TOPSIS METHOD IN THE ENVIRONMENT OF NEUTROSOPHIC SOFT SETS; APPLICATIONS IN THE S-BOXES IMAGE ENCRYPTION ANALYSIS

[1]Muhammad Gulistan, [1]Naveed Ul Haq, [1]Salma Khan
[1]Department of Mathematics and Statistics, Hazara University Mansehra, KP, Pakistan
gulistanmath@hu.edu.pk[1], naveedulhaqabc@gmail.com[1], salmakhan359@gmail.com[1],

Abstract

Decision making implies variety of the most excellent decision from a collection of given options. In various cases, this choice is dependent on past knowledge. To study the past data we analyse the situations and we get a result in these situations. Main objective of the paper is to summarize the strength of S-box of block cipher in image encryption applications with respect to neutrosophic soft set. In this paper we compare two type of topsis methods on the base of neutrosophic set. First method is neutrosophic topsis method and second one is neutrosophic simplified topsis method. By Applying both the methods on the same data we get same result. Similarly we prosposed a new topsis method using neutrosophic soft set which also gives same result like the above mentioned method. In this paper numerical example is presented and then at the end also used this method in s-boxes image encryption analysis.
Keywords: Neutrosophic soft set, s-boxes, image encryption.

1 Introduction

L. A. Zadeh[1] was the first mathematician who introduces the model of fuzzy sets in 1965. Because in uncertain, ambiguous environment L. A. Zadeh applied the fuzzy set. Fuzzy set is a mapping from any universe of discourse to $[0, 1]$. Sometimes it becomes tricky to choose the membership value for a fuzzy value. So to handle such type of problem interval valued fuzzy sets was proposed by Turksen.[2] We must take truth-membership, falsity-membership value for suitable explanation of an item in unsure, unclear atmosphere like in information fusion, expert system, belief system etc. To handle such type of problem we cannot use both interval valued fuzzy sets and the fuzzy sets. Intended for this purpose Atanassov[3] proposed an intuitionistic fuzzy set. These fuzzy sets are appropriate to complete in a row considering the truth-membership value and falsity-membership value. But in belief system vague and unpredictable information exists and to avoid such problem the idea of neutrosophic set was proposed by Smarandache.[4] And neutrosophic sets is appropriate to handle the problem consist of indeterminacy data. Smarandache give the concept of indeterminacy and established the idea of neutrosophic set in 1999. Neutrosophic set have three components truth, indeterminacy and falsity value. And these components are independent of each other. For further study about such problem Molodtsov[6] proposed the soft set theory. The idea about different operations on soft sets see (,[7],[8] and[9]). Its properties and algebra are discussed in.[10] By Maji et al.[11] the perception of fuzzy soft sets anticipated as well as soon after on applied such ideas in decision making problem (,[9,12]). Approach to fuzzy soft based on making decision with the help of soft sets was introduced by Feng et al. in.[13] Soft set theory is rising incredibly quickly since its introduction.[6] The nobel perception of soft set theory acting an vital responsibility as a mathematical tool for dealing with doubts. And finally combining soft set with neutrosophic set we get a new generalization and using this in decision making by Maji.[14]

Procedure for the Order of Foundness by Similarity to Ideal Solution (TOPSIS) Is the multi-criteria method of decision analysis initially developed Von Hwang et al. in 1981.[15] With more improvements by Yoon in 1987[16] and Hwang et al. in 1993.[17] TOPSIS is based on the idea That the preferred alternative should have the shortest geometric distance possible From the positive ideal solution[18] to the longest geometric distance Negative ideal solution (NIS).[18] This is a compensatory selection approach that matches a set of alternatives by isolating weights for each condition, normalizing scores for each criterion and measuring the geometric distance between each alternative and the ideal alternative Definition, which is the superlative score in allcriterions. A supposition of TOPSIS is that the criterion decreases or increases monotonically. Normalization is usually important as the parameters in multi-criteria problems are often inconsistent..[19,20] Compensatory techniques, e.g. TOPSIS, require trade-offs between criteria where a bad outcome can be invalidated by a good outcome in additional criterion. This allows modeling to be used more effectively than non-compensatory approaches that include or reject alternatives based on hard cutoffs.[21] A utilisation model for nuclear power plants is givenin.[22]

TOPSIS Method which we have used in this paper is consist of some steps which is understudy as follows First of all we have set of alternatives and parameters in the form of matrix with corresponding weights. And in second step we compute maximum and minimum ideal solution of neutrosophic soft ideal solution with the help of presented formula. In third step determine the neutrosophic separation measure for every alternative from maximum and minimum neutrsosphic soft ideal solution.And in fourth step we measure relative closeness coefficient to the neutrosophic soft ideal solution neutrosophic soft numbers and at the end we arrange the data in increasing order and select the alternative which have maximum value.

