thus, $x \in L_{(\alpha,\beta,\gamma)}$ and hence, $L_{(\alpha,\beta,\gamma)}$ is a filter of G. Therefore, $L_{(\alpha,\beta,\gamma)} \neq \emptyset$ is filter of G. □

Lemma 4.35. *Let $(G, \cdot, \leq)$ be an ordered groupoid and A a single valued neutrosophic set over G. If $\overline{L}_{(\alpha,\beta,\gamma)} = \{x \in G : N_A(x) = (\alpha, \beta, \gamma)\}$ is a filter of G for all $0 \leq \alpha, \beta, \gamma \leq 1$ then A is a single valued neutrosophic filter of G.*

Proof. Let $x, y \in G$ with $N_A(x \cdot y) = (\alpha, \beta, \gamma)$. Then $x \cdot y \in \overline{L}_{(\alpha,\beta,\gamma)}$. Having $\overline{L}_{(\alpha,\beta,\gamma)}$ a filter of G implies that $x, y \in \overline{L}_{(\alpha,\beta,\gamma)}$. The latter implies that $N_A(x) = N_A(y) = (\alpha, \beta, \gamma)$ and hence, $T_A(x) \wedge T_A(y) = \alpha = T_A(x \cdot y)$, $I_A(x) \wedge I_A(y) = \beta = I_A(x \cdot y)$, and $F_A(x) \vee F_A(y) = \gamma = F_A(x \cdot y)$. Let $x \leq y$ with $N_A(x) = (\alpha, \beta, \gamma)$. Having $x \in \overline{L}_{(\alpha,\beta,\gamma)}$ and $\overline{L}_{(\alpha,\beta,\gamma)}$ a filter of$G$ implies that $y \in \overline{L}_{(\alpha,\beta,\gamma)}$. The latter implies that $T_A(x) = \alpha \leq T_A(y)$, $I_A(x) = \beta \leq I_A(y)$, and $F_A(x) = \gamma \geq F_A(y)$. Therefore, A is a single valued neutrosophic filter of G. □

5 Some remarks on SVNS in ordered groups

In this section, we apply the definition of SVNS in ordered groupoids to ordered groups, present some remarks and results. The results of this section can be considered as a base for new results on SVNS in ordered groups.

Definition 5.1. Let $(G, \cdot, \leq)$ be an ordered group and A a SVNS over G. Then A is single valued neutrosophic subgroup of G if for all $x, y \in G$, the following conditions hold:

- $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$;
- $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$;
- $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$;
- $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$, $F_A(x^{-1}) \leq F_A(x)$;
- If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$, and $F_A(x) \leq F_A(y)$.

Proposition 5.2. *Let $(G, \cdot, \leq)$ be an ordered group with identity "e" and A a single valued neutrosophic subgroup of G. Then the following statements hold.*

1. $N_A(x) = N_A(x^{-1})$ *for all* $x \in G$.
2. $T_A(e) \geq T_A(x)$, $I_A(e) \geq I_A(x)$, *and* $F_A(e) \leq F_A(x)$ *for all* $x \in G$.

Proof. The proof is straightforward. □

Proposition 5.3. *Let $(G, \cdot, \leq)$ be an ordered group and A a SVNS over G. Then A is a single valued neutrosophic left/right ideal of G if and only if A is the trivial single valued neutrosophic ideal of G.*

Proof. The proof follows from Proposition 4.8. □

Proposition 5.4. *Let $(G, \cdot, \leq)$ be an ordered group with identity "e" and A a SVNS over G. If e and x are comparable for all $x \in G$ then A is a single valued neutrosophic subgroup of G if and only if A is the trivial single valued neutrosophic subgroup of G.*

Proof. If A is the trivial single valued neutrosophic subgroup of G then we are done.

Let A be a single valued neutrosophic subgroup of G. Since e, x are comparable, it follows that $x \leq e$ or $e \leq x$. If $x \leq e$ then $T_A(x) \geq T_A(e)$, $I_A(x) \geq I_A(e)$, and $F_A(x) \leq F_A(e)$. Proposition 5.2, 2. implies that A is the trivial single valued neutrosophic subgroup of G. If $e \leq x$ then $x^{-1} \leq e$. The latter implies that $T_A(e) \geq T_A(x)$, $I_A(e) \geq I_A(x)$, and $F_A(e) \leq F_A(x)$ and $T_A(x) = T_A(x^{-1}) \geq T_A((e)$, $I_A(x) = I_A(x^{-1}) \geq I_A(e)$, and $F_A(x) = F_A(x^{-1}) \leq F_A(e)$. Thus, A is the trivial single valued neutrosophic subgroup of G. □

Proposition 5.5. *Let $(G, \cdot, \leq)$ be an ordered cyclic group with identity "e" and generator a, and A a SVNS over G. If $e \leq a$ then A is a single valued neutrosophic subgroup of G if and only if A is the trivial single valued neutrosophic subgroup of G.*

Proof. If A is the trivial single valued neutrosophic subgroup of G then we are done.

Let A be a single valued neutrosophic subgroup of G. Since $e \leq a$, it follows that $a^{-1} \leq e$ and hence $T_A(e) \leq T_A(a^{-1}) = T_A(a)$, $I_A(e) \leq I_A(a^{-1}) = I_A(a)$, and $F_A(e) \geq F_A(a^{-1}) = F_A(a)$. The latter and Proposition 5.2, 2. imply that $T_A(e) = T_A(a)$, $I_A(e) = I_A(a)$, and $F_A(e) = F_A(a)$. Having $e \leq a$ implies that $e \leq a^k$ for all $k = 1, 2, \ldots$ and hence, $a^{-k} \leq e$. The latter implies that $T_A(e) = T_A(a^k)$, $I_A(e) = I_A(a^k)$, and $F_A(e) = F_A(a^k)$ for all $k \in \mathbb{Z}$. Therefore, A is the trivial single valued neutrosophic subgroup of G. □

Proposition 5.6. *Let $(G, \cdot, \leq)$ be a finite ordered group with identity "e" and A a SVNS over G. If $e \leq a$ for some $a \in G - \{e\}$ then A is a single valued neutrosophic subgroup of G if and only if A is the trivial single valued neutrosophic subgroup of G.*

Proof. Let $|G| = n$. Then $a^n = e$. Since $e \leq a$, it follows that $a \leq a^k$ for all $k = 1, 2, \ldots$. By setting $k = n$, we get that $a \leq e$. Having $e \leq a$ and $a \leq e$ implies that $a = e$ which is a contradiction. □

6 Conclusion and discussion

This paper contributed to the study of neutrosophic algebraic structures by introducing, for the first time, SVNS in ordered algebraic structures. Several new concepts were defined and studied like single valued neutrosophic subgroupoid, single valued neutrosophic ideal, and single valued neutrosophic filter of groupoids and many interesting examples were presented. Finally an application of this study to ordered groups was presented.

For future work, we will work on SVNS in ordered groups and elaborate more properties about it. Also we will work on SVNS in ordered semigroups.

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