A New Similarity Measure Of Picture Fuzzy Sets And Application in pattern recognition

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Abstract

In this paper, we propose some novel similarity measures between picture fuzzy sets. The novel similarity measure is constructed by combining negative functions of each degree membership of picture fuzzy set. We apply them in several pattern recognition problems. Finally, we apply them to find the fault diagnosis of the steam turbine.

Keywords: Picture fuzzy set, similarity measure, fault turbine.

1.Introduction

In 2014, Cuong introduced the picture fuzzy set [4]. It is a generalization of Zadeh's fuzzy set [27] and intuitionistic fuzzy set [1, 2, 3]. A picture fuzzy set (PFS) consider three degrees: the positive membership function, the neural membership and negative-membership function. From its inception to the present day, PFSs have been proven to be a very effective tool for processing uncertainties in real-world problems: the pattern recognition, the decision making…There are the theoretical results bases on picture fuzzy set as: picture fuzzy database was introduced by Dinh et al [5], some aspects of picture fuzzy sets as $(\alpha,\theta,\lambda)-$cut and the height of a picture fuzzy set was studied by Dutta and Ganju [9], Thao and Dịnh studied rough picture fuzzy set that is combined the picture fuzzy set and the crisp approximations and investigated the picture topologies generated by rough picture fuzzy set [16]. As opposed to fuzzy sets, picture fuzzy set also have broad applications for uncertain data processing such as decision making, medical diagnose, agriculture [6-8, 10-15, 19-21, 23-24].Along with distance measurements, correlation measurements, similar measurements of picture fuzzy sets are also studied and widely used in many areas and now it is a hot topic. The similarity measure of picture fuzzy sets is useful to handle the problems in many areas: decision making, machine learning, pattern recognition [25-26]. Wei [25] proposed the similarity measure between picture fuzzy sets based on the cosine functions and applied it in the MCDM. After that, Wei [26] also developed the similarity measure constructed by the distance measure in multiple attribute decision making and recognition problem… The proposed methods constructed by combining negative functions of each degree membership of picture fuzzy set.

In this paper, we introduce the new similarity measure of picture fuzzy sets which built by using negative functions for all membership, non-membership and neutral membership functions. At the same time, we also apply new measures in the problem of pattern recognition and multi-criteria decision-making. The rest of this paper is organized as follows. In Section 2, we recall the concept of picture fuzzy set and the similarity measure of them. In Section 3, we construct the new similarity measure between the PFSs and give an example demonstrate our measure to recognize the pattern. Finally, we propose a picture fuzzy software quality model in section 4 and give an example to illustrate for the proposed model.

2. Primarily

Let X be a universal set. We have

Definition 1[4]. An picture fuzzy set on X is a defined by form

$$A=\left\{\left(x,\mu_A(x),\eta_A(x),\nu_A(x)\right)\middle| x\in X\right\}$$

in which $\mu_A(x)\in[0,1]$, $\eta_A(x)\in[0,1]$ and $\nu_A(x)\in[0,1]$ are the membership degree, the neural degree and the non-membership degree of the element x in X to A, respectively, and

$$\mu_A(x)+\eta_A(x)+\nu_A(x)\le 1,\forall x\in X.$$

We denote $PFS(\mathrm{X})$ is a collection of picture fuzzy set on X. In which

$$X=\{(\mathrm{x},1,0,0)|\,\mathrm{x}\in \mathrm{X}\}$$

and $\varnothing=\{(\mathrm{x},0,0,1)|\,x\in X\}$.

For convenience in this paper, we call $P=(a,b,c)$ is a picture fuzzy number if $a,b,c\ge 0$ and $a+b+c\le 1$.

For two picture fuzzy sets $A,B\in P\mathrm{F}S(X)$ we have:

- $A\subset B$ if only if $\mu_A(x)<\mu_B(x)$, $\eta_A(x)\le\eta_B(x)$ and $\nu_A(x)>\nu_B(x)$ for all $x\in X$.
- $A=B$ if only if $\mu_A(x)=\mu_B(x)$, $\eta_A(x)=\eta_B(x)$ and $\nu_A(x)=\nu_B(x)$ for all $x\in X$.

Now, we recall the similarity measure in literal.