2 Prelimnaries

In this section we recall some helpful material from the existing literature.

Definition 2.1. [4]Let X be a space of points (objects) , by means of neutrosophic set (NS) in X is express by

A neutrosophic set (NS) in X is express by a truthfull membership function η_T, indeterminate membership function η_I, and a untruthfull membership function η_F. $\eta = \{(\eta_T(x), \eta_I(x), \eta_F(x)), x \in X\}$ where $\eta_T(x), \eta_I(x)$ and $\eta_F(x)$ are actual standard or non standard subsets of $]0, 1^+[$.That is

$\eta_T : X \to]0, 1^+[$

$\eta_I : X \to]0, 1^+[$

$\eta_F : X \to]0, 1^+[$

There is no objection on the sum of $\eta_T(x)$, $\eta_I(x)$ and $\eta_F(x)$ so there sum is lies in between $0 \leq \sup \eta_T(x) + \sup \eta_I(x) + \sup \eta_F(x) \leq 3^+$.

Definition 2.2. [11] Let Y be an universal set, E be an attributive set with respect to universal set Y, and A$\subseteq E$. Then a pair (K,A) is called a fuzzy soft set where K is a mapping such that

$$K : A \to F^Y$$

where F^Y represents the collection of all fuzzy subsets of Y. A pair (K,A) can be expressd as

$$(K, A) = \left\{\langle e, K(e)\rangle \,|e \in A, K(e) \in F^Y\right\}$$

where

$$K(e) = \{\langle y, \mu_F(y)\rangle \,|y \in Y\}$$

and

$$\mu_F(y) \in [0, 1]$$

Example 2.3. Assume that Y=$\{y_1, y_2, y_3, y_4\}$ be a universal set, E=$\{e_1, e_2, e_3, e_4\}$ be an attributive set with respect to Y. Let A=$\{e_1, e_2, e_3\} \subseteq E$ then a fuzzy soft set (K,A) can be defined as

$$(K, A) = \left\{\begin{array}{c} \left\langle e_1, \left\{\left(y_1, \frac{6}{7}\right), \left(y_2, \frac{4}{7}\right), \left(y_3, \frac{3}{5}\right), \left(y_4, \frac{4}{5}\right)\right\}\right\rangle, \\ \left\langle e_2, \left\{\left(y_1, \frac{4}{5}\right), \left(y_2, \frac{3}{6}\right), \left(y_3, \frac{7}{10}\right), \left(y_4, \frac{4}{10}\right)\right\}\right\rangle, \\ \left\langle e_3, \left\{\left(y_1, \frac{6}{10}\right), \left(y_2, \frac{4}{10}\right), \left(y_3, \frac{2}{5}\right), \left(y_4, \frac{3}{5}\right)\right\}\right\rangle \end{array}\right\}$$

Definition 2.4. [5] Let Y be an universal set, E be an attributive set with respect to universal set Y, and A$\subseteq E$. Then a pair (K,A) is called neutrosophic soft set where K is a mapping such that

$$K : A \to N^Y$$

where N^Y represents the collection of all neutrosophic subsets of Y. A pair (K,A) can be expressd as

$$(K, A) = \left\{\langle e, K(e)\rangle \,|e \in A, K(e) \in N^Y\right\}$$

where

$$K(e) = \{\langle y, T_N(y), I_N(y), F_N(y)\rangle \,|y \in Y\}$$

3 TOPSIS METHOD IN THE ENVIRONMENT OF NEUTROSOPHIC SOFT SET

Definition 3.1. Chebyshev Distance for neutrosophic soft numbers

Let

$$F(x_1) = \{(x_1, \langle T_{\bar{F}}(e_1)(x_1), I_{\bar{F}}(e_1)(x_1), F_{\bar{F}}(e_1)(x_1)\rangle) | \varrho_i \epsilon A, x \epsilon X\}$$

