Bipolar Neutrosophic Pythagorean Set and its Topological Space

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Abstract

The aim of this paper is to introduce the concept of Bipolar Neutrosophic Pythagorean Set with truth membership function , Indeterminancy membership function IN and false membership function FN with TN and FN as an independent neutrosophic component. Bipolar Neutrosophic pythagorean set is an extension of Bipolar neutrosophic set. We establish some of its relative properties of Bipolar neutrosophic pythagorean set.

Keywords: Neutrosophic set, Bipolar Neutrosophic pythagorean set, Bipolar Neutrosophic set, Bipolar pythagorean set

1.Introduction

The fuzzy set was introduced by Zadeh [14] in 1965. F. Smarandache, a mathematical tool for handling problems involving imprecise, indeterminacy and inaccurate data, introduced the idea of the Neutrosophic package.

Smarandache [11] in neutrosophic sets discussed. The indeterminacy membership function walks along independently of the membership of the reality or the membership of falsity in neutrosophic sets. Neutrosophic theory has been extensively discussed in the treatment of real-life conditions involving uncertainty by researchers for application purposes. While the hesitation margin of neutrosophical theory is independent of membership in truth or falsehood, it still seems more general than intuitionist fuzzy sets. Recently, the relationships between inconsistent intuitionistic fuzzy sets, image fuzzy sets, neutrosophic sets, and intuitionistic fuzzy sets have been examined in Atanassov et al. [1] however, it remains doubtful whether the indeterminacy associated with a particular element exists due to the element's ownership or non-belongingness. The concept of pythagorean sets was initiated by Xindong Peng[13]. Many authors[4-9] have studied related to neutrosophic pythagorean sets.

The degree of dependency between the components of the fuzzy set and neutrosophic sets was first introduced by F. Smarandache in 2016. The key concept of Neutrosophic sets is to define each value statement in a 3D-Neutrosophic space, where each dimension of the space represents the true membership, falsity membership, and indeterminacy respectively, when two components T and F are dependent and I is independent then $T+I+F\leq 2$. If T and F are dependent neutrosophic pythagorean components then $T^2 + F^2 \leq 1$. The concept of bipolar neutrosophic sets was introduced by Irfan Deli.Et.al.[2] and studied some of its properties

In this paper, we must introduce the definition of the introduction of the Bipolar neutrosophic pythagorean set with T, and F as dependent neutrosophic components and I as an independent neutrosophic component and define some of its properties.

2. Preliminaries

Definition 2.1[11]

Let X be a universe. A Neutrosophic set A on X can be defined as follows: $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$

Where $T_A , I_A, F_A: U \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

Definition 2.2[3]

Let X be a universe. A neutrosophic pythagorean set A with T, and F as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, I_A, F_A >: x \in X\}$$

Where $T_A + F_A \leq 1$ and $(T_A)^2 + (I_A)^2 + (F_A)^2 \leq 2$

Here, $T_A(x)$ is the truth membership, $I_A(x)$ is an indeterminancy membership and $F_A(x)$ is the false membership .

Here, $T_A(x)$ and $F_A(x)$are dependent neutrosophic components and $I_A(x)$ is an independent component.

Definition 2.3[2]

Suppose *X* be a non-empty set. A bipolar neutrosophic set (BNS) *A*, over *X* characterizes each element *x* in *X* by a positive truth-membership function T_A^+, a positive indeterminancy membership function I_A^+, a positive falsity membership function F_A^+, a negative truth membership function T_A^-, a negative indeterminancy membership function I_A^-, a negative falsity membership function F_A^-, such that for each $x \in X$, T_A^+ , I_A^+, , $F_A^+ \in [0,1]$, T_A^- , I_A^-, $F_A^- \in$ [0,1],

and $T_A^+ + I_A^+ \ F_A^+ \leq 3, -3 \leq T_A^- + I_A^- + + F_A^- \leq 0$

Definition 2.4[2]

A BNS *B* over *X* is said to be an universe BNS, denoted by 1_X, if and only if its membership values are respectively defined as $T_A^+(x) = 1$, $I_A^+(x) = 0, F_A^+(x) = 0, T_A^-(x) = 0$, $I_A^-(x) = -1$, $F_A^-(x) = -1$, for all $x \in$ [0,1].

