



# Neutrosophic Strongly $b$ -Open Mapping Via Neutrosophic Topological Spaces

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## Abstract

In this paper, we introduce the notion of strongly neutrosophic  $b$ -open function and strongly neutrosophic  $b$ -closed functions in neutrosophic topological spaces. Further, we give some properties of these functions and investigate some relations among them.

**Keywords:** Neutrosophic Set, Neutrosophic Topology, Neutrosophic Strongly  $b$ -Open Set

## 1. Introduction

A pair  $(W, \tau)$ , where  $W$  is a non-empty set and  $\tau$  is a neutrosophic topology on  $W$  is called a neutrosophic topological space. Salama and Alblawi [1] was first introduced the concept of neutrosophic topological space. Salama and Alblawi [2] also studied generalized neutrosophic set and generalized neutrosophic topological spaces. Iswaraya and Bageerathi [15] introduced the concept of neutrosophic semi-open set and neutrosophic semi-closed set. Rao and Srinivasa [17] introduced neutrosophic pre-open set and pre-closed set. Arokiarani, Dhavaseelan, Jafari, and Parimala [13] defined some new functions in neutrosophic topological spaces. Andrijevic [8] introduced the concept of  $b$ -open set in topological space. Thereafter Tripathy and Sarma [6] studied on  $b$ -locally open sets in bitopological space. The concept of strongly  $b$ -open function and strongly  $b$ -closed function in topological space was introduced by Mustafa [14]. Afterwards Tripathy and Sarma [4] studied the concept of pairwise strongly  $b$ -open function and pairwise strongly  $b$ -closed function in bitopological spaces. In this paper, we introduce some properties of neutrosophic  $b$ -open function, strongly neutrosophic  $b$ -open function and strongly neutrosophic  $b$ -closed function in neutrosophic topological spaces.

## 2. Preliminaries

In this section we discuss some basic existing definitions and notions those are defined by many researchers.

**Definition 2.1** [12] An neutrosophic set  $E$  over a non-empty set  $W$  is denoted as follows:

$E = \{(y, T_E(y), I_E(y), F_E(y)) : y \in W, \text{ and } T_E(y), I_E(y), F_E(y) \in ]0, 1^+[ \}$ , where  $T_E(y), I_E(y), F_E(y)$  denotes the degree of truthness, indeterminacy, and falseness of each  $y \in W$ .

There is no restriction on the sum of  $T_E(y), I_E(y), F_E(y)$ , so

$$0 \leq T_E(y) + I_E(y) + F_E(y) \leq 3^+.$$

**Definition 2.2** [13] Let  $p, q, r$  be real standard and non-standard subsets of  $]0,1^+[$ . A neutrosophic set  $x_{p,q,r}$  is said to be a neutrosophic point (in short NP) in a non-empty set  $W$  given by

$x_{p,q,r}(y) = \begin{cases} (p, q, r), & \text{if } x = y \\ (0, 0, 1), & \text{if } x \neq y \end{cases}$ , for  $y \in x$  is called the support of  $x_{p,q,r}$ ; where  $p, q, r$  denotes the degree of truth, indeterminacy and falsity membership values of  $x_{p,q,r}$ .

**Definition 2.3** [1] A family  $\tau$  of neutrosophic sets (in short NS) over a non-empty set  $W$  is said to be a neutrosophic topology (in short NT) on  $W$  if the following properties holds:

- (i)  $0_N, I_N \in \tau$
- (ii)  $T_1, T_2 \in \tau \Rightarrow T_1 \cap T_2 \in \tau$
- (iii)  $\cup T_i \in \tau$ , for every  $\{T_i: i \in \Delta\} \subseteq \tau$ .

In that case, the pair  $(W, \tau)$  is called a neutrosophic topological space (in short NTS). Each element of  $\tau$  are called a neutrosophic open set (in short NOS) and the complement of each neutrosophic open set are called a neutrosophic closed set (in short NCS).

**Remark 2.1** [2] The collection of all neutrosophic open set and neutrosophic closed set in a NTS  $(W, \tau)$  may be denoted by  $\text{NOS}(W)$  and  $\text{NCS}(W)$  respectively.

