



Novel Idea of NG_δ - α -Locally Continuous Functions

R.Narmada Devi, Department of Mathematics

Vel Tech Rangarajan Dr. Sagunthala R& D Institute of Science and Technology, Chennai

Email: narmadadevi23@gmail.com.

Abstract

The new concepts of a neutrosophic G_δ set and neutrosophic G_δ - α -locally closed sets are introduced. Also, a neutrosophic εG_δ - α -locally quasi neighbourhood, neutrosophic G_δ - α -locally continuous function, neutrosophic G_δ - α -local T_2 space, neutrosophic G_δ - α -local Urysohn space, neutrosophic G_δ - α -local connected space and neutrosophic G_δ - α -local compact space are discussed and some interesting properties are established.

Keywords: NG_δ set, NG_δ - α -lcs, $N\varepsilon G_\delta$ - α -lqnbnd, NG_δ - α -lcf, NG_δ - α -local T_2 space, NG_δ - α -local Urysohn space, NG_δ - α -local connected space and NG_δ - α -local compact space.

1 Introduction

The fuzzy sets was introduced by Zadeh [13]. After the introduction of fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy set, interval-valued fuzzy sets, etc. Ganster and Relly used locally closed sets [6] to define LC-continuity and LC-irresoluteness. G.Balasubramanian [2] introduced and studied the concept of fuzzy G_δ set in a fuzzy topological space. After the introduction of intuitionistic fuzzy sets and its topological spaces by Atanassov[1] and Coker[3, 4], the concept of imprecise data called neutrosophic sets was introduced by Smarandache[5]. The concept of neutrosophic topological space was introduced [12]. Later R.Narmada Devi[7,8,9,10,11] was introduced the concepts of intuitionistic fuzzy G_δ sets, intuitionistic fuzzy G_δ - α -locally continuous function and its properties are introduced. In this paper, the new concepts of a neutrosophic G_δ set and neutrosophic G_δ - α -locally closed sets are introduced. Also, a neutrosophic G_δ - α -locally continuous function, neutrosophic G_δ - α -local T_2 space, neutrosophic G_δ - α -local Urysohn space, neutrosophic G_δ - α -local connected space and neutrosophic G_δ - α -local compact space are studied and some interesting properties are established. In this connection, some interesting properties among these functions along with the spaces are discussed and provided with the necessary examples.

2 Preliminaries

Definition 2.1. [3] An IFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Definition 2.2. [2]

Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X . Then λ is called fuzzy G_δ if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$. The complement of fuzzy G_δ is fuzzy F_σ .

Definition 2.3. [5] A subset A of a space (X, τ) is called locally closed (briefly lc) if $A = C \cap D$, where C is open and D is closed in (X, τ) .

3 Characteristics of NG_δ - α -locally continuous functions

Definition 3.1. Let A be NS on $NTS (X, \tau)$. Then A is said be a

- (i) NG_δ set if $A = \bigcap_{i=1}^\infty A_i$, where A_i is a NOS.
- (ii) Neutrosophic locally closed set ($Nlcs$) if $A = B \cap C$, where B is a NOS and C is a NCS.
- (iii) NG_δ -locally closed set ($NG_\delta-lcs$) if $A = B \cap C$, where B is NG_δ set and C is NCS.
- (iv) NG_δ - α -locally closed set (NG_δ - α -lcs) if $A = B \cap C$, where B is a NG_δ set and C is $N\alpha$ -closed set.

Note 3.1. The complement of a NG_δ set (resp, $Nlcs$, NG_δ - α -lcs and NG_δ - α -lcs) is a NF_σ set (resp. $Nlcs$, NG_δ - α -lcs and NG_δ - α -los).

Definition 3.2. The NG_δ - α -locally closure and NG_δ - α -locally interior of A are given by

- (i) NG_δ - α -lcl(A)= $\bigcap\{B: B$ is a NG_δ - α -lcs and $A \subseteq B\}$.
- (ii) NG_δ - α -lint(A)= $\bigcup\{B: B$ is a NG_δ - α -los and $B \subseteq A\}$.

Remark 3.1. (i) NG_δ - α -lcl(A) = A if and only if A is a NG_δ - α -lcs.

