



Neutrosophic Bi-topological Group

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Abstract

Neutrosophic topological groups are neutrosophic groups in an algebraic sense together with neutrosophic continuous group operations. In this article, we have presented neutrosophic bi-topological groups with illustrative examples. We have also defined eight new models of neutrosophic bi-topological groups. Neutrosophic bi-topological group that depends on two neutrosophic topologies group is more general than the neutrosophic topological group. Finally, Some basic properties of neutrosophic bi-topological groups were studied.

Keywords: Neutrosophic topological groups, neutrosophic bi-topological groups, neutrosophic groups.

1. Introduction

The concept of neutrosophic set was first time appeared by F. Smarandache [2,3]. As a generalization of a fuzzy set defined by Zadeh [1], and intuitionistic fuzzy set, via this new logic[4], A.A. Salama and S.A. Alblowi [5] presented neutrosophic topological space.

In recent time, the theory of neutrosophic theory has becomes very widespread among scientists around the world.

Agboola et al. presented the concept of neutrosophic group and neutrosophic ring in[6,7].

Also, In 2015, he presented the concept of refined neutrosophic algebraic structures in[8], and introduced refined neutrosophic groups. Recently. In 2020, Adeleke et al. in [9,10] studied several refined concepts as refined neutrosophic rings and introduced their basic properties, refined neutrosophic ideals and refined neutrosophic homomorphisms in details.

Resently, In 2020, Q. Imran, et al. in [11] studied several types of neutrosophic topological groups and introduced their basic properties. Also, Sumathi et al. in [12] introduced the concept of neutrosophic topological groups.

This paper is devoted to the study of neutrosophic bi-topological groups for the first time, and investigate its basic properties. Also, we provide many new definitions, important Results, some theories and examples.

2. Definition

DOI: <https://doi.org/10.54216/IJNS.170104>

Received: September 27, 2021 Accepted: December 01, 2021

In this section, $\tilde{N}\text{-}\check{T}\check{S}$ ($\tilde{N}\text{-}\check{T}\hat{G}$) = neutrosophic topological space (grups). $\tilde{N}\text{-}\check{O}\check{S}$ ($\tilde{N}\text{-}\check{C}\check{S}$) = neutrosophic open (closed) sets. Now, we recall some basic definitions as neutrosophic group, and neutrosophic ring,...etc. which are useful in sequel.

Definition 2.1. [10]

Let $(\hat{G}, *)$ be any group, the neutrosophic group is generated by I and \hat{G} under $*$ denoted by $N(\hat{G}) = \{ \langle \hat{G} \cup I \rangle, * \}$.

Definition 2.2. [12]

A neutrosophic topological group $(\tilde{N}\text{-}\check{T}\hat{G})$ is a set \hat{G} which carries a group structure and a neutrosophic topology with the following two postulates:

- (i) The operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$, given as $\phi(g, h) = g.h$ is a N-continuous.
- (ii) The inversion function $\psi: \hat{G} \rightarrow \hat{G}$, given as $\psi(g) = g^{-1}$ is a N-continuous.

Definition 2.3. [14]

Let $f: (\check{X}, \check{T}_1) \rightarrow (\hat{Y}, \check{T}_2)$ be a function on $\tilde{N}\text{-}\check{T}\check{S}$. Then f is neutrosophic continuous (N-continuous) if the preimage of each open neutrosophic set in \hat{Y} is open neutrosophic set in \check{X} .

Definition 2.4.

Let $f: (\check{X}, \check{T}_1) \rightarrow (\hat{Y}, \check{T}_2)$ be a function on $\tilde{N}\text{-}\check{T}\check{S}$, then f is called:

- (i) Neutrosophic α -continuous (N- α -continuous) iff for each $A \tilde{N}\text{-}\check{O}\check{S}$ in \hat{Y} , then $f^{-1}(A)$ is a $N\alpha$ OS in \check{X} [14].
- (ii) Neutrosophic α -irresolute (N- α -irresolute) iff for each A $N\alpha$ OS in \hat{Y} , then $f^{-1}(A)$ is a $N\alpha$ OS in \check{X} .

Proposition 2.5. [15]

Every N-continuous function is a N- α -continuous, but the opposite is not valid in general.

