Neutrosophic Soft Tri-Topological Spaces

Hasan Dadas [1] *, Sibel Demiralp[2]
[1]Department of Mathematics, Faculty of Arts and Sciences, Kastamonu University, Turkey;
Email: hsn.dad@gmail.com
[2]Department of Mathematics, Faculty of Arts and Sciences, Kastamonu University, Turkey;
Email sdemiralp@kastamonu.edu.tr
* Correspondence: hsn.dad@gmail.com

Abstract

In this study, the concept of neutrosophic soft tri-topological space is defined as a generalization of neutrosophic soft bitopological space. Then neutrosophic soft tri-open and tri-closed sets are defined and in this space. Also, some basic properties of these new types of open and closed sets are investigated and supported by many examples to further clarify the study.

Keywords: Neutrosophic set; Neutrosophic soft set; Soft bitopological space; Neutrosophic soft bitopological space; Neutrosophic soft tri-topological space.

1. Introduction

Our world contains a lot of ambiguity, neutrality, and complexity, as looking at any issue in this universe is characterized by lies sometimes, truthfulness at times, and ambiguity and neutrality at other times, and this matter is not studied by normal classical logic. For this reason, a new logic was defined that evaluates only the cases that are examined as true (1) or false (0) as true, indeterminate or false, each of which takes a value in the range of [0,1] by Smarandache[15] in 1999, He presented the concept of neutrosophic logic as an extension of the theory of logic which was defined by Zadeh[17] in 1965.

Molodtsov [24], carried out several successful types of research in many different branches of mathematics using the theory of soft sets in 1999. Many researchers have also showed great interest in the soft of the elastic group and many different types of research have been conducted on this theory.

Neutrosophic topological soft set theory was defined by Maji [22]. In 2015, for the first time. Later, Deli and Broumi [8] reconfigured this theory. In 2017, Bera [4] provided soft topological spaces. Still researchers show a keen interest in soft group theory.

In 1965 Kelly [13] defined the concept of bitopological space using two independent topological structures defined on a topological space. In 2019, this concept was extended to neutrosophic spaces by Öztürk et al. [27].

In 2020, the concept of bitopology was extended and studied by Dadas and Demiralp [9] to neutrosophic soft topological spaces

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In this study, the concept of neutrosophic soft tri-topological space is defined as a generalization of neutrosophic soft bitopological space. Then new types of open and closed sets have been introduced in neutrosophic soft bitopological spaces.

2. Preliminary

In this section, basic definitions and theorems are given about neutrosophic set theory and neutrosophic soft set theory.

2.1. Neutrosophic sets

Let X be a space of points (objects), with a generic element in X denoted by x, A neutrosophic set A in X is characterized by a truth-membership function T, an indeterminacy-membership function I and a falsity-membership function F [15] . That is: $T, I, F: X \to]-0, 1^{+}[$ where $T(x), I(x)$, and $F(x)$ are real standard or non-standard subsets of$]^{-}0,1^{+}[$. In general, there is no restriction on the sum of $T(x), I(x)$ and $F(x)$, so $-0 \leq T(x) + I(x) + F(x) \leq 3^{+}$. T, I and F are called neutrosophic components, the set of all neutrosophic sets in X is denoted by $N(X)$.

2.1 Definition [15]: Let $R, Q \in N(X)$.

1. Subset: $Q \subset R$ if $T_Q(z) \leq T_R(z), I_Q(z) \leq I_R(z), F_Q(z) \geq F_R(z)$ for all $z \in X$.
2. Equality: $Q = R$ if $Q \subset R$ and $R \subset Q$.
3. Union:
$$Q \cup R = \{< z, max\{T_Q(z), T_R(z)\}, max\{I_Q(z), I_R(z)\}, min\{F_Q(z), F_R(z)\} > : z \in X\}.$$
4. Intersection:
$$Q \cap R = \{< z, min\{T_Q(z), T_R(z)\}, min\{I_Q(z), I_R(z)\}, max\{F_Q(z), F_R(z)\} > : z \in X\}.$$

More generally, the intersection and the union of a collection of neutrosophic sets $\{Q_i\} \in N(X)$ are defined by:
$$\cap_{i \in I} Q_i = \{< z, inf\ inf\ \{T_{Q_i}(z)\}, inf\ inf\ \{I_{Q_i}(z)\}, sup\ sup\ \{F_{Q_i}(z)\} >: z \in X\},$$
$$\cup_{i \in I} Q_i = \{< z, sup\ sup\ \{T_{Q_i}(z)\}, sup\ sup\ \{I_{Q_i}(z)\}, inf\ inf\ \{F_{Q_i}(z)\} >: z \in X\}.$$

5. The neutrosophic set defined as $T_Q(z) = 1, I_Q(z) = 1$ and $F_Q(z) = 0$ for all $z \in X$ is called the universal NS denoted by 1_X. Also, the neutrosophic set defined as $T_Q(z) = 0, I_Q(z) = 0$ and $F_Q(z) = 1$ for all $z \in X$ is called the empty NS denoted by 0_X.
6. Complement: $Q^c = 1_X \backslash Q$

2.2 Definition [14]: Let $\Gamma \subset N(Y)$. Then Γ is named a neutrosophic topology on Y if the following conditions hold:

1. 0_X and 1_X belong to Γ,
2. Union of any number of neutrosophic sets in Γ again belongs to Γ,
3. The intersection of finite number of neutrosophic sets in Γ belongs to Γ.