- (ii) NG_δ - α -lint(A) $\subseteq A \subseteq NG_\delta$ - α -lcl(A).
- (iii) NG_δ - α -lint(1_N) = 1_N
- (iv) NG_δ - α -lint(0_N) = 0_N
- (v) NG_δ - α -lcl(1_N) = 1_N

Definition 3.3. Let A be NS on NTS (X, τ) . Then A is said be a

- (i) $N\epsilon G_\delta$ - α -locally neighbourhood ($N\epsilon G_\delta$ - α -lnbd)of a NP $x_{l,m,n}$ if there exists a NG_δ - α -los B such that $x_{l,m,n} \in B, B \subseteq A$.
- (ii) $N\epsilon G_\delta$ - α -locally quasi neighbourhood ($N\epsilon G_\delta$ - α -lqnbd)of NP $x_{l,m,n}$ if there exists a G_δ - α -los B such that $x_{l,m,n} \in B, B \subseteq A$.

Remark 3.2. (i) The collection of all $N\epsilon G_\delta$ - α -lnbd of $x_{l,m,n}$ is denoted by $N_\epsilon^{NG_\delta-\alpha-l}(x_{l,m,n})$.

- (ii) The collection of all $N\epsilon G_\delta$ - α -lqnbd of $x_{l,m,n}$ is denoted by $N_\epsilon^{NG_\delta-\alpha-lq}(x_{l,m,n})$.

Definition 3.4. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then f is said to be NG_δ - α -locally continuous function(NG_δ - α -lcf), if for each NP $x_{l,m,n}$ and $B \in N_\epsilon f(x_{l,m,n})$, there exists $A \in N_\epsilon^{NG_\delta-\alpha-lq}(x_{l,m,n})$ such that $f(A) \subseteq B$.

Proposition 3.1. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then the following are equivalent.

- (i) f is a NG_δ - α -lcf.
- (ii) $f^{-1}(A)$ is a NG_δ - α -los in X , for each NOS A in Y .
- (iii) $f^{-1}(B)$ is a NG_δ - α -lcs in X , for each NCS B in Y .
- (iv) NG_δ - α -lcl($f^{-1}(A)$) $\subseteq f^{-1}(Ncl(A))$, for each NS A in Y .
- (v) $f^{-1}(Nint(A)) \subseteq NG_\delta$ - α -lint($f^{-1}(A)$), for each NS A in Y .

Proposition 3.2. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic bijective function. Then

- (i) f is a NG_δ - α -lcf iff $Nint(f(A)) \subseteq f(NG_\delta$ - α -lint(A)), for each NS A of X .
- (ii) f is a NG_δ - α -lcf iff $f(NG_\delta$ - α -lcl(A)) $\subseteq Ncl(f(A))$, for each NS A of X .

Proposition 3.3. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic bijective function and NG_δ - α -lcf. If $A \in I^Y$ is NCS, then $f^{-1}(A)=NG_\delta$ - α -lcl ($f^{-1}(A)$).

Proposition 3.4. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be any NG_δ - α -lcf. If $f(X) \subset Z \subset Y$, Then $g : (X, \tau) \rightarrow (Z, \varphi)$ where $R = S/Z$ restricting the range of f is a NG_δ - α -lcf.

Proposition 3.5. Let $(X, \tau), (X_1, \tau_1)$ and (X_2, τ_2) be any NTSs and $P_i : X_1 \times X_2 \rightarrow X_i$ be a neutrosophic Projection of $X_1 \times X_2$ onto X_i . If $f : X \rightarrow X_1 \times X_2$ is a NG_δ - α -lcf. Then $P_i \circ f : X \rightarrow X_i$ is also a NG_δ - α -lcf.

Proposition 3.6. Let (X, τ) and (Y, δ) be any two *NTSSs*. If neutrosophic graph function $g : X \rightarrow X \times Y$ is a $NG_{\delta-\alpha}$ -lcf. Then $f : (X, \tau) \rightarrow (Y, \delta)$ is a $NG_{\delta-\alpha}$ -lcf.

Definition 3.5. Let (X, τ) and (Y, δ) be any two *NTSSs*. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then f is said to be a

- (i) $NG_{\delta-\alpha}$ -locally irresolute function, if for each $NG_{\delta-\alpha}$ -lcs A in Y , $f^{-1}(A)$ is a $NG_{\delta-\alpha}$ -lcs in X
- (ii) N – weakly – $G_{\delta-\alpha}$ -locally function, if for each $NG_{\delta-\alpha}$ -lcs A in Y , $f^{-1}(A)$ is *NCS* in X .

