# Neutrosophic Vague Incidence Graph

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## Abstract

Vague sets gives more intuitive graphical notation of vague data, that devotes better analysis in information relationships, incompleteness and similarity measures. Neutrosophic graphs are used as a mathematical tool to kept an imprecise and unspecified information. In this paper, the neutrosophic vague incidence graphs are introduced. The edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic vague incidence graphs are established. The given results are illustrated with suitable example.
**Keywords:** Neutrosophic vague incidence graph, Edge-connectivity, Vertex-connectivity and Pair-connectivity.

## 1 Introduction

Vague sets are denoted as a higher-order fuzzy sets which develops the solution procedure more complex to obtain the results more accurate than fuzzy but not affecting the complexity on computation time/volume and memory space. The restrictions in vague sets allow only to hold an incomplete data, but the handling of indeterminate information still remains. Can we see an instance, suppose there are 10 patients to check a pandemic during testing. In that time, there are five patients having positive, three will have negative and two are undecided or yet to come. By employing the neutrosophic concepts, it can be expressed as $x(0.5, 0.2, 0.3)$. Hence the neutrosophic field arises to hold the indeterminacy data. It generalizes the aforementioned sets from the philosophical viewpoint. The single-valued neutrosophic set is the generalisation of intuitionistic fuzzy sets and is used expediently to deal with real-world problems, especially in decision support.[1–5,20,21,40] The computation of believe in that element (truth), the disbelieve in that element (falsehood) and the indeterminacy part of that element with the sum of these three components are strictly less than 1. Neutrosophic sets are the base of neutrosophic logic, a multiple value logic that generalizes the fuzzy logic which deals with paradoxes, contradictions, antitheses, antinomies is proposed by Smarandache[34,36] and references therein.

The neutrosophic set is introduced by the author Smarandache in order to use the inconsistent and indeterminate information, and has been studied extensively (see[34–40]). In the definition of neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are defined completely independent with the sum of these values lies between 0 and 3. Neutrosophic set and related notions paid attention by the researchers in many weird domains.[9,10] The combination of neutrosophic set and vague set are introduced by Alkhazaleh in 2015.[11] Single valued neutrosophic graph are established in the papers.[16,17] Some types of neutrosophic graphs and co-neutrosophic graphs are discussed in.[23] Intuitionistic bipolar neutrosophic set and its application to graphs are established in.[31] Al-Quran and Hassan in[8] introduced a combination of neutrosophic vague set and soft expert set to improving the reason-ability of decision making in real life application. Neutrosophic vague graphs are investigated in.[30] Comparative study of regular and (highly) irregular vague graphs with applications are obtained in.[18] Furthermore, some properties of degree of vague graphs, domination number and regularity properties of vague graphs are established by the author Borzooei.[12–14] Authors in[7] presented some properties of single-valued neutrosophic incidence graphs and discussed the edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic incidence graphs. Motivated by papers,[7,11,28,30] we introduce the concept of neutrosophic

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vague incidence graphs. The main contributions of this paper are to introduce the neutrosophic vague incidence graphs, and the edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic vague incidence graphs are discussed in Section 3.

## 2 Preliminaries

In this section, basic definitions and example are given.

**Definition 2.1.** [41] A vague set $\mathbb{A}$ on a non empty set $\mathbb{X}$ is a pair $(T_{\mathbb{A}}, F_{\mathbb{A}})$, where $T_{\mathbb{A}} : \mathbb{X} \to [0,1]$ and $F_{\mathbb{A}} : \mathbb{X} \to [0,1]$ are true membership and false membership functions, respectively, such that

$$0 \leq T_{\mathbb{A}}(x) + F_{\mathbb{A}}(y) \leq 1 \text{ for any } x \in \mathbb{X}.$$

Let $\mathbb{X}$ and $\mathbb{Y}$ be two non-empty sets. A vague relation $\mathbb{R}$ of $\mathbb{X}$ to $\mathbb{Y}$ is a vague set $\mathbb{R}$ on $\mathbb{X} \times \mathbb{Y}$ that is $\mathbb{R} = (T_{\mathbb{R}}, F_{\mathbb{R}})$, where $T_{\mathbb{R}} : \mathbb{X} \times \mathbb{Y} \to [0,1]$, $F_{\mathbb{R}} : \mathbb{X} \times \mathbb{Y} \to [0,1]$ and satisfy the condition:

$$0 \leq T_{\mathbb{R}}(x,y) + F_{\mathbb{R}}(x,y) \leq 1 \text{ for any } x \in \mathbb{X}.$$

