Neutrosophic N-structure's Characteristic Function and its Properties

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Abstract

The notion of neutrosophic N-structure's characteristic function is introduced and derived from some results. The notion of $(\in,\in \vee q_\eta)$ neutrosophic N-subsemigroup is introduced and discusses the results on the intersection and union of $(\in,\in \vee q_\eta)$ neutrosophic N-subsemigroup. The notion of the η -intersection, η -union, and η -composition on $(\in,\in \vee q_\eta)$ neutrosophic N-subsemigroup is introduced and discusses some results.

Keywords: Characteristic function of neutrosophic N-structure; $(\in,\in \vee q_\eta)$ neutrosophic N-subsemigroup; η -intersection, η -union and η -composition on $(\in,\in \vee q_\eta)$ neutrosophic N-subsemigroup.

1. Introduction

Lofti Zadeh introduced the concept of fuzzy set, in which degree of truth membership as specified in the year 1965, the concept of fuzzy has been practised by several researchers to generalize many basic concepts of algebra, graph theory, coding theory and automata theory. Now a day the concept of fuzzy is widely used in artificial intelligence and neural networks also. Rosenfeld bestowed the concept of fuzziness to groups. Kuroki established the concept of fuzzy semigroups and ideal theory in fuzzy semigroups. Florentin Smarandache [1] developed the concept of Neutrosophy and then he extended it to Neutrosophic Probability, Set, and Logic.

In real-life problems, negative membership values are also required in many situations, to tackle that negative membership function was defined in N-fuzzy ideals of ordered semigroups by A.Khan et al. By combining the positive and negative membership values vague set was introduced by Gau and Buehrer. Neutrosophic set was introduced by Smarandache in 1995 and develop various concepts on neutrosophic sets. Neutro means the idea of neutral. Next, the concept of negative structures was applied to neutrosophic set by Madad Khan et al [6].

The neutrosophic N-structures defined on crisp set S to $[-1,0]$ in which negative membership functions namely negative truth T , negative false F and negative indeterminacy member function I such that $-3 \le T(x) + F(x) + I(x) \le 0$, $\forall x \in S$. Further neutrosophic N-structure, neutrosophic N-subsemigroup, union, intersection and composition of neutrosophic N-structures were defined and then characterizations of neutrosophic N-subsemigroup was given by G.Muhiuddin et al [7]. Balasubramanian Elavarasan et al [5] developed Neutrosophic N-ideals of semigroups.

S.K.Bhakat, P.Das[4] introduced the concept of $(\in,\in\vee q)$ -fuzzy subgroup in 1996. Muhammad Shabir et al[2] generalising the notions of $(\in,\in\vee q)$ -fuzzy subgroup and extends to $(\in,\in\vee q_k)$ -fuzzy subsemigroup. Further Faiz Muhammad Khan et al [3] extends it to Ordered Semigroups based on $(\in,\in\vee q_k^{\delta})$ -Fuzzy ideals.

We're in this paper introduced neutrosophic N-structure's characteristic function and some results on this characteristic function. Defined $(\in,\in\vee q_\eta)$ neutrosophic N-subsemigroup, some results on neutrosophic N-subsemigroup, next establish the relation between neutrosophic N-structure, $(\in,\in)$ and (q_η,q_η) -neutrosophic subsemigroup. Then define η -intersection, η -union and η -composition on $(\in,\in\vee q_\eta)$ neutrosophic N-subsemigroup and results on these operations.

2. Neutrosophic N-structure's characteristic function

Definition 2.1: [6] (Definition of Neutrosophic N-structure)

A neutrosophic N-structure over S is a structure defined as $S_X=(S;T_X,I_X,F_X)$ = $\{q;T_X(q),I_X(q),F_X(q))/q\in S\}$ where T_X,I_X,F_X are negative membership functions from S to $[-1,0]$ are the negative truth, negative indeterminacy and the negative false membership functions respectively.

Definition 2.2: [6] (Definition of Neutrosophic N-subsemigroup)

Neutrosophic N-structure $S_X=(S;T_X,I_X,F_X)$ over semigroup S is said to be a Neutrosophic N-subsemigroup if (i) $T_X(qz)\le T_X(q)\vee T_X(z)$,

(ii) $I_X(qz)\ge I_X(q)\wedge I_X(z)$ and

(iii) $F_X(qz)\le F_X(q)\vee F_X(z)$, $\forall q,z\in S$

Definition 2.3: [7] (Definition of Neutrosophic N-structures subset)

Let $S_X=(S;T_X,I_X,F_X)$ and $S_Y=(S;T_Y,I_Y,F_Y)$ be two neutrosophic N-structures over S , S_X is subset of S_Y if (i) $T_X\supseteq T_Y$, (ii) $I_X\subseteq I_Y$ and (iii) $F_X\supseteq F_Y$. It is represented by $S_X\subseteq S_Y$.

Definition 2.4: [6] (Definition of Neutrosophic N-structures intersection)

Let $S_X=(S;T_X,I_X,F_X)$ and $S_Y=(S;T_Y,I_Y,F_Y)$ be two neutrosophic N-structures over S . The intersection of S_X and S_Y is defined as $S_X\cap S_Y=(S;T_X\cap T_Y,I_X\cap I_Y,F_X\cap F_Y)$

where (i) $(T_X\cap T_Y)(q)=T_X(q)\vee T_Y(q)$,

(ii) $(I_X\cap I_Y)(q)=I_X(q)\wedge I_Y(q)$,

(iii) $(F_X\cap F_Y)(q)=F_X(q)\vee F_Y(q),\forall q\in S$

Definition 2.5: [6] (Definition of Neutrosophic N-structures union)

Let $S_X = (S; T_X, I_X, F_X)$ and $S_Y = (S; T_Y, I_Y, F_Y)$ be two neutrosophic N-structures over S. S_X union S_Y is defined as $S_X \cup S_Y = (S; T_X \cup T_Y, I_X \cup I_Y, F_X \cup F_Y)$ where

(i) $(T_X \cup T_Y)(q) = T_X(q) \wedge T_Y(q)$,

(ii) $(I_X \cup I_Y)(q) = I_X(q) \vee I_Y(q)$,

(iii) $(F_X \cup F_Y)(q) = F_X(q) \wedge F_Y(q), \forall q \in S$

Definition 2.6: [6] (Definition of Neutrosophic N-structures composition)

Let $S_X = (S; T_X, I_X, F_X)$, $S_Y = (S; T_Y, I_Y, F_Y)$ be two neutrosophic N-structures over S. The composition $S_X o S_Y$ is defined by $S_X o S_Y = (S; T_X o T_Y, I_X o I_Y, F_X o F_Y)$

where (i) $(T_X o T_Y)(a) = \begin{cases} \underset{a=qz}{\wedge}(T_X(q) \vee T_Y(z)) & if \quad a = qz \quad exists \\ 0 \quad otherwise \end{cases}$,

(ii) $(I_X o I_Y)(a) = \begin{cases} \underset{a=qz}{\vee}(I_X(q) \wedge I_Y(z)) & if \quad a = qz \quad exists \\ 0 \quad otherwise \end{cases}$ and

(iii) $(F_X o F_Y)(a) = \begin{cases} \underset{a=qz}{\wedge}(F_X(q) \vee F_Y(z)) & if \quad a = qz \quad exists \\ 0 \quad otherwise \end{cases}$

Theorem 2.7: ([6])

The intersection of two neutrosophic N-subsemigroups is again a neutrosophic N-subsemigroup.