Definition 2.5[2]

A BNS *B* over *X* is said to be an empty BNS, denoted by 0_X, if and only if its membership values are respectively defined as $T_A^+(x) = 0$, I_A^+(x) $= 1, F_A^+$(x) =1, $T_A^-(x) = -1$, $I_A^-(x) = 0, F_A^-(x) = 0$, for all x ∈ [0,1].

Definition 2.6[2]

The complement of BNS A = $\{T_A^+ , I_A^+, F_A^+, T_A^- , I_A^-, F_A^-\}$ is denoted by A^C and is defined as

$A^C = \{F_A^+ , I_A^+, T_A^+, F_A^- , I_A^-, T_A^-\}$

Definition 2.7[2]

B = $\{T_B^+ , I_B^+, F_B^+, T_B^- , I_B^-, F_B^-\}$ is denoted by A∪B and is defined as follows

A∪B = {max $\{T_A^+, T_B^+\}$, min $\{I_A^+, I_B^+\}$, min $\{F_A^+, F_B^+\}$,min $(T_A^-, T_B^-\}$, max $(I_A^-, I_B^-\}$, max $(F_A^-, F_B^-\}$}

Definition 2.8[2]

The intersection of any two BNS A = $\{T_A^+ , I_A^+, F_A^+, T_A^- , I_A^-, F_A^-\}$ and

B = $\{T_B^+ , I_B^+, F_B^+, T_B^- , I_B^-, F_B^-\}$ is denoted by A∪B and is defined as follows

A∩B = {min $\{T_A^+, T_B^+\}$, max $\{I_A^+, I_B^+\}$, max $\{F_A^+, F_B^+\}$,max $(T_A^-, T_B^-\}$, min $(I_A^-, I_B^-\}$, min $(F_A^-, F_B^-\}$}

3. Bipolar Neutrosophic Neutrosophic Pythagorean Set

Definition 3.1

A Bipolar Neutrosophic Pythagorean set [BNPS] P in R is defined in the form of

P = { p, $[TN_P^+, IN_P^+, FN_P^+, TN_P^-, IN_P^-, FN_P^-]$: p∈ $R\}$,

Where TN_P^+, IN_P^+, FN_P^+ : R → $[0,1]$, TN_P^-, IN_P^-, FN_P^-: R → $[-1,0]$,

$$(TN_P^+)^2 + (IN_P^+)^2 + (FN_P^+)^2 \leq 2 \text{and } (TN_P^-)^2 + (IN_P^-)^2 + (FN_P^-)^2 \geq -2$$

$$TN_P^+ + FN_P^+ \leq 1 \; and \; TN_P^- + FN_P^- \geq -1.$$

In this definition , there $TN_P^+ and\ TN_P^-$ are acceptable and unacceptable in the past [positive truth membership and negative truth membership].Similarly IN_P^+ and IN_P^- are acceptable and unacceptable in future[positive indeterminancy and negative indeterminancy]. FN_P^+ and FN_P^- are acceptable and unacceptable in the present[positive falsity membership and negative falsity membership] respectively.