**Definition 2.2.** [12] Let $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ be a graph. A pair $\mathbb{G} = (\mathbb{J}, \mathbb{K})$ is called a vague graph on $\mathbb{G}^*$, where $\mathbb{J} = (T_{\mathbb{J}}, F_{\mathbb{J}})$ is a vague set on $\mathbb{V}$ and $\mathbb{K} = (T_{\mathbb{K}}, F_{\mathbb{K}})$ is a vague set on $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ such that for each $xy \in \mathbb{E}$,

$$T_{\mathbb{K}}(xy) \leq \min(T_{\mathbb{J}}(x), T_{\mathbb{J}}(y)) \text{ and } F_{\mathbb{K}}(xy) \geq \max(F_{\mathbb{J}}(x), F_{\mathbb{J}}(y)).$$

**Definition 2.3.** [19,34] Let $\mathbb{X}$ be a space of points (objects), with a generic elements in $\mathbb{X}$ denoted by $x$. A single valued neutrosophic set $\mathbb{A}$ in $\mathbb{X}$ is characterised by truth-membership function $T_{\mathbb{A}}(x)$, indeterminacy-membership function $I_{\mathbb{A}}(x)$ and falsity-membership-function $F_{\mathbb{A}}(x)$, For each point $x$ in $\mathbb{X}$, $T_{\mathbb{A}}(x), F_{\mathbb{A}}(x), I_{\mathbb{A}}(x) \in [0,1]$. Also

$$A = \{x, T_{\mathbb{A}}(x), F_{\mathbb{A}}(x), I_{\mathbb{A}}(x)\} \text{ and } 0 \leq T_{\mathbb{A}}(x) + I_{\mathbb{A}}(x) + F_{\mathbb{A}}(x) \leq 3.$$

**Definition 2.4.** [34] A Neutrosophic set $\mathbb{A}$ is contained in another neutrosophic set $\mathbb{B}$, (i.e) $\mathbb{A} \subseteq \mathbb{B}$ if $\forall x \in \mathbb{X}, T_{\mathbb{A}}(x) \leq T_{\mathbb{B}}(x), I_{\mathbb{A}}(x) \geq I_{\mathbb{B}}(x)$and $F_{\mathbb{A}}(x) \geq F_{\mathbb{B}}(x)$.

**Definition 2.5.** [6,17] A neutrosophic graph is defined as a pair $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ where
(i) $\mathbb{V} = \{v_1, v_2, .., v_n\}$ such that $T_1 : \mathbb{V} \to [0,1]$, $I_1 : \mathbb{V} \to [0,1]$ and $F_1 : \mathbb{V} \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

$$0 \leq T_1(v) + I_1(v) + F_1(v) \leq 3,$$

(ii) $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ where $T_2 : \mathbb{E} \to [0,1]$, $I_2 : \mathbb{E} \to [0,1]$ and $F_2 : \mathbb{E} \to [0,1]$ are such that

$$\begin{gathered} T_2(uv) \leq \min\{T_1(u), T_1(v)\}, \\ I_2(uv) \leq \min\{I_1(u), I_1(v)\}, \\ F_2(uv) \leq \max\{F_1(u), F_1(v)\}, \\ \text{and } 0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3, \quad \forall uv \in \mathbb{E}. \end{gathered}$$

**Definition 2.6.** [11] A neutrosophic vague set $\mathbb{A}_{NV}$ (NVS in short) on the universe of discourse $\mathbb{X}$ written as

$$\mathbb{A}_{NV} = \{\langle x, \hat{T}_{\mathbb{A}_{NV}}(x), \hat{I}_{\mathbb{A}_{NV}}(x), \hat{F}_{\mathbb{A}_{NV}}(x)\rangle, x \in \mathbb{X}\},$$

whose truth-membership, indeterminacy membership and falsity-membership function is defined as

$$\hat{T}_{\mathbb{A}_{NV}}(x) = [T^-(x), T^+(x)], \hat{I}_{\mathbb{A}_{NV}}(x) = [I^-(x), I^+(x)] \text{ and } \hat{F}_{\mathbb{A}_{NV}}(x) = [F^-(x), F^+(x)],$$

where $T^+(x) = 1 - F^-(x), F^+(x) = 1 - T^-(x)$, and $0 \leq T^-(x) + I^-(x) + F^-(x) \leq 2$.

**Definition 2.7.** [11] The complement of NVS $\mathbb{A}_{NV}$ is denoted by $\mathbb{A}^c_{NV}$ and it is defined by

$$\begin{gathered} \hat{T}^c_{\mathbb{A}_{NV}}(x) = [1 - T^+(x), 1 - T^-(x)], \\ \hat{I}^c_{\mathbb{A}_{NV}}(x) = [1 - I^+(x), 1 - I^-(x)], \\ \hat{F}^c_{\mathbb{A}_{NV}}(x) = [1 - F^+(x), 1 - F^-(x)]. \end{gathered}$$

**Definition 2.8.** [11] Let $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ be two NVSs of the universe $\mathbb{U}$. If for all $u_i \in \mathbb{U}$,

$$\hat{T}_{\mathbb{A}_{NV}}(u_i) \leq \hat{T}_{\mathbb{B}_{NV}}(u_i), \hat{I}_{\mathbb{A}_{NV}}(u_i) \geq \hat{I}_{\mathbb{B}_{NV}}(u_i), \hat{F}_{\mathbb{A}_{NV}}(u_i) \geq \hat{F}_{\mathbb{B}_{NV}}(u_i),$$

then the NVS, $\mathbb{A}_{NV}$ are included in $\mathbb{B}_{NV}$, denoted by $\mathbb{A}_{NV} \subseteq \mathbb{B}_{NV}$ where $1 \leq i \leq n$.