Given $X=\left\{x_1,x_2,...,x_n\right\}$ is a universal set. And

$$A=\left\{\left(x_i,\mu_A(x_i),\eta_A(x_i),\nu_A(x_i)\right)\middle| x_i\in X\right\},$$

$$B=\left\{\left(x_i,\mu_B(x_i),\eta_B(x_i),\nu_B(x_i)\right)\middle| x_i\in X\right\}$$

are two PFSs on X .

Definition 3: A mapping $\mathrm{S}\colon \mathrm{PF}S(X)\times P\mathrm{F}S(X)\rightarrow[0,1]$ is a similarity measure of two picture fuzzy sets if it satisfies the following conditions:

(S1) $0\le \mathrm{S(A,B)}\le 1,\ \forall\mathrm{A,B}\ \in P\mathrm{F}S(X).$

(S2) $\mathrm{S(A,B)}=\mathrm{S(B,A)},\ \forall\mathrm{A,B}\ \in P\mathrm{F}S(X).$

(S3) $\mathrm{S(A,A)}=1,\ \forall\mathrm{A}\ \in P\mathrm{F}S(X).$

(S4) For all $\mathrm{A,B,\ C}\ \in\ \mathrm{PF}S(X)$ such that $A\subseteq B\subseteq C$ then

$$S(A,C)\le\min\left\{S(A,B),S(B,C)\right\}.$$

3. A new similarity measure of the IFSs

Let $X=\left\{x_1,x_2,...,x_n\right\}$ be a finite set, A and B are two arbitrary PFSs in X .We denote

$$S_i^{\mu}(A,B)=\frac{e^{1-|\mu_A(x_i)-\mu_B(x_i)|}-1}{e-1}$$

$$S_i^{\eta}(A,B)=\frac{e^{1-|\eta_A(x_i)-\eta_B(x_i)|}-1}{e-1}$$

$$S_i^{\nu}(A,B)=1-\left|\nu_A(x_i)-\nu_B(x_i)\right|$$

for all $i=1,2,...,n.$

Definition 4. A mapping $\mathrm{S}\colon \mathrm{PF}S(X)\times P\mathrm{F}S(X)\rightarrow[0,1]$ is defined by

$$\mathrm{S}_0(A,B)=\tfrac{1}{n}\sum_{i=1}^{n}S_i^0(A,B) \qquad (1)$$

where $S_i^0(A,B)=S_i^{\mu}(A,B)\times S_i^{\eta}(A,B)\times S_i^{\nu}(A,B)$

Theorem1. Let A and B be two arbitrary $PFSs$ in X . The function $\mathrm{S}(A,B)$ in Eq.(1) satisfies the following conditions:

(S1) $0\le \mathrm{S}_0(A,B)\le 1,\ \forall\mathrm{A,B}\ \in P\mathrm{F}S(X).$

(S2) $S_0(A,B) = S_0(A,B), \forall A,B \in PFS(X).$

(S3) $S_0(A,A) = 1, \forall A \in PFS(X).$

(S4) For all $A,B, C \in PFS(X)$ such that $A \subseteq B \subseteq C$ then

$$S_0(A,C) \le \min\{S_0(A,B), S_0(B,C)\}.$$

It means that

$S_0(A,B) = \frac{1}{n}\sum_{i=1}^{n} S_i^{\mu}(A,B) \times S_i^{\eta}(A,B) \times S_i^{\nu}(A,B)$ is a similarity measure of two picture fuzzy sets.

Proof.

(S1). We have

$$0 \le S_i^{\mu}(A,B) = \frac{e^{1-|\mu_A(x_i)-\mu_B(x_i)|}-1}{e-1} \le 1,$$

$$0 \le S_i^{\eta}(A,B) = \frac{e^{1-|\eta_A(x_i)-\eta_B(x_i)|}-1}{e-1} \le 1$$

and $0 \le S_i^{\nu}(A,B) = 1 - |\nu_A(x_i) - \nu_B(x_i)| \le 1$ for all $A,B \in PFS(X)$. So that

$$0 \le S_i^{0}(A,B) \le 1.$$

Hence

$$0 \le S_0(A,B) = \frac{1}{n}\sum_{i=1}^{n} S_i^{0}(A,B) \le 1$$

for all $A,B \in PFS(X)$.

(S2). It is obvious.