Then the pair (Y, Γ) is named neutrosophic topology on Y.

2.2 Neutrosophic soft sets

2.3. Definition [22]: Let P be an initial universe set and E be a set of parameters. The pair (L, E) is called a neutrosophic soft set (NSS) over P, where L is a mapping from E to $N(P)$.

The set of all NSS over P is denoted by $NSS(P)$. A neutrosophic soft set (L, E) can be written as:

$$(L, E) = \{(e, \{< x, T_{L(e)}(x), I_{L(e)}(x), F_{L(e)}(x) >\}: x \in P, e \in E\}.$$

2.4 Definition [27]: Let X be an initial universe set and E be a set of parameters. Then the neutrosophic soft set $x^e{}_{(\alpha,\beta,\gamma)}$defined as

$$x^e{}_{(\alpha,\beta,\gamma)}(e')(y) = \{(\alpha, \beta, \gamma)\ if\ \ e = e^{'} and\ x = y\ (0,0,1)\ if\ \ e \neq e^{'} and\ x \neq y$$

for all $x \in X, 0 < \alpha, \beta, \gamma \leq 1, e \in E$, is called a neutrosophic soft point.

2.5 Definition [3]: Let $(L, E), (M, E) \in NSS(P)$. Then for all $x \in P$

1. Subset: $(L, E) \subset (M, E)$ if $T_{L(e)}(x) \leq T_{M(e)}(x), I_{L(e)}(x) \leq I_{M(e)}(x)$ and $F_{L(e)}(x) \geq F_{M(e)}(x)$ for all $e \in E$,
2. Equality: $(L, E) = (M, E)$ if $(L, E) \subset (M, E)$ and $(M, E) \subset (L, E)$,
3. Intersection:

$$(L, E) \cap (M, E) = \{(e, \{< x, min\{T_{L(e)}(x), T_{M(e)}(x)\}, max\{I_{L(e)}(x), I_{M(e)}(x)\}, max\{F_{L(e)}(x), F_{M(e)}(x)\} >\}: e \in E\},$$

4. Union:

$$(L, E) \cup (M, E) = \{(e, \{< x, max\{T_{L(e)}(x), T_{M(e)}(x)\}, min\{I_{L(e)}(x), I_{M(e)}(x)\}, min\{F_{L(e)}(x), F_{M(e)}(x)\} >\}: e \in E\}.$$

More generally, the intersection and the union of a collection of $\{(L_i, E)\} \subset NSS(P)$ are defined by:

$$\cup_{i\in I}(L_i, E) = \left\{(e, \left\{< x, sup\ sup\ \left\{T_{L_{i(e)}}(x)\right\}, sup\ sup\ \left\{I_{L_{i(e)}}(x)\right\}, inf\ inf\ \left\{F_{L_{i(e)}}(x)\right\}\ >\right\}: e \in E\right\}$$

$$\cap_{i\in I}(L_i, E) = \left\{(e, \left\{< x, inf\ inf\ \left\{T_{L_{i(e)}}(x)\right\}, inf\ inf\ \left\{I_{L_i(e)}(x)\right\}, sup\ sup\ \left\{F_{L_i(e)}(x)\right\}\ >\right\}: e \in E\right\}$$

5. The NSS defined as $T_{L(e)}(x) = 1, I_{L(e)}(x) = 1$ and $F_{L(e)}(x) = 0$, for all $e \in E$ and $x \in P$ is called the universal NSS denoted by $1_{(P,E)}$. Also, the neutrosophic set defined as $T_{L(e)}(x) = 0, I_{L(e)}(x) = 0$ and $F_{L(e)}(x) = 1$ for all $e \in E$ and $x \in P$ is called the empty NSS denoted by $0_{(P,E)}$.
6. Complement: $(L, E)^c = 1_{(P,E)} \backslash (L, E) = \{e, \{x, F_{L(e)}(x), 1 - I_{L(e)}(x), T_{L(e)}(x) >\}: e \in E\}$
Clearly, the complements of $1_{(X,E)}$ and $0_{(X,E)}$ are defined:

$$(1_{(P,E)})^c = 1_{(P,E)} \backslash 1_{(P,E)} = \{(e, \{< x, 0,0,1 >\} : e \in E\} = 0_{(P,E)}$$

$$(0_{(P,E)})^c = 1_{(P,E)} \backslash 0_{(P,E)} = \{(e, \{< x, 1,0,0 >\}: e \in E\} = 1_{(P,E)}$$

2.6 Definition [4]: Let $\Gamma \subset NSS(Y)$. Then Γ is named a neutrosophic soft topology on Y if the following conditions hold:

NST1)$0_{(P;E)}$and $1_{(P,E)}$ Belong to Γ.

NST2) Union of any number of NSSs in Γ again belongs to Γ.

NST3) Intersection of a finite number of NSSs in Γ belongs to Γ.

Then the pair (Y, Γ) is named neutrosophic soft topology on Y. Elements of Γ is called as a neutrosophic soft open set. An NSS whose complement is neutrosophic soft open is called a neutrosophic soft closed set.

