Neutrosophic Soft Filters

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Abstract

In this paper, the concept of neutrosophic soft filter and its basic properties are introduced. Later, we set up a neutrosophic soft topology with the help of a neutrosophic soft filter. We also give the notions of the greatest lower bound and the least upper bound of the family of neutrosophic soft filters, neutrosophic soft filter subbase and neutrosophic soft filter base and explore some basic properties of them.
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1 Introduction

We can not solve the problems by using mathematical tools generally in the social life since in mathematics, the concepts are precise and not subjective. To deal with this problem, researchers proposed several methods such as fuzzy set theory [12], rough set theory [7] and soft set theory [6]. Theories of fuzzy sets and rough sets can be considered as tools for dealing with vagueness but both of these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as mentioned by Molodtsov [6] in 1999. Molodtsov initiated a novel concept of soft set theory which is a completely new approach for modeling uncertainities and succesfully applied it into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement and so on. The fundamental concepts of neutrosophic set were introduced by Smarandache [10]. This theory is a generalization of classical sets, fuzzy set theory [12], intuitionistic fuzzy set theory [1], etc. Later some researchers [8,9] studied basic concepts and properties of neutrosophic sets. The notion of neutrosophic soft sets was first defined by Maji [5] and later, Deli and Broumi [3] modified it. Bera [2] introduced the concept of neutrosophic soft topological spaces. Also, neutrosophic soft point concept and neutrosophic soft T_i-spaces were presented by Günüuz Aras et al. [4].

The main purpose of this paper is to introduce neutrosophic soft filters. Later we study some basic properties of neutrosophic soft filters and set up a neutrosophic soft topology with the help of a neutrosophic soft filter. Some new notions in neutrosophic soft filters such as the greatest lower bound and the least upper bound of the family of neutrosophic soft filters, neutrosophic soft filter subbase and neutrosophic soft filter base were introduced. Also, we give some basic properties of these concepts.

2 Preliminaries

In this section, we present the basic definitions and results of neutrosophic soft sets and neutrosophic soft topological spaces that we require in the next sections.

Definition 2.1. [3] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $(\widetilde{F}, E)$ over X is a set defined by a set value function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $(\widetilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs.

$$(\widetilde{F}, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where $T_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}(e)}(x),\ F_{\widetilde{F}(e)}(x) \in [0,1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. Since the supremum of each $T,\ I,\ F$ is 1, the inequality $0 \leq T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.2. [2] Let $(\widetilde{F}, E)$ be a neutrosophic soft set over the universe set X. The complement of $(\widetilde{F}, E)$ is denoted by $(\widetilde{F}, E)^c$ and is defined by:

$$(\widetilde{F}, E)^c = \left\{ \left(e, \left\langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\}.$$

It is obvious that $\left((\widetilde{F}, E)^c \right)^c = (\widetilde{F}, E)$.

Definition 2.3. [5] Let $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ be two neutrosophic soft sets over the universe set X. $(\widetilde{F}, E)$ is said to be a neutrosophic soft subset of $(\widetilde{G}, E)$ if $T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \leq I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$. It is denoted by $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$. $(\widetilde{F}, E)$ is said to be neutrosophic soft equal to $(\widetilde{G}, E)$ if $(\widetilde{F}, E)$ is a neutrosophic soft subset of $(\widetilde{G}, E)$ and $(\widetilde{G}, E)$ is a neutrosophic soft subset of $(\widetilde{F}, E)$. It is denoted by $(\widetilde{F}, E) = (\widetilde{G}, E)$.

Definition 2.4. [4] Let $(\widetilde{F}_1, E)$ and $(\widetilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X. Then their union is denoted by $(\widetilde{F}_1, E) \cup (\widetilde{F}_2, E) = (\widetilde{F}_3, E)$ and is defined by:

$$(\widetilde{F}_3, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where

$$\begin{aligned} T_{\widetilde{F}_3(e)}(x) &= \max\left\{ T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x) \right\}, \\ I_{\widetilde{F}_3(e)}(x) &= \max\left\{ I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x) \right\}, \\ F_{\widetilde{F}_3(e)}(x) &= \min\left\{ F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x) \right\}. \end{aligned}$$