Theorem 2.8: ([6])

Intersection of the family of neutrosophicN-subsemigroups is also a neutrosophic N-subsemigroup.

Theorem 2.9: ([6])

A neutrosophic N-structure S_X over S is neutrosophic N-subsemigroup of S iff $S_X o S_X \subseteq S_X$

Definition 2.10: [8] (Definition of the characteristic function of Neutrosophic N-structure)

Let X be a non-empty subset of semigroup S and $S_M = (S; T_M, I_M, F_M)$ be neutrosophic N-structure over S. The characteristic function χX_M of X defined by $\chi X_M = (X; \chi T_M, \chi I_M, \chi F_M)$ where

(i) $\chi T_M(q) = \begin{cases} -1 & if \quad q \in X \\ 0 & if \quad q \notin X \end{cases}$,

(ii) $\chi F_M(q) = \begin{cases} 0 & if \quad q \in X \\ -1 & if \quad q \notin X \end{cases}$ and

(iii) $\chi I_M(q) = \chi T_M(q) \vee \chi F_M(q), \forall q \in X$

Theorem 2.11:

A non-empty subset X of a semigroup S is subsemigroup iff the characteristic function χX_M is neutrosophic N-subsemigroup of S.

Proof: Assume X is subsemigroup of S.

Let $q,z \in X \Rightarrow qz \in X$

Consider $\chi T_M(qz) = -1 = -1 \vee -1 = \chi T_M(q) \vee \chi T_M(z)$

$\Rightarrow \chi T_M(qz) \leq \chi T_M(q) \vee \chi T_M(z), \forall q,z \in X$

Consider $\chi F_M(qz) = 0 = 0 \vee 0 = \chi F_M(q) \vee \chi F_M(z)$

$\Rightarrow \chi F_M(qz) \leq \chi F_M(q) \vee \chi F_M(z), \forall q,z \in X$

Consider $\chi I_M(qz) = \chi T_M(qz) \vee \chi F_M(qz) = -1 \vee 0 = 0$

and $\chi I_M(q) \wedge \chi I_M(z) = (\chi T_M(q) \vee \chi F_M(q)) \wedge (\chi T_M(z) \vee \chi F_M(z)) = (-1 \vee 0) \wedge (-1 \vee 0) = 0$

$\Rightarrow \chi I_M(qz) \geq \chi I_M(q) \wedge \chi I_M(z), \forall q,z \in X$

therefore χX_M is neutrosophic N-subsemigroup of S.

Conversely, suppose that χX_M is neutrosophic N-subsemigroup of S

If possible suppose $qz \notin X$ whenever $q,z \in X$

but we have $\chi T_M(qz) \leq \chi T_M(q) \vee \chi T_M(z), \forall q,z \in X$

$\Rightarrow 0 \leq -1 \vee -1 = -1$ which is contradiction.

$\Rightarrow qz \in X \;\; \forall q,z \in X$

$\Rightarrow X$ is subsemigroup of S.

Theorem 2.12:

Let X , Y be two non-empty subsets of S. If $X_M = (X; T_M, I_M, F_M)$ and $Y_N = (Y; T_N, I_N, F_N)$ are two neutrosophic N-structures over S then

(a) $\chi X_M \cap \chi Y_N \leq \chi(X_M \cap Y_N)$

(b) $\chi X_M \cup \chi Y_N \geq \chi(X_M \cup Y_N)$

(c) $\chi X_M o \chi Y_N \leq \chi(X_M Y_N)$

Proof: (a) Let $q \in X \cap Y \Rightarrow q \in X$ and $q \in Y$

Consider $(\chi T_M \cap \chi T_N)(q) = \chi T_M(q) \vee \chi T_N(q) = -1 \vee -1 = -1 = \chi T_{M \cap N}(q)$

Suppose $q \notin X \cap Y \Rightarrow q \notin X$ or $q \notin Y$

Consider $(\chi T_M \cap \chi T_N)(q) = \chi T_M(q) \vee \chi T_N(q) = 0 = -1 \vee 0 = 0 = \chi T_{M \cap N}(q)$

Therefore $(\chi T_M \cap \chi T_N)(q) = \chi T_{M \cap N}(q)$

Let $q \in X \cap Y \Rightarrow q \in X$ and $q \in Y$

Consider $(\chi F_M \cap \chi F_N)(q) = \chi F_M(q) \vee \chi F_N(q) = 0 \vee 0 = 0 = \chi F_{M \cap N}(q)$

Suppose $q \notin X \cap Y \Rightarrow q \notin X$ or $q \notin Y$

Consider $(\chi F_M \cap \chi F_N)(q) = \chi F_M(q) \vee \chi F_N(q) = 0 \geq -1 = \chi F_{M \cap N}(q)$

Therefore $(\chi F_M \cap \chi F_N)(q) \geq \chi F_{M \cap N}(q)$

Let $q \in X \cap Y \Rightarrow q \in X$ and $q \in Y$

Consider $(\chi I_M \cap \chi I_N)(q) = \chi I_M(q) \wedge \chi I_N(q)$

$$= (\chi T_M(q) \vee \chi F_M(q)) \wedge (\chi T_N(q) \vee \chi F_N(q))$$

$$= (-1 \vee 0) \wedge (-1 \vee 0) = 0 = -1 \vee 0$$

$$= \chi T_{M\cap N}(q) \vee \chi F_{M\cap N}(q) = \chi I_{M\cap N}(q)$$

Suppose $q \notin X \cap Y \Rightarrow q \notin X$ or $q \notin Y$

Consider $(\chi I_M \cap \chi I_N)(q) = \chi I_M(q) \wedge \chi I_N(q)$

$$= (\chi T_M(q) \vee \chi F_M(q)) \wedge (\chi T_N(q) \vee \chi F_N(q))$$

$$= (0 \vee -1) \wedge (0 \vee -1) = 0 = 0 \vee -1$$

$$= \chi T_{M\cap N}(q) \vee \chi F_{M\cap N}(q) = \chi I_{M\cap N}(q)$$

Therefore $(\chi I_M \cap \chi I_N)(q) = \chi I_{M\cap N}(q)$

Thus from T, I, F we get $\chi X_M \cap \chi Y_N \leq \chi(X_M \cap Y_N)$

(b) Let $q \in X \cup Y \Rightarrow q \in X$ or $q \in Y$ or both

Consider $(\chi T_M \cup \chi T_N)(q) = \chi T_M(q) \wedge \chi T_N(q) = -1 \wedge 0 = -1 = \chi T_{M\cup N}(q)$