Example 3.2

Let R = {p, q}. Then a bipolar neutrosophic pythagorean subset P of R is

$$P = \begin{cases} \{p, [0.5, 0.8, 0.2, -0.1, -0.5, -0.4]\} \\ \{q, [0.3, 0.5, 0.3. -0.2, -0.7, -0.6]\} \end{cases}$$

Definition 3.3

The complement of a Bipolar neutrosophic pythagorean set P on R denoted by P^c and is defined as

$P^c = \{< r, FN_P^+, (1 - IN_P^+), TN_P^+, FN_P^-, (-1 - IN_P^-), TN_P^- >: r \in R\}$

Example 3.4

Let R = {p, q}. Then a bipolar neutrosophic pythagorean subset P of R is

$$P = \begin{cases} \{p, [0.3, 0.6, 0.4, -0.3, -0.2, -0.5]\} \\ \{q, [0.2, 0.4, 0.3. -0.4, -0.5, -0.2]\} \end{cases}$$

The complement of the bipolar neutrosophic pythagorean set is

$$P^C = \begin{cases} \{p, [0.4, 0.4, 0.3, -0.4, -0.5, -0.1]\} \\ \{q, [0.3, 0.6, 0.2. -0.6, -0.3, -0.2]\} \end{cases}.$$

Definition 3.5

A Bipolar neutrosophic pythagorean set P is contained in another bipolar neutrosophic pythagorean set

Q (i.e) P $\subseteq Q$ if

$$TN_P^+ \leq TN_Q^+, IN_P^+ \geq IN_Q^+, FN_P^+ \geq FN_Q^+, TN_P^- \geq TN_Q^-, IN_P^- \leq IN_Q^- \, and \, FN_P^- \leq FN_Q^-.$$

Example 3.6

Let P and Q be two Bipolar neutrosophic pythagorean subsets of R.

$$P = \begin{cases} \{p, [0.3, 0.6, 0.4, -0.2, -0.7, -0.5]\} \\ \{q, [0.2, 0.4, 0.3. -0.4, -0.5, -0.2]\} \end{cases},$$

$$Q = \begin{cases} \{p, [0.6, 0.5, 0.2, -0.3, -0.8, -0.6]\} \\ \{q, [0.4, 0.1, 0.2. -0.4, -0.5, -0.2]\} \end{cases}.$$

Then P ⊆ Q.

Definition 3.7

Let P = { p, $[TN_P^+, IN_P^+, FN_P^+, TN_P^-, IN_P^-, FN_P^-]$: p$\in R\}$ and Q = { q, $[TN_Q^+, IN_Q^+, FN_Q^+, TN_Q^-, IN_Q^-, FN_Q^-]$: p$\in R\}$,

are two bipolar neutrosophic pythagorean sets P and Q in R . Then the union and intersection of two bipolar neutrosophic pythagorean sets can be defined as

$P \cup Q = \{\max(TN_P^+, TN_q^+), \min(IN_P^+, IN_Q^+), \min(FN_P^+, FN_Q^+), \min(TN_P^-, TN_q^-), \max(IN_P^-, IN_Q^-), \max(FN_P^-, FN_Q^-)\}$

$P \cap Q = \{\min(TN_P^+, TN_q^+), \max(IN_P^+, IN_Q^+), \max(FN_P^+, FN_Q^+),$
$\max(TN_P^-, TN_q^-), \min(IN_P^-, IN_Q^-), \min(FN_P^-, FN_Q^-)\}$

Example 3.8

Let R = {p, q}. Then the bipolar neutrosophic pythagorean subsets of P and Q in R can be

P = $\begin{cases}\{p,[0.4,0.6,0.4,-0.2,-0.7,-0.5]\}\\ \{q,[0.3,0.4,0.3.-0.4,-0.5,-0.2]\}\end{cases}$,

Q = $\begin{cases}\{p,[0.5,0.5,0.2,-0.5,-0.7-0.2]\}\\ \{q,[0.2,0.1,0.6.-0.5,-0.5,-0.1]\}\end{cases}$.

Then

$P \cup$ Q = $\begin{cases}\{p,[0.5,0.5,0.4,-0.5,-0.7,-0.2]\}\\ \{q,[0.3,0.1,0.3.-0.5,-0.5,-0.1]\}\end{cases}$,

P ∩ Q = $\begin{cases}\{p,[0.4,0.6,0.4,-0.2,-0.7,-0.5]\}\\ \{q,[0.2,0.4,0.6.-0.4,-0.5,-0.2]\}\end{cases}$.