Definition 2.5. [4] Let $(\widetilde{F}_1, E)$ and $(\widetilde{F}_2, E)$ be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $(\widetilde{F}_1, E) \cap (\widetilde{F}_2, E) = (\widetilde{F}_3, E)$ and is defined by:

$$(\widetilde{F}_3, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}_3(e)}(x), I_{\widetilde{F}_3(e)}(x), F_{\widetilde{F}_3(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where

$$\begin{aligned} T_{\widetilde{F}_3(e)}(x) &= \min\left\{ T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x) \right\}, \\ I_{\widetilde{F}_3(e)}(x) &= \min\left\{ I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x) \right\}, \\ F_{\widetilde{F}_3(e)}(x) &= \max\left\{ F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x) \right\}. \end{aligned}$$

Definition 2.6. [4] A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be a null neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 0$, $I_{\widetilde{F}(e)}(x) = 0$, $F_{\widetilde{F}(e)}(x) = 1$; $\forall e \in E$, $\forall x \in X$. It is denoted by $0_{(X,E)}$.

Definition 2.7. [4] A neutrosophic soft set $(\widetilde{F}, E)$ over the universe set X is said to be an absolute neutrosophic soft set if $T_{\widetilde{F}(e)}(x) = 1$, $I_{\widetilde{F}(e)}(x) = 1$, $F_{\widetilde{F}(e)}(x) = 0$; $\forall e \in E$, $\forall x \in X$. It is denoted by $1_{(X,E)}$.

Clearly, $0^c_{(X,E)} = 1_{(X,E)}$ and $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 2.8. [4] Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau \subseteq NSS(X, E)$. Then τ is said to be a neutrosophic soft topology on X if:

1. $0_{(X,E)}$ and $1_{(X,E)}$ belong to τ,
2. the union of any number of neutrosophic soft sets in τ belongs to τ,
3. the intersection of a finite number of neutrosophic soft sets in τ belongs to τ.

Then (X, τ, E) is said to be a neutrosophic soft topological space over X. Each member of τ is said to be a neutrosophic soft open set. A neutrosophic soft set $(\widetilde{F}, E)$ is called a neutrosophic soft closed set iff its complement $(\widetilde{F}, E)^c$ is a neutrosophic soft open set.

Definition 2.9. [4] Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X. Then neutrosophic soft set $x^e_{(\alpha,\beta,\gamma)}$ is called a neutrosophic soft point, for every $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$, $e \in E$ and is defined as follows:

$$x^e_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & if\ e' = e\ and\ y = x, \\ (0, 0, 1) & if\ e' \neq e\ or\ y \neq x. \end{cases}$$

Definition 2.10. [4] Let $(\widetilde{F}, E)$ be a neutrosophic soft set over the universe set X. We say that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$ read as belonging to the neutrosophic soft set $(\widetilde{F}, E)$ whenever $\alpha \leq T_{\widetilde{F}(e)}(x)$, $\beta \leq I_{\widetilde{F}(e)}(x)$ and $F_{\widetilde{F}(e)}(x) \leq \gamma$.

Definition 2.11. [4] Let (X, τ, E) be a neutrosophic soft topological space over X. A neutrosophic soft set $(\widetilde{F}, E)$ in (X, τ, E) is called a neutrosophic soft neighborhood of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$, if there exists a neutrosophic soft open set $(\widetilde{G}, E)$ such that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \subseteq (\widetilde{F}, E)$.

Theorem 2.12. *[4] Let (X, τ, E) be a neutrosophic soft topological space and $(\widetilde{F}, E)$ be a neutrosophic soft set over X. Then $(\widetilde{F}, E)$ is a neutrosophic soft open set if and only if $(\widetilde{F}, E)$ is a neutrosophic soft neighborhood of its neutrosophic soft points.*

The neighborhood system of a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$, denoted by $U(x^e_{(\alpha,\beta,\gamma)}, E)$, is the family of all its neighborhoods.

Theorem 2.13. *[4] The neighborhood system $U(x^e_{(\alpha,\beta,\gamma)}, E)$ at $x^e_{(\alpha,\beta,\gamma)}$ in a neutrosophic soft topological space (X, τ, E) has the following properties:*

1) If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$, then $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$,

2) If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{F}, E) \subseteq (\widetilde{H}, E)$ then $(\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$,

3) If $(\widetilde{F}, E), (\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ then $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$,

4) If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ then there exists a $(\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{G}, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$ for each $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$.