Definition 2.6. [11]

Let \hat{G} be a set that equips with a group structure and a neutrosophic topology. Then \hat{G} is called:

- (i) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (1) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ are both N- α -continuous.
- (ii) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (2) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ are both N- α -irresolute.
- (iii) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (3) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N- α -continuous and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N-continuous.
- (iv) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (4) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N- α -irresolute and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N-continuous.
- (v) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (5) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N- α -irresolute and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N- α -continuous.
- (vi) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (6) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N- α -continuous and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N- α -irresolute.
- (vii) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (7) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N-continuous, and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N- α -continuous.
- (viii) $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (8) iff the operation function $\phi: \hat{G} \times \hat{G} \rightarrow \hat{G}$ is N-continuous, and the inversion function $\psi: \hat{G} \rightarrow \hat{G}$ is N- α -irresolute.

Remark 2.7. [11]

- (i) Every $\tilde{N}\text{-}\check{T}\hat{G}$ is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (R), where $R=1,3,7$.
- (ii) Every $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (2) is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (5).
- (iii) Every $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (2) is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (6).
- (iv) Every $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (4) is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (3).
- (v) Every $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (4) is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (5).
- (vi) Every $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (R) is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (1), where $R=2,3,\dots,8$.

Definition 2.8. [11]

A function $f: (\check{X}, \check{T}_1) \rightarrow (\hat{Y}, \check{T}_2)$ is said to be M-function iff $f^{-1}(Nint(Ncl(A))) \subseteq Nint(Ncl(f^{-1}(A)))$, for every $N\alpha$ OS A of \hat{Y} .

Remark 2.9. [11]

- (i) A $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (3) with M-function operation (M-FO) μ is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (4).
- (ii) A $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (1) with M-FO ψ and M-FO ϕ is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (2).
- (iii) A $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (1) with M-FO ϕ is a $\tilde{N}\text{-}\check{T}\hat{G}$ of pattern (5).

- (iv) A $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (1) with M-FO ψ is a $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (6).
- (v) A $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (5) with M-FO ψ is a $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (2).
- (vi) A $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (6) with M-FO ϕ is a $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (2).
- (vii) A $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (7) with M-FO ψ is a $\tilde{N}_- \tilde{T} \hat{G}$ of pattern (8).

3. Neutrosophic Bi-Topological Group

In this section, we study neutrosophic bi-topological groups, also, we define new types of neutrosophic bi-topological groups and investigate its basic properties.

Definition 3.1.

Let \hat{G} be a neutrosophic group. Let \tilde{T}_1, \tilde{T}_2 be two neutrosophic topologies on \hat{G} . A neutrosophic group \hat{G} , with two neutrosophic topologies is called a neutrosophic bi-topological group $(\hat{G}, \tilde{T}_1, \tilde{T}_2)$ ($\tilde{N}_{Bi} \tilde{T} \hat{G}$) if and only if the mappings

(1) $\phi : (\hat{G}, \tilde{T}_i) \times (\hat{G}, \tilde{T}_i) \rightarrow (\hat{G}, \tilde{T}_i)$ defined by $\phi(x, y) = x.y$, and

(2) $\psi : (\hat{G}, \tilde{T}_i) \rightarrow (\hat{G}, \tilde{T}_i)$ defined by $\psi(x) = x^{-1}$ for $x \in \hat{G}$ are both \tilde{T}_i -continuous; for $i = 1, 2$.

An equivalent condition to (i) and (ii) is that, the function $h : (\hat{G}, \tilde{T}_i) \times (\hat{G}, \tilde{T}_i) \rightarrow (\hat{G}, \tilde{T}_i)$ defined by $\phi(x, y) = x.y^{-1}$, for $x, y \in \hat{G}$ is \tilde{T}_i -N-continuous; for $i = 1, 2$.

Example 3.2.

Clearly, every neutrosophic group is a $\tilde{N}_{Bi} \tilde{T} \hat{G}$ with the discrete neutrosophic topology and indiscrete neutrosophic topology.

Theorem 3.3.

Let \hat{G} be a neutrosophic group. Let \tilde{T}_1, \tilde{T}_2 be two neutrosophic topologies on \hat{G} , then :

$(\hat{G}, \tilde{T}_1, \tilde{T}_2)$ is a $(\tilde{N}_{Bi} \tilde{T} \hat{G})$ iff both $(\hat{G}, \tilde{T}_1), (\hat{G}, \tilde{T}_2)$ are neutrosophic topological groups.

Proof.

\Rightarrow : Since $(\hat{G}, \tilde{T}_1, \tilde{T}_2)$ is a $\tilde{N}_{Bi} \tilde{T} \hat{G}$, then the mappings

(1) $\phi : (\hat{G}, \tilde{T}_i) \times (\hat{G}, \tilde{T}_i) \rightarrow (\hat{G}, \tilde{T}_i)$ defined by $\phi(x, y) = x.y$, and

(2) $\psi : (\hat{G}, \tilde{T}_i) \rightarrow (\hat{G}, \tilde{T}_i)$ defined by $\psi(x) = x^{-1}$ for $x \in \hat{G}$ are both \tilde{T}_i -continuous; for $i = 1, 2$.