Proposition 3.7. Let (X, τ) and (Y, δ) be any two *NTSSs*. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then the following statements are equivalent

- (i) f is a $NG_{\delta-\alpha}$ -locally irresolute function.
- (ii) for every *NS* A of X , $f(NG_{\delta-\alpha}\text{-lcl}(A)) \subseteq NG_{\delta-\alpha}\text{-lcl}(f(A))$.
- (iii) for every *NS* A of Y , $NG_{\delta-\alpha}\text{-lcl}(f^{-1}(A)) \subseteq f^{-1}(NG_{\delta-\alpha}\text{-lcl}(A))$.

Definition 3.6. Let (X, τ) and (Y, δ) be any two *NTSSs*. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then f is said to be a

- (i) $NG_{\delta-\alpha}$ -locally function, if for each *NCS* A in X , $f(A)$ is a $G_{\delta-\alpha}$ -lcs in Y .
- (ii) N – strongly – $G_{\delta-\alpha}$ -locally function, if for each $NG_{\delta-\alpha}$ -lcs A in X , $f(A)$ is a $NG_{\delta-\alpha}$ -lcs in Y .

Proposition 3.8. Let (X, τ) and (Y, δ) be any two *NTSSs*. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then the following statements are equivalent

- (i) f is a $NG_{\delta-\alpha}$ -locally function.
- (ii) for each *NS* A of Y and each *NCS* B of X with $f^{-1}(A) \subseteq B$, there is a $NG_{\delta-\alpha}$ -lcs D of Y with $A \subseteq D$ such that $f^{-1}(D) \subseteq B$.
- (iii) $f^{-1}(NG_{\delta-\alpha}\text{-lcl}(A)) \subseteq Ncl(f^{-1}(A))$, for each *NS* A of Y .
- (iv) $f(\text{int}(B)) \subseteq NG_{\delta-\alpha}\text{-lint}(f(B))$, for each *NS* B of X .

Definition 3.7. Let (X, τ) and (Y, δ) be any two *NTSSs*. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then f is said to be a $NG_{\delta-\alpha}$ -locally homeomorphism if f is 1 – 1, onto, $NG_{\delta-\alpha}$ -locally irresolute function and N – strongly – $G_{\delta-\alpha}$ -locally function.

Proposition 3.9. Let (X, τ) and (Y, δ) be any two *NTSSs*. If $f : (X, \tau) \rightarrow (Y, \delta)$ is a $NG_{\delta-\alpha}$ -locally homeomorphism. Then the following statements are valid.

- (i) For any *NS* A in X , $NG_{\delta-\alpha}\text{-lcl}(f(A)) = f(NG_{\delta-\alpha}\text{-lcl}(A))$
- (ii) For any *NS* A in X , $f(C(NG_{\delta-\alpha}\text{-lint}(C(A)))) = C(NG_{\delta-\alpha}\text{-lint}(f(C(A))))$
- (iii) For any *NS* A in Y , $NG_{\delta-\alpha}\text{-lcl}(f^{-1}(A)) = f^{-1}(NG_{\delta-\alpha}\text{-lcl}(A))$
- (iv) For any *NS* A in Y , $f^{-1}(C(NG_{\delta-\alpha}\text{-lint}(C(A)))) = C(NG_{\delta-\alpha}\text{-lint}(f^{-1}(C(A))))$

Proposition 3.10. Let (X, τ) , (Y, δ) and (Z, φ) be any three *NTSSs*. If $f : (X, \tau) \rightarrow (Y, \delta)$ and $g : (Y, \delta) \rightarrow (Z, \varphi)$ be any two neutrosophic mappings. Then the following statements are valid.

- (i) If f is a $NG_{\delta-\alpha}$ -locally irresolute function and g is a $NG_{\delta-\alpha}$ -locally continuous function, then $g \circ f$ is a $NG_{\delta-\alpha}$ -lcf.
- (ii) If f is a $NG_{\delta-\alpha}$ -lcf and g is a N – weakly – $G_{\delta-\alpha}$ -locally function, then $g \circ f$ is a $NG_{\delta-\alpha}$ -locally irresolute function.

Definition 3.8. A *NTS* (X, τ) is said to be a

- (i) $NG_{\delta-\alpha}$ -local T_2 space iff for every *NP* $c_{l,m,n}$ and $d_{r,s,t}$ in X and $c \neq d$ there exists a $NG_{\delta-\alpha}$ -los's G and H with $T_G(c) = 0, I_G(c) = 0, F_G(c) = 1, T_H(d) = 1, I_H(d) = 1, F_H(d) = 0$ and $G \cap H = 0_N$.