Suppose $q \notin X \cup Y \Rightarrow q \notin X$ and $q \notin Y$

Consider $(\chi T_M \cup \chi T_N)(q) = \chi T_M(q) \wedge \chi T_N(q) = 0 \wedge 0 = 0 = \chi T_{M\cup N}(q)$

Therefore $(\chi T_M \cup \chi T_N)(q) = \chi T_{M\cup N}(q)$

Let $q \in X \cup Y \Rightarrow q \in X$ or $q \in Y$ or both

Consider $(\chi F_M \cup \chi F_N)(q) = \chi F_M(q) \wedge \chi F_N(q) = -1 \leq 0 = \chi F_{M\cup N}(q)$

Suppose $q \notin X \cup Y \Rightarrow q \notin X$ and $q \notin Y$

Consider $(\chi F_M \cup \chi F_N)(q) = \chi F_M(q) \wedge \chi F_N(q) = -1 \wedge -1 = -1 = \chi F_{M\cup N}(q)$

Therefore $(\chi F_M \cup \chi F_N)(q) \leq \chi F_{M\cup N}(q)$

Let $q \in X \cup Y \Rightarrow q \in X$ or $q \in Y$ or both

Consider $(\chi I_M \cup \chi I_N)(q) = \chi I_M(q) \vee \chi I_N(q)$

$$= (\chi T_M(q) \vee \chi F_M(q)) \vee (\chi T_N(q) \vee \chi F_N(q))$$

$$= (-1 \vee 0) \vee (-1 \vee 0) = 0 = (-1 \vee 0)$$

$$= \chi T_{M\cup N}(q) \vee \chi F_{M\cup N}(q) = \chi I_{M\cup N}(q)$$

Suppose $q \notin X \cup Y \Rightarrow q \notin X$ and $q \notin Y$

Consider $(\chi I_M \cup \chi I_N)(q) = \chi I_M(q) \vee \chi I_N(q)$

$$= (\chi T_M(q) \vee \chi F_M(q)) \vee (\chi T_N(q) \vee \chi F_N(q))$$

$$= (0 \vee -1) \vee (0 \vee -1) = 0 = 0 \vee -1$$

$$= \chi T_{M\cup N}(q) \vee \chi F_{M\cup N}(q) = \chi I_{M\cup N}(q)$$

Therefore $(\chi I_M \cup \chi I_N)(q) = \chi I_{M\cup N}(q)$

Thus from T, I, F we get $\chi X_M \cap \chi Y_N \geq \chi(X_M \cap Y_N)$

(c) Let $b \in S$ and $b \in XY \Rightarrow b = qz$ for some $q \in X$ and $z \in Y$

Consider $(\chi T_M o \chi T_N)(b) = \underset{a=qz}{\wedge}(\chi T_M(q) \vee \chi T_N(z))$

$$= \wedge(-1 \vee -1) = -1 = \chi(T_M T_N)(b)$$

If $b \notin XY$ then we have $b \neq qz, \forall q \in X$ and $z \in Y$

and if $b = rt$ for some $r, t \in S$

then $(\chi T_M o \chi T_N)(b) = \underset{a=rt}{\wedge}(\chi T_M(r) \vee \chi T_N(t))$

$$= \wedge(0 \vee 0) = 0 = \chi(T_M T_N)(b)$$

If $b \neq rt$ for all $r, t \in S$

then $(\chi T_M o \chi T_N)(b) = 0 = \chi(T_M T_N)(b)$

Therefore $(\chi T_M o \chi T_N) = \chi(T_M T_N)$

Let $b \in S$ and $b \in XY \Rightarrow b = qz$ for some $q \in X$ and $z \in Y$

Consider $(\chi F_M o \chi F_N)(b) = \underset{a=qz}{\wedge}(\chi F_M(q) \vee \chi F_N(z))$

$$= \wedge(0 \vee 0) = 0 = \chi(F_M F_N)(b)$$

If $b \notin XY$ then we have $b \neq qz, \forall q \in X$ and $z \in Y$

and if $b = rt$ for some $r, t \in S$

then $(\chi F_M o \chi F_N)(b) = \underset{a=rt}{\wedge}(\chi F_M(r) \vee \chi F_N(t))$

$$= \wedge(-1 \vee -1) = -1 = \chi(F_M F_N)(b)$$

If $b \neq rt$ for all $r, t \in S$

then $(\chi F_M o \chi F_N)(b) = 0 \geq -1 = \chi(F_M F_N)(b)$

Therefore $(\chi F_M o \chi F_N) \geq \chi(F_M F_N)$

Let $b \in S$ and $b \in XY \Rightarrow b = qz$ for some $q \in X$ and $z \in Y$

Consider $(\chi I_M o \chi I_N)(b) = \underset{a=qz}{\vee}(\chi I_M(q) \wedge \chi I_N(z))$

$$= \underset{a=qz}{\vee}\{(\chi T_M(q) \vee \chi F_M(q)) \wedge (\chi T_M(z) \vee \chi F_M(z))\}$$

$$= \vee\{(-1 \vee 0) \wedge (-1 \vee 0)\} = 0 = -1 \vee 0$$

$$= \chi(T_M T_N)(b) \vee \chi(F_M F_N)(b) = \chi(I_M I_N)(b)$$

If $b \notin XY$ then we have $b \neq qz, \forall q \in X$ and $z \in Y$

and if $b = rt$ for some $r, t \in S$

then $(\chi I_M o \chi I_N)(b) = \underset{a=rt}{\vee}(\chi I_M(r) \wedge \chi I_N(t))$

$$= \vee(-1 \wedge -1) = -1 = -1 \vee -1$$

$$= \chi(T_M T_N)(b) \vee \chi(F_M F_N)(b) = \chi(I_M I_N)(b)$$

If $b \neq rt$ for all $r, t \in S$

then $(\chi I_M o \chi I_N)(b) = 0 = 0 \vee -1 = \chi(T_M T_N)(b) \vee \chi(F_M F_N)(b)$

$$= \chi(I_M I_N)(a)$$

Therefore $(\chi I_M o \chi I_N) = \chi(I_M I_N)$

Thus from T, I, F we get $\chi X_M o \chi Y_N \leq \chi(X_M Y_N)$

Notation 2.13: [7]