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2.7 Definition [19]: Let $\Gamma \subset NSS(Y)$. Then Γ is named a neutrosophic soft supra topology on Y if $0_{(Y;E)}, 1_{(Y,E)} \in \Gamma$ and union of any number of NSSs in Γ again belong to Γ.

2.8 Definition [9]: If $(X, \tau_1, E)$and (X, τ_2, E) are two neutrosophic soft topological space, then (X, τ_1, τ_2, E) is named as neutrosophic soft bitopological space. The sets belong to τ_i They are called neutrosophic soft i −open sets for $i = 1,2$.

3. Neutrosophic soft tri-topological spaces

In this part, the concept of neutrosophic soft tri-topological space is defined. Furthermore, new types of open and closed sets have been introduced in neutrosophic soft tri-topological spaces.

3.1 Definition: If $(X, \tau_1, E), (X, \tau_2, E)$ and (X, τ_3, E) are three neutrosophic soft topological spaces, then $(X, \tau_1, \tau_2, \tau_3, E)$ is named as neutrosophic soft tri-topological space. The sets belonging to τ_i are called neutrosophic soft i −open sets for $i = 1,2,3$.

3.2 Example: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space, where $X = \{x_1, x_2, x_3\}, E = \{e_1, e_2\}$ and $\tau_1 = \{0_{(X,E)}, 1_{(X,E)}, (L_1, E), (L_2, E)\}$,

$$\tau_2 = \{0_{(X,E)}, 1_{(X,E)}, (N_1, E), (N_2, E)\},$$

$\tau_3 = \{0_{(X,E)}, 1_{(X,E)}, (Q_1, E), (Q_2, E)\}$. The NSSs are defined as

$$f_{(L_1,E)}(e_1) = \{< x_1, 0.8,0.3,0.2 >, < x_2, 0.4,0.4,0.4 >, < x_3, 0.3,0.4,0.2 >\}$$

$$f_{(L_1,E)}(e_2) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(L_2,E)}(e_1) = \{< x_1, 0.6,0.3,0.4 >, < x_2, 0.3,0.5,0.4 >, < x_3, 0.2,0.5,0.3 >\}$$

$$f_{(L_2,E)}(e_2) = \{< x_1, 0.5,0.4,0.3 >, < x_2, 0.4,0.6,0.2 >, < x_3, 0.3,0.6,0.4 >\}$$

$$f_{(N_1,E)}(e_1) = \{< x_1, 0.2,0.6,0.6 >, < x_2, 0.2,0.6,0.6 >, < x_3, 0.1,0.6,0.4 >\}$$

$$f_{(N_1,E)}(e_2) = \{< x_1, 0.2,0.6,0.5 >, < x_2, 0.2,0.7,0.6 >, < x_3, 0.1,0.5,0.5 >\}$$

$$f_{(N_2,E)}(e_1) = \{< x_1, 0.7,0.2,0.1 >, < x_2, 0.3,0.4,0.4 >, < x_3, 0.2,0.4,0.2 >\}$$

$$f_{(N_2,E)}(e_2) = \{< x_1, 0.5,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(Q_1,E)}(e_1) = \{< x_1, 0.2,0.5,0.6 >, < x_2, 0.2,0.5,0.4 >, < x_3, 0.1,0.5,0.4 >\}$$

$$f_{(Q_1,E)}(e_2) = \{< x_1, 0.4,0.4,0.3 >, < x_2, 0.2,0.6,0.3 >, < x_3, 0.1,0.6,0.4 >\}$$

$$f_{(Q_2,E)}(e_1) = \{< x_1, 0.2,0.4,0.5 >, < x_2, 0.2,0.5,0.4 >, < x_3, 0.1,0.5,0.3 >\}$$

$$f_{(Q_2,E)}(e_2) = \{< x_1, 0.5,0.4,0.3 >, < x_2, 0.2,0.6,0.2 >, < x_3, 0.1,0.5,0.4 >\}.$$

Then $(L_1,E)\cap(L_2,E)=(L_2,E)$, $(L_1,E)\cap(N_1,E)=(N_1,E)$, $(L_1,E)\cap(Q_1,E)=(Q_1,E),(L_1,E)\cap(Q_2,E)=(Q_2,E)$, $(L_1,E)\cap(N_2,E)=(N_2,E)$, $(L_2,E)\cap(N_1,E)=(N_1,E),(N_1,E)\cap(N_2,E)=(N_1,E)$, $(N_1,E)\cap(Q_1,E)=(N_1,E),(N_1,E)\cap(Q_2,E)=(N_1,E)$, $(L_2,E)\cap(N_2,E)=(L_2,E),(L_2,E)\cap(Q_2,E)=(Q_2,E)$, $(L_2,E)\cap(Q_1,E)=(Q_1,E),(Q_1,E)\cap(Q_2,E)=(Q_1,E)$ and $(L_1,E)\cup(L_2,E)=(L_1,E)$, $(L_1,E)\cup(N_1,E)=(L_1,E)$, $(L_1,E)\cup(Q_1,E)=(L_1,E),(L_1,E)\cup(Q_2,E)=(L_1,E)$, $(L_1,E)\cup(N_2,E)=(L_1,E)$, $(L_2,E)\cup(N_1,E)=(L_2,E),(N_1,E)\cup(N_2,E)=(N_2,E)$, $(N_1,E)\cup(Q_1,E)=(Q_1,E),(N_1,E)\cup(Q_2,E)=(Q_2,E)$, $(L_2,E)\cup(N_2,E)=(N_2,E),(L_2,E)\cup(Q_2,E)=(L_2,E),(L_2,E)\cup(Q_1,E)=(L_2,E),(Q_1,E)\cup(Q_2,E)=(Q_2,E)$ Therefore τ_1 ,τ_2 and τ_3 are neutrosophic soft topologies on X and so $(X,\tau_1,\tau_2,\ \tau_3,E)$ is a neutrosophic soft tri-topological space.