and

$$F(x_2) = \{(x_2, \langle T_{\bar{F}}(e_2)(x_2), I_{\bar{F}}(e_2)(x_2), F_{\bar{F}}(e_2)(x_2)\rangle) | \varrho_i \epsilon A, x \epsilon X\}$$

be the two neutrosophic soft numbers. Separation measure between $F(x_1)$ and $F(x_2)$ based on Chebyshev Distance can be defined as follows:

$$\mathrm{D}_{Chebyshev}(F(x_1, x_2)) = \max\left\{\left\langle \begin{array}{c} \left|T_{\bar{F}}(e_1)(x_1) - T_{\bar{F}}(e_2)(x_2)\right|, \left|I_{\bar{F}}(e_1)(x_1) - I_{\bar{F}}(e_2)(x_2)\right|, \\ \left|F_{\bar{F}}(e_1)(x_1) - F_{\bar{F}}(e_2)(x_2)\right| \end{array} \right\rangle\right\}$$

Definition 3.2. Separation measure from maximum neutrosophic soft ideal solution

$$D^{j+}_{Chebyshev}\left(d^{w_j}_{ij}, d^{w_{j+}}_{ij}\right) = \max\left\{\left\langle \begin{array}{c} \left|T^{wj}_{ij}(e_i)(x) - T^{w+}_{ij}(e_i)(x)\right|, \\ \left|I^{wj}_{ij}(e_i)(x) - I^{w+}_{ij}(e_i)(x)\right|, \\ \left|F^{wj}_{ij}(e_i)(x) - F^{w+}_{ij}(e_i)(x)\right| \end{array} \right\rangle\right\} \quad (1)$$

$\forall\, j = 1, 2, ..n.$

and $NS^+ = \sum_{j=1}^{n} \mathrm{D}^{j+}_{Chebyshev}\left(d^{w_j}_{ij}, d^{w_{j+}}_{ij}\right)$ for $i = 1, 2, 3, ..m.$

Definition 3.3. Separation measure from minimum neutrosophic soft ideal solution

$$D^{j+}_{Chebyshev}\left(d^{w_j}_{ij}, d^{w_{j-}}_{ij}\right) = \max\left\{\left\langle \begin{array}{c} \left|T^{wj}_{ij}(e_i)(x) - T^{w-}_{ij}(e_i)(x)\right|, \\ \left|I^{wj}_{ij}(e_i)(x) - I^{w-}_{ij}(e_i)(x)\right|, \\ \left|F^{wj}_{ij}(e_i)(x) - F^{w-}_{ij}(e_i)(x)\right| \end{array} \right\rangle\right\} \quad (2)$$

$\forall\, j = 1, 2, ..n.$

and $NS^- = \sum_{j=1}^{n} D^{j+}_{Chebyshev}\left(d^{w_j}_{ij}, d^{w_{j-}}_{ij}\right)$ for $i = 1, 2, 3, ..m.$

3.1 Extension of neutrosophic soft topsis method

Let we have set of alternatives $A = \{A_1, A_2, A_3, ., ., A_m\}$ and set of parameters $C = \{C_1, C_2, C_3, ., ., C_n\}$ and also its respective weights are given. The performance of alternatives are determined by neutrosophic soft set using the following steps:

step 1 First of all we calculate weighted decision matrix by multiplying corresponding weights to each alternatives. Multiplication is defined as follows

$$D^W = D \otimes W = d_{i_j}^w = w_j \otimes d_{i_j}$$
$$= \left(T^w_{\bar{F}(eij)}, I^w_{\bar{F}(eij)}, F^w_{\bar{F}(eij)} \right)$$

$$w_j \otimes d_{i_j} = \begin{pmatrix} a_j + T_{\bar{F}(eij)} - a_j T_{\bar{F}(e_{ij})}, \\ b_j I_{\bar{F}(e_{ij})}, c_j F_{\bar{F}(e_{ij})} \end{pmatrix} \tag{3}$$