For a given neutrosophic N-structure $S_X = (S; T_X, I_X, F_X)$ over S, $\alpha, \gamma \in [-1,0)$ $\beta \in (-1,0]$, define

(i) $(T_X)_{\in}^{\alpha} = \{g \in S / T_X(g) \le \alpha\}$,

(ii) $(I_X)_{\in}^{\beta} = \{g \in S / I_X(g) \ge \beta\}$,

(iii) $(F_X)_{\in}^{\gamma} = \{g \in S / F_X(g) \le \gamma\}$,

(iv) $(T_X)_{q_\eta}^{\alpha} = \{g \in S / T_X(g) + \alpha + \eta < -1\}$,

(v) $(I_X)_{q_\eta}^{\beta} = \{g \in S / I_X(g) + \beta + \eta > -1\}$,

(vi) $(F_X)_{q_\eta}^{\gamma} = \{g \in S / F_X(g) + \gamma + \eta < -1\}$,

(vii) $(T_X)_{\in \vee q_\eta}^{\alpha} = \{g \in S / T_X(g) \le \alpha \quad or \quad T_X(g) + \alpha + \eta < -1\}$,

(viii) $(I_X)_{\in \vee q_\eta}^{\beta} = \{g \in S / I_X(g) \ge \beta \quad or \quad I_X(g) + \beta + \eta > -1\}$,

(ix) $(F_X)_{\in \vee q_\eta}^{\gamma} = \{g \in S / F_X(g) \le \alpha \quad or \quad F_X(g) + \gamma + \eta < -1\}$

Definition 2.14: (Definition of $(\in, \in \vee q_\eta)$ -neutrosophic N-subsemigroup)

A neutrosophic N-structure $S_X = (S; T_X, I_X, F_X)$ of semigroup S is called $(\in, \in \vee q_\eta)$ -neutrosophic N-subsemigroup if (i) $g \in (T_X)_{\in}^{\alpha_1}, h \in (T_X)_{\in}^{\alpha_2} \Rightarrow gh \in (T_X)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$,

(ii) $g \in (I_X)_{\in}^{\beta_1}, h \in (I_X)_{\in}^{\beta_2} \Rightarrow gh \in (I_X)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2}$ and

(iii) $g \in (F_X)_{\in}^{\gamma_1}, h \in (F_X)_{\in}^{\gamma_2} \Rightarrow gh \in (F_X)_{\in \vee q_\eta}^{\gamma_1 \vee \gamma_2}$, $\forall g, h \in S$

and $\alpha_1, \alpha_2, \gamma_1, \gamma_2 \in [-1,0), \beta_1, \beta_2 \in (-1,0]$.

Theorem 2.15:

Let S_X, S_Y be two $(\in, \in \vee q_\eta)$ -neutrosophic N-subsemigroups of a semigroup S. Then intersection $S_X \cap S_Y$ is also $(\in, \in \vee q_\eta)$ -neutrosophic N-subsemigroup of S.

Proof: Let $g \in (T_X \cap T_Y)_{\in}^{\alpha_1}$, $h \in (T_X \cap T_Y)_{\in}^{\alpha_2}$

$\Rightarrow (T_X \cap T_Y)(g) \le \alpha_1$ and $(T_X \cap T_Y)(h) \le \alpha_2$

$\Rightarrow T_X(g) \vee T_Y(g) \le \alpha_1$, $T_X(h) \vee T_Y(h) \le \alpha_2$

$\Rightarrow T_X(g) \le \alpha_1$ and $T_Y(g) \le \alpha_1 \Rightarrow g \in (T_X)_{\in}^{\alpha_1}$ and $g \in (T_Y)_{\in}^{\alpha_1}$

Also $T_X(h) \le \alpha_2$ and $T_Y(h) \le \alpha_2 \Rightarrow h \in (T_X)_{\in}^{\alpha_2}$ and $h \in (T_Y)_{\in}^{\alpha_2}$

Now $g \in (T_X)_{\in}^{\alpha_1}$, $h \in (T_X)_{\in}^{\alpha_2}$ and T_X is an element of S_X, S_X is $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup.

$\Rightarrow gh \in (T_X)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$ Similarly, we get $gh \in (T_Y)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow gh \in (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$ or $gh \in (T_X)_{q_\eta}^{\alpha_1 \vee \alpha_2}$ and $gh \in (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$ or $gh \in (T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

Case 1: If $gh \in (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$ and $(gh) \in (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow T_X(gh) \le \alpha_1 \vee \alpha_2$ and $T_Y(gh) \le \alpha_1 \vee \alpha_2$

Consider $(T_X \cap T_Y)(gh) = T_X(gh) \vee T_Y(gh) \leq (\alpha_1 \vee \alpha_2) \vee (\alpha_1 \vee \alpha_2) = \alpha_1 \vee \alpha_2$

$\Rightarrow gh \in (T_X \cap T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

Case 2: If $gh \notin (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$ and $(gh) \in (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow gh \in (T_X)_{q_\eta}^{\alpha_1 \vee \alpha_2}$ and $T_Y(gh) \leq \alpha_1 \vee \alpha_2$

$\Rightarrow (T_X)(gh) + (\alpha_1 \vee \alpha_2) + \eta < -1$ and $T_Y(gh) \leq \alpha_1 \vee \alpha_2$

Consider $(T_X \cap T_Y)(gh) = T_X(gh) \vee T_Y(gh) < (-1 - (\alpha_1 \vee \alpha_2) - \eta) \vee (\alpha_1 \vee \alpha_2) < \alpha_1 \vee \alpha_2$

$\Rightarrow gh \in (T_X \cap T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

case 3: If $gh \in (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$ and $(gh) \notin (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

similar to case 2 we get $gh \in (T_X \cap T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

Case 4: If $gh \notin (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$ and $(gh) \notin (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow gh \in (T_X)_{q_\eta}^{\alpha_1 \vee \alpha_2}$ and $gh \in (T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow (T_X)(gh) + (\alpha_1 \vee \alpha_2) + \eta < -1$ and $(T_Y)(gh) + (\alpha_1 \vee \alpha_2) + \eta < -1$