3.3 Theorem: Let $(X,\tau_1,\tau_2,\ \tau_3,E)$ be a neutrosophic soft tri-topological space. Then $\tau_1\cap\tau_2\cap\ \tau_3$ is a neutrosophic soft topology on X.

Proof: NST1 and NST3 are clear. For NST2, let $\{(L_i,E);i\in I\}\in\tau_1\cap\tau_2\cap\ \tau_3$. Then $(L_i,E)\in\tau_1,(L_i,E)\in\tau_2$ and $(L_i,E)\in\tau_3$. Since τ_1 ,τ_2 and τ_3 are neutrosophic soft topologies on X, $\cup_i(L_i,E)\in\tau_1$,$\cup_i(L_i,E)\in\tau_2$ and $\cup_i(L_i,E)\in\tau_3$. Therefore, $\cup_i(L_i,E)\in\tau_1\cap\tau_2\cap\ \tau_3$.

3.4 Remark: Let $(X,\tau_1,\tau_2,\ \tau_3,E)$ be a neutrosophic soft tri-topological space. Then $\tau_1\cup\tau_2\cup\tau_3$ need not be a neutrosophic soft topological space on X.

3.5 Example: Let $(X,\tau_1,\tau_2,\ \tau_3,E)$ be a neutrosophic soft tri-topological space $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$, $\tau_1=\{0_{(X,E)},1_{(X,E)},(L_1,E),(L_2,E),(L_3,E)\}$,

$\tau_1=\left\{0_{(X,E)},1_{(X,E)},(N_1,E),(N_2,E)\right\}$ and $\tau_3=\left\{0_{(X,E)},1_{(X,E)},(Q_1,E),(Q_2,E)\right\}$. The NSSs are defined as

$$f_{(L_1,E)}(e_1)=\{<x_1,0.8,0.3,0.2>,<x_2,0.4,0.4,0.4>,<x_3,0.3,0.4,0.2>\}$$

$$f_{(L_1,E)}(e_2)=\{<x_1,0.6,0.2,0.3>,<x_2,0.5,0.5,0.1>,<x_3,0.5,0.3,0.4>\}$$

$$f_{(L_2,E)}(e_1)=\{<x_1,0.6,0.3,0.4>,<x_2,0.3,0.5,0.4>,<x_3,0.2,0.5,0.3>\}$$

$$f_{(L_2,E)}(e_2)=\{<x_1,0.5,0.4,0.3>,<x_2,0.4,0.6,0.2>,<x_3,0.3,0.6,0.4>\}$$

$$f_{(L_3,E)}(e_1)=\{<x_1,0.4,0.4,0.5>,<x_2,0.2,0.6,0.6>,<x_3,0.1,0.6,0.4>\}$$

$$f_{(L_3,E)}(e_2)=\{<x_1,0.3,0.6,0.4>,<x_2,0.3,0.7,0.3>,<x_3,0.1,0.7,0.5>\}$$

$$f_{(N_1,E)}(e_1)=\{<x_1,0.2,0.6,0.6>,<x_2,0.2,0.6,0.6>,<x_3,0.1,0.6,0.4>\}$$

$$f_{(N_1,E)}(e_2)=\{<x_1,0.2,0.6,0.5>,<x_2,0.2,0.7,0.6>,<x_3,0.1,0.5,0.5>\}$$

$$f_{(N_2,E)}(e_1)=\{<x_1,0.7,0.2,0.1>,<x_2,0.3,0.4,0.4>,<x_3,0.2,0.4,0.2>\}$$

$$f_{(N_2,E)}(e_2)=\{<x_1,0.5,0.2,0.3>,<x_2,0.5,0.5,0.1>,<x_3,0.5,0.3,0.4>\}.$$

$$f_{(Q_1,E)}(e_1)=\{<x_1,0.2,0.5,0.6>,<x_2,0.2,0.5,0.4>,<x_3,0.1,0.5,0.4>\}$$

$$f_{(Q_1,E)}(e_2)=\{<x_1,0.4,0.4,0.3>,<x_2,0.2,0.6,0.3>,<x_3,0.1,0.6,0.4>\}.$$

$$f_{(Q_2,E)}(e_1)=\{<x_1,0.2,0.4,0.5>,<x_2,0.2,0.5,0.4>,<x_3,0.1,0.5,0.3>\}$$

$$f_{(Q_2,E)}(e_2) = \{< x_1, 0.5,0.4,0.3 >, < x_2, 0.2,0.6,0.2 >, < x_3, 0.1,0.5,0.4 >\}.$$