**Definition 2.9.** [11] The union of two NVSs $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ is a NVSs, $\mathbb{C}_{NV}$, written as $\mathbb{C}_{NV} = \mathbb{A}_{NV} \cup \mathbb{B}_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ by

$$\hat{T}_{\mathbb{C}_{NV}}(x) = [\max(T^-_{\mathbb{A}_{NV}}(x), T^-_{\mathbb{B}_{NV}}(x)), \max(T^+_{\mathbb{A}_{NV}}(x), T^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\min(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \min(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\min(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \min(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

**Definition 2.10.** [11] The intersection of two NVSs, $A_{NV}$ and $B_{NV}$ is a NVSs $C_{NV}$, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of $A_{NV}$ and $B_{NV}$ by

$$\hat{T}_{\mathbb{C}_{NV}}(x) = [\min(T^-_{\mathbb{A}_{NV}}(x), T^-_{\mathbb{B}_{NV}}(x)), \min(T^+_{\mathbb{A}_{NV}}(x), T^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\max(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \max(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\max(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \max(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

**Definition 2.11.** [30] Let $\mathbb{G}^* = (\mathbb{R}, \mathbb{S})$ be a graph. A pair $\mathbb{G} = (\mathbb{A}, \mathbb{B})$ is called a neutrosophic vague graph (NVG) on $\mathbb{G}^*$ or a neutrosophic vague graph where $\mathbb{A} = (\hat{T}_{\mathbb{A}}, \hat{I}_{\mathbb{A}}, \hat{F}_{\mathbb{A}})$ is a neutrosophic vague set on $\mathbb{R}$ and $\mathbb{B} = (\hat{T}_{\mathbb{B}}, \hat{I}_{\mathbb{B}}, \hat{F}_{\mathbb{B}})$ is a neutrosophic vague set $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ where

(1)$\mathbb{R} = \{v_1, v_2, ..., v_n\}$ such that $T^-_{\mathbb{A}} : \mathbb{R} \to [0,1], I^-_{\mathbb{A}} : \mathbb{R} \to [0,1], F^-_{\mathbb{A}} : \mathbb{R} \to [0,1]$ which satisfies the condition $F^-_{\mathbb{A}} = [1 - T^+_{\mathbb{A}}]$
$T^+_{\mathbb{A}} : \mathbb{R} \to [0,1], I^+_{\mathbb{A}} : \mathbb{R} \to [0,1], F^+_{\mathbb{A}} : \mathbb{R} \to [0,1]$ which satisfies the condition $F^+_{\mathbb{A}} = [1 - T^-_{\mathbb{A}}]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in \mathbb{R}$, and

$$0 \leq T^-_{\mathbb{A}}(v_i) + I^-_{\mathbb{A}}(v_i) + F^-_{\mathbb{A}}(v_i) \leq 2$$
$$0 \leq T^+_{\mathbb{A}}(v_i) + I^+_{\mathbb{A}}(v_i) + F^+_{\mathbb{A}}(v_i) \leq 2.$$

(2) $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ where

$$T^-_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1], I^-_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1], F^-_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1]$$
$$T^+_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1], I^+_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1], F^+_{\mathbb{B}} : \mathbb{R} \times \mathbb{R} \to [0,1]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in \mathbb{S}$, respectively and such that,

$$0 \leq T^-_{\mathbb{B}}(v_iv_j) + I^-_{\mathbb{B}}(v_iv_j) + F^-_{\mathbb{B}}(v_iv_j) \leq 2$$
$$0 \leq T^+_{\mathbb{B}}(v_iv_j) + I^+_{\mathbb{B}}(v_iv_j) + F^+_{\mathbb{B}}(v_iv_j) \leq 2,$$

such that

$$T^-_{\mathbb{B}}(v_iv_j) \leq \min\{T^-_{\mathbb{A}}(v_i), T^-_{\mathbb{A}}(v_j)\}$$
$$I^-_{\mathbb{B}}(v_iv_j) \leq \min\{I^-_{\mathbb{A}}(v_i), I^-_{\mathbb{A}}(v_j)\}$$
$$F^-_{\mathbb{B}}(v_iv_j) \leq \max\{F^-_{\mathbb{A}}(v_i), F^-_{\mathbb{A}}(v_j)\},$$

and similarly

$$T^+_{\mathbb{B}}(v_iv_j) \leq \min\{T^+_{\mathbb{A}}(v_i), T^+_{\mathbb{A}}(v_j)\}$$
$$I^+_{\mathbb{B}}(v_iv_j) \leq \min\{I^+_{\mathbb{A}}(v_i), I^+_{\mathbb{A}}(v_j)\}$$
$$F^+_{\mathbb{B}}(v_iv_j) \leq \max\{F^+_{\mathbb{A}}(v_i), F^+_{\mathbb{A}}(v_j)\}.$$