Consider $(T_X \cap T_Y)(gh) + (\alpha_1 \vee \alpha_2) + \eta = (T_X(gh) \vee T_Y(gh)) + (\alpha_1 \vee \alpha_2) + \eta$

$$= ((T_X)(gh) \vee (\alpha_1 \vee \alpha_2) + \eta) + (T_Y(gh) \vee (\alpha_1 \vee \alpha_2) + \eta)$$
$$< (-1) + (-1) = -2 < -1$$

$\Rightarrow gh \in (T_X \cap T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

In all the four cases, we get either $gh \in (T_X \cap T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$ or $gh \in (T_X \cap T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

$\Rightarrow gh \in (T_X \cap T_Y)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$

Let $g \in (I_X \cap I_Y)_{\in}^{\beta_1}$, $h \in (I_X \cap I_Y)_{\in}^{\beta_2}$

$\Rightarrow (I_X \cap I_Y)(g) \geq \beta_1$ and $(I_X \cap I_Y)(h) \geq \beta_2 \Rightarrow I_X(g) \wedge I_Y(g) \geq \beta_1$ and $I_X(h) \wedge I_Y(h) \geq \beta_2$

$\Rightarrow I_X(g) \geq \beta_1$, $I_Y(g) \geq \beta_1$, $I_X(h) \geq \beta_2$ and $I_Y(h) \geq \beta_2$

For $I_X(g) \geq \beta_1$, $I_X(h) \geq \beta_2$

$\Rightarrow g \in (I_X)_{\in}^{\beta_1}$, $h \in (I_X)_{\in}^{\beta_2}$

Since I_X is an element of S_X , S_X is $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup.

$\Rightarrow gh \in (I_X)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2}$ Similarly, we get $gh \in (I_Y)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2}$

Now $gh \in (I_X)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2} \Rightarrow gh \in (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ or $gh \in (I_X)_{q_\eta}^{\beta_1 \wedge \beta_2}$

and $gh \in (I_Y)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2} \Rightarrow gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$ or $gh \in (I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

Case 1: If $gh \in (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ and $gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

$\Rightarrow (I_X)(gh) \geq \beta_1 \wedge \beta_2$ and $(I_Y)(gh) \geq \beta_1 \wedge \beta_2$

consider $(I_X \cap I_Y)(gh) = I_X(gh) \wedge I_Y(gh) \geq \beta_1 \wedge \beta_2$

$\Rightarrow gh \in (I_X \cap I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

Case 2: If $gh \notin (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ and $gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

$\Rightarrow gh \in (I_X)_{q_\eta}^{\beta_1 \wedge \beta_2}$ and $gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

$\Rightarrow (I_X)(gh) + (\beta_1 \wedge \beta_2) + \eta > -1,\ (I_Y)(gh) \geq \beta_1 \wedge \beta_2$

Consider $(I_X \cap I_Y)(gh) = I_X(gh) \wedge I_Y(gh) > (-1 - (\beta_1 \wedge \beta_2) - \eta) \wedge (\beta_1 \wedge \beta_2)$

$$= -1 - (\beta_1 \wedge \beta_2) - \eta$$

$\Rightarrow (I_X \cap I_Y)(gh) + (\beta_1 \wedge \beta_2) + \eta > -1$

$\Rightarrow gh \in (I_X \cap I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

Case 3: If $gh \in (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ and $gh \notin (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

Which is similar to case 2, we get $gh \in (I_X \cap I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

Case 4: If $gh \notin (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ and $gh \notin (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

$\Rightarrow (I_X)(gh) + (\beta_1 \wedge \beta_2) + \eta > -1$ and $(I_Y)(gh) + (\beta_1 \wedge \beta_2) + \eta > -1$

consider $I_{X \cap Y}(gh) + (\beta_1 \wedge \beta_2) + \eta = I_X(gh) \wedge I_Y(gh) + (\beta_1 \wedge \beta_2) + \eta$

$$= (I_X(gh) + (\beta_1 \wedge \beta_2) + \eta) \wedge (I_Y(gh) + (\beta_1 \wedge \beta_2) + \eta)$$

$$> -1 + -1 = -1$$

$\Rightarrow gh \in (I_{X \cap Y})_{q_\eta}^{\beta_1 \wedge \beta_2}$

In all the four cases, we get either $gh \in (I_X \cap I_Y)_{\in}^{\beta_1 \wedge \beta_2}$ or $gh \in (I_X \cap I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

$\Rightarrow gh \in (I_X \cap I_Y)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2}$

Similarly, we prove that if $g \in (F_{X \cap Y})_{\in}^{\gamma_1}, h \in (F_X \cap F_Y)_{\in}^{\gamma_2}$ then $gh \in (F_X \cap F_Y)_{\in \vee q_\eta}^{\gamma_1 \vee \gamma_2}$

Therefore $S_X \cap S_Y$ is $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup.

Theorem 2.16:

Let S_X, S_Y be two $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroups. Then $S_X \cup S_Y$ is also a $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup if one is contained in the other.

Proof: Let S_X, S_Y be two $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroups such that $S_X \subseteq S_Y$ or $S_Y \subseteq S_X$

Assume that $S_X \subseteq S_Y$

Let $g \in (T_X \cup T_Y)_{\in}^{\alpha_1}, h \in (T_X \cup T_Y)_{\in}^{\alpha_2}$

$\Rightarrow (T_X \cup T_Y)(g) \leq \alpha_1$ and $(T_X \cup T_Y)(h) \leq \alpha_2$

$\Rightarrow (T_X)(g) \wedge T_Y(g) \leq \alpha_1, (T_X)(h) \wedge T_Y(h) \leq \alpha_2$

Since $S_X \subseteq S_Y$ we have $T_X \supseteq T_Y$

$\Rightarrow T_Y(g) \leq \alpha_1, T_Y(h) \leq \alpha_2$

$\Rightarrow g \in (T_Y)_{\in}^{\alpha_1}$ and $h \in (T_Y)_{\in}^{\alpha_2}$

$\Rightarrow gh \in (T_Y)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$ since S_Y is $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup.

