Equitable Domination in Neutrosophic Graphs

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Abstract:

This paper shows the equitable domination in neutrosophic graphs. In this proposed work, equitable neighbourhood of a vertex and equitabe degree are defined. Minimal neutrosophic dominating sets, minimal and maximal equitable independent sets,strong and weak equitable dominating sets in neutrosophic graphs are likewise settled. A few hypotheses on equitable domination in neutrosophic graphs are inferred with numerical examples.

Keywords:
Neutrosophic set, neutrosophic graph, neutrosophic equitable dominating set, neutrosophic equitable degree, neutrosophic equitable independent set.

Introduction:

Different types of domination in graphs were studied by many researchers[1,4,6]. L.A.Zadeh(1965) introduced the concepts of a fuzzy subset of a set as a way for representing uncertainty. His idea have been applied to a wide range of scientific areas. Fuzzy concepts is also introduced in graph theory. Formally, a fuzzy graph G=(v, σ, μ) is a non empty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all u,v in V, where σ is a fuzzy vertex set of G and μ is a fuzzy edge set of G. S.Somasundaram and A.Somasundaram present the concepts of independent domination, total domination and connected domination of fuzzy graphs[14]. C.Natarajan and S.K.Ayyaswamy introduced the strong (weak) domination in fuzzy graphs[3]. The concept of equitable domination [16] in graphs was introduced by Venkatasubramanian Swaminathan and Kuppusamy markandan Dharmalingam.

The first definition of intuitionistic fuzzy graph was proposed by Atanassov in [2]. T. Different types of dominations in intuitionistic fuzzy graphs were investigated by many researchers in [8,10,16].Fuzzy graph and intuitionistic fuzzy graph approaches are failed in some applications when indeterminacy occurs. Neutrosophic set proposed by smarandache[13] is a powerful tool for dealing incomplete and indeteminate problems in the real world. It is the generalization of fuzzy set and intuitionistic fuzzy set. Neutrosophic set is very useful in solving the problems with indeterminacy. M.Mullai introduced the concept of domination in neutrosophic graphs in [12]. In this paper, the basic concept of equitable domination in neutrosophic graph is developed . Minimal and maximal equitable independent sets, and strong and weak equitable dominating sets in neutrosophic graphs are also established with suitable exambles and theorems.

1. Preliminaries:

Definition 1.1. [5]A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \epsilon V$.

Definition 1.2. [15] The order p and size q of a fuzzy graph G=(σ,μ) are defined to be p =$\sum_{u\in V}\sigma(u)$ and $q=\sum_{uv\in E}\mu(uv)$.

Definition 1.3. [13] The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $deg(u)$.

Definition 1.4. [6] An intuitionistic fuzzy graph(IFG) is of the form G=(V,E) where,
(1). $V=v_1,v_2,......,v_n$ such that $\mu_1:V\rightarrow[0,1]$ and $\gamma_1:V\rightarrow[0,1]$ denote the degree of membership of the element $v_1\in V$,(i=1,2,...n),
(2). $E\subseteq v\times v$ where $\mu_2:V\times V\rightarrow[0,1]$ is such that $\mu_2(v_i,v_j)\leq min[\mu_i(v_i),\mu_1(v_j)]$ and $\gamma_2(v_i,v_j)\geq max[\gamma(v_i,\gamma_1(v_j)]$ and $0\leq\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\leq 1$ for every $(v_i,v_j)\in E$, (i,j=1,2,....,n)

Definition 1.5. [9]Let $G=(V,E)$ be an intuitionistic fuzzy graph. Let $u,v\in V$,we say that u dominates v in G if there exist a strong arc between them. A subset $D\subseteq V$ is said to be dominating set in G if for every $v\in V-D$, there exist u dominates v.

Definition 1.6. [7] A dominating set D of IFG is said to be minimal dominating set if no proper subset of S is a dominating set. Minimum cardinality among all minimal dominating set is called the intuitionistic fuzzy domination number, and is denoted by $\gamma_{if}(G)$.

Definition 1.7. [9] A subset S of V is called dominating set in G if for every $v\in V-S$, there exist $u\in S$ such that u dominates v.

Definition 1.8. [11] A dominating set S of an IFG is said to be minimal dominating set if no proper subset of S is a dominating set.