Definition 3.9

A Bipolar neutrosophic pythagorean set P over the universe R is said to be empty bipolar neutrosophic pythagorean set ∅ with respect to the parameter P if

$TN_P^+ = 0, IN_P^+ = 1, FN_P^+ = 1, TN_P^- = -1, IN_P^- = 0, FN_P^- = 0.$

It is also denoted by 0_R.

Definition 3.10

A Bipolar neutrosophic pythagorean set P over the universe R is said to be universe bipolar neutrosophic pythagorean set ∅ with respect to the parameter P if

$TN_P^+ = 1, IN_P^+ = 0, FN_P^+ = 0, TN_P^- = 0, IN_P^- = -1, FN_P^- = -1.$

It is also denoted by 1_R.

Theorem 3.11

The set-theoretic axioms are satisfied by any Bipolar neutrosophic pythagorean sets as it can be easily verified. Consider Bipolar neutrosophic pythagorean sets *A*, *B*, *C* over the same universe *R*. Then the following properties holds all for Bipolar neutrosophic pythagorean sets over *R*.

(i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$.
(iii) $A \cup (B \cup C) = (A \cup B) \cup C$
(iv) $A \cap (B \cap C) = (A \cap B) \cap C$
(v) $A \cap (A \cup B) = A$
(vi) $A \cup (A \cap B) = A$.
(vii) $(A^c)^c = A$.
(viii) $(A \cup B)^c = A^c \cap B^c$

(ix) $(A \cap B)^c = A^c \cup B^c$
(x) $A \cup A = A \cup A$;
(xi) $A \cap A = A \cap A$.
(xii) $A \cup \emptyset = A$;
(xiii) $A \cap \emptyset = \emptyset$.
(xiv) $A \cup \emptyset = A$

4.Bipolar Neutrosophic Pythagorean Topological Spaces

Definition 4.1

A bipolar neutrosophic pythagorean topology (BNPT) on a non empty set R is of BNP sets satisfying the following axioms.

[1] $0_x, 1_x \in \tau$
[2] $A \cap B \in \tau$ for any a,b $\in \tau$
[3] $\cup A_i \in \tau$ for any arbitrary family$\{A_i \in J\} \in \tau$
The pair (X, τ) is called Bipolar neutrosophic pythagorean topological spaces (BNPTS).

Any BNP set in τ is called as BNP open set in R. The complement of BNP open set is BNP closed set.

Example 4.2

Let R = {p, q}. Then the bipolar neutrosophic pythagorean subsets A and B of R can be given as follows.

$$A = \begin{cases} < \mathrm{p}, (0.5,0.6,0.4, -0.1, -0.5, -0.7) > \\ < q, (0.4,0.7,0.3, -0.2, -0.7, -0.3) > \end{cases}$$

$$B = \begin{cases} < \mathrm{p}, (0.9,0.4,0.5, -0.7, -0.5, -0.1) > \\ < q, (0.5,0.3,0.4, -0.3, -0.3, -0.2) > \end{cases}$$

$$C = \begin{cases} < \mathrm{p}, (0.9,0.4,0.4, -0.7, -0.5, -0.1) > \\ < q, (0.5,0.3,0.3, -0.3, -0.3, -0.2) > \end{cases}$$

Then $\tau = \{0_R, 1_R, A, B, C\}$ is a bipolar neutrosophic pythagorean topology on R. Then (R, τ) is a bipolar neutrosophic pythagorean topological space[BNPTS]

Example 4.3

Let R = {p, q} and A be a bipolar neutrosophic pythagorean set as

$$A = \begin{cases} < \mathrm{p}, (0.5,,0.6,0.4, -0.1, -0.5, -0.7) > \\ < q, (0.4,0.7,0.3, -0.2, -0.7, -0.3) > \end{cases}$$

Then $\tau = \{0_R, 1_R, A\}$ is a bipolar neutrosophic topology on R.