$\Rightarrow gh \in (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$ or $gh \in (T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

Case 1: If $gh \in (T_Y)_{\in}^{\alpha_1 \vee \alpha_2}$ then $T_Y(gh) \leq \alpha_1 \vee \alpha_2$

$\Rightarrow T_X(gh) \wedge T_Y(gh) = T_Y(gh) \leq \alpha_1 \vee \alpha_2$ since $S_X \subseteq S_Y$

$\Rightarrow (T_X \cup T_Y)(gh) \leq \alpha_1 \vee \alpha_2$

$\Rightarrow gh \in (T_{X \cup Y})_{\in}^{\alpha_1 \vee \alpha_2}$

Case 2: If $gh \in (T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$ then $T_Y(gh) + (\alpha_1 \vee \alpha_2) + \eta < -1$

consider $(T_X \cup T_Y)(gh) + (\alpha_1 \vee \alpha_2) + \eta = (T_X(gh) \wedge T_Y(gh)) + (\alpha_1 \vee \alpha_2) + \eta$

$$< T_Y(gh) + (\alpha_1 \vee \alpha_2) + \eta < -1$$

$\Rightarrow gh \in (T_X \cup T_Y)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

From cases 1 and 2 we get $gh \in (T_X \cup T_Y)_{\in \vee q_\eta}^{\alpha_1 \vee \alpha_2}$

Let $g \in (I_X \cup I_Y)_{\in}^{\beta_1}$ and $h \in (I_X \cup I_Y)_{\in}^{\beta_2}$

$\Rightarrow (I_X \cup I_Y)(g) \geq \beta_1,\ (I_X \cup I_Y)(h) \geq \beta_2$

$\Rightarrow I_X(g) \vee I_Y(g) \geq \beta_1, I_X(h) \vee I_Y(h) \geq \beta_2$ since $I_X \subseteq I_Y$

$\Rightarrow I_Y(g) \geq \beta_1, I_Y(h) \geq \beta_2$

$\Rightarrow g \in (I_Y)_{\in}^{\beta_1}$ and $h \in (I_Y)_{\in}^{\beta_2}$ and S_Y is $(\in, \in \vee q_\eta)$ -Neutrosophic N-subsemigroup

$\Rightarrow gh \in (I_Y)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2} \Rightarrow gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$ or $gh \in (I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

If $gh \in (I_Y)_{\in}^{\beta_1 \wedge \beta_2}$ then $I_Y(gh) \geq \beta_1 \wedge \beta_2$

$\Rightarrow I_X(gh) \vee I_Y(gh) \geq I_Y(gh) \geq \beta_1 \wedge \beta_2 \Rightarrow (I_X \cup I_Y)(gh) \geq \beta_1 \wedge \beta_2$

$\Rightarrow gh \in (I_X \cup I_Y)_{\in}^{\beta_1 \wedge \beta_2}$

If $gh \in (I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$ then $I_Y(gh) + (\beta_1 \wedge \beta_2) + \eta > -1$

$\Rightarrow (I_X(gh) \vee I_Y(gh)) + (\beta_1 \wedge \beta_2) + \eta \geq I_Y(gh) + (\beta_1 \wedge \beta_2) + \eta > -1$

$\Rightarrow gh \in (I_X \cup I_Y)_{q_\eta}^{\beta_1 \wedge \beta_2}$

Therefore $gh \in (I_X \cup I_Y)_{\in \vee q_\eta}^{\beta_1 \wedge \beta_2}$

If $g \in (F_{X \cup Y})_{\in}^{\gamma_1}$ and $h \in (F_{X \cup Y})_{q_\eta}^{\gamma_2}$ then $gh \in (F_X \cup F_Y)_{\in \vee q_\eta}^{\gamma_2}$ which is similar to $T_X \cup T_Y$.

Definition 2.17: (Definition of $(\in, \in)$ -neutrosophic N-subsemigroup)

A neutrosophic N-structure $S_X = (S; T_X, I_X, F_X)$ of semigroup S is called $(\in, \in)$ -neutrosophic N-subsemigroup if (i) $a \in (T_X)_{\in}^{\alpha_1}, b \in (T_X)_{\in}^{\alpha_2} \Rightarrow ab \in (T_X)_{\in}^{\alpha_1 \vee \alpha_2}$,

(ii) $a \in (I_X)_{\in}^{\beta_1}, b \in (I_X)_{\in}^{\beta_2} \Rightarrow ab \in (I_X)_{\in}^{\beta_1 \wedge \beta_2}$ and

(iii) $a \in (F_X)_{\in}^{\gamma_1}, b \in (F_X)_{\in}^{\gamma_2} \Rightarrow ab \in (F_X)_{\in}^{\gamma_1 \vee \gamma_2}$,

$\forall a, b \in S, \alpha_1, \alpha_2, \gamma_1, \gamma_2 \in [-1,0), \beta_1, \beta_2 \in (-1,0]$.

Theorem 2.18:

Let S_X be $(\in,\in\vee q_\eta)$ -Neutrosophic N-structures over S. S_X is $(\in,\in)$ -Neutrosophic N-subsemigroup of S iff $S_X o S_X \subseteq S_X$

Proof: Let $a,q,z\in S$ and $a=qz$

Consider $(T_X o T_X)(a)=\underset{a=qz}{\wedge}(T_X(q)\vee T_X(z))$

$$\geq \underset{a=qz}{\wedge} T_X(qz)=T_X(a)\Rightarrow T_X o T_X \supseteq T_X$$

Consider $(I_X o I_X)(a)=\underset{a=qz}{\vee}(I_X(q)\wedge I_X(z))$

$$\leq \underset{a=qz}{\vee} I_X(qz)=I_X(a)\Rightarrow I_X o I_X \subseteq I_X$$

Consider $(F_X o F_X)(a)=\underset{a=qz}{\wedge}(F_X(q)\vee F_X(z))$

$$\geq \underset{a=qz}{\wedge} F_X(qz)=F_X(a)\Rightarrow F_X o F_X \supseteq F_X$$

Therefore $S_X o S_X \subseteq S_X$

Conversely, assume that $S_X o S_X \subseteq S_X$

Let $a\in(T_X)_{\in}^{\alpha_1}, b\in(T_X)_{\in}^{\alpha_2}\Rightarrow T_X(a)\leq\alpha_1$ and $T_X(b)\leq\alpha_2$

Consider $T_X(ab)\leq T_X(a)\vee T_X(b)\leq\alpha_1\vee\alpha_2$

$\Rightarrow ab\in(T_X)_{\in}^{\alpha_1\vee\alpha_2}$

Let $a\in(I_X)_{\in}^{\beta_1}, b\in(I_X)_{\in}^{\beta_2}\Rightarrow I_X(a)\geq\beta_1$ and $I_X(b)\geq\beta_2$

Consider $I_X(ab)\geq I_X(a)\wedge I_X(b)\geq\beta_1\wedge\beta_2$

$\Rightarrow ab\in(I_X)_{\in}^{\beta_1\wedge\beta_2}$

Let $a\in(F_X)_{\in}^{\gamma_1}, b\in(F_X)_{\in}^{\gamma_2}\Rightarrow F_X(a)\leq\gamma_1$ and $F_X(b)\leq\gamma_2$

Consider $F_X(ab)\leq F_X(a)\vee F_X(b)\leq\gamma_1\vee\gamma_2$

$\Rightarrow ab\in(F_X)_{\in}^{\gamma_1\vee\gamma_2}$

Therefore S_X is $(\in,\in)$ -Neutrosophic N-subsemigroup of S.