(S3). Considering two picture fuzzy sets A, B on X. If $A = B$ then $\mu_A(x_i) = \mu_B(x_i), \eta_A(x_i) = \eta_B(x_i),$ $\nu_A(x_i) = \nu_B(x_i)$ according to the Eq.(1) we have

$S_i^{\mu}(A,B) = 1,\ S_i^{\eta}(A,B) = 1$ and $S_i^{\nu}(A,B) = 1$. So that

$$S_0(A,B) = \frac{1}{n}\sum_{i=1}^{n} S_i^{\mu}(A,B) \times S_i^{\eta}(A,B) \times S_i^{\nu}(A,B) = 1.$$

(S4). For all $\mathrm{A,B,C} \in PFS(X)$ such that $\mathrm{A} \subseteq \mathrm{B} \subseteq \mathrm{C}$ then $\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$, $\eta_A(x_i) \le \eta_B(x_i) \le \eta_C(x_i)$, and $\nu_C(x_i) \ge \nu_B(x_i) \ge \nu_A(x_i)$. So that

$$\max\{|\mu_A(x_i) - \mu_B(x_i)|, |\mu_B(x_i) - \mu_C(x_i)|\}$$

$$\le |\mu_A(x_i) - \mu_C(x_i)|,$$

$$\max\{|\eta_A(x_i) - \eta_B(x_i)|, |\eta_B(x_i) - \eta_C(x_i)|\}$$

$$\le |\eta_A(x_i) - \eta_C(x_i)|,$$

and

$$\max\{|\nu_A(x_i) - \nu_B(x_i)|, |\nu_B(x_i) - \nu_C(x_i)|\}$$

$$\le |\nu_A(x_i) - \nu_C(x_i)|.$$

Then we have

$$\min\{-|\mu_A(x_i) - \mu_B(x_i)|, -|\mu_B(x_i) - \mu_C(x_i)|\}$$

$$\ge -|\mu_A(x_i) - \mu_C(x_i)|,$$

$$\min\{-|\eta_A(x_i) - \eta_B(x_i)|, -|\eta_B(x_i) - \eta_C(x_i)|\}$$

$$\ge -|\eta_A(x_i) - \eta_C(x_i)|,$$

and

$$\min\{-|\nu_A(x_i) - \nu_B(x_i)|, -|\nu_B(x_i) - \nu_C(x_i)|\}$$

$$\ge -|\nu_A(x_i) - \nu_C(x_i)|.$$

Since, we obtain

$$S_i^{\mu}(A,C) \le \min\{S_i^{\mu}(A,B), S_i^{\mu}(B,C)\},$$

$$S_i^{\eta}(A,C) \le \min\{S_i^{\eta}(A,B), S_i^{\eta}(B,C)\}$$

and

$$S_i^{\nu}(A,C) \le \min\{S_i^{\nu}(A,B), S_i^{\nu}(B,C)\}.$$

Hence, we have

$S_i^0(A,C) \le \min\{S_i^0(A,B), S_i^0(B,C)\}$.

So that

$S_0(A,C) \le \min\{S_0(A,B), S_0(B,C)\}$.□

Now, we can define the similarity measure of the picture fuzzy sets that assigned with the weight of each element in the universal set.

We assume that each element x_i in the universal $X = \{x_1, x_2, ..., x_n\}$ which assigned with a weight $\omega_i \in [0,1]$ for $i = 1, 2, ..., n$ such that $\sum_{i=1}^{n} \omega_i = 1$.

Definition 5. A mapping $\text{S: IF}S(X) \times \text{IF}S(X) \to [0,1]$ is defined by $\text{S}_0^\omega(A,B) = \sum_{i=1}^{n} \omega_i S_i^0(A,B)$ (2)

Theorem 2. Let A and B be two arbitrary $PFSs$ in X. The function $\text{S}_0^\omega(A,B)$ in Eq.(2) satisfies the following conditions:

(S1) $0 \le \text{S}_0^\omega(\text{A,B}) \le 1,\ \forall \text{A,B} \in PFS(X)$.

(S2) $\text{S}_0^\omega(\text{A,B}) = \text{S}_0^\omega(\text{B,A}),\ \forall \text{A,B} \in \text{PF}S(X)$.

(S3) $\text{S}_0^\omega(\text{A,A}) = 1,\ \forall \text{A} \in \text{PF}S(X)$.

(S4) For all $\text{A,B, C} \in \text{PF}S(X)$ such that $A \subseteq B \subseteq C$ then

$$\text{S}_0^\omega(A,C) \le \min\left\{\text{S}_0^\omega(A,B), \text{S}_0^\omega(B,C)\right\}.$$

It means that

$$\text{S}_0^\omega(A,B) = \sum_{i=1}^{n} \omega_i S_i^0(A,B)$$

is a similarity measure of two picture fuzzy sets.