**Definition 2.4** [2] Assume that  $(W, \tau)$  be a NTS and  $\beta$  be a NS over  $W$ . Then the neutrosophic interior and neutrosophic closure of  $\beta$  are defined by

$$N_{int}(\beta) = \cup \{V : V \text{ is a NOS in } W \text{ and } V \subseteq \beta\};$$

$$N_{cl}(\beta) = \cap \{F : F \text{ is a NCS in } W \text{ and } \beta \subseteq F\}.$$

**Definition 2.5** [13] Assume that  $(W, \tau)$  be a NTS and  $E$  be a NS over  $W$ . Then  $E$  is said to be a neutrosophic neighbourhood of a neutrosophic point  $x_{p,q,r}$  in  $(W, \tau)$  if there exist a neutrosophic open set  $F$  in  $W$  such that  $x_{p,q,r} \in F \subseteq E$ .

**Definition 2.6.** Let  $(W, \tau)$  be a NTS and  $G$  be a NS over  $W$ . Then  $\beta$  is called a

- (i) [13] Neutrosophic  $\alpha$ -open ( $N\alpha$ -O) set if and only if  $\beta \subseteq N_{int}(N_{cl}(N_{int}(\beta)))$ ;
- (ii) [15] Neutrosophic semi-open (NSO) set if and only if  $\beta \subseteq N_{cl}(N_{int}(\beta))$ ;
- (iii) [17] Neutrosophic pre-open (NPO) set if and only if  $\beta \subseteq N_{int}(N_{cl}(\beta))$ .
- (iv) [10] Neutrosophic  $b$ -open (NBO) set if and only if  $\beta \subseteq N_{int}(N_{cl}(\beta)) \cup N_{cl}(N_{int}(\beta))$ .

**Remark 2.2.** The collection of all Neutrosophic  $\alpha$ -open, Neutrosophic semi-open, Neutrosophic pre-open, neutrosophic  $b$ -open sets and neutrosophic  $b$ -closed sets in a neutrosophic topological space  $(W, \tau)$  may be denoted by  $N\alpha$ -O( $W$ ), NSO( $W$ ), NPO( $W$ ), NBO( $W$ ) and NBC( $W$ ) respectively. Clearly  $\text{NOS}(W) \subseteq \text{NBO}(W)$  and  $\text{NCS}(W) \subseteq \text{NBC}(W)$ .

**Definition 2.7** [10] Let  $\beta$  be a NS over  $W$  and  $(W, \tau)$  be a NTS. Then the neutrosophic  $b$ -interior and neutrosophic  $b$ -closure of  $\beta$  is defined by

(i)  $N_{bint}(\beta) = \cup\{E : E \text{ is a NBO set in } (W, \tau) \text{ and } E \subseteq \beta\}$ ;

(ii)  $N_{bcl}(\beta) = \cap\{F : F \text{ is a NBC set in } (W, \tau) \text{ and } \beta \subseteq F\}$ .

**Theorem 2.1** [13] Let  $\xi$  be a function from a NTS  $(W, \tau_1)$  to another NTS  $(M, \tau_2)$ . Then  $\xi$  is called a

(i) neutrosophic open function if  $\xi(K)$  is a neutrosophic open set in  $M$ , whenever  $K$  is a neutrosophic open set in  $W$ .

(ii) neutrosophic  $\alpha$ -open function if  $\xi(K)$  is a neutrosophic  $\alpha$ -open set in  $M$ , whenever  $K$  is a neutrosophic open set in  $W$ .

(iii) neutrosophic pre-open function if  $\xi(K)$  is a neutrosophic pre-open set in  $M$ , whenever  $K$  is a neutrosophic open set in  $W$ .

(iv) neutrosophic semi-open function if  $\xi(K)$  is a neutrosophic semi-open set in  $M$ , whenever  $K$  is a neutrosophic open set in  $W$ .

**Definition 2.8** [13] A function  $\xi: (W, \tau_1) \rightarrow (M, \tau_2)$  is said to be a neutrosophic irresolute function if  $\xi^{-1}(L)$  is neutrosophic closed set in  $W$  for every neutrosophic closed set  $L$  in  $M$ .