Definition 4.4

Let (R, τ) be a BNP topological space and A = $\{TN_A^+, IN_A^+, FN_A^+, TN_A^-, IN_A^-, FN_A^-\}$ be a BNP set in R. Then the closure and interior of A is defined as

BNPInt(A) = $\cup$ {F: F is a BNP open set in R and F $\subseteq$A}

BNPCl (A) = ∩{F: F is a BNP closed in R and F ⊆ A}

Here BNPCl(A) is BNP closed set and BNPInt(A) is a BNP open set in X.

(a) A is BNP open set in X iff BNPInt(A) = A
(b) A is BNP closed set in X iff BNPCl(A) = A

Example 4.5

Let R = {p, q} and $\tau = \{ 0_R, 1_R, A, B, C\}$ where

$$A = \begin{cases} < p, (0.5,,0.6,0.4,-0.1,-0.5,-0.7) > \\ < q, (0.4,0.7,0.3,-0.2,-0.7,-0.3) > \end{cases}$$

$$B = \begin{cases} < p, (0.9,0.4,0.5,-0.7,-0.5,-0.1) > \\ < q, (0.5,0.3,0.4,-0.3,-0.3,-0.2) > \end{cases}$$

$$C = \begin{cases} < p, (0.9,0.4,0.4,-0.7,-0.5,-0.1) > \\ < q, (0.5,0.3,0.3,-0.3,-0.3,-0.2) > \end{cases}$$

Consider the BNP set D of R as

$$D = \begin{cases} < p, (0.7,0.3,0.2,-0.3,-0.2,-0.7) > \\ < q, (0.5,0.4,0.1,-0.4,-0.4,-0.3) > \end{cases}$$

Then BNPInt(D) = A and BNPCl(D) = 1_X.

Theorem 4.6

Let (M, τ_1) and (M, τ_2) be two BNP topological space on M, Then $\tau_1 \cap \tau_2$ is an BNP topology on M where $\tau_1 \cap \tau_2 = \{A_M: A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

Proof :

Obviously $0_M, 1_M \in \tau$.

Let $A_M, B_M \in \tau_1 \cap \tau_2$

Then $A_M, B_M \in \tau_1$ and $A_M, B_M \in \tau_2$

We know that τ_1 and τ_2 are two BNP topological space M.

Then $A_M \cap B_M \in \tau_1$ and $A_M \cap B_M \in \tau_2$

Hence $A_M \cap B_M \in \tau_1 \cap \tau_2$.

Let τ_1 and τ_2 are two BNP topological spaces on M.

Denote $\tau_1 \vee \tau_2$ = $\{A_M \cup B_M \; : A_M \in \tau_1$ and $A_M \in \tau_2\}$

$\tau_1 \wedge \tau_2$ = $\{A_M \cap B_M \; : A_M \in \tau_1$ and $A_M \in \tau_2\}$

Example 4.7

Let A_M *and* B_M be two BNP topological space on M.

Define $\tau_1 = \{0_M, \; 1_M, \; A_M\}$

$\tau_2 = \{0_M, \; 1_M, \; B_M\}$

Then $\tau_1 \cap \tau_2 = \{0_M, \; 1_M\}$ is a BNP topological space on M.

But $\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}$,

$\tau_1 \vee \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\}$ *and*

$\tau_1 \wedge \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}$ are not BNP topological space on M.

Theorem 4.8

Let (M, $\boldsymbol{\tau}$) be a be a Bipolar neutrosophic pythagorean topological space over M and Let $\boldsymbol{A_M}$ in bipolar neutrosophic pythagorean topological space. Then the following properties hold.