**Definition 2.12.** [7] A neutrosophic incidence graph of an incidence graph, $G^* = (V, E, I)$, is an ordered triplet, $\tilde{G} = (A, B, C)$, such that
1. $A$ is a neutrosophic set on $V$,
2. $B$ is a neutrosophic relation on $V$ and
3. $C$ is a neutrosophic subset of $V \times E$ such that

$$\begin{aligned} T_C(x, xy) &\leq \min\{T_A(x), T_B(xy)\}, \\ I_C(x, xy) &\leq \min\{I_A(x), I_B(xy)\}, \\ F_C(x, xy) &\leq \max\{F_A(x), F_B(xy)\}, \text{ for all } xy \in E \end{aligned}$$

# 3 Neutrosophic Vague incidence graph Graphs

In this section, the definition of NVIGs are introduced. Some properties on edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic vague incidence graphs are established.

**Definition 3.1.** A neutrosophic vague incidence graph of an incidence graph $G = (\mathbb{V}, \mathbb{E}, \mathbb{I})$, is an ordered triplet, $G^* = (Q, R, S)$, such that
1. $Q$ is a neutrosophic vague set on $\mathbb{V}$,
2. $R$ is a neutrosophic vague relation on $\mathbb{V}$ and
3. $S$ is a neutrosophic vague subset of $\mathbb{V} \times \mathbb{E}$ such that

$$\begin{aligned} T_S^-(a, ab) &\leq \min\{T_Q^-(a), T_R^-(ab)\}, \\ I_S^-(a, ab) &\leq \min\{I_Q^-(a), I_R^-(ab)\}, \\ F_S^-(a, ab) &\leq \max\{F_Q^-(a), F_R^-(ab)\}, \end{aligned}$$

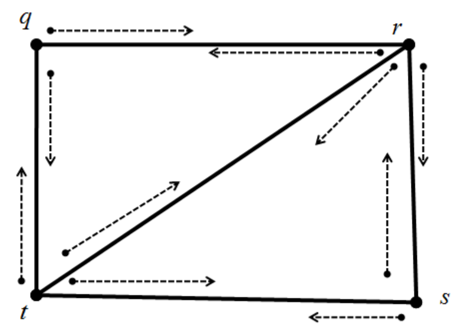
similarly

$$\begin{aligned} T_S^+(a, ab) &\leq \min\{T_Q^+(a), T_R^+(ab)\}, \\ I_S^+(a, ab) &\leq \min\{I_Q^+(a), I_R^+(ab)\}, \\ F_S^+(a, ab) &\leq \max\{F_Q^+(a), F_R^+(ab)\}, \forall a \in \mathbb{V}, ab \in \mathbb{E}. \end{aligned}$$

**Example 3.2.** Consider an incidence graph $G = (\mathbb{V}, \mathbb{E}, \mathbb{I})$ such that $\mathbb{V} = \{q, r, s, t\}$, $\mathbb{E} = \{qr, rs, rt, st, qt\}$ and $\mathbb{I} = \{(q, qr), (r, qr), (r, rs), (s, rs), (r, rt), (t, rt), (s, st), (t, st), (q, qt), (t, qt)\}$, as shown in figure 1
Let $G^* = (Q, R, S)$ be a neutrosophic vague incidence graph associated with $G$, as shown in figure 3, where $q = [0.5, 0.6], [0.4, 0.4], [0.4, 0.5]$, $r = [0.3, 0.3], [0.5, 0.6], [0.7, 0.7]$, $s = [0.6, 0.5], [0.3, 0.4], [0.5, 0.4]$, $t = [0.4, 0.7], [0.5, 0.6], [0.3, 0.6]$
$q^- = (0.5, 0.4, 0.4), q^+ = (0.6, 0.4, 0.5)$ $r^- = (0.3, 0.5, 0.7), r^+ = (0.3, 0.6, 0.7)$
$s^- = (0.6, 0.3, 0.5), q^+ = (0.5, 0.4, 0.4)$ $t^- = (0.3, 0.6, 0.7), q^+ = (0.7, 0.6, 0.6)$