Proof.

(S1) Because

$$0 \le S_i^\mu(A,B), S_i^\eta(A,B), S_i^\nu(A,B) \le 1.$$

We have

$$0 \le \text{S}_0^\omega(A,B) = \sum_{i=1}^{n} \omega_i S_i^0(A,B) \le \tfrac{1}{n} \sum_{i=1}^{n} \omega_i = 1.$$

(S2). It is obviously.

(S3). If $A=B$ then $\mu_A(x_i)=\mu_B(x_i), \eta_A(x_i)=\eta_B(x_i), \nu_A(x_i)=\nu_B(x_i)$ according to the Eq.(2) we have

$S_i^{\mu}(A,B)=1, S_i^{\eta}(A,B)=1, S_i^{\nu}(A,B)=1$. So that

$$S_0^{\omega}(A,B)=\sum_{i=1}^{n}\omega_i S_i^0(A,B)=\tfrac{1}{n}\sum_{i=1}^{n}\omega_i=1.$$

(S4). For all $A,B,C \in PFS(X)$ such that $A\subseteq B\subseteq C$ then

$$S_i^{\mu}(A,C)\le \min\{S_i^{\mu}(A,B),S_i^{\mu}(B,C)\},$$

$$S_i^{\eta}(A,C)\le \min\{S_i^{\eta}(A,B),S_i^{\eta}(B,C)\}$$

and

$$S_i^{\nu}(A,C)\le \min\{S_i^{\nu}(A,B),S_i^{\nu}(B,C)\}.$$

Hence

$$S_i^0(A,C)\le \min\{S_i^0(A,B),S_i^0(B,C)\}$$

So that

$$S_0^{\omega}(A,C)=\sum_{i=1}^{n}\omega_i S_i^0(A,C)$$

$$\le \tfrac{1}{n}\sum_{i=1}^{n}\omega_i \min\{S_i^0(A,B),S_i^0(B,C)\}$$

$$= \min\{S_0^{\omega}(A,B),S_0^{\omega}(B,C)\}. \square$$

Compare with some other similarity measures:

Now, we consider an example to compare our proposed similarity measures and some other similarity measures on picture fuzzy sets which introduced by Wei in [24, 25].

+Cosine similarity measure

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A(x_i)^2+\eta_A(x_i)^2+\nu_A(x_i)^2}\sqrt{\mu_B(x_i)^2+\eta_B(x_i)^2+\nu_B(x_i)^2}}$$

+ Dice similarity measure

$$S_{C1}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\max(\mu_A(x_i)^2+\eta_A(x_i)^2+\nu_A(x_i)^2,\mu_B(x_i)^2+\eta_B(x_i)^2+\nu_B(x_i)^2)}$$

+ Grey similarity measure

$$S_G(A,B)=\frac{1}{3n}\sum_{i=1}^{n}(\frac{\Delta\mu_{\min}+\Delta\mu_{\max}}{\Delta\mu+\Delta\mu_{\max}}+\frac{\Delta\eta_{\min}+\Delta\eta_{\max}}{\Delta\eta+\Delta\eta_{\max}}+\frac{\Delta\nu_{\min}+\Delta\nu_{\max}}{\Delta\nu+\Delta\nu_{\max}})$$

where $\Delta\mu_i=\left|\mu_A(x_i)-\mu_B(x_i)\right|$, $\Delta\mu_{\min}=min\{\Delta\mu_i\}$, $\Delta\mu_{\max}=\max\{\Delta\mu_i\}$, $\Delta\eta_i=\left|\eta_A(x_i)-\eta_B(x_i)\right|$, $\Delta\eta_{\min}=min\{\Delta\eta_i\}$, $\Delta\eta_{\max}=max\{\Delta\eta_i\}$, $\Delta\nu_i=\left|\nu_A(x_i)-\nu_B(x_i)\right|$, $\Delta\nu_{\min}=min\{\Delta\nu_i\}$.