Step 2 Compute maximum and minimum ideal solution of neutrosophic soft ideal solution with the help of following formula

$$A_N^+ = \left(d_1^{w+}, d_2^{w+}, d_3^{w+}, .,., d_n^{w+}\right) \tag{4}$$
$$d_j^{w+} = \left(T^{w+}_{\bar{F}(ej)}, I^{w+}_{\bar{F}(ej)}, F^{w+}_{\bar{F}(ej)} \right)$$
$$\text{where } T^{w+}_{\bar{F}(ej)} = \left\{ \max_i \left(\left\{ T^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$
$$I^{w+}_{\bar{F}(ej)} = \left\{ \max_i \left(\left\{ I^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$
$$F^{w+}_{\bar{F}(ej)} = \left\{ \min_i \left(\left\{ F^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$

$$A_N^- = \left(d_1^{w-}, d_2^{w-}, d_3^{w-}, .,., d_n^{w-}\right) \tag{5}$$
$$d_j^{w-} = \left(T^{w-}_{\bar{F}(ej)}, I^{w-}_{\bar{F}(ej)}, F^{w-}_{\bar{F}(ej)} \right)$$
$$T^{w-}_{\bar{F}(ej)} = \left\{ \min_i \left(\left\{ T^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$
$$I^{w-}_{\bar{F}(ej)} = \left\{ \min_i \left(\left\{ I^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$
$$F^{w-}_{\bar{F}(ej)} = \left\{ \max_i \left(\left\{ F^{wj}_{\bar{F}(eij)} \mid j = 1, ..., n \right\} \right) \right\}$$

Step 3 gives the neutrosophic separation measure for every alternatives from maximum and minimum neutrsosphic soft ideal solution.

We proposed new distance measure of two neutrosophic soft number in place of Manhattan distance or Euclidean distance.

Step 4: And in this step we calculate relative closeness coefficient to neutrosophic soft ideal solution neutrosophic soft numbers. formula is given below

$N_{T_i} = \frac{N_{S_i^-}}{N_{S_i^-} + N_{S_i^+}}$ where $i = 1, 2, 3, ...m$

and finally the decision will be taken on the base of highest value of N_{T_i}.

3.2 Numerical problem

Let we have four different alternatives with given parameters and also corresponding weights which are given below in the form of table and we choose the best alternative.**Step1**

TABLE 1 CRITERIA WEIGHTS

	C_1	C_2	C_3
W_i	$(0.755, 0.222, 0.217)$	$(0.887, 0.113, 0.107)$	$(0.765, 0.226, 0.182)$

	C_4	C_5	C_6
W_i	$(0.692, 0.277, 0.251)$	$(0.788, 0.200, 0.180)$	$(0.700, 0.272, 0.244)$

TABLE 2 NEUTROSOPHIC SOFT DECISION MATRIX

	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$

	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_1	$0.744, 0.256, 0.204$	$0.652, 0.348, 0.293$	$0.608, 0.392, 0.336$
A_1	$0.779, 0.256, 0.204$	$0.811, 0.189, 0.109$	$0.850, 0.150, 0.092$
A_1	$0.727, 0.273, 0.221$	$0.791, 0.209, 0.148$	$0.808, 0.192, 0.127$

Step 2 Now we compute neutrosophic weighted decision matrix by using equation 3 we get table 3 given as follows

TABLE 3 WEIHGTED DECISION MATRIX

d_{ij}^{w}	C_1	C_2	C_3
A_1	$(0.9666, 0.0301, 0.0175)$	$(0.9833, 0.0166, 0.0098)$	$(0.953, 0.0452, 0.0273)$
A_2	$(0.9184, 0.0739, 0.0601)$	$(0.9691, 0.0308, 0.0234)$	$(0.9217, 0.0752, 0.0504)$
A_3	$(0.9706, 0.0266, 0.0145)$	$(0.9872, 0.0127, 0.0068)$	$(0.9609, 0.0375, 0.0203)$
A_4	$(0.9184, 0.0739, 0.0601)$	$(0.9700, 0.0299, 0.0208)$	$(0.9454, 0.0524, 0.0327)$

d_{ij}^{w}	C_4	C_5	C_6
A_1	$(0.9088, 0.0819, 0.0604)$	$(0.9624, 0.0354, 0.0221)$	$(0.9592, 0.0369, 0.0197)$
A_2	$(0.9211, 0.0709, 0.0512)$	$(0.9262, 0.0696, 0.0527)$	$(0.8824, 0.1066, 0.0819)$
A_3	$(0.9319, 0.0709, 0.0512)$	$(0.9599, 0.0378, 0.0196)$	$(0.955, 0.0408, 0.0224)$
A_4	$(0.9159, 0.0756, 0.0554)$	$(0.9556, 0.0418, 0.0266)$	$(0.9424, 0.0522, 0.0309)$