Here $\tau_1 \cup \tau_2 \cup \tau_3 = \{0_{(X,E)}, 1_{(X,E)}, (L_1, E), (L_2, E), (L_3, E), (N_1, E), (N_2, E), (Q_1, E), (Q_2, E)\}$ is not a neutrosophic soft topology on $X, since$

$$(L_3, E) \cup (N_1, E) = \{(e_1, \{< x_1, 0.4,0.4,0.6 >, < x_2, 0.2,0.6,0.6 >, < x_3, 0.1,0.7,0.5 >\}),$$

$$(e_2, \{< x_1, 0.3,0.6,0.6 >, < x_2, 0.3,0.7,0.4 >, < x_3, 0.1,0.7,0.6 >\})\}$$

is not in $\tau_1 \cup \tau_2 \cup \tau_3$.

3.6 Definition: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space. Then an NSS

$$(N, E) = \{(e, \{< x, T_{N(e)}(x), I_{N(e)}(x), F_{N(e)}(x) >\}): x \in U, e \in E\}$$

is called a neutrosophic soft tri-open set if there exists a neutrosophic soft open set (N_1, E) in τ_1 , a neutrosophic soft open set (N_2, E) in τ_2 and a neutrosophic soft open (N_3, E) in $\tau_3$such that for all $x \in U$

$$(N, E) = (N_1, E) \cup (N_2, E) \cup (N_3, E)$$

$$= \{(e, \{< x, max\{T_{N_1(e)}(x), T_{N_2(e)}(x), T_{N_3(e)}(x)\}, min\{I_{N_1(e)}(x), I_{N_2(e)}(x), I_{N_3(e)}(x)\}, min\{F_{N_1(e)}(x), F_{N_2(e)}(x), F_{N_3(e)}(x)\} >\}): e \in E\}$$

The set of all neutrosophic soft tri-open sets in $(X, \tau_1, \tau_2, \tau_3, E)$ is denoted by NSTO$(X, \tau_1, \tau_2, \tau_3, E)$.

3.7 Definition: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space. Then an NSS

$$(N, E) = \{(e, \{< x, T_{N(e)}(x), I_{N(e)}(x), F_{N(e)}(x) >\}): x \in U, e \in E\}$$

is called a neutrosophic soft tri-closed set if $(N, E)^c$ is a neutrosophic soft tri-open set. It is clear that (N, E) is a neutrosophic soft tri-closed set if there exists a neutrosophic soft closed set (N_1, E) in τ_1 and a neutrosophic soft closed set (N_2, E) in τ_2 and a neutrosophic soft closed set (N_3, E) in τ_3 such that for all $x \in U$

$$(N, E) = (N_1, E) \cap (N_2, E) \cap (N_3, E)$$

$$= \{(e, \{< x, min\{T_{N_1(e)}(x), T_{N_2(e)}(x), T_{N_3(e)}(x)\}, max\{I_{N_1(e)}(x), I_{N_2(e)}(x), I_{N_3(e)}(x)\}, max\{F_{N(e)}(x), F_{G(e)}(x)\} >\}): e \in E\}$$

The set of all neutrosophic soft tri-closed sets in $(X, \tau_1, \tau_2, \tau_3, E)$ is denoted by NSTC$(X, \tau_1, \tau_2, \tau_3, E)$.

3.8 Example: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space. $X = \{x_1, x_2, x_3\}$,$E = \{e_1, e_2\}$, $\tau_1 = \{0_{(X,E)}, 1_{(X,E)}, (L_1, E)\}$, $\tau_2 = \{0_{(X,E)}, 1_{(X,E)}, (L_2, E)\}$,

$\tau_3 = \{0_{(X,E)}, 1_{(X,E)}, (L_3, E)\}$. The NSSs are defined as

$$f_{(L_1,E)}(e_1) = \{< x_1, 0.8,0.3,0.2 >, < x_2, 0.4,0.4,0.4 >, < x_3, 0.3,0.4,0.2 >\}$$

$$f_{(L_1,E)}(e_2) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(L_2,E)}(e_1) = \{< x_1, 0.6,0.2,0.4 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(L_2,E)}(e_2) = \{< x_1, 0.6,0.3,0.4 >, < x_2, 0.3,0.5,0.4 >, < x_3, 0.2,0.5,0.3 >\}.$$

$$f_{(L_3,E)}(e_1) = \{< x_1, 0.2,0.5,0.6 >, < x_2, 0.2,0.5,0.4 >, < x_3, 0.1,0.5,0.4 >\}$$

$$f_{(L_3,E)}(e_2) = \{< x_1, 0.4,0.4,0.3 >, < x_2, 0.2,0.6,0.3 >, < x_3, 0.1,0.6,0.4 >\}.$$