+ The similarity measures based on cosine function

$$S_{COS1}(A,B)=\frac{1}{n}\sum_{i=1}^{n}COS_{1,i}(A,B)$$

where

$$COS_{1,i}(A,B)=COS(\frac{\pi}{2}\max\begin{Bmatrix}\left|\mu_A(x_i)-\mu_A(x_i)\right|,\\ \left|\eta_A(x_i)-\eta_A(x_i)\right|,\\ \left|\nu_A(x_i)-\nu_A(x_i)\right|\end{Bmatrix});$$

and

$$S_{COS2}(A,B)=\frac{1}{n}\sum_{i=1}^{n}COS_{2,i}(A,B)$$

where

$$COS_{2,i}(A,B)=COS(\frac{\pi}{4}\begin{Bmatrix}\left|\mu_A(x_i)-\mu_A(x_i)\right|+\\ \left|\eta_A(x_i)-\eta_A(x_i)\right|+\\ \left|\nu_A(x_i)-\nu_A(x_i)\right|\end{Bmatrix})$$

Example 1. Suppose that A and B are two PFSs in $X=\{x\}$ where $A=\{(x,0,0,0)\}$ and $B=\{(x,0,0,0)\}$. We easy see that $S_C(A,B)=Null$, $S_{C1}(A,B)=Null$, $S_G(A,B)=Null$. This result is not suitable to the condition (S3) in the definition 3. Our similarity measure satisfy this condition $S_0(A,B)=1$.

Example 2. Suppose that A and B are two pattern PFSs in $X=\{x_1,x_2\}$ where

$A=\{(x_1,0.1,0,0.1),(x_2,0,0.4,0.4)\}$,

$B=\{(x_1,0.1,0.1,0),(x_2,0.1,0,0.1)\}$

and if we have sample PFSs in $X=\{x_1,x_2\}$

$C=\{(x_1,0,0.1,0.1),(x_2,0,0.2,0.2)\}$.

Question: Which pattern does C belong to?

Using the above similarity measures, we obtain

$S_0(A,C)=0.7873, S_0(B,C)=0.7023$ so that C belong to the class of pattern B.

This result is coincide with the result using the similarity S_G, specifically $S_G(A,C)=0.77, S_G(B,C)=0.86$. If using the similarity measure S_{COS1} and S_{COS2} we cannot put C belong to the class of A or B because $S_{COS1}(A,C)=S_{COS1}(B,C)=0.97$ and $S_{COS1}(A,C)=S_{COS1}(B,C)=0.97$. These examples are showed the meaningful of new similarity measures.

4. Application the picture similarity measure in the fault diagnosis of steam turbine

In this section, we apply our proposed similarity measure to predict the fault diagnosis of steam turbine. Data is cited in [21] (see Table 1). In this data, we see that each error pattern has the form of a picture fuzzy set on the set of attributes. Moreover, a picture fuzzy set is also called a standard neutrosophic set [17, 18]. So that it is motivation for us to apply our proposed similarity measure on the picture fuzzy set to diagnose errors for steam turbines.

There are a set of ten fault patterns

$A=\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10}\}$

in which A_1: Unbalance; A_2: Pneumatic force couple; A_3: Offset center; A_4: Oil-membrane oscillation; A_5: Radial impact friction of rotor; A_6: Symbiosis looseness; A_7: Damage of antithrust bearing; A_8: Surge; A_9: Looseness of bearing block; A_{10}: Non-uniform bearing stiffness.

Each fault pattern is a picture fuzzy set on a set of nine frequency ranges for different frequency spectrum

$$C=\left\{\begin{matrix} C_1\left(0.01-0.39f\right), C_2\left(0.4-0.49f\right), C_3\left(0.5f\right), \\ C_4\left(0.51-0.99f\right), C_5(f), C_6(2f), C_7(3-5f), \\ C_8(\text{Odd times of } f), C_9\left(\text{High frequency} > 5\text{ f}\right) \end{matrix}\right\}$$

There are two fault sample are picture fuzzy sets B_1 and B_2 on the set of attributes C as in Table 2.

Question: Which fault pattern do B_1 and B_2 belong to?

To solve this problem, we do it in two steps

Step 1.Determine the weight of each attribute, we assume that the weight of each attribute C_j is $\omega_j=\frac{1}{9}$ for all $j=1,2,...,9$.

Step 2. Compute the similarity measure $s(B_i, A_j)$ between $B_i\left(i=1,\ 2\right)$ and $A_j\left(j\ =\ 1,2,\ \ldots,\ 10\right)$ by using eq.(1) to rank order of all faults. For the sample B_i we have $A_j \succ A_k$ if $s(B_i, A_j) > s(B_i, A_k)$ where $A_j \succ A_k$ means that A_j having the higher order than A_k. The computing results is shown in Table 3.