Step 3 now by using equation 4 and 5 we calculate maximum and minimum neutrosophic soft ideal solution given below

TABLE 4 MAXIMUM NEUTROSOPHIC SOFT IDEAL SOLUTION

	C_1	C_2	C_3
d_i^{w+}	$\begin{pmatrix} 0.9706, 0.0739, \\ 0.0145 \end{pmatrix}$	$\begin{pmatrix} 0.9872, 0.0308, \\ 0.0068 \end{pmatrix}$	$\begin{pmatrix} 0.9609, 0.0752, \\ 0.0203 \end{pmatrix}$
	C_4	$C5$	C_6
d_i^{w+}	$\begin{pmatrix} 0.9319, 0.0819, \\ 0.0512 \end{pmatrix}$	$\begin{pmatrix} 0.9624, 0.0696, \\ 0.0196 \end{pmatrix}$	$\begin{pmatrix} 0.9592, 0.1066, \\ 0.0197 \end{pmatrix}$

Graphical presentation of table 4 is given

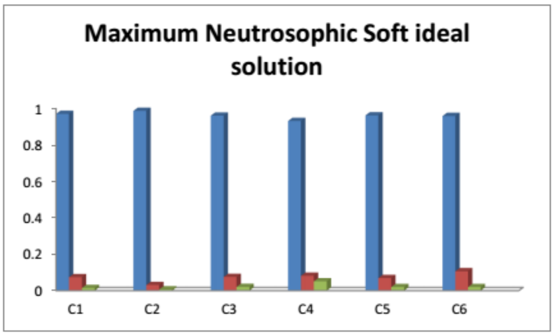
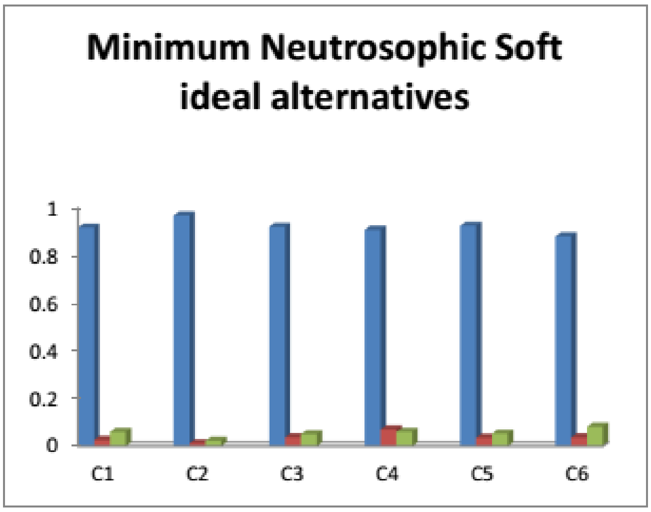


TABLE 5 MINIMUM NEUTROSOPHIC SOFT IDEAL SOLUTION

	C_1	C_2	C_3
d_i^{w-}	$\begin{pmatrix} 0.9184, 0.0266, \\ 0.0601 \end{pmatrix}$	$\begin{pmatrix} 0.9691, 0.0127, \\ 0.0234 \end{pmatrix}$	$\begin{pmatrix} 0.9217, 0.0375, \\ 0.0504 \end{pmatrix}$
	C_4	C_5	C_6
d_i^{w-}	$\begin{pmatrix} 0.9088, 0.0709, \\ 0.0604 \end{pmatrix}$	$\begin{pmatrix} 0.9262, 0.0354, \\ 0.0527 \end{pmatrix}$	$\begin{pmatrix} 0.8824, 0.0369, \\ 0.0819 \end{pmatrix}$

Graphical view of table 5 is given as follows

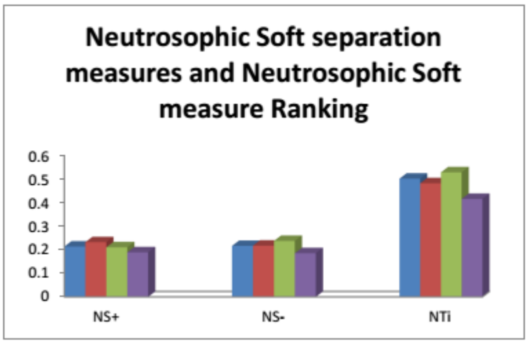


Step 4 determine neutrosophic soft separation measures for every alternatives from the maximum neutrosophic soft ideal solution and manimum neutrosophic soft ideal solution. For this purpuse we use equation **??** and **??** so we get following table