Definition 2.19: (Definition of (q_η,q_η) -neutrosophic N-subsemigroup)

A neutrosophic N-structure $S_X=(S;T_X,I_X,F_X)$ of semigroup S is called (q_η,q_η) -neutrosophic N-subsemigroup if (i) $a\in(T_X)_{q_\eta}^{\alpha_1}, b\in(T_X)_{q_\eta}^{\alpha_2}\Rightarrow ab\in(T_X)_{q_\eta}^{\alpha_1\vee\alpha_2}$,

(ii) $a\in(I_X)_{q_\eta}^{\beta_1}, b\in(I_X)_{q_\eta}^{\beta_2}\Rightarrow ab\in(I_X)_{q_\eta}^{\beta_1\wedge\beta_2}$ and

(iii) $a\in(F_X)_{q_\eta}^{\gamma_1}, b\in(F_X)_{q_\eta}^{\gamma_2}\Rightarrow ab\in(F_X)_{q_\eta}^{\gamma_1\vee\gamma_2}$,

$\forall a,b\in S$, $\alpha_1,\alpha_2,\gamma_1,\gamma_2\in[-1,0), \beta_1,\beta_2\in(-1,0]$

Theorem 2.20:

Let S_X be $(\in,\in\vee q_\eta)$ -neutrosophic N-structures over S. S_X is (q_η,q_η) -neutrosophic N-subsemigroup of S

iff $S_X o S_X \subseteq S_X$

Proof: Necessary part is obvious from theorem 2.19

Conversely, let $S_X o S_X \subseteq S_X$

Let $a \in (T_X)_{q_\eta}^{\alpha_1}, b \in (T_X)_{q_\eta}^{\alpha_2} \Rightarrow T_X(a) + \alpha_1 + \eta < -1$ and $T_X(b) + \alpha_2 + \eta < -1$

Let $ab = c$

Consider $T_X(ab) = T_X(c) \leq (T_X o T_X)(c) = \underset{c=ab}{\wedge}(T_X(a) \vee T_X(b)) \leq (T_X(a) \vee T_X(b))$

$$< (-1-\alpha_1-\eta) \vee (-1-\alpha_2-\eta) = (-1-\eta) - (\alpha_1 \vee \alpha_2)$$

$\Rightarrow T_X(ab) + (\alpha_1 \vee \alpha_2) + \eta < -1$

$\Rightarrow ab \in (T_X)_{q_\eta}^{\alpha_1 \vee \alpha_2}$

Let $a \in (I_X)_{q_\eta}^{\beta_1}, b \in (I_X)_{q_\eta}^{\beta_2} \Rightarrow I_X(a) + \beta_1 + \eta > -1$ and $I_X(b) + \beta_2 + \eta > -1$

Let $ab = c$

Consider $I_X(ab) = I_X(c) \geq (I_X o I_X)(c) = \underset{c=ab}{\vee}(I_X(a) \wedge I_X(b)) \geq (I_X(a) \wedge I_X(b))$

$$> (-1-\beta_1-\eta) \wedge (-1-\beta_2-\eta) = (-1-\eta) - (\beta_1 \wedge \beta_2)$$

$\Rightarrow I_X(ab) + (\beta_1 \wedge \beta_2) + \eta > -1$

$\Rightarrow ab \in (I_X)_{q_\eta}^{\beta_1 \wedge \beta_2}$

similarly we prove that if $a \in (F_X)_{q_\eta}^{\gamma_1}, b \in (I_X)_{q_\eta}^{\gamma_2}$ then $ab \in (F_X)_{q_\eta}^{\gamma_1 \wedge \gamma_2}$

Therefore S_X is (q_η, q_η) -Neutrosophic N-subsemigroup of S.

Definition 2.21: [2] (Definition of η -membership functions of $(\in, \in \vee q_\eta)$ -neutrosophic N-structures)

Let $S_X = (S; T_X, I_X, F_X)$ be $(\in, \in \vee q_\eta)$ -neutrosophic N-structures over S. We define

(i) $T_{X\eta}(a) = T_X(a) \vee (\frac{-1-\eta}{2})$,

(ii) $I_{X\eta}(a) = I_X(a) \wedge (\frac{-1-\eta}{2})$ and

(iii) $F_{X\eta}(a) = F_X(a) \vee (\frac{-1-\eta}{2})$ $\forall a \in S$ and $\eta \in [-1, 0)$

Definition 2.22: [2] (Definition of η -intersection of $(\in, \in \vee q_\eta)$ -neutrosophic N-structures)

Let $S_X = (S; T_X, I_X, F_X)$ and $S_Y = (S; T_Y, I_Y, F_Y)$ be two $(\in, \in \vee q_\eta)$ -neutrosophic N-structures over S. The η -intersection of S_X, S_Y is $S_X \cap_\eta S_Y = (S; T_X \cap_\eta T_Y, I_X \cap_\eta I_Y, F_X \cap_\eta F_Y)$ defined as

(i) $(T_X \cap_\eta T_Y)(a) = (T_X \cap T_Y)(a) \vee (\frac{-1-\eta}{2})$,

(ii) $(I_X \cap_\eta I_Y)(a) = (I_X \cap I_Y)(a) \wedge (\frac{-1-\eta}{2})$ and

(iii) $(F_X \cap_\eta F_Y)(a) = (F_X \cap F_Y)(a) \vee (\frac{-1-\eta}{2})$, $\forall a \in S$ and $\eta \in [-1,0)$

Definition 2.23: [2] (Definition of η -union of $(\in, \in \vee q_\eta)$ -neutrosophic N-structures)

Let $S_X = (S; T_X, I_X, F_X)$ and $S_Y = (S; T_Y, I_Y, F_Y)$ be two $(\in, \in \vee q_\eta)$ -neutrosophic N-structures over S. The η -union of S_X, S_Y is $S_X \cup_\eta S_Y = (S; T_X \cup_\eta T_Y, I_X \cup_\eta I_Y, F_X \cup_\eta F_Y)$ defined as (i) $(T_X \cup_\eta T_Y)(a) = (T_X \cup T_Y)(a) \wedge (\frac{-1-\eta}{2})$,