Step 3. Chose the fault having the highest rank. For the sample B_1 we put it belong to the fault pattern A_7. For the sample B_2 we put it belong to the fault pattern A_9 (See Table 3).

Table 1. The fault patterns with picture fuzzy number values

	A_1	A_2	A_3	A_4	A_5
C_1	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.09,0.02,0.89>$	$<0.09,0.03,0.88>$
C_2	$<0,0,1>$	$<0.28,0.03,0.69>$	$<0,0,1>$	$<0.78,0.04,0.18>$	$<0.09,0.02,0.89>$
C_3	$<0,0,1>$	$<0.09,0.03,0.88>$	$<0,0,1>$	$<0,0,1>$	$<0.08,0.04,0.88>$
C_4	$<0,0,1>$	$<0.55,0.15,0.3>$	$<0,0,1>$	$<0.08,0.03,0.89>$	$<0.09,0.03,0.88>$
C_5	$<0.85,0.15,0>$	$<0,0,1>$	$<0.03,0.28,0.42>$	$<0,0,1>$	$<0.18,0.03,0.79>$

C_6	$<0.04,0.02,0.94>$	$<0,0,1>$	$<0.04,0.22,0.38>$	$<0,0,1>$	$<0.08,0.05,0.87>$
C_7	$<0.04,0.03,0.93>$	$<0,0,1>$	$<0.08,0.05,0.87>$	$<0,0,1>$	$<0.08,0.05,0.87>$
C_8	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.08,0.04,0.88>$
C_9	$<0,0,1>$	$<0.08,0.05,0.87>$	$<0,0,1>$	$<0,0,1>$	$<0.08,0.04,0.88>$

Table 1. *The fault patterns with picture fuzzy number values (cont.)*

	A_6	A_7	A_8	A_9	A_{10}
C_1	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.85,0.08,0.07>$	$<0,0,1>$
C_2	$<0,0,1>$	$<0,0,1>$	$<0.27,0.05,0.68>$	$<0,0,1>$	$<0,0,1>$
C_3	$<0,0,1>$	$<0.08,0.04,0.88>$	$<0.08,0.04,0.88>$	$<0,0,1>$	$<0,0,1>$
C_4	$<0,0,1>$	$<0.86,0.07,0.07>$	$<0.54,0.08,0.38>$	$<0,0,1>$	$<0,0,1>$
C_5	$<0.18,0.04,0.78>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$
C_6	$<0.12,0.05,0.83>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.77,0.06,0.17>$
C_7	$<0.37,0.08,0.55>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.19,0.04,0.77>$
C_8	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0.08,0.04,0.88>$	$<0,0,1>$
C_9	$<0.22,0.06,0.72>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$	$<0,0,1>$

Table 2. Two fault sample with picture fuzzy number values

	B_1	B_2
C_1	(0,0,1)	(0.39,0,0.61)
C_2	(0,0,1)	(0.07,0,0.93)
C_3	(0.1,0,0.9)	(0,0,1)
C_4	(0.9,0,0.1)	(0.06,0,0.94)
C_5	(0,0,1)	(0,0,1)
C_6	(0,0,1)	(0.13,0,0.87)
C_7	(0,0,1)	(0,0,1)
C_8	(0,0,1)	(0,0,1)
C_9	(0,0,1)	(0.35,0,0.65)

Table 3. The similarity measures of $B_i\left(i=1,\ 2\right)$ *and* $A_j\left(j\ =\ 1,2,\ \ldots,\ 10\right)$

$s_0(B_i, A_j)$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
B_1	0.732	0.7714	0.6429	0.6863	0.6457	0.6137	**0.8886**	0.7963	0.6808	0.6602
B_2	0.6349	0.6041	0.5702	0.6304	0.6426	0.6522	0.6308	0.5927	**0.6551**	0.5876

5.Conclusion

In this article we have provided the new similarity measures between picture fuzzy sets. Next, we compare our proposed measure with some existing similarity measures. Finally, we apply them in the pattern recognition problem to find fault turbine. In future, we will find the new measures for picture fuzzy sets and apply it to the real problems.

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REFERENCES

[1] Atanassov, K., (1986). Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), 87-96.

[2] Atanassov, K., (2012). On Intuitionistic Fuzzy Sets Theory, Springer, Berlin, 2012.

[3] Atanassov,K. Gargov,G. , (1989) Interval valued intuitionistic fuzzy sets, *Fuzzy sets and systems*, *31*(3), 343-349.