TABLE 6 NEUTROSOPHIC SOFT SEPARATION MEASURES AND NEUTROSOPHIC SOFT MEASURE RANKING

	NS_i^+	NS_i^-	NT_i
A_1	0.215	0.2177	0.5031
A_2	0.2335	0.2188	0.4837
A_3	0.2117	0.2389	**0.5301**
A_4	0.1904	0.1864	0.4173

Graphical view of final ranking is given as To observe the above table we get the result that $A_3 \succ A_1 \succ$



$A_2 \succ A_4$ So we get the same result like Neutrosophic Topsis[23] and Simplified-TOPSIS method and Neutrosophics.

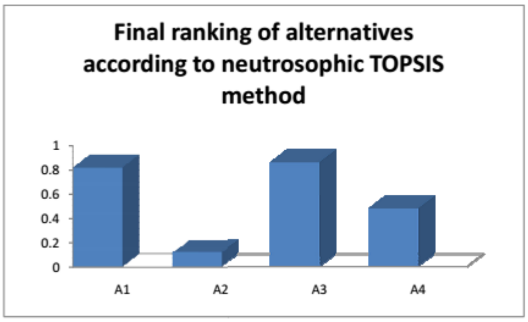
4 Comparison Analysis

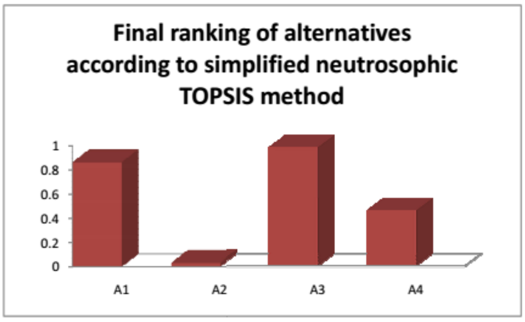
We compare our presented method to other two already proposed method named as single valued neutrosophic TOPSIS method and simplified neutrosophic TOPSIS method. Graphical comaprison is already presented in this section which is given below.

Alternatives	$M(A_i)$
A_1	$0.8190, 0.8645, 0.5031$
A_2	$0.1158, 0.0352, 0.4837$
A_3	$0.8605, 0.9802, 0.5301$
A_4	$0.4801, 0.4598, 0.4173$

Grades based on Method-1	Method-2	Method-3
A_3	A_3	A_3
A_1	A_1	A_1
A_4	A_4	A_4
A_2	A_2	A_2

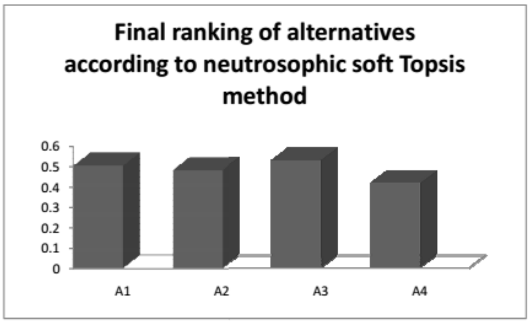
Graphical view of grading based on Method-1





Graphical view of grading based on Method-2

Graphical view of grading based on Method-3



5 Application of S-boxes image encryption in the environment of neutrosophic soft set

In this section we will be able to know that how and which one block cipher s-box image encryption is best to choose among the other under study s-boxes which proctect the data in safe mode.The procedure for the selection step wise is given as follows.