Definition 2.14. Let (X, τ, E) be a neutrosophic soft topological space and Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ be a family of some neutrosophic soft neighborhoods of neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$. If, for each neutrosophic soft neighborhood $(\widetilde{G}, E)$ of $x^e_{(\alpha,\beta,\gamma)}$, there exists a $(\widetilde{H}, E)$ $\in$Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{H}, E) \subseteq (\widetilde{G}, E)$, then we say that Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ is a neutrosophic soft neighborhood base at $x^e_{(\alpha,\beta,\gamma)}$.

Theorem 2.15. *If for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ there corresponds a family $U(x^e_{(\alpha,\beta,\gamma)}, E)$ such that the properties 1) - 4) in Theorem 2.13 are satisfied, then there is a unique τ neutrosophic soft topological structure over X such that for each $x^e_{(\alpha,\beta,\gamma)}$, $U(x^e_{(\alpha,\beta,\gamma)}, E)$ is the family of τ-neutrosophic soft neighborhoods of $x^e_{(\alpha,\beta,\gamma)}$.*

Proof. Let $\tau = \left\{(\widetilde{G}, E) \in NSS(X, E) : x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \Longrightarrow (\widetilde{G}, E) \in \mathrm{U}(x^e_{(\alpha,\beta,\gamma)}, E)\right\}$. It is clear that, τ is a neutrosophic soft topology over X. The family τ certainly satisfies axioms 2. and 3. in Definition 2.8: for 3., this follows immediately from 2) in Theorem 2.13 and for 2., from 3) in Theorem 2.13. The axiom 1. in Definition 2.8 is a result of 2) and 3) in Theorem 2.13. It remains to show that, in the neutrosophic soft topology defined by τ, $U(x^e_{(\alpha,\beta,\gamma)}, E)$ is the set of τ-neutrosophic soft neighborhoods of $x^e_{(\alpha,\beta,\gamma)}$ for each $x^e_{(\alpha,\beta,\gamma)}$. It follows from 2) in Theorem 2.13 that every τ-neutrosophic soft neighborhood of $x^e_{(\alpha,\beta,\gamma)}$ belongs to $U(x^e_{(\alpha,\beta,\gamma)}, E)$. Conversely, let $(\widetilde{G}_1, E)$ be a neutrosophic soft set belonging to $U(x^e_{(\alpha,\beta,\gamma)}, E)$ and let $(\widetilde{G}_2, E)$ be the neutrosophic soft set of neutrosophic soft points $y^{e'}_{(\alpha',\beta',\gamma')}$ such that $(\widetilde{G}_1, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$. If we can show that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}_2, E)$, $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$ and $(\widetilde{G}_2, E) \in \tau$, then the proof will be complete. Since for every neutrosophic soft point $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$ belongs to $(\widetilde{G}_1, E)$ by reason of 1) in Theorem 2.13 and the hypothesis $(\widetilde{G}_1, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$, we obtain $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$. Since $(\widetilde{G}_1, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$, we have $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}_2, E)$. It remains to show that $(\widetilde{G}_2, E) \in \tau$, i.e. that $(\widetilde{G}_2, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$ for each $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$. If $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$ then by 4) in Theorem 2.13 there is a neutrosophic soft set $(\widetilde{G}_3, E)$ such that for each $z^{e''}_{(\alpha'',\beta'',\gamma'')} \in (\widetilde{G}_3, E)$ we have $(\widetilde{G}_1, E) \in U(z^{e''}_{(\alpha'',\beta'',\gamma'')}, E)$. Since $(\widetilde{G}_1, E) \in U(z^{e''}_{(\alpha'',\beta'',\gamma'')}, E)$ means that $z^{e''}_{(\alpha'',\beta'',\gamma'')} \in (\widetilde{G}_2, E)$, it follows that $(\widetilde{G}_3, E) \subseteq (\widetilde{G}_2, E)$ and therefore, by 2) in Theorem 2.13, that $(\widetilde{G}_2, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$. □

3 Neutrosophic soft filters

Definition 3.1. Let $\aleph \subseteq NSS(X, E)$, then $\aleph$ is called a neutrosophic soft filter on X if $\aleph$ satisfies the following properties:

($\aleph_1$) $0_{(X,E)} \notin \aleph$,
($\aleph_2$) $\forall(\widetilde{F}, E),\ (\widetilde{G}, E) \in \aleph \Longrightarrow (\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph$,
($\aleph_3$) $\forall(\widetilde{F}, E) \in \aleph$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E) \Longrightarrow (\widetilde{G}, E) \in \aleph$.