Definition 1.9. [13] Let X be a space of points (objects) with generic elements in X denoted by X, then the neutrosophic sets A(NS A) is an object having the form $A=<x:T_A(x),I_A(x),F_A(x)>,x\in X$ where the functions $T,I,F:X\rightarrow]0^-,1_+[$ define respectively the truth membership function, and a falsity membership function of the element $x\in X$ to the set A with the condition $\sigma\leq T_A(x)+I_A(x)\leq 3+$ The function $T_A(x),I_A(x)andF_A(x)$ are real standard or nonstandard subsets of $]o^-,1^+[$.

Definition 1.10. [14] The minimum cardinality of a dominating set in a neutrosophic graph G is called the domination number of G and is denoted by $\gamma^N(G)$ (or) γN.

Definition 1.11. [15] The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $deg(u)$.

Definition 1.12. If D is an fuzzy equitable dominating set then any super set of D is a fuzzy equitable dominating set.

Definition 1.13. [18] A vertex $u\in V$ is said to be degree equitable fuzzy graph with a vertex $v\epsilon V$ if $\mid deg(u)-deg(v)\mid\leq 1$ and $\mu(uv)\leq\sigma(u)\wedge\sigma(v)$.

2. EQUITABLE DOMINATION IN NEUTROSOPHIC GRAPHS

Definition 2.1. Let $G=(V,E)$ be a single valued neutrosophic graph. A subset D^N of V is called an dominating set if for every $v\in V-D^N$, there exists a vertex $u\in D^N$ such that $uv\in E(G)$ and
$\mid deg(u)-deg(v)\mid$ = 1 and
$\mu_2(v_i,v_j)=\mu_1(v_i)\wedge\mu_1(v_j)$
$\gamma_2(v_i,v_j)=\gamma_1(v_i)\vee\gamma_1(v_j)$
$\sigma_2(v_i,v_j)=\sigma_1(v_i)\vee\sigma_1(v_j)$

Definition 2.2. Let $G=(V,E)$ be a neutrosophic graph. A vertex $u\in V$ is said to be equitable degree in G with a vertex $v\in V$ if $\mid deg(u)-deg(v)\mid\leq 1$ and $\mu_2(v_i,v_j)=\mu_1(v_i)\vee\gamma_1(v_j)$.

Definition 2.3. A neutrosophic equitable dominating set D^N is said to be a minimal neutrosophic equitable dominating set if no proper subset of D^N is an neutrosophic equitable dominating set.

Definition 2.4. Let $u \in V$. A neutrosophic equitable neighbourhood of u denoted by $N_{eif}(u)$ is defined as
$N_{eif}(u) = v \in V \mid v \in N(u), \mid deg(u) - deg(v) \mid \leq 1,$
$\mu^N(v_i, v_j) = \mu^N(v_i) \wedge \mu^N(v_j)$, $\gamma^N(v_i, v_j) = \gamma^N(v_i) \vee \gamma^N(v_j)$ and $\sigma^N(v_i, v_j) = \sigma^N(v_i) \vee \sigma^N(v_j)$
and $u \in I_e \Leftrightarrow N_{eif}(u) = \phi$.
The cardinality of $N_{eif}(u)$ is called an neutrosophic equitable degree of u and it is denoted by $d_{eif}(u)$.

Definition 2.5. The maximum and minimum neutrosophic equitable degrees of a vertex in G are denoted respectively by $\triangle^N_{eif}(G)$ and $\delta^N_{eif}(G)$. That is
$\triangle^N_{eif}(G) = max_{u \in V(G)} \mid N_{eif}(u) \mid$ and $\delta^N_{eif}(G) = min_{u \in V(G)} \mid N_{eif}(u) \mid$.

Theorem 1. *A dominating set D^N of a neutrosophic graph G is a minimal neutrosophic equitable dominating set iff for each $d \epsilon D^N$, one of the following two conditions holds:*
(1) $N_{eif}(d) \cap D^N = \phi$
(2) There is a vertex $c \in V - D^N$ such that $N_{eif}(c) \cap D^N = d$
Proof:
Let D^N be a minimal neutrosophic equitable dominating set and $d \in D^N$.
Then, $D^N_d = D^N \setminus d$ is not an neutrosophic equitable dominating set and hence there exists $x \in V \setminus D^N_d$ such that x is not dominated by any element of D^N_d.
If $x = d$, then we get (1) and if $x \neq d$, then we get (2).
The converse is obvious.