Therefore both $(\hat{G}, \tilde{T}_1), (\hat{G}, \tilde{T}_2)$ are $\tilde{N}_- \tilde{T} \hat{G}$.

\Leftarrow : Since (\hat{G}, \tilde{T}_1) is a $\tilde{N}_- \tilde{T} \hat{G}$, then the mappings

(1) $\phi : (\hat{G}, \tilde{T}_1) \times (\hat{G}, \tilde{T}_1) \rightarrow (\hat{G}, \tilde{T}_1)$ defined by $\phi(x, y) = x.y$, and

(2) $\psi : (\hat{G}, \tilde{T}_1) \rightarrow (\hat{G}, \tilde{T}_1)$ defined by $\psi(x) = x^{-1}$ for $x \in \hat{G}$ are both \tilde{T}_1 -continuous.

Since (\hat{G}, \tilde{T}_2) is a $\tilde{N}_- \tilde{T} \hat{G}$, then the mappings

(1) $\phi : (\hat{G}, \tilde{T}_2) \times (\hat{G}, \tilde{T}_2) \rightarrow (\hat{G}, \tilde{T}_2)$ defined by $\phi(x, y) = x.y$, and

(2) $\psi : (\hat{G}, \tilde{T}_2) \rightarrow (\hat{G}, \tilde{T}_2)$ defined by $\psi(x) = x^{-1}$ for $x \in \hat{G}$ are both \tilde{T}_2 -continuous.

Therefore $(\hat{G}, \tilde{T}_1, \tilde{T}_2)$ is a $\tilde{N}_{Bi} \tilde{T} \hat{G}$.

Definition 3.4.

Let \hat{G} be a set that equips with a group structure and a neutrosophic topology. Then \hat{G} is called:

(i) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (1) ($\tilde{N}_{Bi} \check{T}\hat{G}(1)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ are both \check{T}_i -N- α -continuous, for $i = 1, 2$.

(ii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2) ($\tilde{N}_{Bi} \check{T}\hat{G}(2)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ are both \check{T}_i -N- α -irresolute, for $i = 1, 2$.

(iii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (3) ($\tilde{N}_{Bi} \check{T}\hat{G}(3)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -continuous and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N-continuous, for $i = 1, 2$.

(iv) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4) ($\tilde{N}_{Bi} \check{T}\hat{G}(4)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -irresolute and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N-continuous, for $i = 1, 2$.

(v) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5) ($\tilde{N}_{Bi} \check{T}\hat{G}(5)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -irresolute and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -continuous, for $i = 1, 2$.

(vi) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (6) ($\tilde{N}_{Bi} \check{T}\hat{G}(6)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -continuous and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -irresolute, for $i = 1, 2$.

(vii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (7) ($\tilde{N}_{Bi} \check{T}\hat{G}(7)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N-continuous, and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -continuous, for $i = 1, 2$.

(viii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (8) ($\tilde{N}_{Bi} \check{T}\hat{G}(8)$) iff the operation function $\phi: (\hat{G}, \check{T}_i) \times (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N-continuous, and the inversion function $\psi: (\hat{G}, \check{T}_i) \rightarrow (\hat{G}, \check{T}_i)$ is \check{T}_i -N- α -irresolute, for $i = 1, 2$.

Theorem 3.5.

Let \hat{G} be a set that equips with a neutrosophic group structure and two neutrosophic topologies \check{T}_1, \check{T}_2 . Then $(\hat{G}, \check{T}_1, \check{T}_2)$ is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i) iff both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), $i=1,2,3,\dots,8$.

Proof.

Proof follows from Definition 3.4.

Theorem 3.6.

(i) $\tilde{N}_{Bi} \check{T}\hat{G} \Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), where $i=1,3,7$.

(ii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2) $\Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

(iii) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2) $\Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (6).

(iv) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4) $\Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (3).

(v) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4) $\Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

(vi) $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i) $\Rightarrow \tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (1), where $i=2,3,\dots,8$.

Proof.

(i) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$, then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$. By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are $\tilde{N}_{Bi} \check{T}\hat{G}$ are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), where $i=1,3,7$.

Hence, \hat{G} is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), where $i=1,3,7$.

(ii) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2). By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

Hence, \hat{G} is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

(iii) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (2). By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (6).

Hence, \hat{G} is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (6).

(iv) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4). By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (3).

Hence, \hat{G} is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (3).