(ii) $(I_X \cup_\eta I_Y)(a) = (I_X \cup I_Y)(a) \vee (\frac{-1-\eta}{2})$,

(iii) $(F_X \cup_\eta F_Y)(a) = (F_X \cup F_Y)(a) \wedge (\frac{-1-\eta}{2})$, $\forall a \in S$ and $\eta \in [-1,0)$

Definition 2.24: [2] (Definition of η -composition of $(\in, \in \vee q_\eta)$ -neutrosophic N-structures)

Let $S_X = (S; T_X, I_X, F_X)$ and $S_Y = (S; T_Y, I_Y, F_Y)$ be two $(\in, \in \vee q_\eta)$ -neutrosophic N-structures over S. The η -composition of S_X and S_Y is $S_X o_\eta S_Y = (S; T_X o_\eta T_Y, I_X o_\eta I_Y, F_X o_\eta F_Y)$

(i) $(T_X o_\eta T_Y)(a) = (T_X o T_Y)(a) \vee (\frac{-1-\eta}{2})$,

(ii) $(I_X o_\eta I_Y)(a) = (I_X o I_Y)(a) \wedge (\frac{-1-\eta}{2})$,

(iii) $(F_X o_\eta F_Y)(a) = (F_X o F_Y)(a) \vee (\frac{-1-\eta}{2})$, $\forall a \in S$ and $\eta \in [-1,0)$

Theorem 2.25:

Let $S_X = (S; T_X, I_X, F_X)$ and $S_Y = (S; T_Y, I_Y, F_Y)$ be two $(\in, \in \vee q_\eta)$ -neutrosophic N-structures over S. Then

a) $T_X \cap_\eta T_Y = T_{X\eta} \cap T_{Y\eta}$ b) $I_X \cap_\eta I_Y = I_{X\eta} \cap I_{Y\eta}$ c) $F_X \cap_\eta F_Y = F_{X\eta} \cap F_{Y\eta}$

d) $T_X \cup_\eta T_Y = T_{X\eta} \cup T_{Y\eta}$ e) $I_X \cup_\eta I_Y = I_{X\eta} \cup I_{Y\eta}$ f) $F_X \cup_\eta F_Y = F_{X\eta} \cup F_{Y\eta}$

g) $T_X o_\eta T_Y = T_{X\eta} o T_{Y\eta}$ h) $I_X o_\eta I_Y = I_{X\eta} o I_{Y\eta}$ i) $F_X o_\eta F_Y = F_{X\eta} o F_{Y\eta}$

Proof: a) Consider $(T_X \cap_\eta T_Y)(a) = (T_X \cap T_Y)(a) \vee (\frac{-1-\eta}{2})$

$$= T_X(a) \vee T_Y(a) \vee (\frac{-1-\eta}{2})$$

$$= (T_X(a) \vee (\frac{-1-\eta}{2})) \vee (T_Y(a) \vee (\frac{-1-\eta}{2}))$$

$$= T_{X\eta}(a) \vee T_{Y\eta}(a) = (T_{X\eta} \cap T_{Y\eta})(a)$$

b) Consider $(I_X \cap_\eta I_Y)(a) = (I_X \cap I_Y)(a) \wedge (\frac{-1-\eta}{2})$

$$= I_X(a) \wedge I_Y(a) \wedge (\frac{-1-\eta}{2})$$

$$= (I_X(a) \wedge (\frac{-1-\eta}{2})) \wedge (I_Y(a) \wedge (\frac{-1-\eta}{2}))$$

$$= I_{X\eta}(a) \wedge I_{Y\eta}(a) = (I_{X\eta} \cap I_{Y\eta})(a)$$

c) Proof of $F_X \cap_\eta F_Y = F_{X\eta} \cap F_{Y\eta}$ which is similar to (a)

d) Consider $(T_X \cup_\eta T_Y)(a) = (T_X \cup T_Y)(a) \wedge (\frac{-1-\eta}{2})$

$$= T_X(a) \wedge T_Y(a) \wedge (\frac{-1-\eta}{2})$$

$$= (T_X(a) \wedge (\frac{-1-\eta}{2})) \wedge (T_Y(a) \wedge (\frac{-1-\eta}{2}))$$

$$= T_{X\eta}(a) \wedge T_{Y\eta}(a) = (T_{X\eta} \cup T_{Y\eta})(a)$$

e) Consider $(I_X \cup_\eta I_Y)(a) = (I_X \cup I_Y)(a) \vee (\frac{-1-\eta}{2})$

$$= I_X(a) \vee I_Y(a) \vee (\frac{-1-\eta}{2})$$

$$= (I_X(a) \vee (\frac{-1-\eta}{2})) \vee (I_Y(a) \vee (\frac{-1-\eta}{2}))$$

$$= I_{X\eta}(a) \vee I_{Y\eta}(a) = (I_{X\eta} \cup I_{Y\eta})(a)$$

f) Proof of $F_X \cup_\eta F_Y = F_{X\eta} \cup F_{Y\eta}$ which is similar to (d).

g) Consider $(T_X o_\eta T_Y)(a) = (T_X o T_Y)(a) \vee (\frac{-1-\eta}{2})$

$$= \wedge_{a=qz} (T_X(q) \vee T_Y(z) \vee (\frac{-1-\eta}{2})$$

$$= \wedge_{a=qz} \{(T_X(q) \vee (\frac{-1-\eta}{2})) \vee (T_Y(z) \vee (\frac{-1-\eta}{2}))\}$$

$$= \wedge_{a=qz} (T_{X\eta}(q) \vee T_{Y\eta}(z)) = (T_{X\eta} o T_{Y\eta})(a)$$

h) Consider $(I_X o_\eta I_Y)(a) = (I_X o I_Y)(a) \wedge (\frac{-1-\eta}{2})$

$$= I_X(a) \wedge I_Y(a) \wedge (\frac{-1-\eta}{2})$$

$$= (I_X(a) \wedge (\frac{-1-\eta}{2})) \wedge (I_Y(a) \wedge (\frac{-1-\eta}{2}))$$

$$= I_{X\eta}(a) \wedge I_{Y\eta}(a) = (I_{X\eta} \cap I_{Y\eta})(a)$$

i) Proof of $F_X \cap_\eta F_Y = F_{X\eta} \cap F_{Y\eta}$ which is similar to (g).

3. Conclusions

The purpose of this paper is to establish the relation between the characteristic function of neutrosophic N-subsemigroup and semigroup S, further, we proved some of the relations using η-intersection, η-union and η-composition of $(\in, \in \vee q_\eta)$ neutrosophic N-subsemigroup.

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