Theorem 2. *Let G be a neutrosophic graph of order P, then*
(1) $\gamma^N_{eif}(G) \leq \gamma^N_{seif}(G) \leq p - \triangle_{eif}(G)$
(2) $\gamma^N_{eif}(G) \leq \gamma^N_{weif}(G) \leq P - \delta_{eif}(G)$.
Proof:
Let G(V,E) be a neutrosophic graph. Every strong neutrosophic equitable dominating set is an neutrosophic equitable dominating set of G.
(i.e) $\gamma^N_{eif}(G) \leq \gamma^N_{seif}(G)$.
Similarly, every weak neutrosophic equitable dominating set in an neutrosophic equitable dominating set of G.
(i.e) $\gamma^N_{eif}(G) \leq \gamma^N_{weif}(G)$.
Let $u, v \epsilon V$.
If, $d_{eif}(u) = \triangle^N_{eif}(G)$ and $d_{eif}(v) = \delta^N_{eif}(G)$.
Clearly, $V - N_{eif}(u)$ is a strong neutrosophic equitable dominating set and $V - N_{eif}(v)$ and is a weak neutrosophic dominating set.
Therefore,
$\gamma^N_{seif}(G) \leq \mid V - N_{eif}(u) \mid_{eif}$
and
$\gamma^N_{weif}(G) \leq \mid V - N_{eif}(v) \mid_{eif}$.
i.e
$\gamma^N_{seif}(G) \leq p - \triangle_{eif}(G)$
and
$\gamma^N_{weif}(G) \leq p - \delta^N_{eif}(G)$.

Theorem 3. *Let G be a neutrosophic graph without isolated vertices. Let D^N be a minimal equitable dominating set of G. The $V \setminus D^N$ is a neutrosophic equitable dominating set of G.*
Proof:
Let d be a any vertex in D^N. There exists a vertex $x \epsilon N(d)$ such that $c \epsilon V \setminus D^N$ from theorem 3.7.
Thus, every vertex of D^N is dominated by some vertex of $V \setminus D^N$.

Theorem 4. *If D^N is a neutrosophic equitable independent dominating set of G then D^N is a both minimal neutrosophic equitable dominating set and a maximal neutrosophic equitable independent set. Conversely any maximal neutrosophic equitable independent set D^N in G is a neutrosophic equitable independent dominating set of G.*
Proof:

If D^N is a neutrosophic equitable independent dominating set of G.
$D^N = D^N \setminus d$ is not a neutrosophic equitable dominating set $d \in D^N$ and $D^N \cup x$ is not a neutrosophic equitable independent for every $v \neq D^N$ so that D^N is a minimal neutrosophic equitable dominating set and a maximal neutrosophic equitable independent set.
Conversely,
Let D^N be a maximal neutrosophic equitable independent set in G.
Then, for every $x \in V \setminus D^N$, $D^N \cup x$ is not a neutrosophic equitable independent and hence x is dominated by some element of D^N.
Thus, D^N is a neutrosophic equitable dominating set of G.

Theorem 5. *A neutrosophic equitable independent set S is a maximal neutrosophic equitable independent set iff it is a neutrosophic equitable independent set and neutrosophic equitable dominating set.*
Proof:
Suppose, a neutrosophic equitable independent set S is maximal neutrosophic equitable independent set.
Then, for every vertex u in V, the set $S \cup u$ is not a neutrosophic graph equitable independent set, that is for every vertex $u \in V - S$ there is a vertex v in S such that u is adjacent to v. Thus, S is a neutrosophic equitable dominating set. Hence, S is both neutrosophic equitable independent and neutrosophic graph equitable dominating set.
Conversely,
Suppose, that a set S is both neutrosophic equitable independent and neutrosophic equitable dominating set. We have to show that it is maximal neutrosophic graph equitable independent set. Suppose, S is not maximal neutrosophic equitable independent set.
Then, there exists a vertex u in $V - S$ such that $S \cup u$ is a neutrosophic equitable independent set.
But, if $S \cup u$ is a neutrosophic equitable independent set, then no vertex in S is adjacent to u.
Hence, S is not a neutrosophic equitable dominating set ,which is contradiction.
Therefore, S is maximal neutrosophic equitable independent set.

Conclusion:

In this proposed work, we presented the idea of equitable dominating set, minimal equitable dominating set, strong (weak) equitable dominating set, total dominating set in neutrosophic graphs are introduced. Likewise equitable independent set in neutrosophic graphs with some theorems are developed. The utilizations of equitable dominating set in neutrosophic graphs will be created in future.

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