(i) BNPInt $(A_M) \subseteq A_M$

(ii) $A_M \subseteq B_M$ implies BNPInt $(A_M) \subseteq$ BNPInt (B_M) .

(iii) BNPInt $(A_M) \in \tau$.

(iv) A_M is a BNP open set implies BNPInt $(A_M) = A_M$.

(v) BNPInt (BNPInt (A_M)) = BNPInt(A_M)

(vi) BNPInt $(0_M) = 0_M$, BNPInt $(1_M) = 1_M$.

Proof:

(i) and (ii) are obviously true.

(iii) obviously $\cup \{B_M \in \tau: B_M \subseteq A_m\} \in \tau$

Note that $\cup \{B_M \in \tau: B_M \subseteq A_m\}$ = BNPInt (A_M)

$\therefore$ BNPInt $(A_M) \in \tau$

(iv) Necessity: Let A_M be a BNP open set. ie., $A_M \in \tau$. By (i) and (ii) BNPInt $(A_M) \subseteq A_m$.

Since $A_M \in \tau$ and $A_M \subseteq A_m$

Then $A_M \subseteq \cup \{B_M \in \tau: B_M \subseteq A_m\}$= BNPInt (A_M)

$A_M \subseteq$ BNPInt (A_M)

Thus BNPInt = A_m.

Sufficiency: Let BNPInt $(A_m) = A_m$

By (iii) PNInt $(A_m) \in \tau$, ie., A_m is a BNP open set.

(v) To prove BNPInt (BNPInt (A_m)) = BNPInt (A_m)

By (iii) BNPInt $(A_m) \in \tau$.

By (iv) BNPInt (BNPInt (A_m)) = BNPInt (A_m).

(vi) We know that 0_M and 1_M are in τ

By (iv) BNPInt $(0_M) = 0_M$, BNPInt $(1_M) = 1_M$.Hence the result.

Theorem 4.9

Let (M, τ) be a be a Bipolar Neutrosophic Pythagorean Topological space over M and Let A_M is in the Bipolar Neutrosophic Pythagorean topological space. Then the following properties hold.

(i) $A_M \subseteq$ BNPCl(A_M)
(ii) $A_M \subseteq B_M$ implies BNPCl $(A_M) \subseteq$ BNPCl (B_M).
(iii) BNPCl $(A_M)^c \in \tau$.
(iv) A_M is a BNP closed set implies BNPCl $(A_M) = A_M$.
(v) BNPCl (BNPCl (A_M)) = BNPCl (A_M)
(vi) BNPCl $(0_M) = 0_M$, BNPCl $(1_M) = 1_M$.

Proof:

(i) and (ii) are obviously true.

(iii) By theorem, BNPInt $({A_M}^c) \in \tau$.

Therefore BNPCl $(A_M)]^c = (\cap\{B_M \in \tau^c: B_M \subseteq A_m\})^c$

$= \cup\{B_M \in \tau: B_M \subseteq A_m{}^c\}$ = BNPInt $(A_M{}^c)$

$\therefore$ BNPCl $(A_M)]^c \in \tau$

(iv) Necessity:

By theorem, $A_M \subseteq$ BNPCl(A_M)

Let A_M be a BNP closed set. ie., $A_M \in \tau^c$.

Since $A_M \in \tau$ and $A_M \subseteq A_m$

BNPCl $(A_M)= \cap\{B_M \in \tau^c: A_M \subseteq B_m\} \subseteq \{B_M \in \tau^c: A_M \subseteq A_m\}$

BNPCl $(A_M) \subseteq A_m$

Thus A_m =NBPCl(A_m)

Sufficiency: This is obviously true by (iii)

(v) and (vi) can be proved by (iii) and (iv)

Theorem 4.10

Let (M, τ) be a be a Bipolar Neutrosophic Pythagorean topological space over M and Let A_M, B_M are in BNP topological space M. Then the following properties hold.