- (ii) $NG_{\delta-\alpha}$ -local Urysohn space iff for every NPS $c_{r,s,t}$ and $d_{l,m,n}$ in X and $c \neq d$, there exists a $NG_{\delta-\alpha}$ -los's G and H with $T_G(c) = 0, I_G(c) = 0, F_G(c) = 1, T_H(d) = 1, I_H(d) = 1, F_H(d) = 0$ and $NG_{\delta-\alpha}\text{-lcl}(G) \cap NG_{\delta-\alpha}\text{-lcl}(H) = 0_N$.

Proposition 3.11. Let (X, τ) and (Y, δ) be any two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be 1-1 and $NG_{\delta-\alpha}\text{-lcf}$. If (Y, δ) is a NT_2 space, then (X, τ) is a $NG_{\delta-\alpha}$ -local T_2 space.

Proposition 3.12. Let (X, τ) and (Y, δ) be any two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be 1-1, N -weakly- $G_{\delta-\alpha}\text{-lcf}$. If (Y, δ) is a $G_{\delta-\alpha}$ -local T_2 space, then (X, τ) is a NT_2 space.

Proposition 3.13. Every $NG_{\delta-\alpha}$ -local Urysohn space is a $NG_{\delta-\alpha}$ -local T_2 space.

4 Interrelation Among $NG_{\delta-\alpha}$ -locally continuous functions

Definition 4.1. Let (X, τ) and (Y, δ) be two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a neutrosophic mapping. Then f is said to be a

- (i) neutrosophic locally continuous function ($Nlcf$), if for each NCS A in Y , $f^{-1}(A)$ is a $Nlcs$ in X .
- (ii) NG_{δ} -locally continuous function ($NG_{\delta}\text{-lcf}$), if for each NCS A in Y , $f^{-1}(A)$ is a $NG_{\delta}\text{-lcs}$ in X .

Proposition 4.1. Let (X, τ) and (Y, δ) be two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a $Nlcf$. Then f is a $NG_{\delta}\text{-lcf}$.

Remark 4.1. The converse of the Proposition 4.1 need not true as shown in Example 4.1.

Example 4.1. Let $X = \{a\}$. We define the NSs $A_n = \langle x, T_{A_n}, I_{A_n}, F_{A_n} \rangle, n = 0, 1, 2, \dots$ by $T_{A_n}(x) = \frac{n}{9n+1}, I_{A_n}(x) = \frac{n}{9n+1}$ and $F_{A_n}(x) = 1 - \frac{n}{9n+1}$. Then the family $\tau = \{0_N, 1_N, A_n : n = 0, 1, \dots\}$ is a NTS on X . Let $D = \langle a, 0.9, 0.9, 0.1 \rangle$ be NS in X . Then the family $\delta = \{0_N, 1_N, D\}$ is a NTS on X . Define a function $f : (X, \tau) \rightarrow (Y, \delta)$ be an identity function. Now, f is a $NG_{\delta}\text{-lcf}$. But f is not $Nlcf$.

Proposition 4.2. Let (X, τ) and (Y, δ) be two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a $Nlcf$. Then f is a $NG_{\delta-\alpha}\text{-lcf}$.

Remark 4.2. The converse of the Proposition 4.2 need not true as shown in Example 4.2.

Proposition 4.3. Let (X, τ) and (Y, δ) be two $NTSs$. Let $f : (X, \tau) \rightarrow (Y, \delta)$ be a $NG_{\delta}\text{-lcf}$. Then f is a $NG_{\delta-\alpha}\text{-lcf}$.

Remark 4.3. The converse of the Proposition 4.3 need not true as shown in Example 4.2.

Example 4.2. Let $X = \{a, b\}$ be a nonempty set. Let $A = \{\langle a, (0.3, 0.4, 0.2) \rangle, \langle b, (0.5, 0.6, 0.3) \rangle\}$ and $B = \{\langle a, (0.5, 0.5, 0.2) \rangle, \langle b, (0.6, 0.6, 0.3) \rangle\}$ be NSs of X . Then the family $\tau = \{0_N, 1_N, A, B\}$ is a NTS on X . Let $D = \{\langle a, (0.7, 0.7, 0.3) \rangle, \langle b, (0.5, 0.5, 0.2) \rangle\}$ be NS of X . Then the family $\delta = \{0_N, 1_N, D\}$ is a NTS on X . Define a function $f : (X, \tau) \rightarrow (X, \delta)$ as $f(a) = b$ and $f(b) = a$. Now, f is a $NG_{\delta-\alpha}\text{-lcf}$. But f is not a $Nlcf$ and $NG_{\delta}\text{-lcf}$.