Let we have set of s-boxes that is $S = \{B_1, B_2, B_3, B_4, B_5, B_6\}$ where B_1 is stand for Plain Image B_2 is stand for AES similarly $B_{3,} B_{4,} B_5, B_6$ is stand for APA, Lui, Gray, and Prime respectively. We make the selection on the basis of set of criteria i.e $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ where C_1 shows entropy, C_2 tells about contrast and C_3, C_4, C_5, C_6 is used for an average correlation, energy, and homogeneity respectively. Decision matrix is given below

Step 1

TABLE 1 DECISION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
B_1	$(0.1, 0.2, 0.3)$	$(0.2, 0.3, 0.4)$	$(0.1, 0.9, 0.7)$	$(0.6, 0.4, 0.2)$	$(0.4, 0.1, 0.2)$	$(0.6, 0.5, 0.1)$
B_2	$(0.2, 0.5, 0.4)$	$(0.2, 0.7, 0.9)$	$(0.5, 0.1, 0.3)$	$(0.5, 0.2, 0.3)$	$(0.3, 0.1, 0.7)$	$(0.9, 0.8, 0.7)$
B_3	$(0.1, 0.6, 0.9)$	$(0.7, 0.8, 0.2)$	$(0.4, 0.1, 0.7)$	$(0.2, 0.2, 0.1)$	$(0.6, 0.8, 0.9)$	$(0.7, 0.3, 0.2)$
B_4	$(0.2, 0.3, 0.6)$	$(0.5, 0.4, 0.3)$	$(0.1, 0.8, 0.4)$	$(0.5, 0.2, 0.6)$	$(0.7, 0.4, 0.3)$	$(0.7, 0.9, 0.5)$
B_5	$(0.5, 0.3, 0.5)$	$(0.6, 0.7, 0.4)$	$(0.9, 0.6, 0.2)$	$(0.3, 0.4, 0.5)$	$(0.6, 0.1, 0.4)$	$(0.8, 0.1, 0.4)$
B_6	$(0.1, 0.1, 0.3)$	$(0.3, 0.2, 0.7)$	$(0.4, 0.6, 0.8)$	$(0.2, 0.6, 0.4)$	$(0.5, 0.2, 0.4)$	$(0.4, 0.8, 0.5)$

TABLE 2 CRITERIA WEIGHTS

	C_1	C_2	C_3	C_4	C_5	C_6
W_i	$\begin{pmatrix} 0.4, 0.6, \\ 0.8 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.5, \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.7, \\ 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.9, \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.9, 0.2, \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.1, \\ 0.2 \end{pmatrix}$

Step 2 Now we compute neutrosophic weighted decision matrix by using equation 3 we get table 3 given as follows

TABLE 3 WEIGHTED DECISION MATRIX

	C_1	C_2	C_3
B_1	$(0.46, 0.12, 0.24)$	$(0.36, 0.15, 0.12)$	$(0.55, 0.63, 0.63)$
B_2	$(0.52, 0.30, 0.32)$	$(0.36, 0.35, 0.63)$	$(0.75, 0.07, 0.63)$
B_3	$(0.46, 0.36, 0.72)$	$(0.76, 0.40, 0.06)$	$(0.7, 0.07, 0.63)$
B_4	$(0.52, 0.18, 0.48)$	$(0.6, 0.20, 0.09)$	$(0.55, 0.56, 0.36)$
B_5	$(0.7, 0.18, 0.40)$	$(0.68, 0.35, 0.12)$	$(0.95, 0.63, 0.18)$
B_6	$(0.46, 0.06, 0.24)$	$(0.44, 0.10, 0.21)$	$(0.7, 0.42, 0.72)$

	C_4	C_5	C_6
B_1	$(0.88, 0.36, 0.08)$	$(0.94, 0.02, 0.06)$	$(0.76, 0.05, 0.02)$
B_2	$(0.85, 0.18.0.12)$	$(0.93, 0.02, 0.21)$	$(0.94, 0.08, 0.14)$
B_3	$(0.76, 0.18, 0.05)$	$(0.96, 0.16, 0.27)$	$(0.82, 0.03, 0.04)$
B_4	$(0.85, 0.18, 0.24)$	$(0.97, 0.08, 0.09)$	$(0.82, 0.03, 0.04)$
B_5	$(0.79, 0.36, 0.20)$	$(0.96, 0.02, 0.12)$	$(0.88, 0.01, 0.08)$
B_6	$(0.76, 0.54, 0.16)$	$(0.95, 0.04, 0.12)$	$(0.64, 0.05, 0.16)$

Step 3 now by using equation 4 and 5 we calculate maximum and minimum neutrosophic soft ideal solution given below