Remark 3.2. It follows from ($\aleph_1$) and ($\aleph_2$) that every finite intersections of neutrosophic soft sets of $\aleph$ are not $0_{(X,E)}$.

Proposition 3.3. *The condition ($\aleph_2$) is equivalent to the following two conditions:*

($\aleph_{2a}$) The intersection of two members of $\aleph$ belongs to $\aleph$.
($\aleph_{2b}$) $1_{(X,E)}$ belongs to $\aleph$.

Example 3.4. The family $\aleph = \{1_{(X,E)}\}$ is a neutrosophic soft filter over X.

Theorem 3.5. *Let $0_{(X,E)} \neq (\widetilde{F}, E) \in NSS(X, E)$. Then the family $\aleph_{(\widetilde{F},E)} = \left\{(\widetilde{G}, E) : (\widetilde{F}, E) \subseteq (\widetilde{G}, E) \in NSS(X, E)\right\}$ is a neutrosophic soft filter over X.*

Proof. Since $1_{(X,E)} \in \aleph$ and $0_{(X,E)} \notin \aleph$, $\emptyset \neq \aleph \neq NSS(X, E)$. Suppose $(\widetilde{H}_1, E)$, $(\widetilde{H}_2, E) \in \aleph$, then $(\widetilde{F}, E) \subseteq (\widetilde{H}_1, E)$, $(\widetilde{F}, E) \subseteq (\widetilde{H}_2, E)$. Thus $T_{\widetilde{F}(e)}(x) \leq \min\left\{T_{\widetilde{H}_1(e)}(x), T_{\widetilde{H}_2(e)}(x)\right\}$, $I_{\widetilde{F}(e)}(x) \leq \min\left\{I_{\widetilde{H}_1(e)}(x), I_{\widetilde{H}_2(e)}(x)\right\}$ and $F_{\widetilde{F}(e)}(x) \leq \max\left\{F_{\widetilde{H}_1(e)}(x), F_{\widetilde{H}_2(e)}(x)\right\}$ for all $x \in X$. So $(\widetilde{F}, E) \subseteq (\widetilde{H}_1, E) \cap (\widetilde{H}_2, E)$ and hence $(\widetilde{H}_1, E) \cap (\widetilde{H}_2, E) \in \aleph$. □

Theorem 3.6. *Let (X, τ, E) be a neutrosophic soft topological space over X. The neighborhood system $U(x^e_{(\alpha,\beta,\gamma)}, E)$ is a neutrosophic soft filter for every neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$. Also, it is called neutrosophic soft neighborhoods filter of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$.*

Proof. $(\aleph_1)$ By 1) in Theorem 2.13, since $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E)$, we obtain $0_{(X,E)} \notin U(x^e_{(\alpha,\beta,\gamma)}, E)$.

$(\aleph_2)$ This is clearly seen by 3) in Theorem 2.13.

$(\aleph_3)$ This is clearly seen by 2) in Theorem 2.13. □

Now, we set up a neutrosophic soft topology with the help of a neutrosophic soft filter.

Theorem 3.7. *If, for every $x^e_{(\alpha,\beta,\gamma)}$, there exists a neutrosophic soft filter $\aleph(x^e_{(\alpha,\beta,\gamma)}) = U(x^e_{(\alpha,\beta,\gamma)}, E)$ which satisfies the following two properties, then there exists a unique neutrosophic soft topology τ such that $\aleph(x^e_{(\alpha,\beta,\gamma)})$ consists of the τ-neutrosophic soft neighborhoods of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$.*

(1) Every neutrosophic soft set in the neutrosophic soft filter $\aleph(x^e_{(\alpha,\beta,\gamma)})$ contains the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$,

(2) For every $(\widetilde{G}, E) \in \aleph(x^e_{(\alpha,\beta,\gamma)})$ there exists a $(\widetilde{H}, E) \in \aleph(x^e_{(\alpha,\beta,\gamma)})$ such that for every $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{H}, E)$, $(\widetilde{G}, E) \in \aleph(y^{e'}_{(\alpha',\beta',\gamma')})$.