### 3. Neutrosophic strongly $b$ -open function

In this section, we give some definitions and establish some results on neutrosophic  $b$ -open functions and neutrosophic strongly  $b$ -open functions via neutrosophic topological spaces.

**Definition 3.1.** A function  $\xi: (W, \tau_1) \rightarrow (M, \tau_2)$  is said to be a neutrosophic  $b$ -open mapping if  $\xi(K)$  is a neutrosophic  $b$ -open set in  $M$  for every neutrosophic open set  $K$  in  $W$ .

**Definition 3.2.** A function  $\xi: (W, \tau_1) \rightarrow (M, \tau_2)$  is said to be a strongly neutrosophic  $b$ -open mapping if  $\xi(K)$  is a neutrosophic  $b$ -open set in  $M$  for every neutrosophic  $b$ -open set  $K$  in  $W$ .

**Definition 3.3.** A function  $\xi: (W, \tau_1) \rightarrow (M, \tau_2)$  is said to be a strongly neutrosophic  $b$ -closed mapping if  $\xi(K)$  is a neutrosophic  $b$ -closed set in  $M$  for every neutrosophic  $b$ -closed set  $K$  in  $W$ .

**Theorem 3.1.** Let  $\xi$  be a function from a NTS  $(W, \tau_1)$  to another NTS  $(M, \tau_2)$ . Then the following two statements are equivalent:

(i)  $\xi$  is neutrosophic  $b$ -open function.

(ii) For every neutrosophic point  $x_{p,q,r} \in W$  and for each neutrosophic neighbourhood  $K$  of  $x_{p,q,r}$  in  $W$  there exists a neutrosophic  $b$ -open set  $N$  in  $M$  such that  $\xi(x_{p,q,r}) \in N \subseteq \xi(K)$

**Proof.** (i) $\Rightarrow$ (ii) Let  $\xi$  be a neutrosophic  $b$ -open function from a NTS  $(W, \tau_1)$  to another NTS  $(M, \tau_2)$ . Let  $x_{p,q,r}$  be a neutrosophic point in  $W$  and  $K$  be a neutrosophic neighbourhood of  $x_{p,q,r}$ . Then there exists a neutrosophic open set  $H$  in  $W$  such that  $x_{p,q,r} \in H \subseteq K$ .

Now  $x_{p,q,r} \in H \subseteq K$  implies that  $\xi(x_{p,q,r}) \in \xi(H) \subseteq \xi(K)$ . Since  $\xi$  is a neutrosophic  $b$ -open function so  $\xi(H) = N$  (say) is a neutrosophic  $b$ -open set in  $M$ .

Thus for every neutrosophic point  $x_{p,q,r}$  in  $W$  and for each neutrosophic neighbourhood  $K$  of  $x_{p,q,r}$  in  $W$  there exists a neutrosophic  $b$ -open set  $N = \xi(H)$  in  $M$  such that  $\xi(x_{p,q,r}) \in N \subseteq \xi(K)$

(ii) $\Rightarrow$ (i) Let  $B$  be a neutrosophic open set in  $W$  and  $x_{p,q,r}$  be a neutrosophic point in  $W$  such that  $x_{p,q,r} \in B$ . Therefore  $\xi(x_{p,q,r}) \in \xi(B)$ . We know that every neutrosophic open set is a neutrosophic neighbourhood of each of its neutrosophic point. So  $B$  is the neutrosophic neighbourhood of  $x_{p,q,r}$ . Then by the hypothesis there exists a neutrosophic  $b$ -open set  $A$  in  $M$  such that  $\xi(x_{p,q,r}) \in A \subseteq \xi(B)$ . This implies that, for each neutrosophic point  $\xi(x_{p,q,r})$  in  $\xi(B)$  there exist a neutrosophic  $b$ -open set  $A$  in  $M$  such that  $\xi(x_{p,q,r}) \in A \subseteq \xi(B)$ . Therefore  $\xi(B)$  is a neutrosophic  $b$ -neighbourhood of each of its points. Hence  $\xi(B)$  is a neutrosophic  $b$ -open set in  $M$ . Thus  $\xi$  is a neutrosophic  $b$ -open function.