(v) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (4). By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

Hence, \hat{G} is a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (5).

(vi) Let \hat{G} be a $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (i), $i=2,3,\dots,8$. By Remark 2.7, we have that (\hat{G}, \check{T}_1) and (\hat{G}, \check{T}_2) are both $\tilde{N}_{Bi} \check{T}\hat{G}$ of pattern (1).

DOI: <https://doi.org/10.54216/IJNS.170104>

Received: September 27, 2021 Accepted: December 01, 2021

Hence, \hat{G} is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (1).

Theorem 3.7.

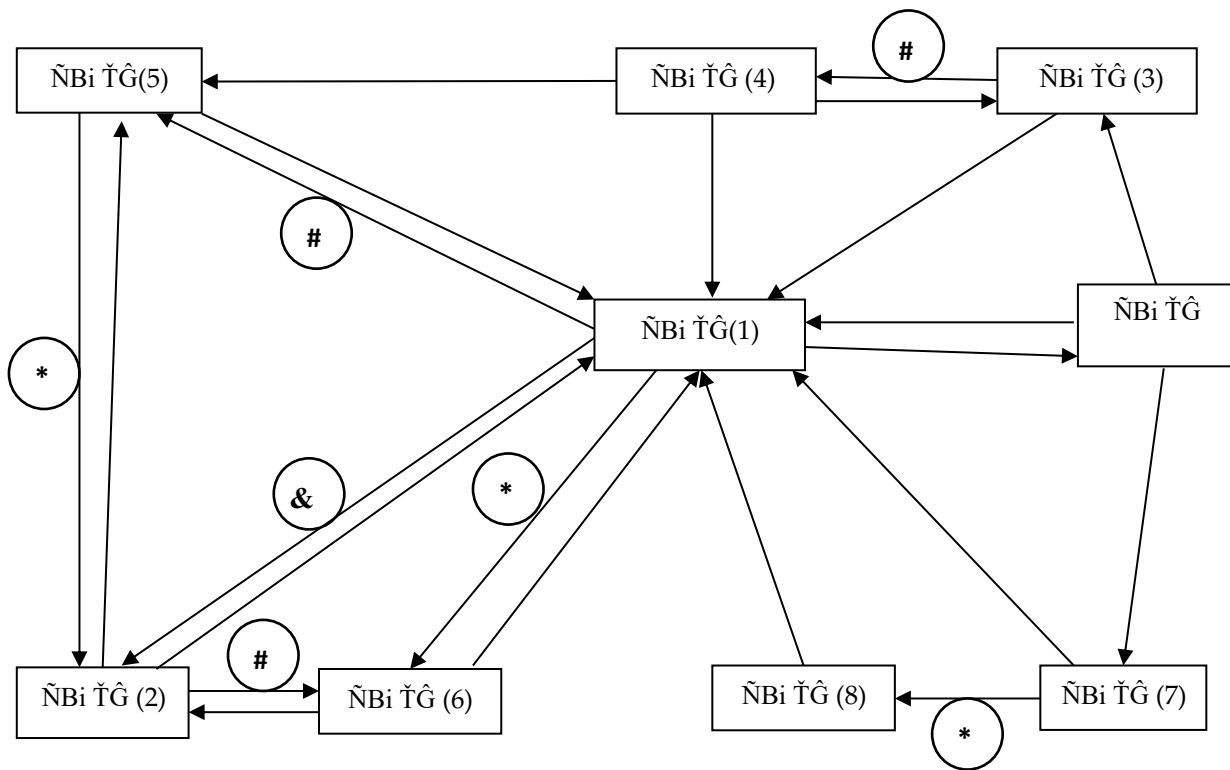
- (i) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (3) with M-FO ϕ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (4).
- (ii) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (1) with M-FO ψ and \mathcal{M} -FO ϕ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (2).
- (iii) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (1) with M-FO ϕ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (5).
- (iv) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (1) with M-FO ψ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (6).
- (v) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (5) with M-FO ψ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (2).
- (vi) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (6) with M-FO ϕ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (2).
- (vii) A $\tilde{N}Bi \check{T}\hat{G}$ of pattern (7) with M-FO ψ is a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (8).

Proof.

- (i) Let \hat{G} be a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (3), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N} \check{T}\hat{G}$ of pattern (3). Since μ is M-FO. So by Theorem 2.9, we get that $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N} \check{T}\hat{G}$ of pattern (4). Hence $\tilde{N}Bi \check{T}\hat{G}$ of pattern (4).
- (ii) Let \hat{G} be a $\tilde{N}Bi \check{T}\hat{G}$ of pattern (1), then both $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N} \check{T}\hat{G}$ of pattern (1). Since μ is M-FO and ψ is M-FO. So by Theorem 2.9, we get that $(\hat{G}, \check{T}_1), (\hat{G}, \check{T}_2)$ are $\tilde{N} \check{T}\hat{G}$ of pattern (2). Hence $\tilde{N}Bi \check{T}\hat{G}$ of pattern (2).