Proof. Since the axioms $(\aleph_1)$, $(\aleph_2)$, $(\aleph_3)$, (1) and (2) are equivalent to the neighborhood axioms $1) - 4)$, by Theorem 2.15, there exists a neutrosophic soft topology τ such that $\aleph(x^e_{(\alpha,\beta,\gamma)})$ consists of the τ-neutrosophic soft neighborhoods of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$. □

Example 3.8. Let (X, τ, E) be a neutrosophic soft topological space and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. Since $(\widetilde{G}, E)$ cannot be an element of Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ for every $(\widetilde{H}, E) \in$Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{H}, E) \subseteq (\widetilde{G}, E)$, then the neutrosophic soft neighborhood base Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ is not a neutrosophic soft filter over X.

4 Comparison of neutrosophic soft filters

Definition 4.1. Let $\aleph_1$ and $\aleph_2$ be neutrosophic soft filters over X. If $\aleph_1 \subseteq \aleph_2$, then $\aleph_2$ is said to be finer that $\aleph_1$ or $\aleph_1$ coarser than $\aleph_2$.

If also $\aleph_1 \neq \aleph_2$, then $\aleph_2$ is strictly finer than $\aleph_1$ or $\aleph_1$ is strictly coarser than $\aleph_2$. If either $\aleph_1 \subseteq \aleph_2$ or $\aleph_2 \subseteq \aleph_1$, then $\aleph_1$ is comparable with $\aleph_2$.

Theorem 4.2. *Let $(\aleph_i)_{i\in I}$ be a family of neutrosophic soft filters over X. Then $\aleph = \underset{i\in I}{\cap} \aleph_i$ is a neutrosophic soft filter over X.*

In fact $\aleph$ is the greatest lower bound of the family $(\aleph_i)_{i\in I}$.

Proof. $(\aleph_1)$ Since $0_{(X,E)} \notin \aleph_i$ for each $i \in I$, then $0_{(X,E)}$ does not belong to $\aleph = \underset{i\in I}{\cap} \aleph_i$.

$(\aleph_2)$ Let $(\widetilde{F}, E)$, $(\widetilde{G}, E) \in \aleph = \underset{i\in I}{\cap} \aleph_i$. Then $(\widetilde{F}, E)$, $(\widetilde{G}, E) \in \aleph_i$ for each $i \in I$. Since $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph_i$ for each $i \in I$, so we obtain $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph = \underset{i\in I}{\cap} \aleph_i$.

$(\aleph_3)$ Let $(\widetilde{F}, E) \in \aleph = \underset{i\in I}{\cap} \aleph_i$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$. Since $(\widetilde{F}, E) \in \aleph_i$ for each $i \in I$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$, we get $(\widetilde{G}, E) \in \aleph_i$ for each $i \in I$. Hence $(\widetilde{G}, E) \in \aleph = \underset{i\in I}{\cap} \aleph_i$. □

Now, we investigate the least upper bound of the family of neutrosophic soft filters over X.

Theorem 4.3. *Let $S \subseteq NSS(X, E)$. Then there exists a neutrosophic soft filter $\aleph$ which contains the family S, if S has the following property: "The all finite intersections of neutrosophic soft sets of S are not $0_{(X,E)}$".*

Proof. Let $S = \left\{ (\widetilde{F_i}, E) : \forall i \in J \; (J \; is \; finite), \; \underset{i\in J}{\cap} (\widetilde{F_i}, E) \neq 0_{(X,E)} \right\}$. Then we give the family which consists of finite intersections of elements of S; $\beta = \left\{ (\widetilde{G}, E) : \forall i \in J \; (J \; is \; finite), \; (\widetilde{F_i}, E) \in S \; and \; (\widetilde{G}, E) = \underset{i\in J}{\cap} (\widetilde{F_i}, E) \right\}$.

Then the family $\aleph(S) = \left\{ (\widetilde{H}, E) : (\widetilde{G}, E) \in \beta \; and \; (\widetilde{G}, E) \subseteq (\widetilde{H}, E) \right\}$ is a neutrosophic soft filter over X.

($\aleph_1$) $0_{(X,E)} \in \beta$, for every $(\widetilde{H}, E) \in \aleph(S)$, $(\widetilde{H}, E) \neq 0_{(X,E)}$ and so $0_{(X,E)} \notin \aleph(S)$.