[4] Cường,B.C., (2014). Picture fuzzy sets, *Journal of Computer Science and Cybernetics* 30.4 , 409-420.

[5] Dinh, NV., Thao,NX., Chau,N.M., (2015).On the picture fuzzy database: theories and application, Journal of Scientist and Development (2015), 13(6), 1028-1035.

[6] Dinh, N.V., Thao, N.X., Chau, N.M., (2017). Some dissimilarity measures of picture fuzzy set. *the 10th Fundamental and Applied IT Research (FAIR'2017)*, 104-109.

[7] Dinh, NV., Thao, N.X., (2018). Some measures of picture fuzzy sets and their application in multi-attribute decision making. *Int. J. Math. Sci. Comput.(IJMSC)*, *4*(3), 23-41.

[8] Dinh, N.V, Thao, N. X., & Chau, N. M. (2019). DISTANCE AND DISSIMILARITY MEASURE OF PICTURE FUZZY SETS. *PROCEEDING of Publishing House for Science and Technology*.

[9] Dutta, P., & Ganju, S. (2017). Some aspects of picture fuzzy set. *Transactions of A. Razmadze Mathematical Institute 172(2), 164-175*.

[10] Hoa, N. D., & Thong, P. H. (2017. Some Improvements of Fuzzy Clustering Algorithms Using Picture Fuzzy Sets and Applications for Geographic Data Clustering. *VNU Journal of Science: Computer Science and Communication Engineering*, *32*(3).

[11] Le, N. T., Van Nguyen, D., Ngoc, C. M., & Nguyen, T. X. (2018). NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS AND APPLICATIONS. *Journal of Computer Science and Cybernetics*, *34*(3), 219-231.

[12] Nguyen, X.T. (2018). Evaluating Water Reuse Applications under Uncertainty: A Novel Picture Fuzzy Multi Criteria Decision Making Medthod. *International Journal of Information Engineering and Electronic Business*, *10*(6), 32-39.

[13] Singh, P.(2015). Correlation coefficients for picture fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *28*(2), 591-604.

[14] Son, LH.(2015). DPFCM: A novel distributed picture fuzzy clustering method on picture fuzzy sets 42(1), 51-66.

[15] Son, L.H.(2016). Generalized picture distance measure and applications to picture fuzzy clustering. *Applied Soft Computing*, *46*(C), 284-295.

[16] Thao, N.X., & Dinh, N. V. (2015). Rough picture fuzzy set and picture fuzzy topologies. *Journal of Computer Science and Cybernetics*, *31*(3), 245.

[17] Thao, N.X., Smarandache, F.(2016). (I, T)-Standard neutrosophic rough set and its topologies properties. V14, pp 65-70.

[18] Thao, N.X., Cuong,B.C., Ali, M., Lan, L.H.(2018). Fuzzy equivalence on standard and rough neutrosophic sets and applications to clustering analysis. In *Information Systems Design and Intelligent Applications* (pp. 834-842). Springer, Singapore.

[19] Thao, N.X., Ali, M., Nhung, L.T., Gianey, H.K., Smarandache, F.(2019). A new multi-criteria decision making algorithm for medical diagnosis and classification problems using divergence measure of picture fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1-12.

[20] Thong, N.T.(2015). HIFCF: An effective hybrid model between picture fuzzy clustering and intuitionistic fuzzy recommender systems for medical diagnosis. *Expert Systems with Applications*, *42*(7), 3682-3701.

[21] Thong, P.H. (2016). Picture fuzzy clustering: a new computational intelligence method. *Soft computing*, *20*(9), 3549-3562.

[22] Ye, J. (2017). Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing*, *21*(3), 817-825.

[23] Wang, C., Zhou, X., Tu, H., Tao, S. (2017). Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. *Italian Journal of Pure and Applied Mathematics*, *37*, 477-492.

[24] Wei, G. (2016). Picture fuzzy cross-entropy for multiple attribute decision making problems. *Journal of Business Economics and Management*, *17*(4), 491-502.

[25] Wei, G., (2017). Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making. *Informatica*, *28*(3), 547-564.

[26] Wei, G.(2018). Some similarity measures for picture fuzzy sets and their applications. *Iranian Journal of Fuzzy Systems*, *15*(1), 77-89.

[27] Zadeh,L. A.(1965). Fuzzy sets, *Information and control* *8*(3), 338-353.