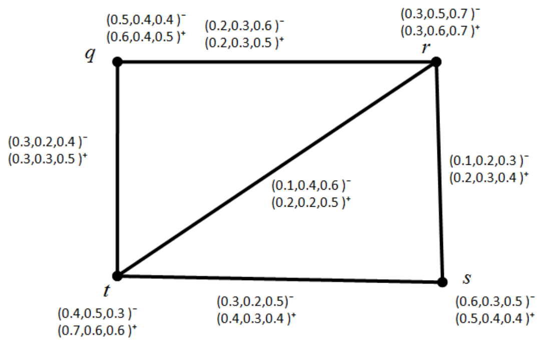
$$\begin{aligned} &Q = \{q^- = (0.5, 0.4, 0.4), q^+ = (0.6, 0.4, 0.5), r^- = (0.3, 0.5, 0.7), r^+ = (0.3, 0.6, 0.7), \\ &\quad s^- = (0.6, 0.3, 0.5), s^+ = (0.5, 0.4, 0.4), t^- = (0.3, 0.6, 0.7), t^+ = (0.7, 0.6, 0.6)\} \\ &R = \{(qr)^- = (0.2, 0.3, 0.6), (qr)^+ = (0.2, 0.3, 0.5), (st)^- = (0.3, 0.2, 0.5), (st)^+ = (0.4, 0.3, 0.4), \\ &\quad (rs)^+ = (0.1, 0.2, 0.3), (rs)^+ = (0.2, 0.3, 0.4)(qt)^- = (0.3, 0.2, 0.4), (qt)^+ = (0.3, 0.3, 0.5), \\ &\quad (rt)^- = (0.1, 0.4, 0.6), (rt)^+ = (0.2, 0.2, 0.5)\} \\ &S = \{(q, qr)^- = (0.2, 0.2, 0.5), (q, qr)^+ = (0.2, 0.1, 0.4), (r, qr)^- = (0.1, 0.3, 0.5), (r, qr)^+ = (0.1, 0.2, 0.4), \\ &\quad (r, rs)^- = (0.1, 0.2, 0.6), (r, rs)^+ = (0.1, 0.3, 0.4), (s, rs)^- = (0.1, 0.2, 0.5), (s, rs)^+ = (0.1, 0.3, 0.3), \\ &\quad (r, rt)^- = (0.1, 0.3, 0.6), (r, rt)^+ = (0.2, 0.1, 0.6), (t, rt)^- = (0.1, 0.3, 0.5), (t, rt)^+ = (0.2, 0.1, 0.5), \\ &\quad (s, st)^- = (0.2, 0.1, 0.4), (s, st)^+ = (0.3, 0.2, 0.3)(t, st)^- = (0.2, 0.1, 0.4), (t, st)^+ = (0.3, 0.2, 0.3), \\ &\quad (q, qt)^- = (0.2, 0.1, 0.3), (q, qt)^+ = (0.2, 0.2, 0.4), (t, qt)^- = (0.2, 0.2, 0.3), (t, qt)^+ = (0.2, 0.2, 0.5)\} \end{aligned}$$

**Definition 3.3.** The support of an NVIG $G^* = (Q, R, S)$ is denoted by $G^{**} = (Q^*, R^*, S^*)$ where
$Q^* =$ support of $Q = \{a \in \mathbb{V} : \hat{T}_Q(a) > 0, \hat{I}_Q(a) > 0, \hat{F}_Q(a) > 0\}$
$R^* =$ support of $R = \{ab \in \mathbb{E} : \hat{T}_R(ab) > 0, \hat{I}_R(ab) > 0, \hat{F}_R(ab) > 0\}$
$S^* =$ support of $S = \{(a, ab) \in \mathbb{I} : \hat{T}_S(a, ab) > 0, \hat{I}_S(a, ab) > 0, \hat{F}_S(a, ab) > 0\}$.



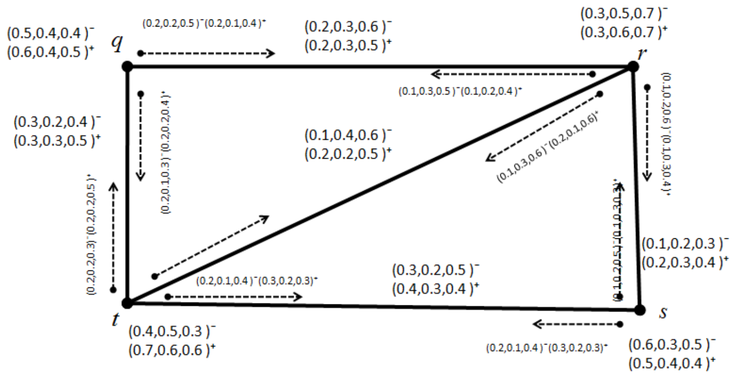

$Figure\,1$

Incidence graph




$Figure\,2$

Neutrosophic Vague Graph




$Figure\,3$

Neutrosophic Vague Incidence Graph

**Definition 3.4.** If $ab \in R^*$, then $ab$ is the edges of the NVIG $G^* = (Q, R, S)$ and if $(a, ab), (b, ab) \in S^*$ then $(a, ab)$ and $(b, ab)$ are called pair of $G^*$.