1. BNPInt $(A_M) \cap$ BNPInt (B_M) = BNPInt $(A_M \cap B_M)$
2. BNPInt $(A_M) \cup$ BNPInt $(B_M) \subseteq$ BNPInt $(A_M \cup B_M)$
3. BNPCl $(A_M) \cup$ BNPCl $(B_M) \subseteq$ BNPCl $(A_M \cup B_M)$
4. BNPCl $(A_M \cup B_M) \subseteq$ BNPCl $(A_M) \cap$ BNPCl (B_M)
5. BNPInt $(A_M))^c$ = BNPCl $(A_M{}^c)$
6. BNPCl $(A_M))^c$ = BNPInt $(A_M{}^c)$

Proof:

(i) Since $A_M \cap B_M \subseteq A_m$ for any m in M

By theorem, BNPInt $(A_M \cap B_M) \subseteq$ BNPInt (A_M)

Similarly, BNPInt ($A_M \cap B_M$) ⊆ BNPInt (B_M)

BNPInt ($A_M \cap B_M$) ⊆ BNPInt (A_M) ∩ BNPInt(B_M)

By theorem, BNPInt (A_M) ⊆ A_M and BNPInt (B_M) ⊆ B_M

Thus BNPInt ($A_M \cap B_M$) ⊆ $A_M \cap B_M$

Therefore, BNPInt (A_M) ∩ BNPInt (B_M) = BNPInt ($A_M \cap B_M$)

Similarly we can prove (ii),(iii) and (iv).

v) (BNPInt (A_M))^c = (∩ {$B_M \in \tau : B_M \subseteq A_m$})^c

= ∩ {$B_M \in \tau^c$: ${A_M}^c \subseteq B_m$}

= BNPCl (${A_M}^c$)

Similarly we can prove (vi)

Example 4.11

Let M = {b_1, b_2} and Let A_M, B_M, C_M be BNP set where

A_M = $\{< b_1, 0.3,0.2,0.3 >< b_2, 0.6,0.2,0.1 > \}$

B_M = $\{< b_1, 0.2,0.5,0.5 >< b_2, 0.6,0.2,0.2 >\}$

C_M = $\{< b_1, 0.3,0.2,0.3 >< b_2, 0.6,0.2,0.1 >\}$

$\tau = \{A_M, B_M, C_M, 0_M, 1_M\}$ is an BNP topology on M.

i) BNPInt (A_M) = 0_M= BNPInt(B_M)

Then $A_M \cup B_M = C_M$

BNPInt (A_M) ∪ BNPInt (B_M) =$0_M \cup 0_M$=0_M

And BNPInt ($A_M \cup B_M$) = BNPInt (C_M)= C_M

BNPInt (A_M) ∪ BNPInt (B_M) ≠ BNPInt ($A_M \cup B_M$)

ii) BNPCl (B_M)^c = (BNPCl (B_M))^c = ${0_M}^c = 1_M$

Similarly, BNPCl (C_M)^c= X_M

BNPCl (A_M)^c ∩ BNPCl (B_M)^c= $1_M \cap 1_M = 1_M$

Similarly, BNPCl (A_M^{c}∩ B_M^{c}) = BNPCl (A_M ∩ B_M)^c

= BNPInt (A_M ∪ B_M)^c

= C_M^{c}

BNPCl (A_M^{c}∩ B_M^{c}) ≠ BNPCl (A_M)^c ∩ (BNPCl (B_M)]^c

Acknowledgment

I thank R. Kathiresan and S. P. Rhea for their guidance and constant support throughout this paper.

IV CONCLUSION

In this paper, we have introduced bipolar neutrosophic pythagorean set and we have put forward some theorems based on this new notion. We have introduced topological structure on bipolar neutrosophic pythagorean set and characterized some of its properties. We hope that this paper will promote in future study on BNP set and BNPTS to carry out a general framework for their application in practical life.

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