Then

$$(L_1, E) \cup (L_2, E) \cup (L_3, E) = \{(e_1, \{\langle x_1, 0.8,0.2,0.2\rangle, \langle x_2, 0.5,0.4,0.1\rangle, \langle x_3, 0.5,0.3,0.2\rangle\}),$$
$$(e_2, \{\langle x_1, 0.6,0.2,0.3\rangle, \langle x_2, 0.5,0.5,0.1\rangle, \langle x_3, 0.5,0.3,0.3\rangle\})\}$$

is a neutrosophic soft tri-open set. Also

$$f_{(L_1,E)^c}(e_1) = \{< x_1, 0.2,0.7,0.8 >, < x_2, 0.6,0.6,0.6 >, < x_3, 0.2,0.6,0.3 >\}$$

$$f_{(L_1,E)^c}(e_2) = \{< x_1, 0.3,0.8,0.6 >, < x_2, 0.1,0.5,0.5 >, < x_3, 0.4,0.7,0.5 >\}$$

$$f_{(L_2,E)^c}(e_2) = \{< x_1, 0.3,0.8,0.6 >, < x_2, 0.1,0.5,0.5 >, < x_3, 0.4,0.7,0.3 >\}$$

$$f_{(L_2,E)^c}(e_1) = \{< x_1, 0.4,0.7,0.6 >, < x_2, 0.4,0.5,0.3 >, < x_3, 0.3,0.5,0.2 >\}$$

$$f_{(L_3,E)^c}(e_1) = \{< x_1, 0.6,0.5,0.2 >, < x_2, 0.4,0.5,0.2 >, < x_3, 0.4,0.5,0.1 >\}$$

$$f_{(L_3,E)^c}(e_2) = \{< x_1, 0.3,0.6,0.4 >, < x_2, 0.3,0.4,0.2 >, < x_3, 0.4,0.4,0.1 >\}.$$

Therefore

$$(L_1, E)^c \cap (L_2, E)^c \cap (L_3, E)^c = \{(e_1, \{\langle x_1, 0.2,0.8,0.8\rangle, \langle x_2, 0.1,0.6,0.6\rangle, \langle x_3, 0.2,0.7,0.3\rangle\})$$
$$(e_2, \{\langle x_1, 0.3,0.8,0.6\rangle, \langle x_2, 0.1,0.5,0.3\rangle, \langle x_3, 0.3,0.7,0.5\rangle\})\}$$

is a neutrosophic soft tri-closed set.

3.9 Theorem: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space. In this case

1. $0_{(X,E)}, 1_{(X,E)} \in$ NSTO $(X, \tau_1, \tau_2, \tau_3, E)$.
2. If $\{(N_i, E) | i \in I\} \subseteq$ NSTO$(X, \tau_1, \tau_2, \tau_3, E)$ then $\bigcup_{i\in I}(N_i, E) \in NSTO(X, \tau_1, \tau_2, \tau_3, E)$.
3. If $\{(G_i, E) | i \in I\} \subseteq NSTC(X, \tau_1, \tau_2, \tau_3, E)$ then $\bigcap_{i\in I}(G_i, E) \in NSTC(X, \tau_1, \tau_2, \tau_3, E)$.

Proof:

1. Since $0_{(X,E)} \cup 0_{(X,E)} \cup 0_{(X,E)} = 0_{(X,E)}$ and $1_{(X,E)} \cup 1_{(X,E)} \cup 1_{(X,E)} = 1_{(X,E)}$ then $0_{(X,E)}$ and $1_{(X,E)}$ are neutrosophic soft tri-closed sets.
2. Since $(N_i, E) \in NSTO(X, \tau_1, \tau_2, \tau_3, E)$, there exist $(N_i^1, E) \in \tau_1$, $(N_i^2, E) \in \tau_2$ and $(N_i^3, E) \in \tau_3$ such that $(N_i, E) = (N_i^1, E) \cup (N_i^2, E) \cup (N_i^3, E)$ for all $i \in I$. Then

   $$\bigcup_{i\in I}(N_i, E) = \bigcup_{i\in I}((N_i^1, E) \cup (N_i^2, E) \cup (N_i^3, E)) = \left(\bigcup_{i\in I}(N_i^1, E)\right) \cup \left(\bigcup_{i\in I}(N_i^2, E)\right) \cup \left(\bigcup_{i\in I}(N_i^3, E)\right).$$

   As τ_1, τ_2 and τ_3 are neutrosophic soft topologies on X, $\bigcup_{i\in I}(N_i^1, E) \in \tau_1$,$\bigcup_{i\in I}(N_i^2, E) \in \tau_2$ *and* $(N_i^3, E) \in \tau_3$.
   Therefore $\bigcup_{i\in I}(N_i, E) \in NSTO(X, \tau_1, \tau_2, E)$.

3. Since $(G_i,E) \in NSTC(X,\tau_1,\tau_2,\tau_3,E)$, there exist $(G_i^1,E)^c \in \tau_1$, $(G_i^2,E)^c \in \tau_2$, $(G_i^3,E)^c \in \tau_3$ such that $(G_i,E) = (G_i^1,E) \cap (G_i^2,E) \cap (G_i^3,E)$ for all $i \in I$. Then

$$\bigcap_{i\in I}(G_i,E) = \bigcap_{i\in I}((G_i^1,E) \cap (G_i^2,E) \cap (G_i^3,E)) = (\bigcap_{i\in I}(G_i^1,E)) \cap (\bigcap_{i\in I}(G_i^2,E) \cap (\bigcap_{i\in I}(G_i^2,E))).$$

Then $\bigcap_{i\in I}(G_i,E) \in NSTC(X,\tau_1,\tau_2,\tau_3,E)$ as $\left(\bigcap_{i\in I}(G_i^1,E)\right)^c \in \tau_1$, $\left(\bigcap_{i\in I}(G_i^2,E)\right)^c \in \tau_2$ and $\left(\bigcap_{i\in I}(G_i^3,E)\right)^c \in \tau_3$.