**Definition 3.5.** Suppose $P = a_0, (a_0, a_0a_1), a_0a_1, (a_1, a_0, a_1), a_1, (a_1, a_1a_2), a_1a_2, (a_2, a_1, a_2), ...a_{n-1}, (a_{n-1}, a_{n-1}a_n), a_{n-1}a_n, (a_n, a_{n-1}, a_n)$ of vertices, edges and pairs in $G^*$ is a walk. It is a closed walk if $a_0 = a_n$. In the above sequence, if all edges are distinct, then it is trail, and if the pairs are distinct, then it is an incidence trail. $P$ is called a path, if the vertices are distinct. A path is called a cycle if the initial and end vertices of the path are same. Any two vertices of $G^*$ are said to be connected, if they are joined by a path.

**Example 3.6.** In the above example, one can see that
$P_1 = q, (q, qr), qr, (r, qr), r, (r, rs), rs, (s, rs), t, (t, tq), tq, (q, tq), q$ is a walk. It is a closed walk since the initial and final vertices are same. (i.e) it is not a path, but it is a trail and incidence trail
$P_2 = q, (q, qr), qr, (r, qr), r, (r, rs), rt, (t, rt), t$. Then, $P_2$ is a walk, path trail and incidence trail.

**Definition 3.7.** Let $G^* = (Q, R, S)$ be an NVIG, then $H = (L, M, N)$ is a neutrosophic vague incidence subgraph of $G^*$, if $L \subseteq Q, M \subseteq R$ and $N \subseteq S$. Also, $H$ is a neutrsophic incidence spanning subgraph of $G^*$, if $L = Q$.

**Definition 3.8.** In an NVIG, the strength of a path, $P$ is an ordered triplet denoted by $\mathbb{S}(P) = (s_1, s_2, s_3)$, where

$$s_1 = \min\{\hat{T}_R(ab) : ab \in P\}, s_2 = \min\{\hat{I}_R(ab) : ab \in P\}, s_3 = \max\{\hat{F}_R(ab) : ab \in P\}.$$

Similarly, the incidence strength of a path, $P$, in an NVG is denoted by $I\mathbb{S}(P) = (is_1, is_2, is_3)$, where

$$is_1 = \min\{\hat{T}_S(ab) : (a, ab) \in P\}, is_2 = \min\{\hat{I}_S(ab) : (a, ab) \in P\}, is_3 = \max\{\hat{F}_S(ab) : (a, ab) \in P\}.$$

**Definition 3.9.** In an NVG, $G^* = (Q, R, S)$ the greatest strength of the path from $l$ to $m$, where $l, m \in Q^* \cup R^*$ is the maximum of strength of all paths from $l$ to $m$.

$$\begin{aligned}\mathbb{S}^\infty(l, m) &= \max\{\mathbb{S}(P_1), \mathbb{S}(P_2), \mathbb{S}(P_3), ...\}\\ &= (s_1^\infty, s_2^\infty, s_3^\infty)\\ &= (\max(s_{11}, s_{12}, s_{13}, ....), \max(s_{21}, s_{22}, s_{23}, ...), \min(s_{31}, s_{32}, s_{33}, ..)),\end{aligned}$$

$\mathbb{S}^\infty(l, m)$ is sometimes called the connectedness between $l$ and $m$.
Similarly, the greatest incidence strength of the path from $l$ to $m$, where $l, m \in Q^* \cup R^*$ is the maximum of incidence strength of all paths from $l$ to $m$.

$$\begin{aligned}I\mathbb{S}^\infty(l, m) &= \max\{I\mathbb{S}(P_1), I\mathbb{S}(P_2), I\mathbb{S}(P_3), ...\}\\ &= (is_1^\infty, is_2^\infty, is_3^\infty)\\ &= (\max(is_{11}, is_{12}, is_{13}, ....), \max(is_{21}, is_{22}, is_{23}, ...), \min(is_{31}, is_{32}, is_{33}, ..)),\end{aligned}$$

where $P_j, j = 1, 2, 3, ...$ are different paths from $l$ to $m$.
$I\mathbb{S}^\infty(l, m)$ is sometimes represented as the incidence connectedness between $l$ to $m$.

**Definition 3.10.** An NVG, $G^* = (Q, R, S)$ is a cycle if and only if, the underlying graph, $G^{**} = (Q^*, R^*, S^*)$ is a cycle.

**Definition 3.11.** An NVG, $G^* = (Q, R, S)$ is a is a neutrosophic vague cycle if and only if, $G^{**} = (Q^*, R^*, S^*)$ is a cycle and there exist no unique edge, $ab \in R^*$ such that

$$\begin{aligned}\hat{T}_R(xy) &= \min\{\hat{T}_R(ab) : ab \in R^*\},\\ \hat{I}_R(xy) &= \min\{\hat{I}_R(ab) : ab \in R^*\},\\ \hat{F}_R(xy) &= \max\{\hat{F}_R(ab) : ab \in R^*\}.\end{aligned}$$