### Theorem 3.2.

(i) Every neutrosophic semi-open (neutrosophic pre-open) function is a neutrosophic  $b$ -open function.

(ii) Every neutrosophic open function is a neutrosophic  $b$ -open function.

(iii) Every neutrosophic  $\alpha$ -open function is a neutrosophic  $b$ -open function.

**Proof.** (i) Let  $\xi$  be a neutrosophic semi-open (neutrosophic pre-open) function from a NTS  $(W, \tau_1)$  to another NTS  $(M, \tau_2)$  and  $P$  be a neutrosophic open set in  $W$ . Since  $\xi$  is a neutrosophic semi open (neutrosophic pre open) function,



**Definition 3.5.** A function  $\xi:(W,\tau_1)\rightarrow(M,\tau_2)$  is said to be a neutrosophic  $b$ -irresolute function if  $\xi^{-1}(L)$  is neutrosophic  $b$ -open (neutrosophic  $b$ -closed) set in  $W$  for every neutrosophic  $b$ -open (neutrosophic  $b$ -closed) set  $L$  in  $M$ .

**Theorem 3.4.** Let  $\xi$  be a function from a NTS  $(W,\tau_1)$  to another NTS  $(M,\tau_2)$ . If  $\xi$  is a bijective mapping then the following three statements are equivalent:

- (i)  $\xi$  is strongly neutrosophic  $b$ -open function.
- (ii)  $\xi$  is strongly neutrosophic  $b$ -closed function.
- (iii)  $\xi^{-1}$  is neutrosophic  $b$ -irresolute function.

**Proof.** (i) $\Rightarrow$ (ii) Let  $\xi$  be a strongly neutrosophic  $b$ -open function and  $K$  be a neutrosophic  $b$ -closed set in  $W$ . Then  $K^c$  is neutrosophic- $b$ -open set in  $W$ . Since  $\xi$  is a strongly neutrosophic  $b$ -open function so  $\xi(K^c) = (\xi(K))^c$  is neutrosophic  $b$ -open set in  $M$  and so  $\xi(K)$  is neutrosophic  $b$ -closed set in  $M$ . Hence  $\xi$  is a strongly neutrosophic  $b$ -closed function.

(ii) $\Rightarrow$ (iii) Let  $\xi$  be a strongly neutrosophic  $b$ -closed function. Let  $L$  be any neutrosophic  $b$ -closed set in  $W$ . By hypothesis  $\xi(L)$  is neutrosophic  $b$ -closed in  $M$ . Thus we have  $(\xi^{-1})^{-1}(L) = \xi(L)$  is neutrosophic  $b$ -closed in  $M$ . Similarly for any neutrosophic  $b$ -open set  $K$  in  $W$  it can be shown that  $(\xi^{-1})^{-1}(K) = \xi(K)$  is a neutrosophic  $b$ -open set in  $M$ . Hence  $\xi^{-1}$  is a neutrosophic  $b$ -irresolute function.

(iii) $\Rightarrow$ (i) Let  $\xi^{-1}$  be neutrosophic  $b$ -irresolute function and  $K$  be any neutrosophic  $b$ -open set in  $W$ . Since  $\xi^{-1}$  is a neutrosophic  $b$ -irresolute function, so  $(\xi^{-1})^{-1}(K) = \xi(K)$  is a neutrosophic  $b$ -open in  $M$ . Therefore for any neutrosophic  $b$ -open set  $K$  in  $W$ ,  $\xi(K)$  is neutrosophic  $b$ -open set in  $M$ . Hence  $\xi$  is a strongly neutrosophic  $b$ -open function.

## 5. Conclusions

In this study, we introduce the notion of strongly neutrosophic  $b$ -open function and strongly neutrosophic  $b$ -closed functions in neutrosophic topological spaces. Further we also give some properties of these functions and investigate some relations among them. By defining strongly neutrosophic  $b$ -open function and strongly neutrosophic  $b$ -closed functions, we provide some remark, theorem on neutrosophic topological spaces. In the future, we hope that based on these notions in NTSSs, many new investigations can be carried out.

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