TABLE 4 MAXIMUM NEUTROSOPHIC SOFT IDEAL SOLUTION

	C_1	C_2	C_3
d_i^{w+}	$0.52, 0.36, 0.24$	$0.76, 0.40, 0.06$	$0.95, 0.63, 0.18$

	C_4	C_5	C_6
d_i^{w+}	$0.88, 0.54, 0.04$	$0.97, 0.16, 0.06$	$0.94, 0.08, 0.02$

Graph of maximum neutrosophic soft ideal solution is given as follows

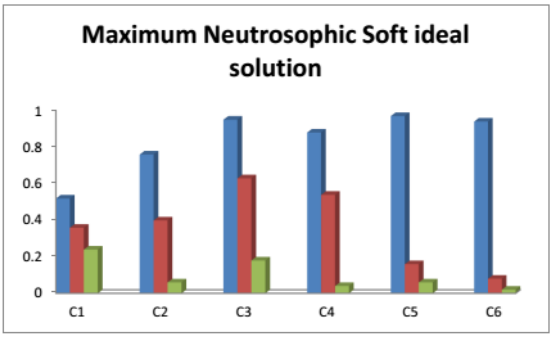
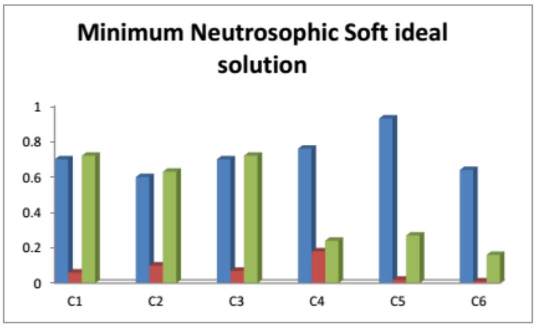


TABLE 5 MINIMUM NEUTROSOPHIC SOFT IDEAL SOLUTION

	C_1	C_2	C_3
d_i^{w-}	$0.7, 0.06, 0.72$	$0.6, 0.10, 0.63$	$0.7, 0.07, 0.72$

	C_4	C_5	C_6
d_i^{w-}	$0.76, 0.18, 0.24$	$0.93, 0.02, 0.27$	$0.64, 0.01, 0.16$

Graph of minimum neutrosophic soft ideal solution is given below **Step 4** determine neutrosophic soft

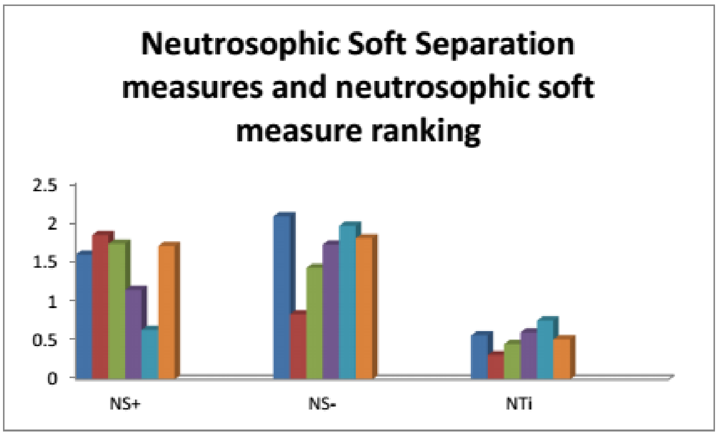


separation measures for every alternatives from maximum neutrosophic soft ideal solution and minimum neutrosophic soft ideal solution. For this purpuse we use equation **??** and **??** so we get following table

TABLE 6 NEUTROSOPHIC SOFT SEPARATION MEASURES AND NEUTROSOPHIC SOFT MEASURE RANKING

	NS_i^+	NS_i^-	NT_i
B_1	1.59	2.08	0.56
B_2	1.84	0.83	0.31
B_3	1.73	1.42	0.45
B_4	1.14	1.72	0.60
B_5	0.63	1.96	$\mathbf{0.75}$
B_6	1.7	1.8	0.51

Final ranking in the form of graph is given below Here we see that $B_5 \succ B_4 \succ B_1 \succ B_6 \succ B_3 \succ B_2$ so



finally we conclude that Gray s-box is best according to the given data.

6 Conclusions

We amalgamate neutrosophic set with soft set and get a more generalized type of neutrosophic set. And in the end that gives us better results than neutrosophic set. We also observe that our proposed "Neutrosophic Soft Topsis Method" when applied on the data gives the same result but calculation and proceedure adopted is more easy as compared to past proposed methods. And finally we apply this method on S-boxes image encryption that helps us to pick the best S-box.

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