**Definition 3.12.** An NVG, $G^* = (Q, R, S)$ is a neutrosophic vague incidence cycle if and only if, $G^{**} = (Q^*, R^*, S^*)$ is a cycle and there exist no unique edge, $ab \in S^*$ such that

$$\begin{aligned}\hat{T}_S(x, xy) &= \min\{\hat{T}_S(a, ab) : ab \in S^*\},\\ \hat{I}_S(x, xy) &= \min\{\hat{I}_S(a, ab) : ab \in S^*\},\\ \hat{F}_S(x, xy) &= \max\{\hat{F}_S(a, ab) : ab \in S^*\}.\end{aligned}$$

**Definition 3.13.** Let $G^* = (Q, R, S)$ be an NVIG. An edge $ab$ in $G$ is called a bridge if and only if, $ab$ is a bridge in $G^{**} = (Q^*, R^*, S^*)$ that is, the removal of $ab$ disconnectes $G^{**}$. An edge, $ab$ is called a neutrosophic vague bridge if

$$\begin{aligned}\mathbb{S}'^\infty(x, y) &< \mathbb{S}^\infty(x, y), \text{ for some } x, y \in Q^*\\ (s_1'^\infty, s_2'^\infty, s_3'^\infty, ) &< (s_1^\infty, s_2^\infty, s_3^\infty, )\\ \Rightarrow s_1'^\infty &< s_1^\infty,\ s_2'^\infty < s_2^\infty,\ s_3'^\infty > s_3^\infty,\end{aligned}$$

where $\mathbb{S}'^{\infty}$ and $\mathbb{S}^{\infty}$ denote the connectedness between $x$ and $y$ in $G' = G^* - ab$ respectively.
An edge $ab$ is called a neutrosophic vague incidence bridge if

$$I\mathbb{S}'^{\infty}(x,y) < I\mathbb{S}^{\infty}(x,y), \text{ for some } x, y \in Q^*$$
$$(is_1'^{\infty}, is_2'^{\infty}, is_3'^{\infty},) < (is_1^{\infty}, is_2^{\infty}, is_3^{\infty},)$$
$$\Rightarrow is_1'^{\infty} < is_1^{\infty},\ is_2'^{\infty} < is_2^{\infty},\ is_3'^{\infty} > is_3^{\infty},$$

where $I\mathbb{S}'^{\infty}$ and $I\mathbb{S}^{\infty}$ denote the connectedness between $x$ and $y$ in $G' = G^* - ab$ respectively.

**Definition 3.14.** Let $G^* = (Q, R, S)$ be an NVG. A vertex $v$ in $G^*$ is a cutvertex if and only if it is a cutvertex in $G^{**} = (Q^*, R^*, S^*)$ that is $G^* - v$ is a disconnected graph.
A vertex $v$ in an NVIG is called a neutrosophic vague cutvertex if the connectedness between any two vertices in $G' = G^* - v$ is less than the conncetedness between the same vertices in $G^*$ that is,

$$\mathbb{S}'^{\infty}(x,y) < \mathbb{S}^{\infty}(x,y), \text{ for some } x, y \in Q^*$$

A vertex $v$ in NVIG $G^*$ is a neutrosophic vague incidence cutvertex if for any pair of vertices, $x, y$ other than $v$ the following condition holds:

$$I\mathbb{S}'^{\infty}(x,y) < I\mathbb{S}^{\infty}(x,y), \text{ for some } x, y \in Q^*$$

where $I\mathbb{S}'^{\infty}$ and $I\mathbb{S}^{\infty}$ denote the connectedness between $x$ and $y$ in $G' = G^* - ab$ respectively.

**Definition 3.15.** Let $G^* = (Q, R, S)$ be an NVIG. A pair $(a, ab)$ is called a cutpair if and only if, $(a, ab)$ is a cutpair in $G^{**} = (Q^*, R^*, S^*)$ that is after removing the pair $(a, ab)$ there is no path between $a$ and $ab$. Let $G^* = (Q, R, S)$ be an NVIG. A pair (a,ab) is called a neutrosophic vague cutpair if deleting the pair $(a, ab)$ reduces the connectedness between $a, ab \in Q^* \cup R^*$ that is

$$\mathbb{S}'^{\infty}(a, ab) < \mathbb{S}^{\infty}(a, ab),$$

where $\mathbb{S}'^{\infty}(a, ab)$ and $\mathbb{S}^{\infty}(a, ab)$ denote the connectedness between $a$ and $ab$ in $G' = G^* - \{(a, ab)\}$ and $G^*$ respectively.
A pair $(a, ab)$ is called neutrosophic vague incidence cutpair if

$$I\mathbb{S}'^{\infty}(a, ab) < I\mathbb{S}^{\infty}(a, ab),$$

for $a, ab \in Q^* \cup R^*$
where $I\mathbb{S}'^{\infty}(a, ab)$ and $I\mathbb{S}^{\infty}(a, ab)$ denotes the connectedness between $a$ and $ab$ in $G' = G^* - \{(a, ab)\}$ and $G^*$ respectively.