3.10 Corollary: Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a neutrosophic soft tri-topological space. Then $NSTC(X,\tau_1,\tau_2,\tau_3,E)$ is a supra neutrosophic soft topology on X. This topology is denoted by τ_{123}.

3.11 Theorem: Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a neutrosophic soft tri-topological space. Then every neutrosophic soft $i-$open set is a neutrosophic soft tri-open set.

Proof: Let (N,E) *be a neutrosophic soft* $i-$ *open set where* $i = 1,2,3$. Since $(N,E) = (N,E) \cup 0_{(X,E)} \cup 0_{(X,E)}$, then $(N,E) \in$ NSTO $(X,\tau_1,\tau_2,\tau_3,E)$.

3.12 Corollary: Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a neutrosophic soft tri-topological space. Then $\tau_1 \cup \tau_2 \cup \tau_3 \subset \tau_{123}$.

The following example shows that the inverse of Theorem 3.11 does not hold.

3.13 Example: Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a neutrosophic soft tri-topological space. $X = \{x_1,x_2,x_3\}$, $E = \{e_1,e_2\}$, $\tau_1 = \{\{0_{(X,E)},1_{(X,E)},(L_1,E)\}$, $\tau_2 = \{0_{(X,E)},1_{(X,E)},(L_2,E)\}$ $\tau_3 = \{0_{(X,E)},1_{(X,E)},(L_3,E)\}$. The NSSs are defined as

$$f_{(L_1,E)}(e_1) = \{< x_1, 0.8,0.3,0.2 >, < x_2, 0.4,0.4,0.4 >, < x_3, 0.3,0.4,0.2 >\},$$

$$f_{(L_1,E)}(e_2) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(L_2,E)}(e_1) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(L_2,E)}(e_2) = \{< x_1, 0.6,0.3,0.4 >, < x_2, 0.3,0.5,0.4 > < x_3, 0.2,0.5,0.3 >\}$$

$$f_{(L_3,E)}(e_1) = \{< x_1, 0.6,0.5,0.2 >, < x_2, 0.4,0.5,0.2 >, < x_3, 0.4,0.5,0.1 >\}$$

$$f_{(L_3,E)}(e_2) = \{< x_1, 0.3,0.6,0.4 >, < x_2, 0.3,0.4,0.8 >, < x_3, 0.4,0.4,0.1 >\}.$$

Then $\tau_{123} = \tau_1 \cup \tau_2 \cup \tau_3 \cup \{(L_1,E) \cup (L_2,E) \cup (L_3,E)\}$ because the neutrosophic soft set $(L_1,E) \cup (L_2,E) \cup (L_3,E)$ Is not a neutrosophic soft i-open set for all i=1,2,3.

3.14 Theorem: Let (X,τ_1,τ_2,E) be a neutrosophic soft tri-topological space. Then every neutrosophic soft $i-$closed set is a neutrosophic soft tri-closed set.

Proof: The proof is similar to the proof of Theorem 3.11.

3.15 Theorem: Let $(X,\tau_1,\tau_2,\tau_3,E)$ Be a neutrosophic soft tri-topological space. If $\tau 1 \subset \tau 3$ *and* $\tau 2 \subset \tau 3$, then $\tau_{123} = \tau_3$.

Proof: Let $\tau 1 \subset \tau 3$ and $\tau 2 \subset \tau 3$ and $(N, E) \in \tau_{123}$. Then there exists a neutrosophic soft open set (N_1, E) in τ_1, a neutrosophic soft open set (N_2, E) in τ_2 and a neutrosophic soft open set (N_3, E) in τ_3 such that $(N, E) = (N_1, E) \cup (N_2, E) \cup (N_3, E)$. Since $\tau 1 \subset \tau 3$ and $\tau 2 \subset \tau 3$, $(N_1, E) \in \tau_3$ and $(N_2, E) \in \tau_3$. Therefore $\tau_{123} \subset \tau_2 \subset \tau_3$. From Theorem 3.11, $\tau_3 \subset \tau_{123}$.

3.16 Definition: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space and $(N, E) \in NSS(X)$. The neutrosophic soft tri-closure of (N, E), denoted by $cl_T^{NSS}(N, E)$, is the intersection of all neutrosophic soft tri-closed sets containing (N, E), i.e.,

$$cl_T^{NSS}(N, E) = \cap \{(L, E) \in PNSC(X) | (N, E) \subset (L, E)\}.$$

It is clear that $cl_T^{NSS}(N, E)$ is the smallest neutrosophic soft tri-closed set containing (N, E).