($\aleph_2$) Let $(\widetilde{H_1}, E), \; (\widetilde{H_2}, E) \in \aleph(S)$. There exist neutrosophic soft sets $(\widetilde{G_1}, E), \; (\widetilde{G_2}, E) \in \beta$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{H_1}, E)$ and $(\widetilde{G_2}, E) \subseteq (\widetilde{H_2}, E)$. From the definition of β, $0_{(X,E)} \neq (\widetilde{G_1}, E) \cap (\widetilde{G_2}, E) \in \beta$. Since $(\widetilde{G_1}, E) \cap (\widetilde{G_2}, E) \subseteq (\widetilde{H_1}, E) \cap (\widetilde{H_2}, E)$, we obtain $(\widetilde{H_1}, E) \cap (\widetilde{H_2}, E) \in \aleph(S)$.

($\aleph_3$) Let $(\widetilde{H_1}, E) \in \aleph(S)$ and $(\widetilde{H_1}, E) \subseteq (\widetilde{H_2}, E)$. Then there exists a neutrosophic soft set $(\widetilde{G}, E) \in \beta$ such that $(\widetilde{G}, E) \subseteq (\widetilde{H_1}, E)$. Since $(\widetilde{H_1}, E) \subseteq (\widetilde{H_2}, E)$, we obtain $(\widetilde{H_2}, E) \in \aleph(S)$. □

Remark 4.4. The neutrosophic soft filter $\aleph(S)$ in Theorem 4.3 is said to be generated by S and S is said to be neutrosophic soft filter subbase of $\aleph(S)$. It is clear that $S \subseteq \aleph(S)$.

Theorem 4.5. *The neutrosophic soft filter $\aleph(S)$ which is generated by S is the coarsest neutrosophic soft filter which contains S.*

Proof. Suppose that $S \subseteq \aleph_1$. By Theorem 4.3, $S \subseteq \beta \subseteq \aleph_1$. By Remark 4.4, for every $(\widetilde{H}, E) \in \aleph(S)$ there exists a $(\widetilde{G}, E) \in \beta$ such that $(\widetilde{G}, E) \subseteq (\widetilde{H}, E)$. Since $\beta \subseteq \aleph_1$, then $(\widetilde{G}, E) \in \aleph_1$. Since $\aleph_1$ is a neutrosophic soft filter, $(\widetilde{H}, E) \in \aleph_1$ by ($\aleph_3$) in Definition 3.1. Hence we obtain $\aleph(S) \subseteq \aleph_1$. □

Theorem 4.6. *The family $(\aleph_i)_{i\in I}$ of neutrosophic soft filters over X has a least upper bound if and only if for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G_i}, E) \in \aleph_i$ $(1 \leq i \leq n)$, $(\widetilde{G_1}, E) \cap \; ... \; \cap (\widetilde{G_n}, E) \neq 0_{(X,E)}$.*

Proof. $\Longrightarrow$: If there exists a least upper bound of the family $(\aleph_i)_{i\in I}$, by ($\aleph_1$) and ($\aleph_2$) in Definition 3.1, for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G_i}, E) \in \aleph_i$ $(1 \leq i \leq n)$, the intersection $(\widetilde{G_1}, E) \cap \; ... \; \cap (\widetilde{G_n}, E) \neq 0_{(X,E)}$.

$\Longleftarrow$: Let $(\widetilde{G_1}, E) \cap \; ... \; \cap (\widetilde{G_n}, E) \neq 0_{(X,E)}$ for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G_i}, E) \in \aleph_i$ $(1 \leq i \leq n)$. Then the neutrosophic soft filter $\aleph(S)$ generated by

$$S = \underset{i\in I}{\cup} \aleph_i = \left\{ (\widetilde{F}, E) : (\exists i \in I) \; (\widetilde{F}, E) \in \aleph_i \right\}$$

is the least upper bound of the family $(\aleph_i)_{i\in I}$ by Theorem 4.5. □

Definition 4.7. Let $\beta \subseteq NSS(X, E)$, then β is said to be a neutrosophic soft filter base on X if

(β_1) $\beta \neq \emptyset$ and $0_{(X,E)} \notin \beta$.
(β_2) The intersection of two members of β contain a member of β.

Remark 4.8. β which is in Theorem 4.3 is a neutrosophic soft filter base.

Remark 4.9. It is clear that, every neutrosophic soft filter is a neutrosophic soft filter base.

Example 4.10. Let (X, τ, E) be a soft topological space and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. The neutrosophic soft neighborhood base $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ is a neutrosophic soft filter base over X.