**Theorem 3.16.** *Let $G^* = (Q, R, S)$ be a NVIG. If $ab$ is a neutrosophic bridge, then $ab$ is not a weakest edge in any cycle.*

*Proof.* Let $ab$ be a neutrosophic vague bridge and suppose, on the contrary that $ab$ is the weakest edge of a cycle. Then, in this cycle, we can find an alternative path, $P_1$ from $a$ to $b$ that does not contain the edge $ab$ and $\mathbb{S}P_1$ is greater than of equal to $\mathbb{S}P_2$, where $P_2$ is the path involving the edge $ab$. Thus, removal of the edge $ab$ from $G^*$ does not affect the connectedness between $a$ and $v - a$ contradiction to our assumption. Hence, $ab$ is not the weakest edge in any cycle. □

**Theorem 3.17.** *If $(a, ab)$ is a neutrosophic vague incidence cutpair, then $(a, ab)$ is not the weakest pair any cycle.*

*Proof.* Let $(a, ab)$ be a neutrosophic vague incidence cutpair in $G^*$. On contrary, suppose that $(a, ab)$ is a weakest pair of a cycle. Then we can find an alternative path from $a$ and $ab$ having incidence strength greater than or equal to that of the path involving the pair $(a, ab)$. Thus, removal of the pair $(a, ab)$ does not affect the incidence connectedness between $a$ and $ab$, but this is a contradiction to our assumption that $(a, ab)$ is a neutrosophic vague incidence cutpair. Hence $(a, ab)$ is not a weakest pair in any cycle. □

**Theorem 3.18.** *Let $G^* = (Q, R, S)$ be a NVIG. If $ab$ is a neutrosophic vague bridge in $G^*$, then*

$$\mathbb{S}^{\infty}(a,b) = (s_1^{\infty}, s_2^{\infty}, s_3^{\infty}) = (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab))$$

*Proof.* Let $G^*$ be an NVIG and $ab$ is a neutrosophic vague bridge in $G^*$. On the contrary, suppose that

$$\mathbb{S}^{\infty}(a,b) > (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab))$$

Then, there exists a $a-b$ path, $P$ with

$$\mathbb{S}(P) > (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab))$$

and

$$(\hat{T}_R(xy), \hat{I}_R(xy), \hat{F}_R(xy)) > (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab))$$

for all edges on path $P$. Now, $P$ together with the edge, $ab$ forms a cycle in which $ab$ is the weakest edge, but it is a contradiction to the fact that $ab$ is a neutrosophic vague bridge. Hence

$$\mathbb{S}^{\infty}(a,b) = (s_1^{\infty}, s_2^{\infty}, s_3^{\infty}) = (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab))$$

. □

**Theorem 3.19.** *If $(a, ab)$ is a neutrosophic vague incidence cutpair in an NVIG $G^* = (Q, R, S)$ then*

$$I\mathbb{S}^{\infty}(a,ab) = (is_1^{\infty}, is_2^{\infty}, is_3^{\infty}) = (\hat{T}_S(a,ab), \hat{I}_S(a,ab), \hat{F}_S(a,ab))$$

.

*Proof.* The proof is on the same line as the above theorem. □

**Theorem 3.20.** *Let $G^* = (Q, R, S)$ be an NVIG and $G^{**}$ is a cycle. then an edge $ab$ is a neutrosophic vague bridge of $G^*$ if and only if it is an edge common to two neutrosophic vague incidence cutpairs.*

*Proof.* Suppose that $ab$ is a neutrosophic vague bridge of $G^*$. Then there exist vertices $a$ and $b$ with the $ab$ edge lying on every path with the greatest incidence strength between $a$ and $b$. Consequently, there exists only one path, $P(say)$ between $a$ and $b$ which contains a $ab$ edge and has the greatest incidence strength. Any pair on $P$ will be a neutrosophic vague incidence cutpair, since the removal of any one of them will disconnect $P$ and reduce the incidence strength. Conversely, let $ab$ be an edge common to two neutrosophic vague incidence cutpairs $(a, ab)$ and $(b, ab)$. Thus both $(a, ab)$ and $(b, ab)$ are not the weakest cutpair of $G^*$. Now, $G^{**}$ being a cycle, there exists only two paths between any two vertices. Also the path $P_1$ from the vertex $a$ and $b$ not containing the pairs $(a, ab)$ and $(b, ab)$ has less incidence strength than the path containing them. Thus, the path with the greatest incidence strength from $a$ to $b$ is

$$P_2 : a, (a, ab), ab, (b, ab), b.$$

Also,

$$\mathbb{S}^{\infty}(a,b) = \mathbb{S}(P_2) = (\hat{T}_R(ab), \hat{I}_R(ab), \hat{F}_R(ab)).$$

Therefore, $ab$ is a neutrosophic vague bridge. □

# 4 Conclusion

In this work, the neutrosophic vague incidence graphs have been introduced. The edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic vague incidence graphs have been established. The given results are illustrated with suitable example. In future, intuitionistic neutrosophic incidence graphs and neutrosophic soft incidence graphs with their properties will be developed.

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