3.17 Example: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be the same as in Example 3.13 and

$$(G, E) = \{(e_1, \{< x_1, 0.3, 0.6, 0.7 >, < x_2, 0.3, 0.4, 0.4 >, < x_3, 0.2, 0.4, 0.4 >\})$$
$$(e_2, \{< x_1, 0.2, 0.5, 0.6 >, < x_2, 0.1, 0.3, 0.7 >, < x_3, 0.3, 0.3, 0.4\rangle\})$$

be a neutrosophic soft set over X. Now, we need to determine neutrosophic soft tri-closed sets in $(X, \tau_1, \tau_2, \tau_3, E)$ to find $cl_T^{NSS}(G, E)$. Then,

$$f_{(L_2, E)}(e_1) = \{< x_1, 0.6, 0.3, 0.4 >, < x_2, 0.3, 0.5, 0.4 >, < x_3, 0.2, 0.5, 0.3 >\}$$
$$\{< x_1, 0.5, 0.4, 0.3 >, < x_2, 0.4, 0.6, 0.2 >, < x_3, 0.3, 0.6, 0.4 >\}$$

and

$$(L_2, E)^c = \{(e_1, \{< x_1, 0.4, 0.7, 0.6 >, < x_2, 0.4, 0.5, 0.3 >, < x_3, 0.3, 0.5, 0.2 >\}),$$
$$(e_2, \{< x_1, 0.3, 0.6, 0.5 >, < x_2, 0.2, 0.4, 0.6 >, < x_3, 0.4, 0.4, 0.3 >\})\}.$$

The neutrosophic soft tri-closed sets which contain (G, E) are $(L_2, E)^c$ and $1_{(X,E)}$. Therefore

$$cl_T^{NSS}(G, E) = (L_2, E)^c \cap 1_{(X,E)} = (L_2, E)^c.$$

3.18 Theorem: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space and $(N, E), (L, E) \in NSS(X)$.

1. $cl_T^{NSS}(0_{(X,E)}) = 0_{(X,E)}$ and $cl_T^{NSS}(1_{(X,E)}) = 1_{(X,E)}$.
2. $(N, E) \subseteq cl_T^{NSS}(N, E)$.
3. (N, E) is a neutrosophic soft tri-closed set if $cl_T^{NSS}(N, E) = (N, E)$.
4. $cl_T^{NSS}(N, E) \subseteq cl_T^{NSS}(L, E)$ if $(N, E) \subseteq (L, E)$.
5. $cl_T^{NSS}(N, E) \cup cl_T^{NSS}(L, E) \subset cl_T^{NSS}((N, E) \cup (L, E))$.
6. $cl_T^{NSS}(cl_T^{NSS}(N, E)) = cl_T^{NSS}(N, E)$, i.e., $cl_T^{NSS}(N, E)$ is neutrosophic soft tri-closed set.

Proof. It is obvious.

3.19 Theorem: Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a neutrosophic soft tri-topological space and $(N, E) \in NSS(X)$. Then, $x^e{}_{(\alpha,\beta,\gamma)} \in cl_T^{NSS}(H, E)$ if and only if for all $U_{x^e{}_{(\alpha,\beta,\gamma)}} \in \tau_{123}(x^e{}_{(\alpha,\beta,\gamma)})$ where $U_{x^e{}_{(\alpha,\beta,\gamma)}}$ is any neutrosophic soft tri-open set that contains $x^e{}_{(\alpha,\beta,\gamma)}$ and $\tau_{12}(x^e{}_{(\alpha,\beta,\gamma)})$ is the family of all neutrosophic soft tri-open sets contains $x^e{}_{(\alpha,\beta,\gamma)}$,

$$U_{x^e{}_{(\alpha,\beta,\gamma)}} \cap (N, E) \neq 0_{(X,E)}.$$

Proof: Let $x^e{}_{(\alpha,\beta,\gamma)} \in cl_T^{NSS}(N, E)$ and suppose that there exists $U_{x^e{}_{(\alpha,\beta,\gamma)}} \in \tau_{123}(x^e{}_{(\alpha,\beta,\gamma)})$ such that $U_{x^e{}_{(\alpha,\beta,\gamma)}} \cap (N, E) = 0_{(X,E)}$. Then $(N, E) \subset \left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c$. Thus $cl_T^{NSS}(N, E) \subset cl_T^{NSS}\left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c = \left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c$ which implies $cl_T^{NSS}(N, E) \cap U_{x^e{}_{(\alpha,\beta,\gamma)}} = 0_{(X,E)}$, a contradiction.

Conversely, assume that $x^e{}_{(\alpha,\beta,\gamma)} \notin cl_T^{NSS}(N, E)$, then $x^e{}_{(\alpha,\beta,\gamma)} \in \left(cl_T^{NSS}(N, E)\right)^c \in \tau_{123}(x^e{}_{(\alpha,\beta,\gamma)})$. Therefore, by hypothesis, $\left(cl_T^{NSS}(N, E)\right)^c \cap (N, E) \neq 0_{(X,E)}$, a contradiction.

4. Conclusions and future work

In this study, the concept of tritopological is expanded to neutrosophic soft set theory. Neutrosophic soft tri-open and neutrosophic soft tri-closed sets are given. Also, supra neutrosophic soft topology is defined by neutrosophic soft tri-open sets. For future work, neighborhood structures will be study and some separation axioms will be given on neutrosophic soft tri-topological spaces.

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