Remark 3.8.

In the following diagram we show the relationships between $\tilde{N}Bi \check{T}\hat{G}$ of pattern (i) and $NBi-T\hat{G}, i=1,2,\dots,8$.



* means ψ is M-function, # means ϕ is M-function, & means ϕ, ψ are M-function.

Remark 3.9.

Let $A \square$ be an neutrosophic open set in a $\tilde{N} \check{T}\hat{G} (\hat{G}, \check{T}_1) \square \square$ Then A^{-1} is neutrosophic open set in (\hat{G}, \check{T}_1) .

Proof.

DOI: <https://doi.org/10.54216/IJNS.170104>

Received: September 27, 2021 Accepted: December 01, 2021

Let $f: (\hat{G}, \check{T}_1) \rightarrow (\hat{G}, \check{T}_1)$ be a mapping defined by $f(x) = x$, for each $x \in \hat{G}$.

Since f is neutrosophic homeomorphism, therefore $f^{-1}(A)$ is neutrosophic open set in (\hat{G}, \check{T}_1) .

Remark 3.10.

Let A be an neutrosophic open set in a $\tilde{N}\text{-}\check{T}\hat{G}(\hat{G}, \check{T}_2)$. Then A^{-1} is neutrosophic open set in (\hat{G}, \check{T}_2) .

Proof.

Let $f: (\hat{G}, \check{T}_2) \rightarrow (\hat{G}, \check{T}_2)$ be a mapping defined by $f(x) = x$, for each $x \in \hat{G}$.

Since f is neutrosophic homeomorphism, therefore $f^{-1}(A) = A^{-1}$ is neutrosophic open set in (\hat{G}, \check{T}_2) .

Conclusion

In this work, we have introduced neutrosophic bi-topological groups and new types of neutrosophic bi-topological groups. Finally, This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

References

- [1] F. Smarandache; "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability". American Research Press, Rehoboth, NM, 1999.
- [2] F. Smarandache; "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM 87301, USA 2002.
- [3] L. A. Zadeh; "Fuzzy Sets". Inform. Control, Vol. 8, 338-353, 1965.
- [4] K. Atanassov; "intuitionistic fuzzy sets". fuzzy sets and systems, Vol. 20, 87-96, 1986.
- [5] A. A. Salma; S.A. Alblowi; "Neutrosophic set and neutrosophic topological spaces". IOSR J. Math., Vol. 3(4), 31-35, 2012
- [6] A. A. A. Agboola; A.D. Akinola and O.Y. Oyebola; "Neutrosophic Rings I", Int. J. of Math. Comb., vol. 4, 1-14, 2011.
- [7] A. A. A. Agboola; Akwu A.O. and Y.T. Oyebo; "Neutrosophic Groups and Neutrosophic Subgroups", Int. J. of Math. Comb., vol. 3, 1-9, 2012.
- [8] A. A. A. Agboola; "On Refined Neutrosophic Algebraic Structures", Neutrosophic Sets and Systems, vol.10, 99-101, 2015.
- [9] E. O. Adeleke; A.A.A. Agboola and F. Smarandache; "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science (IJNS), vol. 2(2), pp. 77-81, 2020. (DOI:10.5281/zenodo.3728222)
- [10] E.O. Adeleke; A.A.A. Agboola and F. Smarandache; "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science (IJNS), vol. 2(2), pp. 89-94, (2020). (DOI:10.5281/zenodo.3728235).
- [11] Q. H. Imran; A. H. Al-Obaidi; F. Smarandache; "On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets". Neutrosophic Sets Syst, Vol. 33, 426-433, 2020.
- [12] R. Sumathi; I. Arockiarani; "Topological Group Structure of Neutrosophic set". Journal of Advanced Studies in Topology, Vol. 7, 12-20, 2016.
- [13] W. B. Vasantha Kandasamy; and F. Smarandache; "Neutrosophic Rings", Neutrosophic Sets and Systems, 2006.
- [14] A. A. Salama, F. Smarandache and V. Kroumov, Neutrosophic Closed Set and Neutrosophic Continuous Functions. Neutrosophic Sets and Systems, 4(2014), 2-8.

- [15] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, Vol.16, 2017, pp.16-19.