(β_1) Clearly, $Ç(x^e_{(\alpha,\beta,\gamma)}, E) \neq \emptyset$. For every $(\widetilde{H}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$, $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{H}, E)$. Then $(\widetilde{H}, E) \neq 0_{(X,E)}$. Hence we obtain $0_{(X,E)} \notin Ç(x^e_{(\alpha,\beta,\gamma)}, E)$.

(β_2) Let $(\widetilde{G}, E), \; (\widetilde{H}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$. Since $(\widetilde{G}, E), \; (\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$, we get $(\widetilde{G}, E) \cap (\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. By Definition 2.14, there exists a $(\widetilde{K}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{K}, E) \subseteq (\widetilde{G}, E) \cap (\widetilde{H}, E)$. Hence we get $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ is a neutrosophic soft filter base of neutrosophic soft neighborhoods filter $U(x^e_{(\alpha,\beta,\gamma)}, E)$ by Definition 4.7.

Theorem 4.11. *Let $\aleph$ be a neutrosophic soft filter over X and $\beta \subseteq \aleph$. Then β is a base of $\aleph$ if and only if every member of $\aleph$ contains a member of β.*

Proof. It is obvious from Theorem 4.3. □

Definition 4.12. Two neutrosophic soft filter bases β_1 and β_2 over X are equivalent if and only if every member of β_1 contains a member of β_2 and every member of β_2 contains a member of β_1.

Remark 4.13. Two equivalent neutrosophic soft filter bases generate the same neutrosophic soft filter.

Theorem 4.14. *Let (X, τ, E) be a soft topological space and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. If $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are different neutrosophic soft neighborhood bases of $x^e_{(\alpha,\beta,\gamma)}$, then $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are two equivalent neutrosophic soft filter bases.*

Proof. For each $(\widetilde{F_1}, E) \in Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$, by Example 4.10, $(\widetilde{F_1}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. Also, since $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E) \subseteq U(x^e_{(\alpha,\beta,\gamma)}, E)$ there exists a $(\widetilde{F_2}, E) \in Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{F_2}, E) \subseteq (\widetilde{F_1}, E)$. Similarly, for each $(\widetilde{F_2}, E) \in Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$, by Example 4.10, $(\widetilde{F_2}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. Since $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E) \subseteq U(x^e_{(\alpha,\beta,\gamma)}, E)$, there exists a $(\widetilde{F_1}, E) \in Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{F_1}, E) \subseteq (\widetilde{F_2}, E)$. Hence we obtain $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are equivalent by Definition 4.12. □

Theorem 4.15. *Let β_1, β_2 be neutrosophic soft filter bases and $\aleph_1$, $\aleph_2$ be neutrosophic soft filters over X such that $\beta_1 \subseteq \aleph_1$ and $\beta_2 \subseteq \aleph_2$. Then $\aleph_2 \subseteq \aleph_1$ if and only if every member of β_2 contains a member of β_1.*

Proof. $\Longrightarrow$: Let $\aleph_2 \subseteq \aleph_1$ and $(\widetilde{G_2}, E) \in \beta_2$. Since $\beta_2 \subseteq \aleph_2 \subseteq \aleph_1$, then $(\widetilde{G_2}, E) \in \aleph_1$. Since $\beta_1 \subseteq \aleph_1$, there exists a $(\widetilde{G_1}, E) \in \beta_1$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{G_2}, E)$ by Theorem 34.

$\Longleftarrow$: Let $(\widetilde{F_2}, E) \in \aleph_2$. From Theorem 34, there exists a $(\widetilde{G_2}, E)$ such that $(\widetilde{G_2}, E) \subseteq (\widetilde{F_2}, E)$. By hypothesis, there exists a $(\widetilde{G_1}, E) \in \beta_1$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{G_2}, E)$. Then we obtain $(\widetilde{G_1}, E) \subseteq (\widetilde{F_2}, E)$. Since $\beta_1 \subseteq \aleph_1$, $(\widetilde{F_2}, E) \in \aleph_1$ by Definition 30. Hence we obtain $\aleph_2 \subseteq \aleph_1$. □

5 Conclusion

In the present study, we have introduced neutrosophic soft filters which are defined over an initial universe with a fixed set of parameters. We set up a neutrosophic soft topology with the help of a neutrosophic soft filter. We further investigate some essential features and basic concepts of neutrosophic soft filters. We expect that results in this paper will be helpfull for future studies in neutrosophic soft sets.

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