5 $NG_{\delta-\alpha}$ -local connected and $NG_{\delta-\alpha}$ -local Compact Spaces

Definition 5.1. NTS (X, τ) is said to be a $NG_{\delta-\alpha}$ -local connected iff the only NSs which are both $NG_{\delta-\alpha}$ -los and $NG_{\delta-\alpha}\text{-lcs}$ are 0_N and 1_N .

Proposition 5.1. Let (X, τ) and (Y, δ) be any two $NTSs$. If $f : (X, \tau) \rightarrow (Y, \delta)$ is a onto, $NG_{\delta-\alpha}\text{-lcf}$ and (X, τ) is a $NG_{\delta-\alpha}$ -local connected space then (Y, δ) is a neutrosophic connected space.

Definition 5.2. Let (X, τ) be NTS .

- (i) If a family $\{G_j : j \in J\}$ of a $NG_{\delta-\alpha}\text{-los}$'s in X satisfies the condition $\bigcup\{G_j : j \in J\} = 1_N$, then it is called as a $NG_{\delta-\alpha}$ -locally open cover of X .
- (ii) A NTS (X, τ) is said to be a $NG_{\delta-\alpha}$ -local compact if every $NG_{\delta-\alpha}$ -locally open cover of $\{A_j : j \in J\}$ of X , there exists a finite subfamily $J_o \subset J$ such that $\bigcup\{A_j : j \in J_o\} = 1_N$.

Proposition 5.2. Let (X, τ) and (Y, δ) be any two $NTSs$. If $f : (X, \tau) \rightarrow (Y, \delta)$ is a $NG_{\delta-\alpha}\text{-lcf}$, bijective function and (X, τ) is a $NG_{\delta-\alpha}$ -local compact space then (Y, δ) is a $NG_{\delta-\alpha}$ -local compact space.

References

- [1] K.T.Atanassov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, 20, pp.87–96,1986.
- [2] G.Balasubramanian, Maximal Fuzzy Topologies, *KYBERNETIKA*, 31,pp.459–465,1995.
- [3] D.Coker, An Introduction to Intuitionistic Fuzzy Topological Spaces,*Fuzzy Sets and Systems*, 88,no.1, pp.81–89,1997.
- [4] D.Coker and M.Demirci, On Intuitionistic Fuzzy Points, *Notes IFS* 1,no.2, pp.79–84,1995.
- [5] Florentin Smarandache, Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic set, Probability and Statistics*, University of New Mexico, Gallup, NM 87301, USA(2002).
- [6] M.Ganster and I.L Relly, Locally Closed Sets and LC-Continuous Functions, *Internet. J.Math. Math.Sci.*, 12, pp.417–424, 1989.
- [7] R.Narmada Devi, E. Roja and M.K.Uma, On Some Applications of Intuitionistic Fuzzy G_δ - α -locally closed sets,*The Journal of Fuzzy Mathematics*, 21, No. 2, pp.85–98,2013
- [8] R.Narmada Devi, E. Roja and M.K.Uma, Intuitionistic Fuzzy G_{δ} - α -locally Continuous Functions,*Annals of Fuzzy Mathematics and Informatics*, 5, No. 2, pp.399–416, 2013.
- [9] R.Narmada Devi, E.Roja and M.K.Uma, Intuitionistic Fuzzy Exterior Spaces Via Rings, *Annals of Fuzzy mathematics and Informatics*,6,2,pp.554–559,2014.
- [10] R.Narmada Devi, E.Roja and M.K.Uma, Basic Compactness and Extremal Compactness in Intuitionistic Fuzzy Structure Ring Spaces,*The Journal of fuzzy Mathematics*,23,6,pp.643–660,2015
- [11] R.Narmada Devi, Neutrosophic Complex \mathcal{N} -continuity,*Annals of Fuzzy Mathematics nad Informatics*,13,1,pp.109–122, 2017.
- [12] A. A. Salama and S. A. Alblowi, Neutrosophic Set ans Neutrosophic Topological Spaces,*ISOR Journal of Mathematics*,3, No.6,pp.31–35, 2012.
- [13] L.A.Zadeh, Fuzzy Sets, *Infor. and Control*, 9, pp.338–353,1965